ABSTRACT: "Historical back analysis" consists in analyzing not only the present failure of a slope, but also its passed stability when it was submitted to conditions which were more critical than those at the time of failure. The method is used to analyze the failure of three rock compartments in limestone cliffs of the Grenoble area. Their probable life times are estimated using a kinematic model of cliff retreat, based on rock fall frequency. The probable seismic accelerations they have undergone, are estimated from a seismic hazard map. The analysis allows estimating the rate of decrease of slope stability and then the rate of decrease of the rock bridges which held the compartments before failure occurs. The linear rate of decrease obtained is about 0.1 mm/year. It could be explained by limestone solution.

1 INTRODUCTION

Rock fall hazard assessment needs evaluating the failure probability of a rock compartment in a given period of time (Fell et al. 2005, Picarelli et al. 2005). In other words the time to failure has to be evaluated in a probabilistic way. This needs to know the present state of stability of the slope and to model the stability decrease it will undergo (Hantz et al. 2003a). In the case of rock slopes, even the present state of stability is difficult to estimate because it strongly depends on the presence and the extension of rock bridges (Einstein et al. 1983, Kemeny 2003). For slope design, continuous joints (without rock bridges) are usually assumed, but this conservative hypothesis would lead to underestimate the stability of presently stable existing slopes (whose calculated stability factors can be lower than 1 even for presently stable slopes!). Back analyses of rock falls in limestone cliffs, involving rock bridge failure, were undertaken by Paronuzzi and Serafini (2005) but they did not consider the passed underwent solicitations. Consideration of rock bridges in stability analysis needs to know the presence and the extension of rock bridges. Recently, ground penetrating radar has been tested for structural survey of rock cliffs (Deparis et al. 2007).

Methods have been proposed and used to predict the time to failure, based on monitoring of slowly moving slopes for periods of some days to several years ("short term" prediction). But hazard assessment for land use planning needs "long term" prediction of failure for slopes which may be presently stable. In this context, the prediction concerns periods of at least one century and testing the prediction methods needs to consider such long periods. The aim of this paper is to introduce and use the principal of "historical back analysis" to test models of stability decrease of a rock slope.

2 PRINCIPLE OF HISTORICAL BACK ANALYSIS

Classical back analysis of a slope failure consists in searching a failure mechanism and values of the mechanical parameters which explain an observed failure under known conditions (for example known water level or seismic acceleration). It is assumed that the safety factor of the slope was 1 when failure occurred. This analysis allows for estimating the values of the strength parameters when the failure occurred ("present" values). In historical back analysis, one considers also the conditions the slope underwent during its "life". For example, a rock compartment which has been exposed in a cliff for a given period (its life) has probably been subjected to a minimal seismic acceleration which depends of this period. This acceleration can be derived from seismic hazard maps, for a given confidence level (95% for example). The life period of rock compartments can be estimated by cosmic ray exposure dating of the rock surface or using a kinematic model of rock cliff retreat, fitted with a rock fall inventory (Hantz et al. 2002, 2003b, Hantz & Frayssines 2007).
3 KINEMATIC MODEL OF CLIFF RETREAT

The rate of retreat of a cliff can be derived from the rock fall frequencies associated to different volume ranges (Hantz et al. 2002, 2003b). We assume a power law distribution of the rock fall volumes (Figure 1):

\[ f(V) = aV^{-b} \]

where \( f(V) \) is the mean number of rock falls with volume greater than \( V \), which occur each century; \( a \) is the mean number of rock falls greater than 1 m\(^3\) which occur each century; \( b \) is a positive constant.

The volumetric erosion rate \( W_t \) corresponds to the hatched area on the Figure 1:

\[ W_t = \int_{0}^{V_{\text{max}}} V df = \frac{a}{(1-b)^{1-b}} V^{(1-b)} \]

(2)

where \( V_{\text{max}} \) is the maximal possible volume for a rock fall or rock avalanche.

Figure 1. Cumulated rock fall frequency \( f \) as a function of the rock fall volume \( V \) (equation 1), and volumetric rock fall rate (equation 2).

Figure 2. Conceptual two-dimensional models (view from above) with three rock fall volume classes \( (V_1, V_2, V_3) \), corresponding respectively to cliff surfaces \( A_1, A_2, A_3 \). In the b configuration, the surfaces \( A_1, A_2, A_3 \) correspond to cliff sectors which vary with time. E is the linear rate of retreat of the cliff.

Figure 3. Assuming the displayed compartments will fall in one century, the total surface \( B_i \) of the rock fall scars for the volume class \( V_i \), which appears in one century, is represented by the thick lines. The average life expectancy of a scar is given by \( A_i / B_i \).

