# Granular Segregation and Finger Formation

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COMPLEX

## Abstract

*Simulations of granular flows with hydrodynamic interactions in 2D is presented. The grains are modeled as discrete, frictionless particles. The gas is described by the pressure only, and the inertia is neglected. The permeability of the porous media is given by Carman-Kozeny, and the pressure drop by Darcy's law. Two different systems are explored with this model, both in a gravity field: Segregation in bidisperse packings, and finger formation in monodisperse packings. For the segregation problem an order parameter is introduced to measure the segregation, and results for different flux rates are presented. The finger formation is discussed as a granular version of the Rayleigh-Taylor instability, and a preliminary theory for the finger formation is presented.*

## EQUATIONS

The pressure $P$ of the interparticle air is given by

$$\phi\,(\partial_t + \boldsymbol{u}\cdot\boldsymbol{\nabla}P) = \boldsymbol{\nabla}\cdot(P\,\kappa/\mu\boldsymbol{\nabla}P) - P\,\boldsymbol{\nabla}\cdot\boldsymbol{u}\,.$$

The derivation is found in [1] and is based on conservation of granular volume fraction $\rho_s = 1-\phi$, where $\phi$ is the porosity, conservation of air density $\rho_a$, and the ideal gas equation of state. The permeability $\kappa$ is given by the Carman-Cozeny relation for spheres $\kappa = d^2/180\ (1-\rho_s)^3/\rho_s^2$, where $d$ is the grain diameter. The viscosity of the gas is $\mu$, and $\boldsymbol{u}$ is the granular velocity.

The equation of motion for the individual grains in a *monodisperse* packing is given by

$$m\frac{d\boldsymbol{v}}{dt} = m\boldsymbol{g} + \boldsymbol{F}_I - m\frac{\boldsymbol{\nabla}P}{\rho_s\rho_g}\,,$$

where $\rho_s\rho_g/m = \rho$, the number density, and $\boldsymbol{F}_I$ is the interparticle force. Thus the pressure force is distributed equally onto the grains in the volume. For a *bidisperse* packing we need two equations of motion

$$m_i\,\boldsymbol{a}_i = m_i\,\boldsymbol{g} + \boldsymbol{F}_I - m_i\,f(d_i)\,\frac{\boldsymbol{\nabla}P}{\rho_{s,i}\,\rho_g}\,,$$

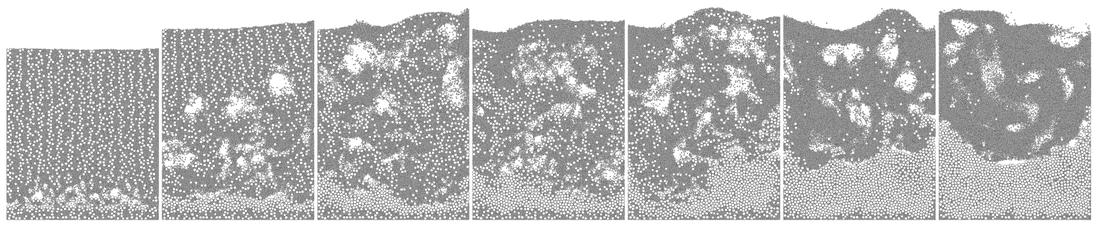
where the index $i = 1,2$ represents small and big grains respectively, and $\rho_{s,i}$ is the volume fraction of grains of size $i$. The function $f(d_i)$, where $d_i$ is the grain diameter, is a weight function that determines the total force that the small and big grains get from the gas. This is because we do not know the exact relation between the force and the grain size, just that there is some dependence. The weight function used in the segregation simulations is given as

$$f(d_i) = \frac{N_i d_i^\alpha}{N_1 d_1^\alpha + N_2 d_2^\alpha}$$

where $N_1$ and $N_2$ is the total number of small and big grains respectively. The $\alpha$ determine the coupling between the grain size and the gas force.

