Deformation of the crack front during propagation in some disordered medium: theoretical and experimental studies

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Abstract: In heterogeneous disordered materials, a straight crack front experiences toughness fluctuations during its propagation that generate geometric fluctuations. Their long time statistical behavior has been studied by Lazarus et al. (JMPS, 2008) using Bueckner-Rice weight function theory. In particular, the evolution of the auto-correlation function, power spectrum and variance of the front fluctuations have been derived. The aim here is to compare these results to some experiments performed on transparent plexiglas blocks with the same apparatus as in Schmittbuhl and Maloy (PRL, 1997) by measuring the amplitude evolution of the crack front fluctuations in addition to the self-affinity roughness parameters.

In a perfectly ideal homogeneous material, an initial straight crack front remains straight during propagation. But in an heterogeneous disordered materials, it becomes rough. The aim of the present paper is to derive an analytical description of the evolution of this roughness and to compare it to experimental results. The assumption of quasi-static brittle crack propagation will be done.

Among the experimental works, one may cite on the one hand, the pioneer work of Daguier *et al.* [2] in which the crack front is obtained postmortem, the crack surface being marked by ink and on the other hand, the works of Delaplace, Maloy and Schmittbuhl [8, 3] in transparent plexiglas in which the crack front can be observed in situ during its evolution. They deal mainly with the universal self-affine character of the crack front. The roughness exponent ξ was measured between 0.5 and 0.6. Here, we have again used the experimental framework of [8, 3] to measure the time evolution of the fluctuations in addition to its roughness.

All the theoretical studies of the statistical properties of the crack front performed in quasi-static, use Bueckner[1]-Rice[7] weight function theory, also called line elastic models, to evaluate the stress intensity factors along the perturbed crack front. Among them, one may distinguish two groups depending on the type of the advance law used. The first ones [9, 10, 6] deal with crack advance governed

by *brittle* fracture Irwin's criterion with a slightly heterogeneous toughness. This criterion is a *threshold* type one: the crack propagates only if the stress intensity factor becomes equal to the local toughness. In particular, by a first order analysis the roughness exponent was derived and found to be $\xi = 0.37$ or $\xi = 0.5$ depending on the papers. This apparent discrepancy will be considered further. The second group deals with crack advance governed by Paris' law (*fatigue* or *sub-critical* fracture). It is a *time dependent* type criterion: the rate of crack advance is proportional to a power law of the stress intensity factor. Lazarus, Leblond and coauthors have performed the study of a tensile tunnel-crack [4] and of a tensile semi-infinite interfacial crack [6]. Contrary to the case of threshold advance law, their first order study in crack advance was not sufficient to obtain the crack front roughness. However Adda-Bedia and Katzav [5] performed the second order study for a semi-infinite crack and obtained $\xi = 0.5$.

Here, the work of Pindra, Lazarus and Leblond [6] is applied to experiments made with the same framework as Delaplace, Maloy and Schmittbuhl [8, 3]. For Irwin's advance law, using Bueckner-Rice formulation for a semi-infinite crack subjected to line loading on its faces, the evolution of the variance and power spectrum, so as the roughness exponent are derived and compared to previous theoretical results of Schmittbuhl, Vilotte and coauthors [9, 10]. Then, comparison with experiments are performed, not solely on the roughness exponent as in previous papers but also on the evolution of the crack front amplitude.
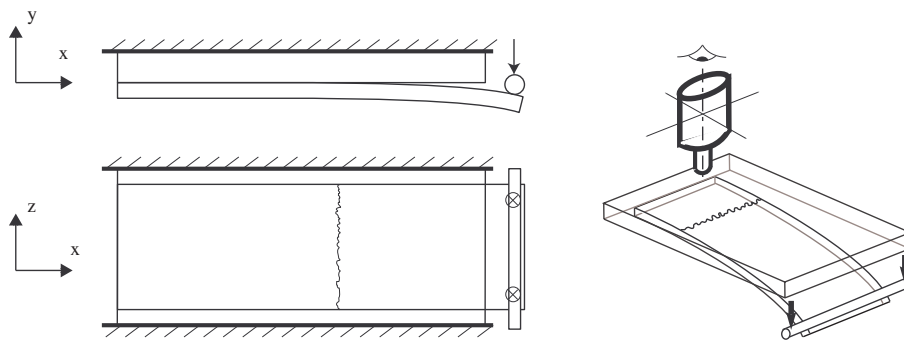
1 Experiments

Figure 1: Two PMMA plates sintered together: a thick clamped one (21 cm long, 11 cm wide, and 1 cm thick) and a thin one (23 cm × 9 cm × 0.6 cm) subjected to a normal displacement.

