Anisotropic scaling of tectonic stylolites: a fossilized signature of the stress field?

Marcus Ebner¹, Renaud Toussaint², Jean Schmittbuhl²,³, Daniel Koehn¹ & Paul Bons⁴

¹ Tectonophysics, Institute of Geosciences, Johannes Gutenberg University, Becherweg 21,  D-55128 Mainz, Germany;
² EOST, University of Strasbourg, France
³ Institut de Physique du Globe de Strasbourg, UMR CNRS 7516, 5 rue Descartes, F-67084 Strasbourg Cedex, France
⁴ Institute for Geosciences, Eberhard Karls University Tübingen, Sigwartstr. 10, D-72076 Tübingen

*Corresponding author: ebnerm@uni-mainz.de
   phone: +49/(0)6131 3926612
   fax: +49/(0)6131 3923863
Abstract

Vertical stylolites are pressure solution features, which are considered to be caused by horizontal tectonic loading with the largest principal compressive stress being (sub) parallel to the earth surface. In the present study we analyze the roughness of such tectonic stylolites from two different tectonic settings in southern Germany and north-eastern Spain aiming to investigate their scaling properties with respect to the stress during formation. High resolution laser profilometry has been carried out on opened stylolite surfaces of nine samples. These datasets were then analyzed using 1D and 2D Fourier power spectral approaches. We found that tectonic stylolites show two self-affine scaling regimes separated by a distinct crossover-length \( L \), as known for bedding parallel stylolites. In addition tectonic stylolites exhibit a clear in-plane scaling anisotropy which modifies \( L \). Since the largest and smallest crossover-lengths are oriented with the sample vertical and horizontal directions (i.e. \( \sigma_2 \) and \( \sigma_3 \)) and \( L \) is a function of the stress field during formation as analytically predicted we conclude that the scaling anisotropy of tectonic stylolites is possibly a function of the stress field. Knowledge of this crossover-length anisotropy would enable the reconstruction of the full 3D stress tensor if independent constraints of the depth of formation can be obtained.

1. Introduction

The intriguing variety of pressure solution features and its wide-spread occurrence in monomineralic rock types provoked a continuous interest and attention in various geoscience disciplines over the past decades [Tada and Siever, 1989]. One of the most prominent and complex pressure solution features are stylolites, which are rough dissolution interfaces that can be found in a large variety of sedimentary rocks [Buxton and Sibley, 1981; Dunnington, 1954; Heald, 1955; Park and Schot, 1968; Railsback, 1993; Rutter, 1983; Stockdale, 1922; Tada and Siever, 1989]. Until recently stylolite morphology has been described qualitatively by the use of a descriptive terminology, which grouped stylolites into generic classes. One
classification uses the orientation of the stylolite plane relative to bedding. Bedding-parallel stylolites are supposed to have formed due to the layer-normal overburden pressure, while tectonic stresses cause the formation of stylolites oblique or perpendicular to bedding [Park and Schot, 1968; Railsback and Andrews, 1995]. A second classification is based on the orientation of the stylolite teeth relative to the stylolite plane. Here the term "stylolite" is used for teeth at a high angle to the plane, and ‘slickolite’ for dissolution surfaces where the teeth are distinctly oblique to the dissolution plane [Bretz, 1940; Gratier et al., 2005; Simon, 2007]. Finally the shape of the characteristic teeth-like asperities and spikes along the interface has been used to characterize stylolites [Guzzetta, 1984; Park and Schot, 1968].

More recently, stylolites have been subjected to more rigorous quantitative analyses to characterise the roughness of the stylolite surface [Brouste et al., 2007; Drummond and Sexton, 1998; Ebner et al., 2009a; Ebner et al., 2009b; c; Gratier et al., 2005; Karcz and Scholz, 2003; Koehn et al., 2007; Renard et al., 2004; Schmittbuhl et al., 2004]. It was demonstrated that the 1D stylolite roughness obeys a fractal scaling invariance [Drummond and Sexton, 1998; Karcz and Scholz, 2003]. Investigation of the rough interface of opened stylolite surfaces by means of laser profilometry revealed that the stylolite morphology shows two self-affine scaling regimes with two distinct roughness exponents on their respective scales, which are separated by a characteristic crossover length at the millimeter scale [Renard et al., 2004; Schmittbuhl et al., 2004] for bedding parallel stylolites. Self-affine surfaces define a group of fractals, which remain statistically unchanged by the transform: $\Delta x \rightarrow b \cdot \Delta x$, $\Delta y \rightarrow b \cdot \Delta y$, $\Delta z \rightarrow b^H \cdot \Delta z$, where $b$ is a transformation factor, which can take any real value and $H$ is the Hurst or roughness exponent [Barabasi and Stanley, 1995], which is a quantitative measure for the roughness of the signal.

Analytical and numerical investigations demonstrated that the growth of the stylolite roughness is induced by heterogeneities in the host rock that pin the interface and is slowed down by two stabilizing forces, the elastic and surface energies. The elastic energy dominates...
on larger scales and is represented by a small roughness exponent of 0.3 to 0.5 whereas the surface energy is dominant on small scales with a roughness exponent of about 1 [Koehn et al., 2007; Renard et al., 2004; Schmittbuhl et al., 2004]. The characteristic crossover length \( L \) that separates these two scaling regimes is a function of the principal normal stress [Renard et al., 2004; Schmittbuhl et al., 2004] on the interface of a bedding parallel stylolite. This analytical predictions were successfully tested by Ebner et al. [2009b], who demonstrated on a set of 13 bedding parallel stylolites from varying stratigraphic depth out of a cretaceous succession that this crossover-length decreases with increasing depth (and normal stress) and thus exhibit the analytically predicted behaviour. The 1D scaling of stylolites with two self-affine scaling invariance regimes can be described as the height difference \( h \) of points along the surface separated by a distance \( \Delta x \) as [Ebner et al., 2009b]

\[
h(\Delta x) \approx A \Delta x^{H_S} g(\Delta x / L) \quad \text{with} \quad g(u) = \begin{cases} u^0 & \text{if } u << 1 \\ u^{H_S - H_L} & \text{if } u >> 1 \end{cases}
\]

where \( A \) is a scaling factor, \( g \) is a scaling function and \( u \) is the ratio \( \Delta x / L \) with \( L \) being a crossover-length. \( H_S, H_L \) correspond to the roughness exponents for small and large scales, respectively. Numerical simulations also demonstrate that the crossover-length is very robust with regard to the kind and amount of quenched noise (heterogeneities initially present) in the rock [Ebner et al., 2009a]. Hence, the use of bedding parallel stylolites as a quantitative stress gauge under the assumption of uniaxial strain (zero horizontal displacement) seems to be verified. Investigations of the surface morphology of bedding parallel stylolites showed that their scaling is isotropic within the plane defined by the stylolite [Renard et al., 2004; Schmittbuhl et al., 2004]. This implies that any arbitrary section through the stylolite interface that contains the principal stress direction (i.e. normal to the plane) fully characterizes the complex self-affine roughness of bedding parallel stylolites. A second mechanism claimed to be responsible for the formation of the characteristic roughness is a stress induced roughening instability along an initially flat solid-solid interfaces [Angheluta et al., 2008] or a solid-fluid-
solid interface [Bonnetier et al., 2009]. In both cases the instability is triggered by elastic stresses acting normal on the interface.

