Viscous fingering in porous media

Renaud Toussaint (Institut de Physique du Globe de Strasbourg)
Jean Schmittbuhl (IPGS)
Knut Jørgen Måløy (University of Oslo: UiO)
Grunde Løvoll (UiO)
Yves Méheust (University of Trondheim, NTNU)

Phys. Rev. E **70**, 026301, 2004
cond-mat/0501047, submitted to Phys. Rev. Lett.

Réseau Alsace de Laboratoires
en Ingénierie et Sciences
pour l'Environnement

Meeting REALISE, January 2005

Analog model for biphasic flow in porous medium

Hele-Shaw cell: two parallel plates, cell with rectangular section, Stokes flow.

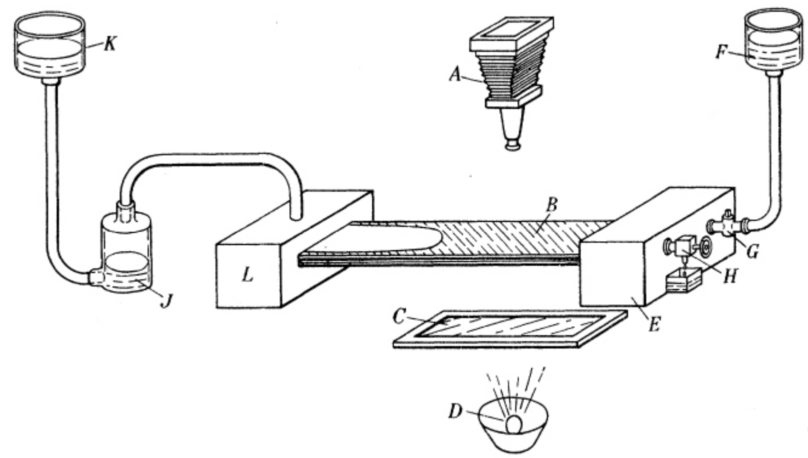


FIGURE 10. Arrangement of long Hele-Shaw cell for photographing fingers.

Saffman and Taylor, 1958, Proc. Roy Soc. London A.

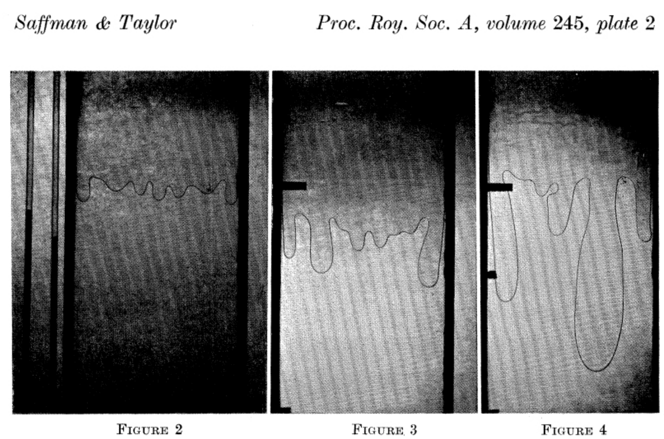
The penetration of a fluid into a porous medium
or Hele-Shaw cell containing a more
viscous liquid

BY P. G. SAFFMAN AND SIR GEOFFREY TAYLOR, F.R.S.
Cavendish Laboratory, University of Cambridge

(*Received* 17 *January* 1958—*Read* 17 *April* 1958)

Saffman Taylor instability

Fluid displacing a more viscous fluid:



Involved forces:

viscous forces $v = -(\kappa/\mu)\nabla P$, capillary forces $\delta P_c = -\gamma/r$.

Saffman Taylor solutions

Family of curves satisfying Darcy law $v = -(\kappa/\mu)\nabla P$, with neglected surface tension along boundary.

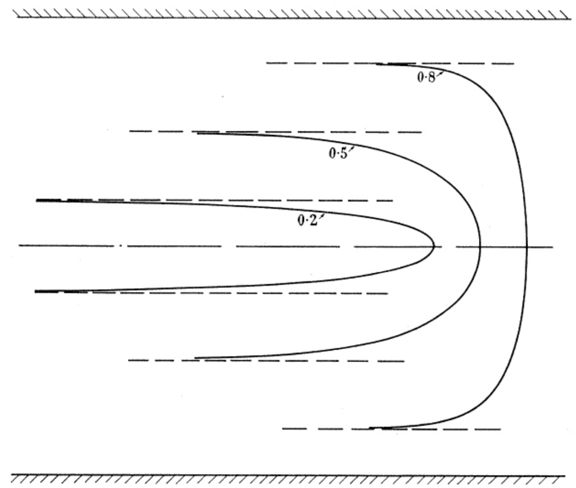


FIGURE 7. Calculated profiles for $\lambda = 0{\cdot}2$, $0{\cdot}5$ and $0{\cdot}8$.