The experimental framework is the one still used by Delaplace, Maloy and Schmittbuhl [8, 3]. Two PMMA transparent plates are sandblasted and annealed together by increasing temperature to obtain one block with a weak plane where the crack will propagate. The larger plate is clamped. A normal displacement is applied to

the smaller one that creates a stable propagating crack in the weak plane of the specimens (fig. 1). The crack front is observed with a microscope at several stages of the propagation.
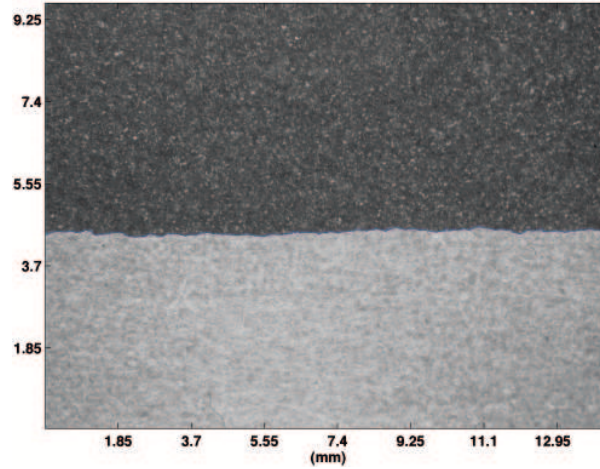

Figure 2: Experimental picture of the crack front for one equilibrium position. In blue the extracted crack front

By image treatment, one extracts the position of the crack fronts at different time steps (fig. 2). The front becomes rough because of the toughness fluctuations introduces by the sandblasting.

2   Theoretical determination of the roughness

2.1   Evolution of the perturbation of the crack fronts

Since [6] is a general paper (an interfacial crack, several type of loadings, fatigue and fragile fracture are considered), and hence is quiet complicated, we give here the reasoning in the particular case of an homogeneous medium ($\varepsilon = 0$), the particular line loading of the experiments, brittle fracture, although most of the results can be directly (but not painlessly) derived from the paper.

Let us consider a semi-infinite plane crack in an infinite homogeneous linear elastic medium. Line tractions $\pm P\vec{e}_y$ are applied on the crack faces at a distance a of the crack front (fig. 3). We suppose that for each equilibrium position of the crack front, Irwin's criterion is satisfied, that is:

$$K(M) = K_c(M) \tag{1}$$

at each point M of the crack front. Here $K(M)$ is the stress intensity factor (SIF) and $K_c(M)$ the toughness, both at point M.
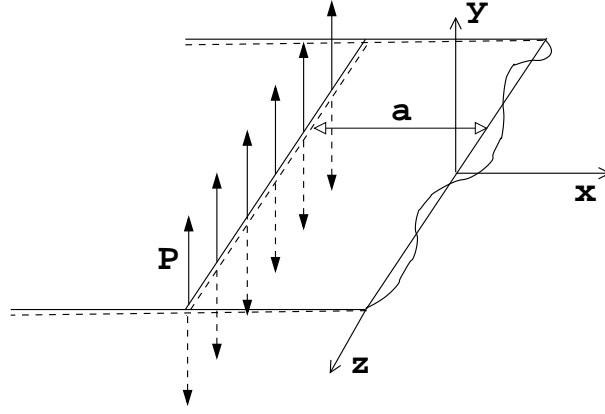
Figure 3: Semi-infinite plane crack with a slightly perturbed crack front and line tractions at a distance a of the crack front

If the toughness is uniform, the crack front remains straight during propagation. The SIF along the crack front are then :

$$K = P\sqrt{\frac{2}{\pi a}} \qquad (2)$$

so that the loading corresponding to position a of the front verifies :

$$P = K_c\sqrt{\frac{\pi a}{2}} \qquad (3)$$

Notice that the propagation is stable, in the sense that the loading has to be increased for the crack to advance (a increases).