Up to now no study has quantitatively investigated the 3D topography of tectonic stylolites, which formed due to (sub-)horizontal compression resulting in a vertical stylolite plane. Tectonic stylolites differ in two major characteristics from bedding parallel stylolites. First, the stress field during the formation of tectonic stylolites is non-isotropic i.e. the in-plane normal stresses differ (i.e. $\sigma_{zz} > \sigma_{xx}$) whereas bedding parallel stylolites often have equal in-plane normal stresses $\sigma_{xx} = \sigma_{yy}$ (Figure 1). This would imply that the scaling of tectonic stylolites is not invariant within the plane, since the crossover-length should scale with the (non-isotropic) stress field as was shown analytically [Schmittbuhl et al., 2004]. A second common feature of tectonic stylolites are oblique/tilted teeth with respect to the mean stylolite plane due to overprinting of pre-existing planes of anisotropy such as joints, bedding planes and other interfaces. Tilting of the teeth with respect to the stylolite plane also influences the morphology because it leads to the dominance of long grooves and ridges [Simon, 2007]. These features could lead to an anisotropic scaling of the stylolite interface in addition to variations of the in-plane stresses.

The present study investigates such tectonic stylolites which formed in a vertical orientation. We mainly concentrate on the influence of (i) the orientation of the dissolution surface with respect to the displacement direction and (ii) the formation stress on the scaling properties of natural stylolites in limestones. To accomplish this task we use laser profilometry data of opened interfaces of tectonic stylolites from flat lying Jurassic limestones of the Swabian Alb in southern Germany and from a Tertiary fold and thrust belt of the Iberian Chain of north-eastern Spain.

2. Geological setting
In the following section we give a brief introduction of the investigated field areas in southern Germany and north-eastern Spain, which both expose upper Jurassic limestones. The Swabian Alb of southern Germany forms a region of flat-lying mainly marine Jurassic deposits [Geyer and Gwinner, 1991]. The studied sections are located 10 km south of the city of Tübingen and comprise upper Jurassic (Oxfordian to Kimmeridgian) limestones. The basal part of the sections (UTM 32U; E 0515212 m; N 5362240 m) are made up of well bedded Oxfordian limestones whereas the upper part of the profile contains massive Kimmeridgian limestones representing a rift facies with sponges and algae being the main rock forming species [Etzold et al., 1996; Geyer and Gwinner, 1991]. The bedding is (sub-) horizontal, dipping slightly (<5°) to the SE on a regional scale. The principal structural features are ENE-WSW striking graben structures, which exhibit a mixed mode displacement with a major normal and a subordinate dextral component [Etzold et al., 1996; Geyer and Gwinner, 1991] and can be attributed to a later compressional phase (see below). The investigated stylolites (Samples: Sa6/1a, Sa6/1b, Sa9/2) form vertical planes which trend WNW-ESE with teeth pointing parallel to the surface normal direction, hence recording a NNE-SSW compression (Figure 2a). Additionally small scale reverse-faults and NNE-SSW trending joints confirm the same kinematic framework. A younger subordinate set of stylolites not investigated in this study form NE-SW trending vertical stylolite planes which can be related to the prominent dextral graben structures found in the area [Geyer and Gwinner, 1991; Kley and Voigt, 2008].

Our relative chronological sequence of deformation events is in agreement with data reported by Kley and Voigt [2008], demonstrating a change in the stress field from NNE-SSW directed compression in the late Cretaceous to a NW-SE directed compression in the Neogene. This second compression phase neither altered the shape nor the orientation of the investigated stylolites, since layer parallel shortening did not cause any orientational change and deformation was restricted to stylolite interfaces.
The Iberian Chain of north-eastern Spain is located south of the Ebro-basin and trends roughly NW-SE. The succession is composed of up to 6000 m of Mesozoic, mainly Jurassic and Cretaceous sediments \[\text{Capote et al., 2002}\], although the sequence is significantly reduced to only 300-400 m in the investigated area. The investigated area belongs to the Maestrazgo structural domain which forms the transition zone between the NW-SE striking fold and thrust belt of the Aragon Branch and the NE-SW striking Catalanian Coastal Ranges.

A regional NNW-SSE compression in the sampling area between the small towns of Molinos and Ejulve is indicated by ENE-WSW striking 100-1000 m scale fold trains with top to the NNW kinematics. The onset of deformation is estimated to be around Early to Middle Eocene, whereas the deformational peak is assumed to be during the Oligocene \[\text{Capote et al., 2002; Casas et al., 2000; Liesa and Simón, 2009}\]. Liesa and Simón \[2009\] report stylolite data which they argue to be attributed to Betic and Guadarrama compressions both having their deformational peaks during the Oligocene. The investigated section \((UTM 30T; E 07111963 m; N 4518336 m)\) comprises well bedded limestones in an upper Jurassic upright antiform which contains several smaller synforms that plunges 25° to the NW. Stylolites were investigated in a shallow ENE dipping limb \((set A\) in Figure 2b) and from an overturned limb which dips steeply to the SE \((set B\) in Figure 2b). In the eastwards-dipping limb of the fold the stylolites \(\text{Samples: M4/1, M4/2, M4/3, M4/4}\) track the far field shortening direction \((\text{SSE-NNW})\) confirmed from field measurements in other outcrops in the area. In the overturned and steeply dipping fold-limb the stylolites \(\text{Samples: M4c/1, M4c/3}\) are rotated around the fold axis into a shallow dipping orientation \((\text{i.e. a counter-clockwise or clockwise rotation of 65° around the fold axis would transform the stylolite orientation from one limb into the orientation of the stylolites in the other limb of the fold})\). Hence, the stylolite formation in this outcrop predates the folding event. In addition the angle between the stylolite plane and the bedding (not shown) is consistent in both positions of the fold thus corrugating the evidence that stylolitization predates the folding event. It has to be noted that stylolites in set A and B
both form in a vertical orientation. Another important feature to notice is that the stylolite teeth are somewhat oblique (\(\sim 10^\circ\)) to the mean stylolite plane, which we interpret as a result of pressure-solution overprint of a pre-existing joint-set which strikes NE-SW, sub-parallel to the stylolite planes.

3. Methodology

The samples collected in the locations described above were all taken oriented in the outcrop to reconstruct the spatial position of the 3D stylolite morphology after laser profilometry. For analysis only “closed” specimens were considered. Stylolite surfaces that were already open in the outcrop and were subjected to an unknown amount of weathering were ignored. The sampled specimens were opened mechanically along the two opposing interfaces of the stylolite. This method causes some negligible damage to the surface due to the interlocking of asperities. The split surfaces were cleaned from any clay material, i.e. the residuum of the dissolved rock, with a soft brush and distilled water. Areas which did not exhibit visual mechanical damage were chosen for profilometry.