Table 1. Application of the rock fall erosion model to the limestone cliffs of the Grenoble area.

<table>
<thead>
<tr>
<th>Volume range ((V_i, V_{i+1})) ( (\text{m}^3) )</th>
<th>0-10(^3)</th>
<th>10(^3)-10(^4)</th>
<th>10(^4)-10(^5)</th>
<th>10(^5)-10(^6)</th>
<th>10(^6)-10(^7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation period</td>
<td>1935-</td>
<td>1935-</td>
<td>1935-</td>
<td>1800-</td>
<td>1600-</td>
</tr>
<tr>
<td>Observed frequency (per century)</td>
<td>51</td>
<td>14</td>
<td>9</td>
<td>1.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Fitted mean frequency (per century)</td>
<td>65</td>
<td>18</td>
<td>5</td>
<td>1.5</td>
<td>0.6</td>
</tr>
<tr>
<td>Volumetric erosion rate for the volume range ((V_i, V_{i+1})) ( (\text{m}^3/\text{century}) )</td>
<td>10,893</td>
<td>19,807</td>
<td>55,825</td>
<td>157,336</td>
<td>443,433</td>
</tr>
<tr>
<td>Total volumetric erosion rate ( (\text{m}^3/\text{century}) )</td>
<td>3.5 \times 10^6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total cliff area ( (\text{m}^2) )</td>
<td>24 \times 10^6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear rate of retreat ( (\text{m/century}) )</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cliff area ( A_i ) which is subjected to rock falls in the volume range ((V_i, V_{i+1})) ( (\text{m}^3) )</td>
<td>0.07 \times 10^6</td>
<td>0.1 \times 10^6</td>
<td>0.4 \times 10^6</td>
<td>10^6</td>
<td>3 \times 10^6</td>
</tr>
<tr>
<td>Scar appearance rate ( B_i ) for rock falls in the volume range ((V_i, V_{i+1})) ( (\text{m}^3/\text{century}) )</td>
<td>27,264</td>
<td>14,240</td>
<td>21,677</td>
<td>33,000</td>
<td>50,235</td>
</tr>
<tr>
<td>Mean life ( T_i ) of the rock compartments in the volume range ((V_i, V_{i+1})) ( (\text{year}) )</td>
<td>272</td>
<td>948</td>
<td>1,755</td>
<td>3,249</td>
<td>6,015</td>
</tr>
<tr>
<td>Undergone acceleration for 95% confidence level ( (\text{m/s}^2) )</td>
<td>1.12</td>
<td>1.79</td>
<td>2.26</td>
<td>2.85</td>
<td>3.60</td>
</tr>
</tbody>
</table>
The linear rate of retreat is:

\[ E = \frac{W_t}{S} \]  

(3)

where \( S \) is the area of the cliff.

The volumetric erosion rates \( W_i \) corresponding to different volume ranges \((V_i, V_{i+1})\) can also be calculated (Hantz et al. 2002, 2003b). As illustrated by the Figure 2, the cliff area which is subjected to rock falls with a volume between \( V_i \) and \( V_{i+1} \) is

\[ A_i = \frac{W_i}{E} \]  

(4)

The scar area \( B_i \) which appears each century due to rock falls in the volume range \((V_p, V_{i+1})\), highlighted in the Figure 3, can be derived assuming a power law distribution of the scar areas. Hence the mean life \( T_i \) of the scars and of the rock compartments in the volume range \((V_p, V_{i+1})\) is:

\[ T_i = \frac{A_i}{B_i} \]  

(5)

4 TEST SITES

The model has been applied to the limestone cliffs of the Grenoble area (French Alps), for which a rock fall data base was available and the rock fall volume distribution and frequency have been analyzed (Hantz et al. 2003a). The cliffs belong to the Subalpine Ranges and are made of limestone (Figure 4). They overlie more gentle slopes made of marl or alternate marl and limestone beds.

The results are given in the Table 1. The calculated linear rate of retreat of the limestone cliff is 1.5 \( \times 10^{-3} \) m/year. This rate is of the same order of magnitude as the long term rates reported by Hoffmann and Schrott (2002) for rockwall retreat in Alpine valleys.

The life time for the different volume ranges varies from about 300 years to 13,000 years, with a weighted mean of 11,000 years, which corresponds to an average age of 5,500 years. This order of magnitude has been recently confirmed by cosmic ray surface exposure dating (method already used to date Holocene rock slides, see for example Ivy-Ochs et al. 2009) of a limestone cliff near Grenoble, which gave four ages of respectively 2,400, 5,000, 11,000 and 14,000 years.