Now introduce some small fluctuations of the toughness :

$$K_c(z, x) = \overline{K}_c(1 + \kappa(z, x)), \quad |\kappa| \ll 1 \qquad (4)$$

It produces small fluctuations $\delta a(z, a)$ and $\delta K(z, a)$ of the crack front position $a(z, a)$ and of the SIF $K(z, a)$ (see fig. 3) so that :

$$\left\{ \begin{array}{llll} a(z, a) & \equiv & a + \delta a(z, a), & |\delta a(z, a)| \quad \ll \quad a \\ K(z, a) & \equiv & K(a) + \delta K(z), & |\delta K(z, a)| \quad \ll \quad K(a) \end{array} \right. \qquad (5)$$

where a and $K(a)$ denote their mean values.

Expanding Irwin's criterion (1) to first order and identifying terms of order 0 and 1, one gets :

$$\left\{ \begin{array}{l} K(a) = K_c \\ \\ \dfrac{\delta K(z, a)}{K(a)} = \kappa(z, a) \end{array} \right. \qquad (6)$$

4

Now we use Bueckner [1]-Rice [7] formalism, expanded to line tractions as in [6], to express the perturbed SIF to first order in δa :

$$\frac{\delta K(z, a)}{K(a)} = -\frac{1}{2a}\delta a(z, a) + \frac{1}{2\pi}\mathrm{PV}\int_{-\infty}^{\infty}\frac{\delta a(z', a) - \delta a(z, a)}{(z' - z)^2}\mathrm{d}z' \qquad (7)$$

Inserting equation (7) into (6.2) and taking the Fourier transform of the equation, one gets:

$$\delta\hat{a}(k, a) = -\frac{2a\hat{\kappa}(k, a)}{1 + |ka|} \qquad (8)$$

2.2   Statistical study of the deformation of the front

We consider a large number of random possible realizations of the heterogeneous medium and the crack geometry. Statistical invariance of the functions $\kappa(z, x)$ in the direction z and x, and $\delta a(z, a)$ in the direction z being assumed, the two point auto-correlation functions of these functions depend only on the relative position of the points considered:

$$E\left[\kappa(z_1, a)\kappa(z_2, a)\right] = \mathcal{K}(z_2 - z_1) \qquad (9)$$
$$E\left[\delta a(z_1, a)\delta a(z_2, a)\right] = \mathcal{A}(z_2 - z_1, a) \qquad (10)$$

where $E[X]$ denotes the mathematical expectation of any quantity X. The functions $\mathcal{K}(z)$ and $\mathcal{A}(z, a)$ can be identified with the average values of $\kappa(z', a)\kappa(z' + z, a)$ and $\delta a(z', a)\delta(z' + z, a)$ over the crack front, provided an ergodic hypothesis is made. The function $\mathcal{A}(z, a)$ and its z-Fourier transform $\widehat{\mathcal{A}}(k, a)$ (the spectral density of the perturbation) provide statistical information about the geometry of the crack front.

Using the property that

$$\widehat{\mathcal{A}}(k, a) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E\left[\delta\hat{a}(k_1, a)\delta\hat{a}(-k, a)\right]\mathrm{d}k_1 \qquad (11)$$

and equation (8), one obtains the power spectrum of the perturbation of the crack front :

$$\widehat{\mathcal{A}}(k, a) = \frac{4a^2}{(1 + |ka|)^2}\widehat{\mathcal{K}}(k) \qquad (12)$$

For $a \gg a_0$ this yields :

$$\begin{cases} \widehat{\mathcal{A}}(k, a) = \dfrac{4}{k^2}\widehat{\mathcal{K}}(k), & \text{for } k \neq 0 \\[3mm] \widehat{\mathcal{A}}(0, a) = 4\widehat{\mathcal{K}}(0)a^2 \end{cases} \qquad (13)$$

Taking the inverse Fourier transform of equation (12), one gets for $a \gg a_0$:

$$\mathcal{A}(z, a) \sim \frac{4}{\pi}a\widehat{\mathcal{K}}(0) \qquad (14)$$

5

The squared fluctuation $\sigma(z,a) \equiv E\left[(\delta a(z,a) - \delta a(0,a))^2\right]^{1/2}$ can also be derived. For $a \gg a_0$ and $z \gg b$, b being the correlation length of κ ($b = 0$ for white noise):

$$\sigma(z,a)^2 \sim 4\widehat{\mathcal{K}}(0)z \tag{15}$$

2.3 Roughness exponent

Suppose that the toughness fluctuation are given by a white noise: $\widehat{\mathcal{K}}(k) = \widehat{\mathcal{K}}_0 = Cst.$. Equations (13) or (15) gives the value of the so-called wandering or roughness exponent :