Width selection

fraction λ of system size invaded, selected by surface tension, no matter how small.

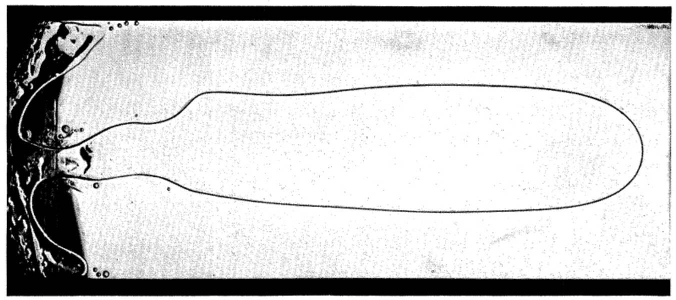


FIGURE 8

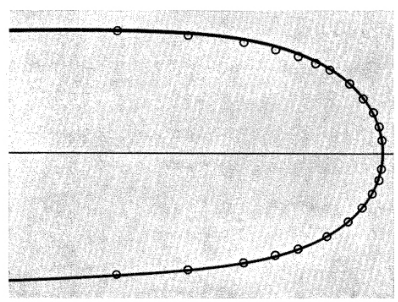


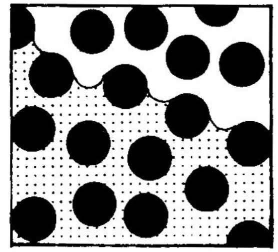
FIGURE 9

(*Facing p.* 318)

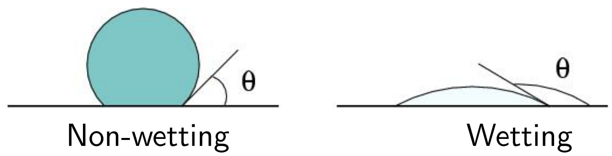
At large speed: $\lambda = 0.5$

Influence of pore-scale disorder

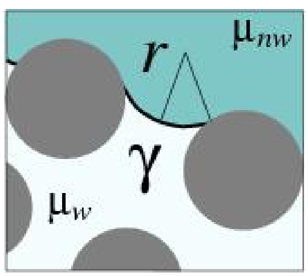
- *Porous medium:*



- *Wettability:*

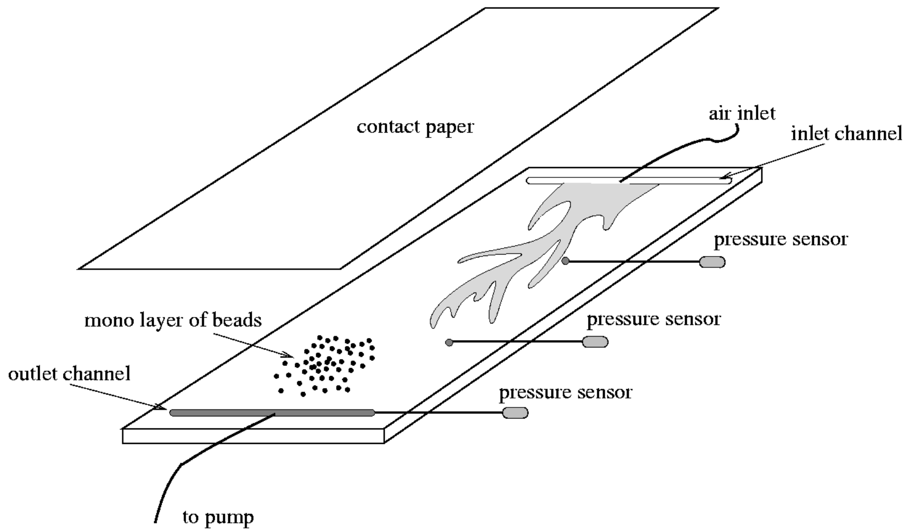


- *Drainage:* the non-wetting fluid displaces the wetting fluid.
- *Capillary pressure:* $P_c = \frac{2\gamma}{r}$ $\implies$ larger pores are more easily invaded



r	–	radius of curvature
γ	–	interface tension
μ_{nw}	–	viscosity (non-wetting)
μ_w	–	viscosity (wetting)