Optical profilometry is based on a laser triangulation of the surface similar to previous studies [Renard et al., 2004; Schmittbuhl et al., 2004; Schmittbuhl et al., 2008]. The triangulation technique uses a laser beam that is focused on the surface of the object, which is monitored by a nearby CCD sensor. The distance between the object and the sensor changes as a function of changes of the angle under which the point of consideration is observed. The distance between the object and the laser-head is then calculated from angular relationships [Schmittbuhl et al., 2008]. Before every individual measurement a test run was made to calibrate voltage fluctuations of the laser beam (volt-height relationship is virtually linear in the chosen range, which gives the estimate of the vertical resolution – small distortions of the profile height, less than 1%, can be expected.). The laser beam is 30 \(\mu\)m wide and horizontal steps between measurement points were \(\Delta x = \Delta y = 25\mu m\) with a horizontal precision of 1\(\mu m\).
The vertical resolution is 2µm. Maps were constructed by movement of the laser-head along parallel profiles over the specimen (Figure 3). Eight samples have been measured at high resolution (Δx=Δy=25µm) with map sizes of 1200x1200 (Samples: M4/1, M4/4), 1600x1600 (Samples: Sa6/1a, Sa6/1b, M4/2,M4/3, M4c/1, M4c/3) & 2000x2000 measurement points (Sample: Sa9/2), which corresponds to square maps with physical side lengths of 30, 40 and 50 mm. The x- and y-directions are arbitrary choices parallel to the principal axis of the profilometer. The sample is usually oriented in a way to fit the biggest square map on the respective stylolite interface. Care was taken that from the orientation of map x/y direction the sample orientation could be reconstructed.

Additionally Sample Sa6/1 was measured twice where the second measurement (Sa6/1b) was rotated 32° clockwise around a vertical axis with respect to the first measurement (Sa6/1a). This was done to test the robustness of the measurements used against possible noise arising from the measurement procedure along discrete profiles. An image registration [Goshtasby, 1986; 1988] of the two measurements in spatial domain revealed the same amount of rotation of 32° with an uncorrelated noise in the height difference between the two images that arises from the discreteness of the two maps (not shown). This height difference is less than 5% (i.e. the ratio of the standard deviation σ of the height difference is 0.063 mm to σ of the height of the surface 1.477 mm). Hence, there seems to be no significant error introduced by the measurement procedure.

4. Data analysis

Before we analyzed the 2D maps in detail the raw data from the laser profilometry was subjected to a series of pre-treatments (Figure 4). First a mean plane calculated from a least square fit was subtracted from the raw data (Figure 4a), i.e. the x/y-plane is adjusted to a global trend and the vertical (z) axis is set to have zero mean height (Figure 4b). To increase the quality of our Fourier transforms (described below) we used a Hanning window technique
Karcz and Scholz, 2003; Press et al., 2007] to force our data to taper to zero at the boundaries (Figure 4c) in order to reduce spectral leakage (compare Figure 3). This is a standard technique in signal processing, which does not modify the frequency and amplitude of the original signal.

4.1. 1D analysis

From the 2D height-field a 1D profile can be extracted either along the x or y-direction or in any arbitrary direction. For an arbitrary 1D profile $f(x)$ the Fourier transform $F(k)$ can be calculated and the power spectrum $P(k) \sim |F(k)|^2$ of the transform can be plotted as a function of the wavenumber $k=2\pi/\lambda \ [m^{-1}]$, which scales inversely to the wavelength $\lambda$ [Renard et al., 2004; Schmittbuhl et al., 1995; Schmittbuhl et al., 2004]. In Figure 5 the averaged spectra of Sample M4/3 along the x and y direction of the measured map are shown. The averaged spectra are found from calculating the mean of $P(k)$ for every $k$-value over all 1D profiles in one direction [Renard et al., 2004; Schmittbuhl et al., 2004]. This averaging procedure reduces the noise attached to an individual 1D profile. A linear slope of the power spectra confirms a self-affine scaling invariance. The power spectrum of a self-affine signal behaves as

$$ P(k) \sim k^{-D-2H}, $$

where $D$ is the topological dimension of the signal ($D=1$ for 1D profiles) and $H$ the Hurst exponent. The Hurst exponent can thus be calculated from the slope of the power spectra.

When we study the averaged 1D spectra of a tectonic stylolite along specific directions (Figure 5a) we see that the signal exhibits two slopes, which are separated by a crossover-length ($L$) in agreement with observations on bedding parallel stylolites [Ebner et al., 2009b; Renard et al., 2004; Schmittbuhl et al., 2004]. The two observed scaling regimes have typical Hurst exponents of $H_S \sim 0.5$ and $H_L \sim 1.1$ for the small and large scale (large and small wavenumber), respectively. These observations indicate that the scaling of bedding parallel
stylolites (Eq. 1) can be extended to tectonic stylolites (compare Figure 5a). To enable a more
detailed comparison of the power spectra of our tectonic stylolites from two different
(orthogonal) directions we normalize the power spectra along the x-direction with the power
spectrum of the y direction at k=1[mm⁻¹] i.e. \( P_x(k)/P_y(1[mm^{-1}]) \) as shown in the inset of Figure
5a. This normalization yields a collapse of the large k-values (small scales), but a notable
difference for the small k-values (large scales) of the scaling functions. This is basically the
expression of the shift in cut-off between the two linear sub-branches, which is the crossover-
length \( L \). Figure 5b shows that the calculated cut-off between the scaling regimes and thus
crossover length differs between them. With 1.22 and 0.62 mm for the x and y-directions the
crossover-length changes by 0.6 mm (Figure 5b). The non-linear fitting plotted as a solid line
in both panels of Figure 5b is a linear-by-parts least square fit in logarithmic space with a
weighting function that changes from the small scale to the large scale fraction of the scaling
law [for details compare Ebner et al., 2009b]. This non-linear model uses a minimization
algorithm to find the least square fit for the crossover-length. The differences found between
the two directions also include a discrepancy in the scaling pre-factor, i.e. a vertical shift of
the power spectra, which is clearly higher for all scales in the y-direction.

To fully quantify rough surfaces it is necessary to characterise this pre-factor of the
scaling function and thus obtain a full description of the surface morphology. In the following
we use the height-height correlation function, to calculate the scaling prefactor. The height-
height correlation function [Barabasi and Stanley, 1995], which is defined for a function \( h(x) \)
over the spatial variable \( x \) by,

\[
C(\Delta x) = \left( \langle (h(x) - h(x + \Delta x))^2 \rangle \right)^{1/2},
\]

where \( \langle \cdot \rangle \) denotes average over the range of \( x \), which estimates the average height difference between two points of the profile
separated by a distance \( \Delta x \). For a self-affine profile, the correlation-function follows a power-
law such that \( C(\Delta x) \sim \Delta x^{2H}, \) where \( H \) is the Hurst exponent and \( t \) is the scaling prefactor.
The prefactor can be designed as \( C(t)=t \), and thus denotes a length scale, also known as the
The topothesy corresponds physically to the length scale for which the slope of the rough profile is equal to 1. In other words, $t$ is the theoretical length scale over which the rough profile has a mean slope of 45°. The smaller $t$, the flatter the profile appears on a macroscopic scale.