The three rock slides which were back-analyzed are located in the Figure 4. They took place in a cliff which does not belong to the inventory area, but which is in a similar geomorphological context. In all cases, the bedding planes dip less than 10° and the rock mass is cut by two subvertical joint sets. The cliff surface is more or less defined by one of these joint sets. The dispersion of the joint orientations in the set allows for sliding configurations as described in the Figure 5, involving thin rock slabs. The sliding planes dip between 75 and 85°. They were closely observed by roping down and the rock bridge rupture areas were directly measured on each scar. The proportion of rock bridge in the sliding surfaces is 0.2-0.3%. The volumes of the fallen compartments are given in the Table 2. The failures were not triggered by a significant earthquake or climatic event (Frayssines & Hantz 2006).

![Figure 4](image)

Figure 4. Geological scheme of the Grenoble area. Bold line: cliffs considered for the rock fall inventory. Star: location of the 3 analyzed rock slides.

![Figure 5](image)

Figure 5. Scheme of a rock slide. The bold line represents a rock bridge. W: weight of the block. N, T: normal and tangential reactions.

5 BACK ANALYSES

5.1 Classical back analysis

The mechanical parameters of the limestone constituting the rock bridges were determined by laboratory tests with 40 mm diameter samples: tensile strength = 7 MPa; uniaxial compressive strength = 142 MPa; cohesion = 23 MPa; friction angle = 54° (from triaxial compression tests with confining pres-
sure varying from 0 to 10 MPa). As the friction angle is less affected by scale effect than the cohesion, the friction angle given by the laboratory tests was assumed to be representative of the rock bridge. The rock bridge cohesion was determined by a classical back-analysis assuming the safety factor equals 1 at failure. The cohesion values obtained are given in the Table 2. They are 2-6 times lower than the laboratory values. This difference can be explained by a scale effect (Frayssines 2005).

5.2 Historical back analysis

The accelerations which were probably undergone by the rock compartments according to their life time were determined from the seismic hazard map of the France. Assuming a power law for the distribution of the seismic accelerations and a Poisson law for their time distribution, the acceleration whose exceedance probability is 0.95, was calculated for each life time $T_i$ (last line in the Table 1).

### Table 2. Main data and results of the back analysis of three rock slides.

<table>
<thead>
<tr>
<th>Rock fall name</th>
<th>Vierge du Vercors</th>
<th>Chalimont</th>
<th>Pas du Fouillet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume (m$^3$)</td>
<td>117</td>
<td>48</td>
<td>24</td>
</tr>
<tr>
<td>Cohesion from back analysis (MPa)</td>
<td>4</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>Rock bridge area at failure (m$^2$)</td>
<td>0.6</td>
<td>0.1</td>
<td>0.08</td>
</tr>
<tr>
<td>Initial rock bridge area (m$^2$)</td>
<td>&gt;0.98</td>
<td>&gt;0.14</td>
<td>&gt;0.11</td>
</tr>
<tr>
<td>Life time of the fallen compartment (year)</td>
<td>948</td>
<td>272</td>
<td>272</td>
</tr>
<tr>
<td>Solution rate (mm/year)</td>
<td>&gt;0.05</td>
<td>&gt;0.07</td>
<td>&gt;0.07</td>
</tr>
</tbody>
</table>

When this acceleration occurred, the rock compartment did not fall. At that time its safety factor was greater than 1 despite the acceleration undergone. Then the potential resisting shear force was higher than today. This decrease can be explained by different physical processes: (a) decrease of the rock strength due to subcritical microcracks growth inside the rock bridge; (b) decrease of the rock bridge area due to stress concentration at its tip (crack propagation as described by fracture mechanics theory); (c) decrease of the rock bridge area due to limestone soluation. Processes (a) and (b) were discussed by Frayssines (2005). It is assumed here that process (c) is the more efficient one in limestone cliffs.

The minimal value of the rock bridge area which explains that the rock compartment resisted to the seismic acceleration was calculated for each rock slide.

Assuming the rock bridge area can only decrease with time, it ensues that this value is a default estimation of the initial rock bridge area (area when the compartment began to be exposed). The rock bridge area has decreased from this initial value to the value at failure, during the life time of the compartment. A default estimation of the linear solution rate can then be calculated. The values obtained for the three rock slides are given in the table 2. They are compatible with the values given in the literature, which vary between 0.01 and 0.1 mm/year.

6 CONCLUSION

Historical back analysis of three rock slides allows a default estimation of the stability decrease of the rock compartments. In the context of rock fall hazard assessment, the maximal life time of a compartment could be estimated if its present state of stability is known. If this life time is of the same order of magnitude than the considered period for hazard assessment, the failure probability must be qualified as high. Such a quantitative analysis can be considered as a progress if compared with the usually used expert judgment.

REFERENCES