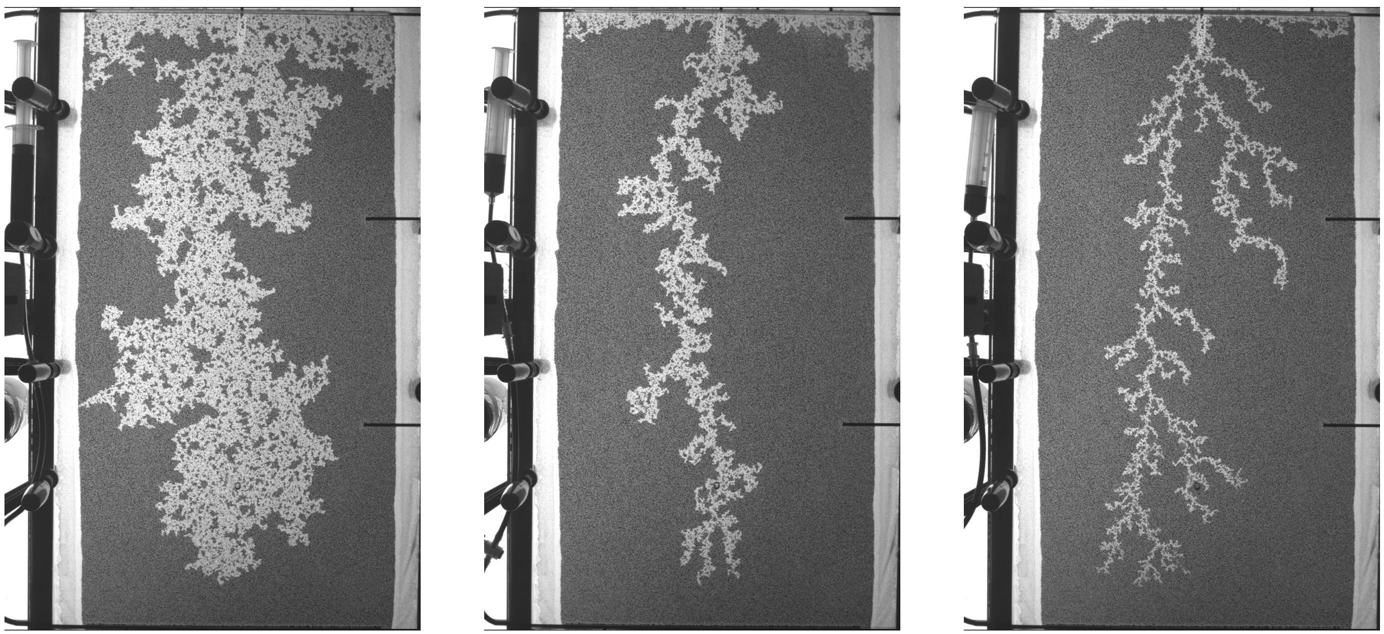
Experimental setup: a model of 2D random porous medium



- Mono-layer of $a = 1$mm glass beads "sandwiched" between two horizontal plates
- air displacing glycerol/water
- Three model widths to check size dependencies (L x W = 840mm x 430mm or 840mm x 215mm or 840mm x 110mm)
- Pictures of the structure taken at regular intervals

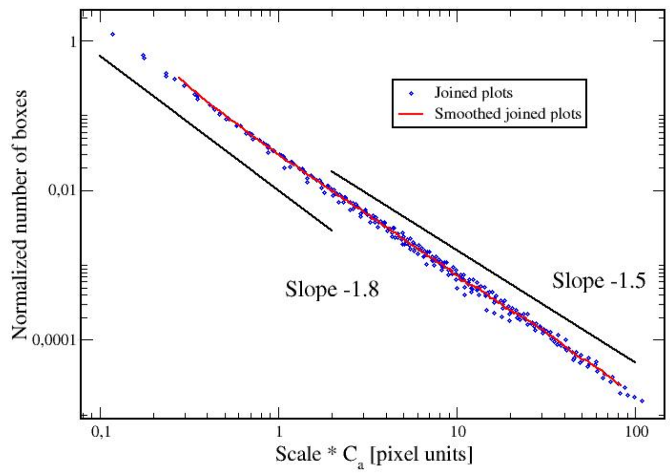
Invasion structures

- We see a continuous transition from capillary fingering to viscous fingering, as function of $\mathrm{Ca} = \frac{\mu v a^2}{\gamma \kappa}$. . .