## SEGREGATION

### Simulations



A series of simulations on fluidized bidisperse packings were performed. Three different values of $\alpha$ = 0.5,1.0,1.5 was used to see how this affects the segregation. Since the simulations are 2D the possible values of $\alpha$ ranges from 0-2. If $\alpha$ = 0 all grains get the same portion of the force regardless of their size. If $\alpha$ = 2 the force is proportional to the volume, and $\alpha$ = 1 gives a force proportional to the surface. When $\alpha$ is increased the big grains are lifted more by the gas.

### Order parameter

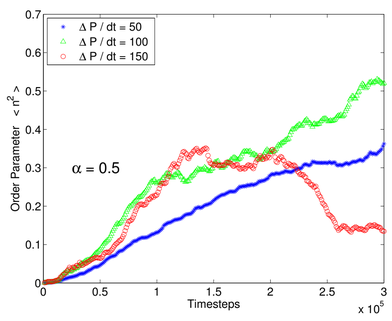
To measure the segregation the box is divided into a number of equally sized cells and the number of small and big grains is counted within each cell. A simple order parameter is introduced

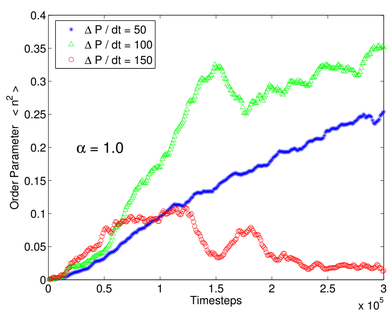
$$n = \frac{N_{1,j} - N_{2,j}}{N_{1,j} + N_{2,j}} - \frac{N_1 - N2}{N_1 + N_2}$$

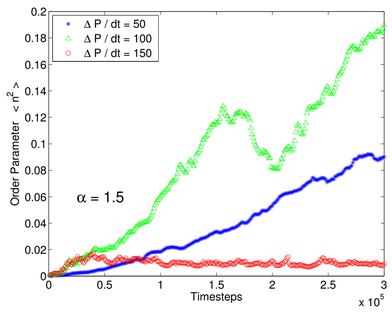
where $N_1$ and $N_2$ is the total number of small and big grains in the box, and $N_{1,j}$ and $N_{2,j}$ is the number of grains in the individual cells. The order parameter $n$ is first squared and then averaged over all the cells to give a number that quantify the segregation for the whole packing.

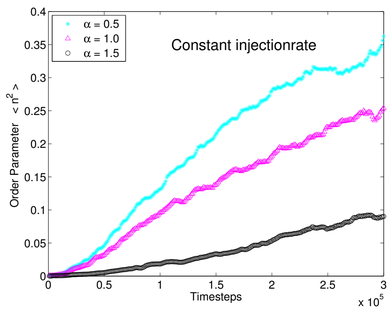
### Preliminary results

The averaged order parameter is plotted as a function of time for three different fluidization levels and three different $\alpha$. The injectionrate of the air is given as $\Delta P/dt$ in units of $\mathrm{Pa}$ per timestep. In the lower right plot the injectionrate is kept constant and the $\alpha$ is changed. It is evident that as the $\alpha$ increase the segregation decrease. When the injectionrate is increased and $\alpha$ is constant, the segregation also increases for the two lowest rates. For the highest rate the grains tend to mix rather than segregate. To determine at which rate this crossover occurs more simulations are needed.




### Conclusions and prospects

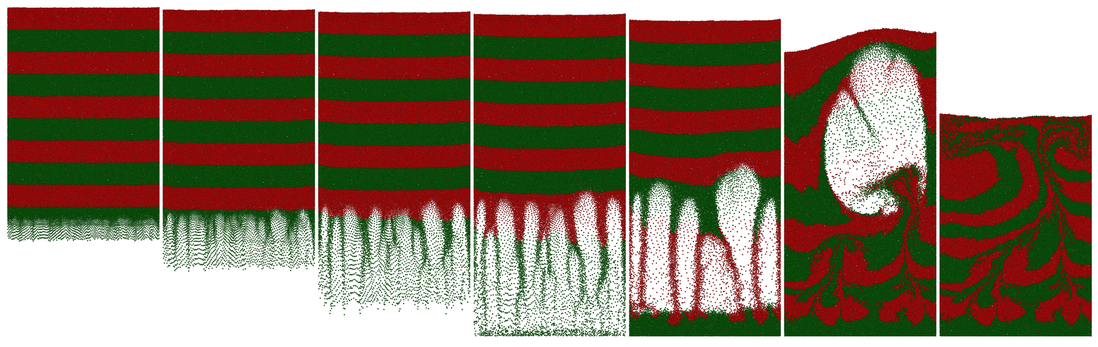
The segregation is quite sensitive to $\alpha$, the coupling between grain size and the force from the air. The $\alpha$ is an input parameter in this model, and to settle the question of its value, we need to look for answers in theory or experiments. The segregation is also strongly dependent on the injectionrate. To establish a flux rate optimal for segregation, further simulations are needed.

