Viscous fingering in porous media

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Analog model for biphasic flow in porous medium

Hele-Shaw cell: two parallel plates, cell with rectangular section, Stokes flow.


The penetration of a fluid into a porous medium or Hele-Shaw cell containing a more viscous liquid

By P. G. Saffman and Sir Geoffrey Taylor, F.R.S.
Cavendish Laboratory, University of Cambridge

(Received 17 January 1958—Read 17 April 1958)
Saffman Taylor instability

Fluid displacing a more viscous fluid:

Involved forces:

viscous forces $v = -\frac{\kappa}{\mu} \nabla P$, capillary forces $\delta P_c = -\frac{\gamma}{r}$. 
Saffman Taylor solutions

Family of curves satisfying Darcy law \( v = -\left(\frac{\kappa}{\mu}\right) \nabla P \), with neglected surface tension along boundary.

\[ \text{Figure 7. Calculated profiles for } \lambda = 0.2, 0.5 \text{ and } 0.8. \]
Width selection

Fraction $\lambda$ of system size invaded, selected by surface tension, no matter how small.

At large speed: $\lambda = 0.5$
Influence of pore-scale disorder

- **Porous medium:**

- **Wettability:**

- **Drainage:** the non-wetting fluid displaces the wetting fluid.

- **Capillary pressure:** \( P_c = \frac{2\gamma}{r} \) \( \Rightarrow \) larger pores are more easily invaded

\( r \) – radius of curvature
\( \gamma \) – interface tension
\( \mu_{nw} \) – viscosity (non-wetting)
\( \mu_w \) – viscosity (wetting)
Experimental setup: a model of 2D random porous medium

- Mono-layer of \( a = 1 \) mm glass beads “sandwiched” between two horizontal plates
- Air displacing glycerol/water
- Three model widths to check size dependencies (L x W = 840mm x 430mm or 840mm x 215mm or 840mm x 110mm)
- Pictures of the structure taken at regular intervals
Invasion structures

- We see a continuous transition from capillary fingering to viscous fingering, as function of $Ca = \frac{\mu va^2}{\gamma \kappa}$. . .

$Ca \approx 0.015$  
$Ca \approx 0.060$  
$Ca \approx 0.20$
Mass fractal dimension: Box counting
Collapse with scaled size $s \cdot Ca$

- $D=1.8$ corresponds to capillary fingering, or invasion percolation.

- $D=1.5$ is smaller than the Diffusion Limited Aggregation result, $D=1.71$. In the absence of granular material at high injection speed in Hele-Shaw cells, only a large scale exponent $D=1.70$ is found in experiments (Moore et al., Phys. Rev. E, 2002)
Average geometry

underlying statistically stationary process: in the reference frame of most advanced tip, analysis of invader’s occupancy probability
Selected width

Occupancy function thresholded at half-maximum: selected width: $\lambda = 0.4$
Conclusion

Presence of disorder in pore structure modifies the large scale structure:
characteristic width of most occupied zone,
large scale fractal dimension.

Can be explained by probabilistic approach of the invasion process.

Perspectives

Numerical and experimental (transparent models) study of the impact of the probability distribution of the pore sizes on the large scale flow.

Extension to biphasic flow in nonsaturated situations.
For each capillary number $C_a$ the speed of the most advances fingertip could be treated as constant.
Average mass of invasion cluster

- Since $\phi(z)$ indep of $Ca$, $v_{tip} \sim constant$, the mass of the invasion cluster could be scaled with $n_{Ca} = \frac{W_\gamma \kappa}{\mu_w a^4} \frac{Ca}{v_{tip}}$

- And the scaled mass is also equal to the cumulative invasion probability distribution $n(z)/n_{Ca} = \Phi(z) = \int_0^z \phi(y)dy$
Ca \textit{dependence in} \( m_\infty \) \textit{and} \( v_{\text{tip}} \)

- The saturation mass and the speed of the fingertip is capillary number dependent.
Transition scale from capillary to viscous fingering: $w_f$

- The characteristic finger width $w_f$ is measured as the average intersection length of cuts perpendicular to the flow direction with trapped clusters removed.

- $w_f$ is the cutoff scale between capillary and viscous fingering, characteristic of maximum loop size.
Transition scale \( w_f \) depending on \( Ca \)

\( w_f \cdot \nabla P \sim \Delta P_c \) width of threshold distribution.
Along a given boundary, \( \nabla P \propto \nabla P(-\infty) \propto Ca \)
Thus, \( w_f \sim 1/Ca \)
Large \( Ca \), minimum loop size \( \sim \) few pore size \( a \), and only branched structure.
Small \( Ca \), whole structure = gigantic loop (lowly branched), and \( w_f \) saturates around the lateral structure extent, \( \sim \lambda W \).

\[
\log_{10}(Ca) = -1.83, \quad -1.22, \quad -0.70
\]
Mass fractal dimension: Box counting

- $D=1.8$ corresponds to capillary fingering, or invasion percolation.