Figure 6a shows a scaling of the correlation function with two linear sub-branches separated by a crossover-length similar to the scaling of the power spectra shown in Figure 5a with only the slopes being different. The correlation function shows, similar to the power spectra, two linear sub-branches separated by a distinct crossover-length. We use the same nonlinear fitting approach as described above (with fixed Hurst exponents of 0.6 and 0.3). The different scaling exponents compared to the power spectral approach is inline with reliability of self-affine measurements performed on synthetic signals [Candela et al., 2009; Schmittbuhl et al., 1995]. These authors have demonstrated that the correlation function underestimates the input Hurst exponents and thus shows lower values than the power spectra. The scaling prefactor and thus the topothesies $t_s$ and $t_l$ for the small and large scale regimes can be found by intersection of the two sub-branches of the scaling function with the 1/1 line (Figure 6a). We estimated the topothesy for all orientations on the surfaces (Figure 6b & c) and found that there is a weak anisotropy in the scaling pre-factor, which shows a correlation with the highest topothesies being parallel to the horizontal direction in the sample orientation (Figure 6b) for most samples but this is only visible in the small scale regime. This observation is similar to what we found from investigation of the power spectra where the small scale regime is shows very consistent results but the large scale regime reveals a higher degree of variability e.g. compare inset in Figure 5a. The small scale topothesy are shown in Figure 6c. The average topothesies range between 0.05-0.15 mm and 0.15-0.3 mm for small and large scales, respectively.

Both the power spectra (i.e. the cut-off length between the linear sub-branches) and topothesy of a 1D signal show a considerable degree of anisotropy which is often obscured
due to the noise associated with an individual 1D profile. We conclude that to account for this in-plane variation a 1D signal fails to capture all scaling characteristics of tectonic stylolites and the choice of the investigated profile is not arbitrary as for bedding parallel stylolites. Hence, tectonic stylolites have a measurable in-plane anisotropy which we want to characterize in detail with a 2D approach.

4.2. 2D analysis

For the 2D analysis we used the processed data as described in section 4 (Figure 4c). First a 2D Fourier transform i.e. a discrete Fourier transform (DFT) was calculated from the data points of the 2D height-field with the Fast Fourier Transform (FFT) algorithm [Cooley and Tukey, 1965] implemented in Matlab®. Then the DFT is shifted so that the zero-frequency component lies in the centre of the spectra and the 2D power spectrum \( P(k_x,k_y) \) is again calculated as the square of the absolute magnitude of the Fourier transform. Figure 7a displays a map of the 2D power spectra \( P(k_x,k_y) \) in which the absolute magnitude squared is shown as greyscale values and \( k_x \) and \( k_y \) range from \(-((n/2)\Delta x)^{-1}\) to \((n/2)\Delta x)^{-1}\) where \( n \) is the number of measurement points in one direction of the map and \( \Delta x=\Delta y \) is the step size. To investigate the power law behaviour located in the 1D signals the 2D power spectra had to be transformed to a double logarithmic space originating from the centre of the map i.e. the zero frequency component or the smallest wavenumber. This is accomplished by translating every value pair \((k_x,k_y)\) by \( \log(\sqrt{k_x^2+k_y^2}) \) along the direction defined by the direction cosine of the position vector \((k_x,k_y)\) with the x-axis of the coordinate system and plotting \( \log(P(k_x,k_y)) \) on the newly formed logarithmic grid. The central point in this case corresponds to the system size, which imposes the smallest non-zero \( k \). Figure 7b illustrates such a double log-plot of sample M3/4, in which the power spectra are displayed as a 3D surface. Notice that the view direction is along the \( k_x \)-axis. The slopes of the surface, which roughly describe an elliptical
cone clearly exhibit two linear branches and a distinct crossover region ($L$) marked by the arrow in Figure 7b. Thus the 3D representation is consistent with the scaling behaviour found from the analysis of the 1D signal.

For further analysis of the anisotropy we resample the 3D representation (Figure 7b) with a 2D logarithmic binning (along $k_x$ and $k_y$ direction), to get a constant density of grid points in double-logarithmic representation (Figure 8a). For this reason a fixed grid that covers the 2D power spectra with a constant bin size ($bs$) in logarithmic space ($log(bs) = 0.4$) in the x and y direction is used to find all $k_x,k_y$-value pairs that fall into a certain bin, and the mean of all power spectra that belong to these $k_x,k_y$-value pairs in this bin is then used to define the binned power spectrum. This procedure allows analyzing the data with an equal importance for the long and small scales, respectively. In addition this method smoothes the data by removing the local fluctuations without an alteration of the overall geometry of the 3D representation, that is characterized by the two scaling regimes and the distinct crossover.

We use isopach/contour maps of the binned 2D power spectra to quantify the degree of anisotropy. Isotropic signals should reveal concentric circular contour lines, which define the same $log(P(k_x,k_y)$ value. Concentric circular contour lines would imply that the crossover length, which separates the self-affine scaling regimes for small and large scales are the same in every direction. Figures 8 show that this is clearly not the case for tectonic stylolites (also compare Figure 7a) where the contour lines reveal an elliptical shape (Figure 8a,b). This shape is clearly different from the circular concentric contours found in bedding parallel stylolites (compare e.g. to Figure 4 of Schmittbuhl et al., 2004). We use a least square criterion to estimate the best fit ellipse of the individual contour lines. From the best fitted ellipse, we calculate the aspect ratio of the principal axis (i.e. $a/b$; where $a$ and $b$ are the semi-major and semi-minor axis of the best fit ellipse) to get a quantitative measure of the anisotropy of the 2D binned power spectra (Figure 8c). For the direction of the anisotropy we utilize the angle $\Theta$ between the long axis ($a$) of the fitted ellipse and the x-direction of the
coordinate system (Figure 8d). For all investigated samples we recognized an increased
ellipticity toward the centre of the 2D power spectra but only a moderate or no significant
change in orientation of the asymmetry with respect to the position in the power spectra. Note
that in this representation (Figure 8a) high contour lines (small wavenumbers) correspond to
large physical length-scales whereas low contour lines (large wavenumbers) correspond to
small length-scales.

The fact that the large wavenumbers display an isotropic power spectrum i.e. aspect
ratio close to 1 (Figure 8c), whereas the small ones show an anisotropic one, is very consistent
with the result of the 1D data analysis (see previous section). This observation is also in
agreement with the physical interpretation of the mechanism of stylolite formation and
morphogenesis [Ebner et al., 2009b; Koehn et al., 2007; Renard et al., 2004; Schmittbuhl et
al., 2004]: At small scales (large wavenumbers), the balance between surface tension and
disorder is controlling the shape of stylolites. Both are a priori isotropic along the stylolite. In
contrast, the large scale morphologies (small wavenumbers) are normally physically
interpreted as resulting from a balance between the elastic field and the material disorder is
controlling the shape of the stylolites. The fact that an anisotropy is observed at large scales is
thus the signature of an in-plane anisotropy of the stress. Since stylolite teeth are normally
parallel to largest stress direction associated with $\sigma_1$, this large scale anisotropy should be
associated to a difference between the two principal values of the in-plane stress components,
$\sigma_2$ and $\sigma_3$.

The orientation of the long axis of the fitted ellipse relative to the vertical orientation
of the sample is shown in rose diagrams (Figure 9) for all samples. The long axes of the
contours of the power spectrum are associated with a shorter crossover-length $L$ (i.e.
reciprocal to the wavenumber) between the large $k$ isotropic scaling and the small $k$
anisotropic one (Fig 9j). We will see in the next sections that this can be interpreted as a
variation of the difference between the largest principal stress (normal to the stylolite plane)
and the two in-plane stress components. The principal stress associated with the direction of the long axis should thus be the smallest one, i.e. $\sigma_3$. Arrows show the orientation of a vertical line projected onto the stylolite plane in its original outcrop orientation. From this representation (Figure 9) it is evident that the vertical direction is roughly normal to the long axis of the anisotropy for all samples except M4c/1 and M4c/3 which formed vertically (compare chapter 2 for details) but were subsequently rotated into a shallow dipping (non vertical) orientation plane due to folding (Figure 9h,i). They thus serve as a cross check to our findings since the vertical direction in these samples does not coincide with the vertical direction during stylolite formation and the anisotropy is therefore not normal to the present vertical direction in these samples as for samples of the upright limb.