$$\xi = 0.5 \tag{16}$$

However, numerical simulations performed in [10] using also Bueckner-Rice formula (7) yield two different values: $\xi = 0.37$ as in [9] and $\xi = 0.5$, the first value being obtained if in their simulations $\delta a_{max} > b$ and the second if $\delta a_{max} < b$. The difference hence is due to the numerical parameters used. May be that for $\delta a_{max} > b$, the results are in fact illicit since Bueckner-Rice formula, that supposes the perturbation and all is derivatives small, is then no more valid for $\frac{\partial \delta a}{\partial z} \sim \frac{\delta a_{max}}{b} > 1$. Moreover, physically it may be more pertinent to suppose that $\delta a_{max} < b$ when the crack front is trapped, as in the experiments by fluctuations of thoughness (roughness smaller than the typical size of the zones of higher thoughness). Whatever, further investigations are under study to clarify this point.

3 Comparison experiments and theoretical results

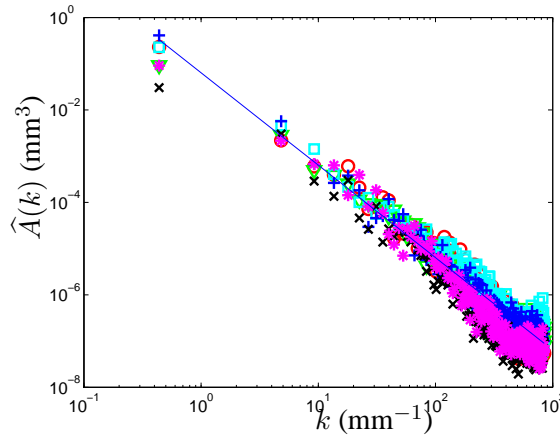## 3.1 Power spectrum evolution

Figure 4: Power spectrum. Marker lines: for several crack fronts with a between 35 mm and 100 mm. Continuous line: $\widehat{\mathcal{A}}(k) \propto k^{-1-2\xi}$, $\xi = 0.5$.

In figure 4, the experimental power spectrum $k \to \widehat{\mathcal{A}}(k, a)$ evolution is given for several crack positions a. Several remarks can be done:

1. For all the fronts considered, $\widehat{\mathcal{A}}(k, a)$ is independent of a. This is in agreement with the behavior for large a obtained in eq. (13).

2. The roughness exponent as in previous measures is near $\xi = 0.5$ in agreement with our theoretical prediction.
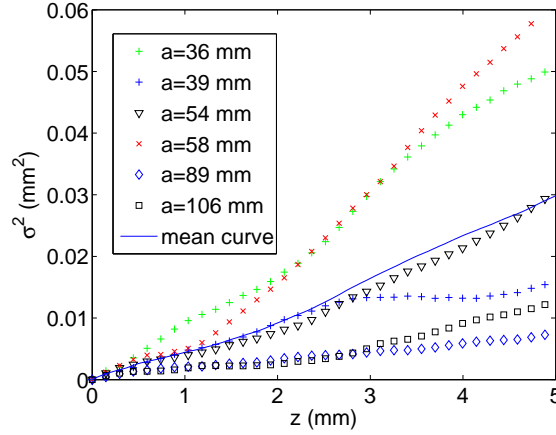
3.2 Variance evolution

Figure 5: Variance. Marker lines: for several crack front with a between 36 mm and 58 mm. Continuous line: the mean of all these values

In figure 5, the experimental evolution of the variance is given. One notices that:

1. The values for several front oscillates around the mean value during propagation. This corresponds to fluctuations of each position of the front around a steady state situation, due to the heterogeneities.

2. The steady state situation observed is also in agreement with the long time behavior of the variance found in equation (15).

3. The mean value varies linearly with z as predicted by equation (15). Hence one retrieves a roughness exponent of $\xi = 0.5$.

4   Conclusion

We considered a semi-infinite crack embedded in an infinite elastic medium subjected to traction line loading on its faces. Using Bueckner-Rice weight function theory for this geometry (elastic line model), the power spectrum and the variance

of the crack fluctuations arising from small toughness fluctuations has been derived analytically. In particular, a roughness exponent of $\xi = 0.5$ has been found in contradiction with some previous numerical works but in agreement with the results found by [10] in the case of small roughness amplitude toward the correlation length of the thoughness fluctuations.

In spite of the approximations made in the theoretical modelization (finite geometry replaced by infinite one, uniform displacement loading along the line replaced by uniform force loading), one recovers at least qualitatively the experimental behavior in particular for the steady state situation and for the value of the roughness exponent.

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