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Collapse, scaled size $s \cdot Ca$

Average collapsed
Large scale: width selection $\lambda = 0.4$

Definition of average density map over exps and time

density: air=1, oil=0,
tip: $z=0$

average over time and 5 experiments.

$Ca = 0.060$
Large scale: width selection $\lambda = 0.4$

Average density map over experiments and time

$Ca = 0.058$
Large scale: width selection $\lambda = 0.4$

Contour at half-maximum average density

$Ca = 0.058$
Large scale: width selection $\lambda = 0.4$

Comparison with Saffman-Taylor finger generic shape

Some similarities with Saffman-Taylor finger shape, selected width $0.35 < \lambda < 0.45$

(conjecture of Arneodo et al, 1996: for DLA, average half-density profile = SF finger, $\lambda = 0.62$ Somfai et al 2003: confirmation for selected width, difference in the detailed head shape.

Similar conclusion for present experiment, selected width: $0.35 < \lambda < 0.45$
Large scale: width selection $\lambda = 0.4$

Result independent of capillary number $Ca = 0.20$

density: air=1, oil=0,

average over time and 5 experiments.
Large scale: width selection $\lambda = 0.4$

Average density map over experiments and time

$Ca = 0.22$
Large scale: width selection $\lambda = 0.4$

Contour at half-maximum average density

$Ca = 0.22$
Large scale: width selection $\lambda = 0.4$

Comparison with Saffman-Taylor finger generic shape

$p_f$ depends on $Ca$, not the width selected $0.35 < \lambda < 0.45$ as long as $Ca > 0.03$
Normalized density along transverse-to-flow direction

$$\rho(x) \propto n(x),$$ normalized density (direction perp. to previous plots), cumulated on all $z$ at $W$ or more from tip or inlet, at $Ca = 0.058$ and $0.22$: 
Relationship between Saffman-Taylor $\lambda = 0.4$, and poroviscous flow through lateral pressure field

Comparison of the pressure scaled with $Ca$, measured in experiments at three positions (17% of lateral extent from side boundary), and calculated in Saffman-Taylor problem with finger of width $\lambda = 0.4$. 

![Graph showing pressure scaled with Ca compared to Saffman-Taylor problem results.](image)
$v_{\text{tip}}$ revisited and $w_f$

- The characteristic finger width $w_f$ is measured as the average intersection length of cuts perpendicular to the flow direction with trapped clusters removed.

- $w_f$ is the cutoff scale between capillary and viscous fingering.

**Finger tip speed $v_{\text{tip}}$**

**Finger width $w_f$**
Selection of $\lambda$ and $D = 1.5$

Laplace problem: $\nabla^2 P = 0$

Boundary condition ahead: $\nabla P \propto Ca$

Boundary condition along the invasion cluster: influence of the capillary threshold lower cutoff Laplacian Growth: $v \propto \nabla P$

DLA: Growth proba $\propto \nabla P$

$\eta$-DBM: Growth proba $\propto (\nabla P)^2$
Here, interface speed along a given cluster, averaged over possible values of the next throat threshold:

\[ v(x, z) = \frac{2\kappa}{\mu} \left( \frac{P_0 - P(x, z) - P_t(x, z)}{a} \right) \]  

\( N(P_t(x, z)) \) capillary pressure distribution. Assume a flat capillary pressure distribution with lower limit \( P_{t_{\text{min}}} \), upper limit \( P_{t_{\text{max}}} \) and width \( W_c \). Expectational value of the interface velocity (average value over the capillary threshold distribution)

\[ \langle v(x, z) \rangle = \int_{P_{t_{\text{min}}}}^{P_0-P(x,z)} \frac{2\kappa}{a\mu} (P_0 - P(x, z) - P_t(x, z)) \]  

\[ \cdot \text{He} \left( P_0 - P(x, z) - P_t(x, z) \right) dP_t/W_c \]  

\[ = \frac{\kappa}{a\mu W_c} \left( P_0 - P(x, z) - P_{t_{\text{min}}} \right)^2. \]

Analogy to Dielectric Breakdown Model, \( \eta = 2 \).
<table>
<thead>
<tr>
<th>Model</th>
<th>Fractal dimension</th>
<th>Width selected</th>
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<tbody>
<tr>
<td>DLA, DBM $\eta = 1$</td>
<td>$D=1.71$</td>
<td>$\lambda = 0.62$</td>
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<td>(Somfai 2003)</td>
<td>(Somfai 2003, Arneodo 1996)</td>
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<tr>
<td>viscous fingering</td>
<td>$D=2$, $D=1.70$</td>
<td>$\lambda = 0.50 - 0.60$</td>
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<td>(empty Hele Shaw)</td>
<td>(slow, ST)(fast)</td>
<td>(Moore-Swinney 2003)</td>
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<td>DBM $\eta = 2$</td>
<td>$D=1.4-1.5$</td>
<td>$\lambda &lt; 0.5$</td>
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<td>(Pietronero, Mathiesen 2003)</td>
<td>(Somfai 2003)</td>
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<tr>
<td>viscous fingering</td>
<td>$D=1.5-1.6$</td>
<td>$\lambda = 0.40$</td>
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<td>in porous medium</td>
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