To estimate the crossover length ($L$) and thus get quantitative information on the stresses during stylolite formation we again use the elliptical fit as a simplified representation of the 2D Fourier transform of our data. We assume that the crossover is located at the position of the biggest change in the local slope of the 2D Fourier transform (compare Figure 7b). We calculate the local slope $s$ as the difference between the long and short axis $(a,b)$ of the best fit ellipse for succeeding $\log(P(k_x,k_y))$-contours $s=(\Delta a+\Delta b)/2$. A plot of the $\log(P(k_x,k_y))$-contours as a function of the local slope $s$ is shown in Figure 10a. The crossover is defined to lie at the minimum local slope in this representation and the crossover is calculated from the principal axis of the best-fit ellipse at this minimum (Figure 10b). It can be noticed that the maximum crossover-length coincides quite well with the vertical direction (indicated by arrow in Figure 10b) this is in agreement with our previous observations that the anisotropy of the power spectra is also oriented (normal) with respect to the sample vertical orientation (compare Figure 9).

Before we discuss the orientation of the anisotropy and the determined crossover length-scales in relation to the stress tensor that was present during stylolite growth, we want to investigate the influence of tilted teeth on the scaling results.
4.3. Synthetic data analysis

It is important to prove that the large scale anisotropy we found in the investigated samples is really related to the stress field during formation and thus exclude the influence of other factors which might as well cause a scaling anisotropy. The second important characteristic of tectonic stylolites, as stated in the introduction, is the occurrence of inclined teeth i.e. slickolites. It is easy to imagine that the ridge and groove morphology of slickolites with highly inclined teeth can cause a difference in the scaling parallel or transverse to these elongated morphological features and thus an anisotropy. To systematically investigate the influence of a tilt of the asperities or teeth we construct synthetic isotropic self-affine surfaces and tilt the teeth around one arbitrary axis. Tilted or inclined asperities are a common feature of slickolites [Simon, 2007] and it is commonly assumed that these structures formed when a stylolite overprinted a pre-existing plane of anisotropy in the host-rock. In this case the principal stresses are oriented oblique to the pressure solution surface, which has recently been proven numerically by Koehn et al. (2007). Synthetic self-affine surfaces can be created following the approaches found in the literature [Meheust and Schmittbuhl, 2001; Turcotte, 1997]. We follow the method described in Meheust and Schmittbuhl (2001) who construct square white noise maps of size $n=512$. The self-affine correlation is then introduced by multiplying the modulus of the 2D Fourier transform of the white noise by the modulus of the wavenumber raised to the power of $-1-H$, where $H$ is the roughness exponent. The self-affine surface is obtained from the inverse Fourier transform. The synthetic surface shown in Figure 11a is constructed with a Hurst exponent of $H=0.5$ and its 2D Fourier transform has a true isotropic self-affine behaviour (compare inset in Figure 11a). A pre-defined tilt of the roughness is then attained from adding a linear trend along the x-axis of the map which corresponds to a tilt angle $\alpha$ and a subsequent back-rotation around $\alpha$ i.e. multiplying the data with a 3D rotation matrix of $-\alpha$. Various tilt angles ranging from 1-50° were realised from the
map shown in Figure 11a. To analyse single valued functions (with no overhangs) the tilted surfaces are projected on a plane defined by the mean surface. The data were then analyzed as described in the previous chapter (section 4.2). The degree (aspect ratio) and orientation (slope) of the anisotropy is displayed in Figure 11b & c. It is evident that the original data-set is isotropic with aspect-ratios for $\log(P(k_x,k_y))$ contours close to 1. With small tilt angles $\alpha < 10^\circ$ an anisotropy for the low $\log(P(k_x,k_y))$ contours and thus large wavenumbers and small scales exists, which decreases with increasing $\alpha$. In addition there is a general increase in the anisotropy in all scales with tilt angles of $\alpha \geq 20^\circ$ (Figure 11b) whereas the orientation is more and more aligned with the rotation/tilt axis (Figure 11c) with increasing tilt angle. The topothesy of the synthetic surfaces do not exhibit a directional anisotropy but reveal a general decrease of the average topothesy with increasing tilt angle from a $t \sim 0.22$ for the original data down to $t \sim 0.09$ for a tilt angle of 50°.

5. Discussion

We have shown that the tectonic stylolites investigated in this study, i.e. stylolites which form when the principal compressive stress direction is horizontal, differ fundamentally from bedding parallel stylolites since they show anisotropic scaling relations. Two self affine scaling regimes (with Hurst exponents of ~0.5 and ~1.1 for the small and large scale, respectively), which are separated by a crossover-length at the millimeter scale can be found in bedding parallel and tectonic stylolites. The crossover-length $L$ scales inversely with the formation stress $L \sim \sigma^{-2}$ for bedding parallel stylolites [Ebner et al., 2009b]. The analytical solution of Schmittbuhl et al. [2004] relates the crossover length ($L$) to the stress-field during stylolite formation. Their stress term is a product of mean and differential stress and can be used to calculate the stress magnitudes in addition to the determination of principal stress directions. The analytical solution shows that
\[ L = \frac{\gamma E}{\beta \sigma_m \sigma_d}, \quad (3) \]

where \( E \) is the Young’s Modulus, \( \gamma \) is the solid-fluid interfacial energy, \( \beta = \nu(1-2\nu)/\pi \) is a dimensionless constant with \( \nu \) the Poisson’s ratio, \( \sigma_m \) and \( \sigma_d \) are the mean and differential stresses respectively. Since for bedding parallel stylolites perfect confinement can be assumed (that is uniaxial strain or zero horizontal displacement) the stresses and thus the crossover length \( L \) is independent of the orientation within the stylolite surface (Figure 1a). For a tectonic stylolite with a vertical stylolite plane the scenario is different (Figure 1b) and it can be assumed that the in-plane stresses are dissimilar. One in-plane principal stress component should be dependent on the amount of overburden and should be oriented vertically whereas the second stress component should have a horizontal orientation. Since the crossover-length \( L \) scales inversely with the product of mean and differential stress and the mean stress should be constant, variations of the crossover should reflect variations of the differential stress \( |\sigma_1-\sigma_{\text{inplane}}| \) [compare to Schmittbuhl et al., 2004]. Therefore the crossover-length has to increase from a minimum in the direction of the least principal stress \( \sigma_3 \) (x-axis in Figure 1b) and thus the direction of the largest differential stress \( |\sigma_1-\sigma_\text{inplane}| \) to a maximum in an in-plane orientation normal to this direction, which corresponds to the direction of the largest inplane stress \( \sigma_2 \) (the vertical direction in Figure 1b), and the smallest differential stress \( |\sigma_1-\sigma_2| \). In conclusion it can be assumed that the orientation of largest and smallest crossover-length coincide with the vertical and horizontal direction (i.e. \( \sigma_{xx} < \sigma_{zz} \)) respectively.