$\mathrm{Ca} \simeq 0.015$ $\mathrm{Ca} \simeq 0.060$ $\mathrm{Ca} \simeq 0.20$

Mass fractal dimension: Box counting
Collapse with scaled size $s \cdot Ca$

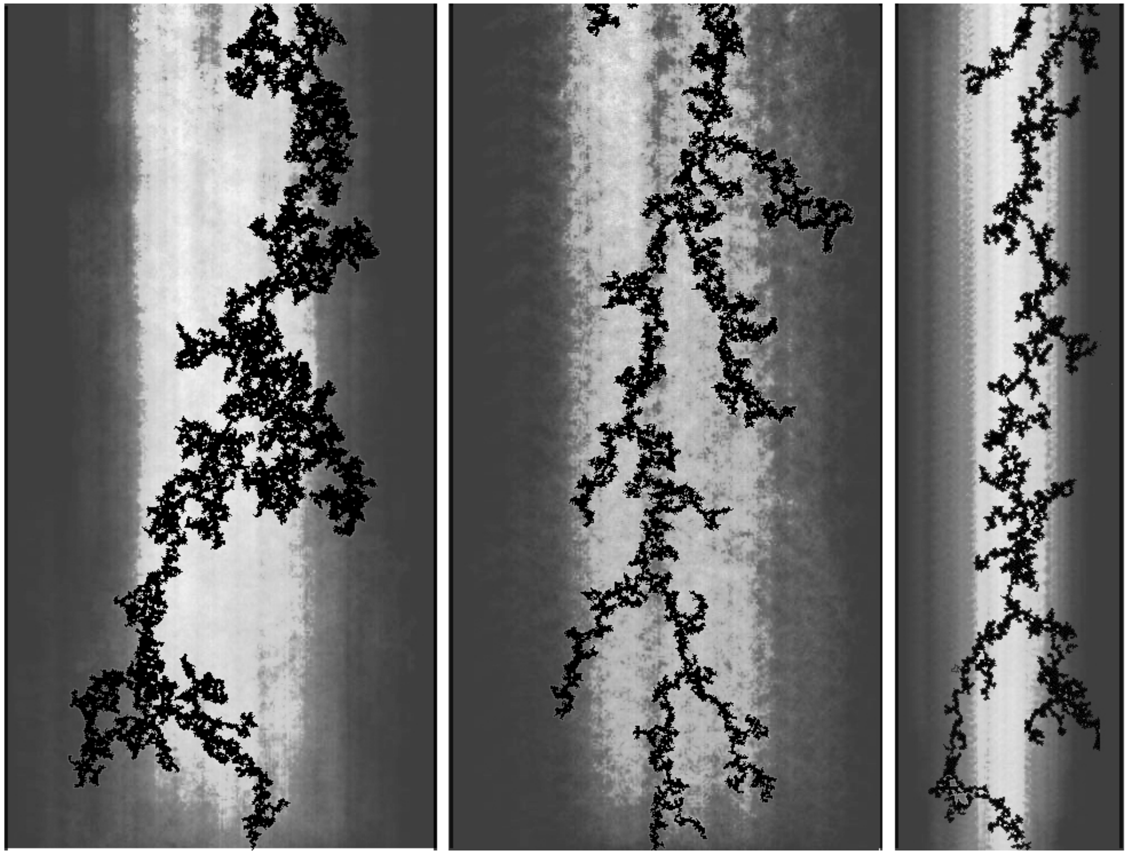


- D=1.8 corresponds to capillary fingering, or invasion percolation.

- D=1.5 is smaller than the Diffusion Limited Aggregation result, D=1.71. In the absence of granular material at high injection speed in Hele-Shaw cells, only a large scale exponent D=1.70 is found in experiments (Moore et al., Phys. Rev. E, 2002)