### References

[1] Sean McNamara, Eirik G. Flekkøy, and Knut Jørgen Måløy. Grains and gas flow: Molecular dynamics with hydrodynamic interactions. *PRE*, 61:4054, April 2000.

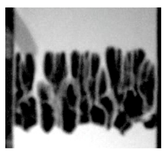
## FINGER FORMATION

### Simulations



The images above are snapshots of a simulation where a monodisperse packing of 40000 grains, initially at rest at the bottom of the box, is turned upside down. As the grains fall from the dense upper part of the packing they form vertical bands, or fingers, separated by more dilute regions. It seems that there is some characteristic distance between the fingers, if the separation of two fingers become too large, a new finger will form between the two. This can be seen on the second image from the right.

### Experiment



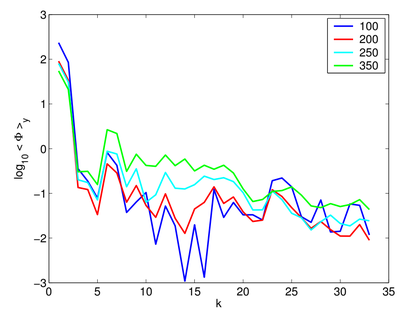
A simple experiment was performed where glass beads of approximately 100 $\mu$m in diameter was confined between two glass plates about $1\mathrm{mm}$ apart. The cell was turned upside down and a series of images were taken. It was difficult to get good images of the very early stages where the fingers are formed. Comparing the image on the left to the simulations above it seems that the simulations at least captures the key features.

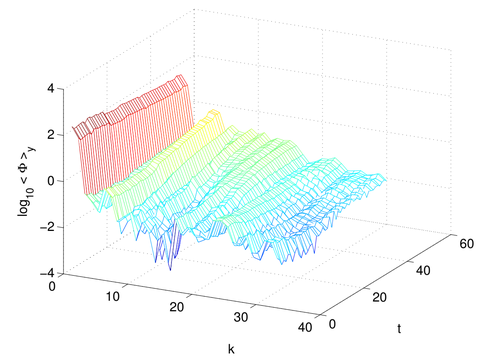
### Linear stability analysis

A theory for the finger formation has been discussed by Flekkøy, Toussaint and Dragos-Victor Anghel using linear stability analysis. The system is described as two connected slabs, one with the closed packed porosity, and one just beneath it where the grains fall freely without contacts and with constant velocity. The intrinsic density disorder in the upper granular packing will serve as seeds from which unstable modes in the bottom slab will grow. Considering small density perturbations in the lower slab, this theory predicts that the density variations reach a maximum at the bottom of the falling slab, which agrees well both with the simulations.

### Preliminary results

To determine the growing modes, the discrete Fourier transform has been applied to the granular density in the moving, bottom part of the packing. The transform is taken in the $x$-direction and then averaged in the $y$-direction to get a better signal. The to plots below give the logarithm of the power spectrum $\Phi$ as a function of the wavenumber $k$. Positive $k$ corresponds to growing modes, and negative $k$ to decreasing modes. The zeroth wavenumber describes the average density and the evolution is due to the expansion of the free falling grains. The plot to the right shows the modes for four different snapshots, while the left plot also shows the time evolution.




### Conclusions and prospects

There seems to be a signal in the power spectrum indicating that the wavenumber around 7 is a fast growing mode. But the signal is rather noisy and better techniques are needed to extract it. A series of experiments with a high-speed camera are in planning to further investigate this problem.