Indeed we found a scaling anisotropy in our data, which shifts the crossover-length accordingly (Figure 9). The 1D analysis (Figure 5) and the 2D data analysis (Figure 9 & 10) reveal that the long axis of the detected anisotropy is normal to the vertical direction with a crossover-length maximum in this direction implying that \( \sigma_2 \) has a vertical orientation. This observation holds for both investigated areas although there is a slight deviation of up to \( \pm 10^\circ \) for some samples. Only the samples (M4c/1, M4c/3 from the overturned fold limb) which
formed vertically but experienced a passive rotation subsequently to stylolite formation due to folding (compare Figure 2b and Figure 9h,i) differ significantly. This can be explained by the fact that the stylolite formation was prior to folding as can be concluded from the structural relationships in the field data (Figure 2). Thus the present orientation of the samples in the overturned fold limb does not coincide with the orientation during formation of the stylolites.

We noticed a small difference (<10°) between the orientation of the stylolite teeth and the pole of the mean stylolite plane for the samples from north eastern Spain. This is due to the fact that the stylolites overprint a pre-existing joint set that is subnormal to the principal shortening direction, which influenced stylolite formation as stated above. To investigate the effect of the tilt of the stylolite teeth and its contribution to the observed scaling anisotropy we used synthetic self-affine surfaces which were systematically tilted to get slickolite similar structures as explained above (Figure 11). The effect of the tilt of the teeth with respect to the mean plane of the stylolite can be characterized by (i) an anisotropy for the large wavenumbers i.e. on the scale of individual teeth or asperities for small tilt angles (<10°) and (ii) a general homogeneous increase of the anisotropy for all scales with an increase of the tilt angle for angles >10°. This anisotropy caused by the imposed tilt of the asperities differs significantly from the anisotropy of real stylolites. Therefore we conclude that the 3D formation stress is the dominant force that influences the scaling anisotropy of the investigated samples. However one has to note that tilted teeth imply that the principal stress components are not necessarily oriented within the stylolite plane. Therefore only tectonic stylolites with plane-perpendicular teeth should be used to recalculate principal stress orientations and magnitudes.

The analytical solution [Schmittbuhl et al., 2004] is only strictly valid for 2D stress cases where the principal stresses parallel to the stylolite plane are invariant along the third direction, which is truly the case for bedding parallel stylolites as discussed by Ebner et al. [2009b]. But since a solution for the 3D case is not available we argue that the above equation
(Eq. 3) could serve as an ersatz, of a first approximation to calculate the order of magnitude and the difference between the principal stresses for such tectonic stresses. We assume that the crossover-length in a specific direction is mainly a function of the stresses in the plane normal to the stylolite surface along the direction of investigation and that the out of plane stresses are invariant. This would imply that the differential stresses for the vertical and horizontal directions could be defined as $\sigma_{dv} = \sigma_{yy} - \sigma_{zz}$ and $\sigma_{dh} = \sigma_{yy} - \sigma_{xx}$ and Eq. 3 could be solved if the depth of stylolite formation and the material properties during stylolite formation are known. For the stylolites from the Swabian Alb with a vertical crossover of 0.95 mm and a horizontal crossover of 0.7 mm, assuming a Poisson’s ratio of 0.25, a surface free energy of calcite of 0.27 J/m$^2$, a Young’s Modulus of 14 GPa [Ebner et al., 2009b] and a vertical stress component ($\sigma_z$) of 6 MPa (assuming a vertical load of 220 m of sediments with a density of 2.7 g/cm$^3$ in agreement with sedimentological constraints) the tectonic stress component ($\sigma_t$) is about 17.7 MPa and the horizontal in-plane stress ($\sigma_3$) component is 1.8 MPa. See appendix for details of the calculation. The theoretical stresses of stylolite formation calculated here can not serve as realistic values since we unjustifiably borrow from the analytical solution for the isotropic case but should give a first order estimate under the limiting assumptions stated above. Nevertheless we would expect stresses during tectonic stylolite formation to be close to the compressive lithospheric strength, i.e. $\sigma_t - \sigma_3 \sim 14$ MPa [Banda and Cloetingh, 1992] but much smaller than uniaxial compressive strength of laboratory measurements for limestones, which are in the range of ~50-200 MPa [Pollard and Fletcher, 2005]. Utilizing the solution given in the appendix the resulting stress magnitudes are surprisingly close to expected values.

For our samples in Spain we do not calculate the stresses because the principal stresses are quite likely not aligned with the stylolite plane as discussed above. We argue that even if it would be possible to calculate the stresses for tectonic stylolites in a fold and thrust belt like in northeastern Spain the stresses deduced from stylolites might be completely different form
that of the folding event. The main reason is that stylolites probably form rather quick, in the order of hundreds of years [Schmittbuhl et al., 2004]. This would allow several generations of stylolites to form (revealing different finite orientation) during a single folding event the analysis of a single set of stylolites would thus result in a snapshot from the geologic history not necessarily revealing the full picture. Even if the stylolites can be attributed to the same kinematic framework as the folding event both most likely have a rather diverse history in terms of stress.

6. Conclusions

Vertical tectonic stylolites investigated in this study show a 1D scaling invariance that resembles those of bedding parallel stylolites investigated in previous studies [Ebner et al., 2009b; Renard et al., 2004; Schmittbuhl et al., 2004]. They have a self-affine scaling invariance, which is characterized by a Hurst exponent of 1.1 for long and 0.5 for short scales and a distinct crossover-length at the millimeter scale that separates these two scaling regimes.

High resolution laser profilometry of tectonic stylolites provides quantitative 3D information of these pressure solution surfaces that enables a 2D analysis of the surface morphology. We demonstrate that our samples of tectonic stylolites have an anisotropic scaling that is not independent of the orientation of the investigated section within the plane of the stylolite. This anisotropy’s main characteristic is a systematic shift of the crossover length that separates the scaling regimes. The presented analysis also confirms that the anisotropy observed in our vertical samples is oriented with respect to the horizontal and vertical direction and thus coincides with the principal stress directions within the stylolite plane for vertical stylolites e.g. $\sigma_2$ & $\sigma_3$ as depicted in Figure 2b. The long axis of the anisotropy and thus the smallest crossover length consistently coincides with the horizontal direction in the stylolite plane, whereas the largest crossover-length is found in a vertical
section. This observation is consistent with the fact that the horizontal in-plane stress is
generally smaller than the vertical in-plane stress, which should be the case for tectonic
stylolites (Figure 1b). They are also both smaller than the normal stress orientated
perpendicular to the stylolite plane, which should be oriented horizontally. Therefore the
crossover-length should be smaller in a horizontal section than in a vertical section (Eq. 3)
using analytical considerations [Schmittbuhl et al., 2004].

In addition we studied the influence of inclined teeth and asperities on the scaling
behavior of stylolites. Using synthetic ‘slickolites’ with various tilt angles we found that the
evolving anisotropy is negligible and clearly different from the anisotropy we observed in the
investigated samples. We thus conclude that the scaling anisotropy of the investigated vertical
tectonic stylolites can be related to the 3D formation stress.