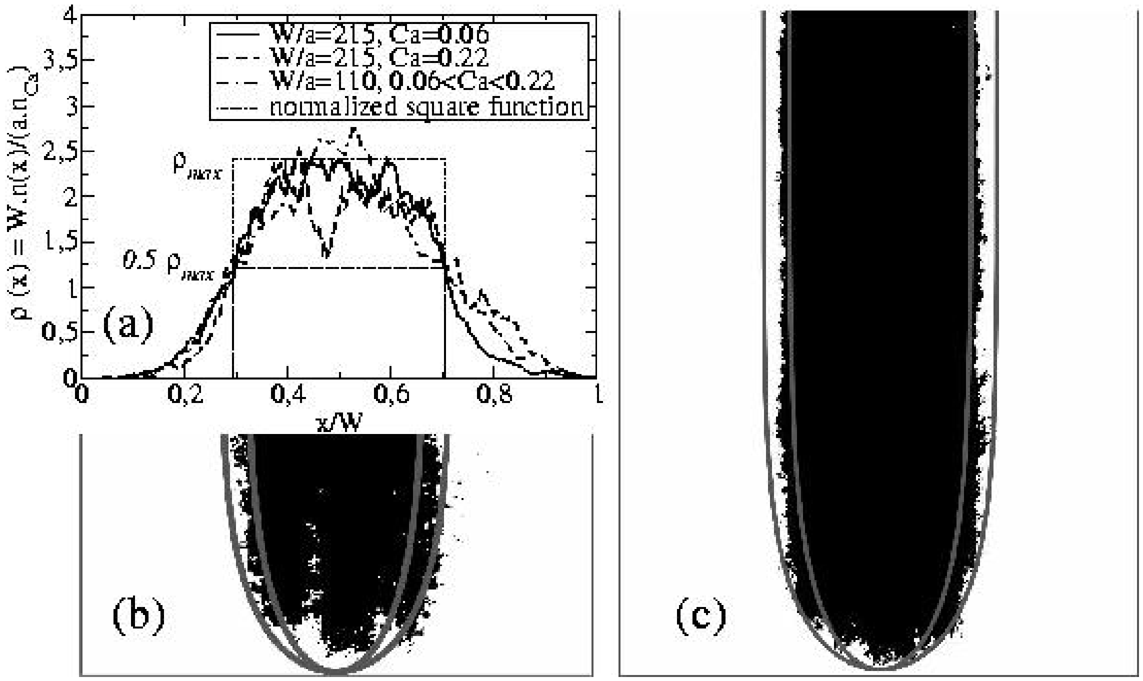
Average geometry

underlying statistically stationary process: in the reference frame of most advanced tip, analysis of invader's occupancy probability



Selected width

Occupancy function thresholded at half-maximum: selected width: $\lambda = 0.4$



Conclusion

Presence of disorder in pore structure modifys the large scale structure:

characteristic width of most occupied zone,

large scale fractal dimension.

Can be explained by probabilistic approach of the invasion process.