7. Appendix: Stress Calculation

Part I

In this appendix we will show how the tectonic stress ($\sigma_1$) and the smaller in-plane stress
component ($\sigma_3$) can be calculated if the vertical stress component can be approximated using
vertical loading conditions. According to equation (4) the vertical and horizontal crossovers
($L_v$ and $L_h$) can be calculated by [Schmittbuhl et al., 2004]

$$\begin{align*}
L_v &= \frac{\gamma E}{\beta} \frac{1}{\sigma_m} \frac{1}{\sigma_{dv}} \\
L_h &= \frac{\gamma E}{\beta} \frac{1}{\sigma_m} \frac{1}{\sigma_{dh}}
\end{align*}$$

(A1)

where $E$ is the Young’s Modulus, $\gamma$ is the solid-fluid interfacial energy, $\beta = \nu(1-2\nu)/\pi$ is a
dimensionless constant with $\nu$ the Poisson’s ratio, $\sigma_m$ and $\sigma_{dv/dh}$ are the mean and differential
stresses respectively. Since the mean stress is the same for both directions we can reformulate
equation A1 to

$$\begin{align*}
\sigma_m &= \frac{\gamma E}{\beta} \frac{1}{L_v} \frac{1}{\sigma_{dv}} \\
\sigma_m &= \frac{\gamma E}{\beta} \frac{1}{L_h} \frac{1}{\sigma_{dh}}
\end{align*}$$

(A2)
and join both equations so that

\[ L_v \sigma_{dv} = L_h \sigma_{dh}. \]  \hspace{1cm} \text{(A3)}

If we now define the differential stresses using the main principal stress components with \( \sigma_1 = \sigma_{yy}, \) i.e. acting normal to the stylolite plane; \( \sigma_2 = \sigma_{zz}, \) i.e. the vertical in plane stress component and \( \sigma_3 = \sigma_{xx}, \) i.e. the horizontal in plane stress component (compare Figure 1b); as

\[ \sigma_{dv} = \sigma_{yy} - \sigma_{zz} \quad \text{and} \quad \sigma_{dh} = \sigma_{yy} - \sigma_{xx} \]

equation A3 becomes

\[ \frac{L_h}{L_v} = \frac{\sigma_{yy} - \sigma_{zz}}{\sigma_{yy} - \sigma_{xx}} \]  \hspace{1cm} \text{(A4)}

and solving for the \( \sigma_{xx} \) component

\[ \sigma_{yy} - \sigma_{xx} = \frac{L_v}{L_h} (\sigma_{yy} - \sigma_{zz}), \]

\[ \sigma_{xx} = \sigma_{yy} - \frac{L_v}{L_h} (\sigma_{yy} - \sigma_{zz}) = \sigma_{yy} - \frac{L_v}{L_h} \sigma_{yy} + \frac{L_v}{L_h} \sigma_{zz} \]  \hspace{1cm} \text{(A5)}.

Part II

For simplification we substitute all material parameters of Equation 4 which are assumed to be constant, according to

\[ a = \frac{\gamma E}{\beta}. \]

Then we use equation 4 for the horizontal cross-over

\[ L_h = a \frac{1}{\sigma_m \sigma_{dh}}, \]

or

\[ \sigma_m \sigma_{dh} = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3} (\sigma_{yy} - \sigma_{xx}) = a \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{L_h}, \]

and

\[ (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})(\sigma_{yy} - \sigma_{xx}) = 3 \frac{a}{L_h} \]  \hspace{1cm} \text{(A6)}
Now we include equation A5 into equation A6 and solve for $\sigma_{yy}$.

$$\left(2\sigma_{yy} + \sigma_{zz} - \sigma_{yy} \frac{L_v}{L_h} + \sigma_{zz} \frac{L_v}{L_h}\right)\left(\sigma_{yy} \frac{L_v}{L_h} - \sigma_{zz} \frac{L_v}{L_h}\right) = \frac{3a}{L_h}$$ \hspace{1cm} (A7)

and multiplying the components gives

$$2\sigma_{yy}^{2} \frac{L_v}{L_h} - \sigma_{yy} \left(\frac{L_v}{L_h}\right)^2 + 2\sigma_{yy} \sigma_{zz} \left(\frac{L_v}{L_h}\right)^2 - \sigma_{yy}^{2} \frac{L_v}{L_h} - \sigma_{zz}^{2} \left(\frac{L_v}{L_h}\right)^2 - 3\frac{a}{L_h} = 0.$$ \hspace{1cm} (A8)

Rearranging equation A8 in order to solve a binomial formula gives

$$\sigma_{yy}^2 + \frac{2\sigma_{zz} \left(\frac{L_v}{L_h}\right)^2 - \sigma_{zz} \frac{L_v}{L_h}}{2 \frac{L_v}{L_h} - \left(\frac{L_v}{L_h}\right)^2} - \frac{\sigma_{zz}^2 \left(\frac{L_v}{L_h}\right)^2 - 3\frac{a}{L_h}}{2 \frac{L_v}{L_h} - \left(\frac{L_v}{L_h}\right)^2} = 0$$ \hspace{1cm} (A9)

and the solution of the binomial formula is then

$$\sigma_{yy_{1,2}} = -0.5 \frac{2\sigma_{zz} \left(\frac{L_v}{L_h}\right)^2 - \sigma_{zz} \frac{L_v}{L_h}}{2 \frac{L_v}{L_h} - \left(\frac{L_v}{L_h}\right)^2} \pm \sqrt{0.25 \left(2\sigma_{zz} \left(\frac{L_v}{L_h}\right)^2 - \sigma_{zz} \frac{L_v}{L_h}\right)^2 - \left(-\sigma_{zz}^{2} \left(\frac{L_v}{L_h}\right)^2 - 3\frac{a}{L_h}\right)^2} \left(2 \frac{L_v}{L_h} - \left(\frac{L_v}{L_h}\right)^2\right)^2.}$$ \hspace{1cm} (A10)

$\sigma_{xx}$ can be derived from equation A5.

8. Acknowledgements

J. Nebelsick, M. Maisch, O. W. Vonderschmidt and T. Sachau are thanked for valuable discussion and support in the field. M. Ebner and D. Koehn acknowledge financial support through the DFG project KO2114/5 and the MWFZ of Mainz and the Geocycles Cluster funded by the state of Rhineland-Palatinate. R. Toussaint and J. Schmittbuhl acknowledge the support of a FORPRO grant. We thank both reviewers for thorough reviews and suggestions that helped to improve the manuscript.
Reference list:

Angheluta, L., et al. (2008), Stress-Driven Phase Transformation and the Roughening of Solid-Solid Interfaces, Physical Review Letters, 100(9), 096105.


Renard, F., et al. (2006), High resolution 3D laser scanner measurements of a strike-slip fault quantify its morphological anisotropy at all scales, *Geophysical Research Letters*, 33(4), -.


**Figure captions:**

**Figure 1:** Schematic drawing of the formation stress state for (a) bedding parallel and (b) tectonic stylolites. The largest compressive stress direction ($\sigma_1$) is indicated by a white arrow.
Below the sketch map an idealized graph of the in-plane differential stress is plotted as a function of the orientation within the stylolite plane. For bedding parallel stylolites (a) the horizontal normal stresses are equal and thus the differential stress is equal in every direction. For tectonic stylolites (b) the in-plane normal stresses are dissimilar and $\sigma_{zz}$ is generally larger than $\sigma_{xx}$. Thus the in-plane differential stress scales inversely with the magnitudes of the $\sigma_{xx}$ and $\sigma_{zz}$ directions having a maximum along the x-axis.