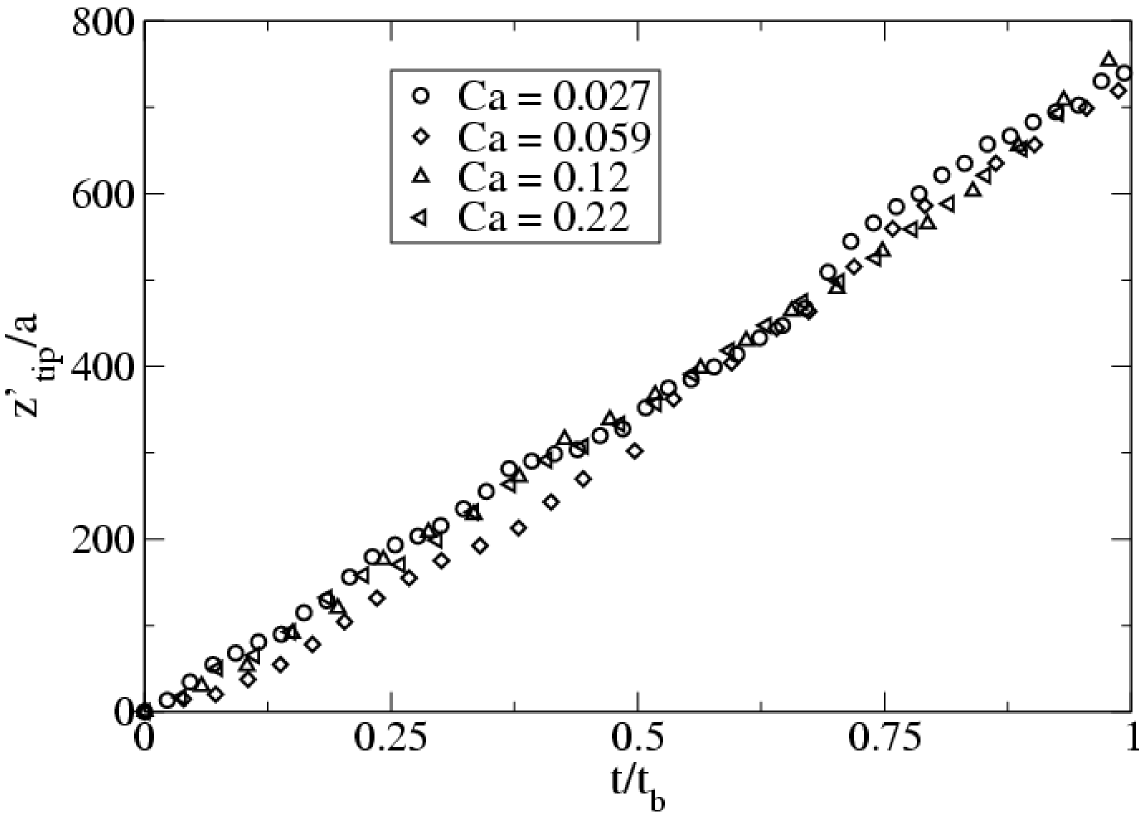
Perspectives

Numerical and experimental (transparent models) study of the impact of the probability distribution of the pore sizes on the large scale flow.

Extension to biphasic flow in nonsaturated situations.

The speed of the fingertip

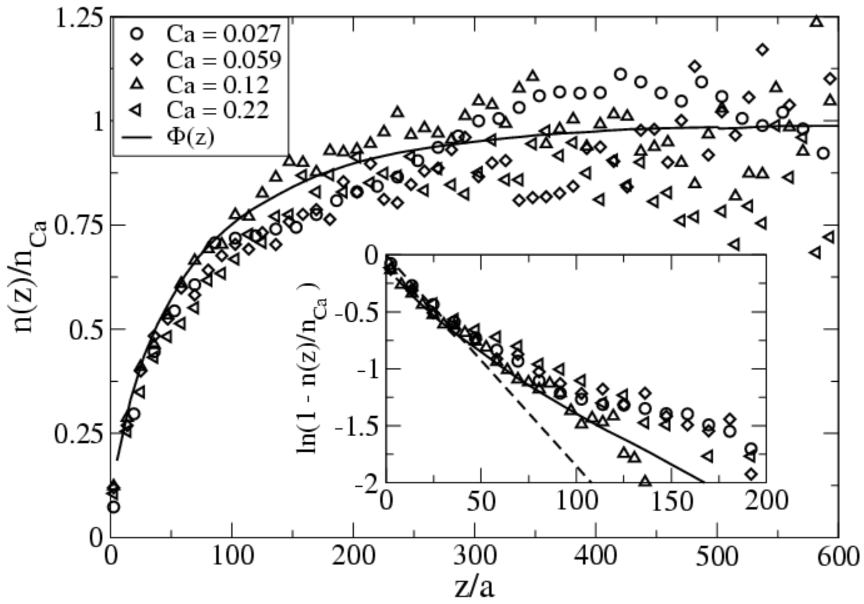
- For each capillary number Ca the speed of the most advances fingertip could be treated as constant



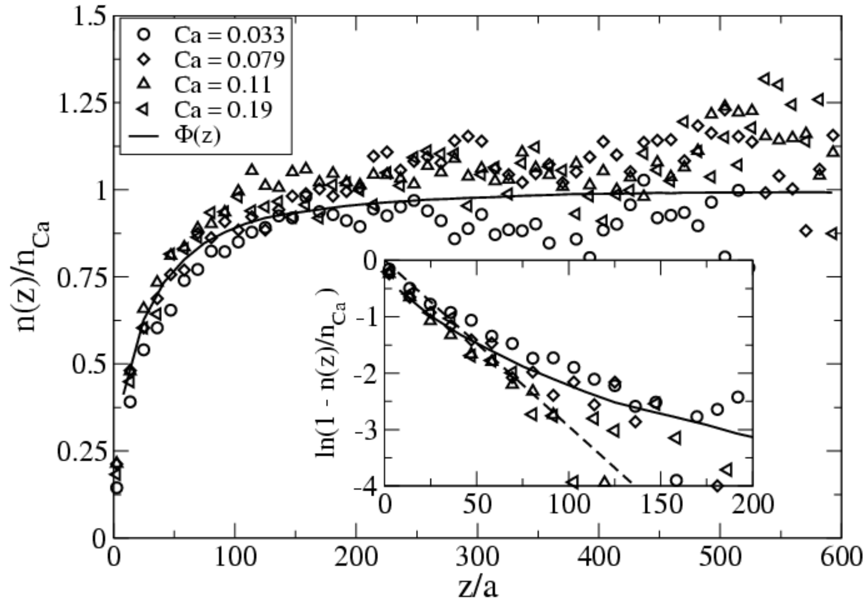
Average mass of invasion cluster

- Since $\phi(z)$ indep of Ca, $v_{tip} \sim constant$, the mass of the invasion cluster could be scaled with $n_{\mathrm{Ca}} = \frac{W\gamma\kappa}{\mu_w a^4} \frac{\mathrm{Ca}}{v_{\mathsf{tip}}}$
- And the scaled mass is also equal to the cumulative invasion probability distribution $n(z)/n_{\mathrm{Ca}} = \Phi(z) = \int_0^z \phi(y)dy$

Wide model



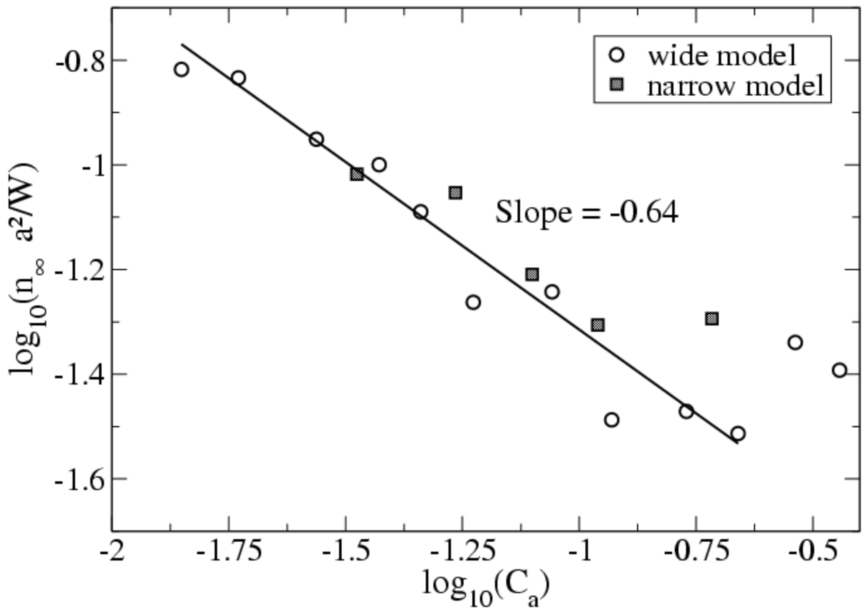
Narrow model



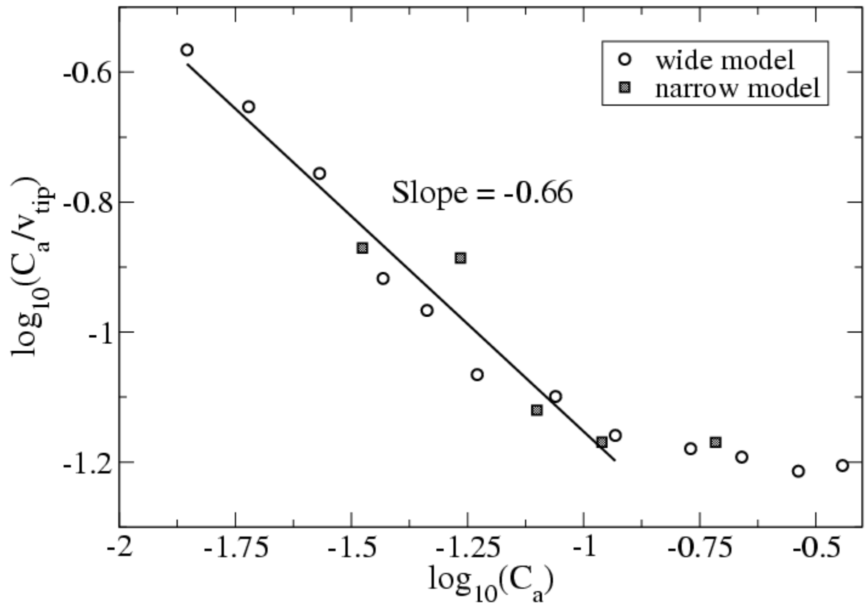
Ca **dependence in** m_∞ **and** v_{tip}

- The saturation mass and the speed of the fingertip is capillary number dependent