**Figure 2:** Lower hemispheric equal area projection (Schmidt’s net) of the field data and schematic cross-sections of the investigated outcrops. (a) The Swabian Alb of southern Germany (n=22). Right panel shows the flat lying Jurassic strata with vertical stylolites limited to individual beds (b) Iberian Chain of north-eastern Spain (n=32). Right panel shows a cross-section of NE plunging fold and the position of set a and set b within the fold. All samples are taken from well bedded Jurassic strata. In the overlying massif Jurassic limestones (vertical stripes) and conglomerates (circles) no stylolites were found. Notice that in (a) only the poles to the stylolite planes are displayed since the shortening direction is normal to that plane. In panel (b) two populations are shown which correspond to the two investigated fold limbs. Poles to planes (circles) diverge slightly from the orientation of the long axis of the teeth (triangle); See text for detailed explanation.

**Figure 3:** Oblique view of the 3D morphology of the surface of an opened stylolite (sample M4/4) reconstructed from optical profilometry. A linear trend is removed from the raw data (compare Figure 4 for details).

**Figure 4:** Greyscale maps of sample M4/3 where (a) shows the raw data from profilometry (notice a general trend from the top left to bottom right); (b) detrended data i.e. linear trend is removed and mean height is set to be zero; (c) detrended data which is modified with a
Hanning window technique where the data is forced to taper off to zero at the boundaries (for explanation see text). Light colours correspond to peaks and ridges and dark colours represent local depressions.

**Figure 5:** 1D data-analysis of sample M4/3; (a) shows the averaged Power spectra $P(k)$ (solid line) and the respective binned spectra (circles) plotted as a function of the wavenumber along the x and the y direction of the measured map. The inset in (a) again shows the power spectra for both directions but the x direction is now normalized with respect to the y direction $P_x(k)/P_y(1\text{mm}^{-1})$. This yields a collapse of the large k-values (small scales), notice that for the small k-values (large scales) the scaling functions deviate considerably (b) non-linear fit of the binned spectra for both directions used to estimate the crossover length $L$ (triangle). Along the x-direction the crossover-length is larger ($L=1.22$) than along the y-direction ($L=0.62$). The slope of the branches of the non-linear model corresponds to Hurst exponents of 1.1 and 0.5 for small and large scales, respectively.

**Figure 6:** 1D analysis of the scaling prefactor i.e. the topothesy of tectonic stylolites. (a) A loglog plot of the correlation function $C(\Delta x)$ of a 1D slice of sample M4/3 oriented parallel to the x direction of the analyzed surface with the nonlinear fit (compare text for details) and the topothesies $t_s$ and $t_l$ for small and large scale sub-branches. The topothesy is constructed from the intersection of the linear sub-branches with the 1/1 line. (b) The topothesies $t_s$ and $t_l$ of sample M4/3 plotted as a function of $\theta$ i.e. the counter clockwise angle from the x-direction of the map. Note that the correlation functions are averaged over 5° intervals. Arrow indicates the vertical direction projected onto the stylolite plane. Note that only the $t_s$ shows a clear correlation with the sample orientation. (c) The small scale topothesy $t_s$ for all samples plotted as a function of $\theta$. 
**Figure 7:** 2D data-analysis of sample M4/3; (a) 2D Fourier transform plotted on a regular grid as a function of $k_x$ and $k_y$ which range from $-((n/2)\Delta x)^{-1}$ to $(n/2)\Delta x)^{-1}$ where $n$ is the number of measurement points in one direction of the map and $\Delta x$ is the step size. (notice that the zero frequency component lies in the centre of the map). A clear anisotropy of the data can be observed sub-parallel to the $k_y$-axis (vertical axis). To investigate the power law scaling exhibited by the 1D analysis the 2D Fourier transform is converted to a double log-space where $\log(k_x, k_y)$ is plotted as a function of the logarithm of the power spectra (b); the 2D power spectra are plotted as a surface whose height corresponds to $\log(P(k_x, k_y))$. The 3D surface is viewed along the $k_x$-direction and the arrow indicates the crossover-length $L$, which separates the two scaling regimes i.e. the two linear subparts of the slope of the cone.

**Figure 8:** Quantification of the 2D scaling anisotropy of sample M4/3; (a) oblique 3D view of the binned 2D power spectra (grey mesh) with an overlay of coloured contour lines of constant $\log(P(k_x, k_y))$-values. (b) Map view of the contours calculated from the conic 2D power spectra. These contours were utilized to calculate best-fitting ellipses using a least squares approach; (c) Aspect ratio ($a/b$) of the fitted ellipse for every $\log(P(k_x, k_y))$-contour. An increasing aspect ratio towards the centre of the map is characteristic for all samples investigated. (d) Slope (i.e. the counter clockwise angle from the x-direction of the measured map) of the long axis of the fitted ellipse plotted for the contour intervals.

**Figure 9:** Rose diagrams of all samples i.e. a histogram with a constant bin size of $10^\circ$ plotting the relative orientation of the long axis of the fitted ellipse to the vertical direction of each sample. Arrow in each panel shows the intersection of the vertical direction of the oriented sample with the mean stylolite plane. (a) sample Sa6/1a, (b) sample Sa6/1b, (c) sample Sa9/2, (d) sample M4/1, (e) sample M4/2, (f) sample M4/3, (g) sample M4/4, (h) sample M4c/1, (i) sample M4c/3; Notice that for all samples the long axis and thus the
direction with the smallest crossover length is roughly normal to the vertical direction (except for h & i; for explanation see text). This direction corresponds typically to the largest differential stress, which is also the smallest in-plane stress (v and h correspond to the vertical and horizontal directions, respectively). (j) Schematic drawing of the relationship between the wavenumber contour [mm⁻¹] (compare Figure 8), the crossover-length $L$ [mm], the principal in-plane stresses and the sample orientation i.e. horizontal and vertical direction. Refer to text for detailed explanation.

**Figure 10:** Crossover length from the contour data of the maps for sample M4/3 and Sa6/1a. (a) Slope of the 2D power spectra calculated as the mean difference between the principal axis of the fitted ellipse (a,b). The biggest change in slope (arrow) is assumed to be the contour at which the crossover is located. (b) The crossover-length plotted as a function of the counter clockwise angle from the x-direction of the measured map. The vertical direction in the stylolite plane is indicated for both samples and roughly corresponds to the largest crossover-length i.e. the smallest differential stress as shown in Figure 1.

**Figure 11:** Greyscale map (a) of a synthetic self affine square surface with a side-length of 512 and a Hurst exponent of 0.5. Inset displays a 2D Fourier transform of that map, which clearly exhibits isotropy with respect to its centre, similar to bedding parallel stylolites. This dataset is then utilized to construct slickolites i.e. stylolites with oblique teeth and asperities (see text), with various tilt angles (e.g. 10° correspond to oblique asperities that are rotated 10° counter clockwise around the x-direction with respect to the mean plane of the synthetic surface). (b) Aspect ratio of elliptical fit of synthetic data set. For small tilt angles an anisotropy on small scales (i.e. large wavenumbers and low $\log(P(k_x, k_y))$-contours) can be observed. For large tilt angles a general increase of the aspect ratio over all scales can be found. (c) Orientation of the long axis of the fitted ellipse (compare Figure 8d). Notice an
increasing alignment of the long axis of the fitted ellipse towards higher $\log(P(k_x, k_y))$-contours with increasing tilt angles.