Saturation mass



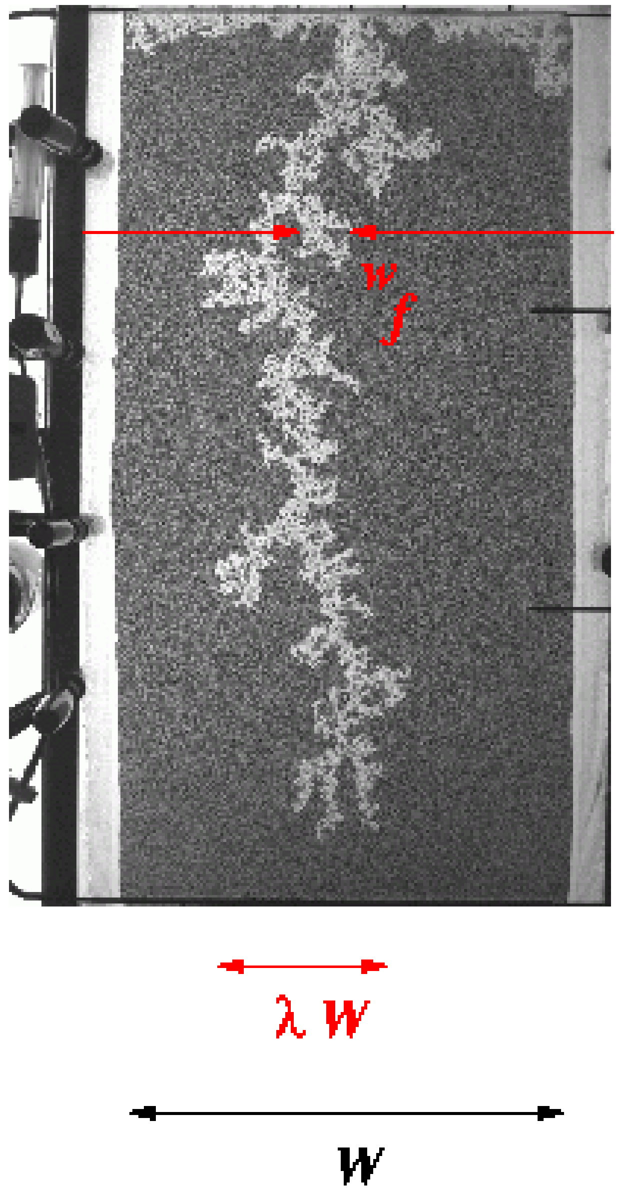
Fingertip speed



Transition scale from capillary to viscous fingering: w_f

- The characteristic finger width w_f is measured as the average intersection length of cuts perpendicular to the flow direction with trapped clusters removed

- w_f is the cutoff scale between capillary and viscous fingering, characteristic of maximum loop size



Transition scale w_f depending on Ca

$w_f \cdot \nabla P \sim \Delta P_c$ width of threshold distribution.

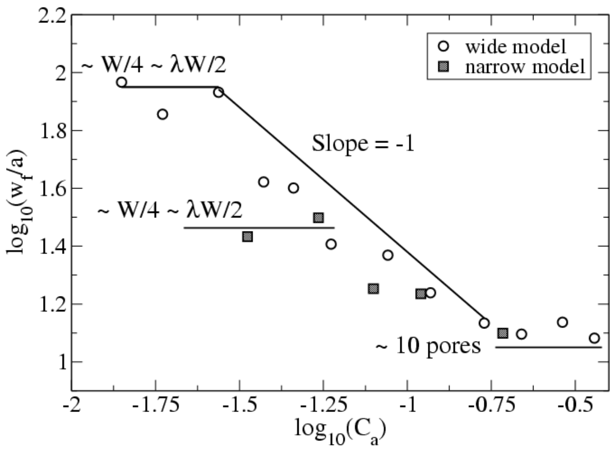
Along a given boundary, $\nabla P \propto \nabla P(-\infty) \propto Ca$

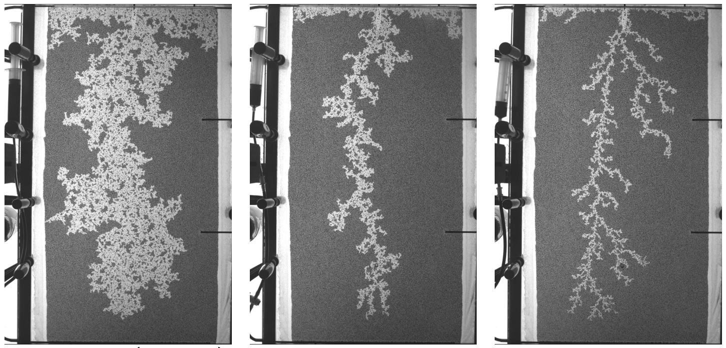
Thus, $w_f \sim 1/Ca$

Large Ca, minimum loop size $\sim$ few pore size a, and only branched structure.

Small Ca, whole structure $=$ gigantic loop (lowly branched), and w_f saturates around the lateral structure extent, $\sim \lambda W$.

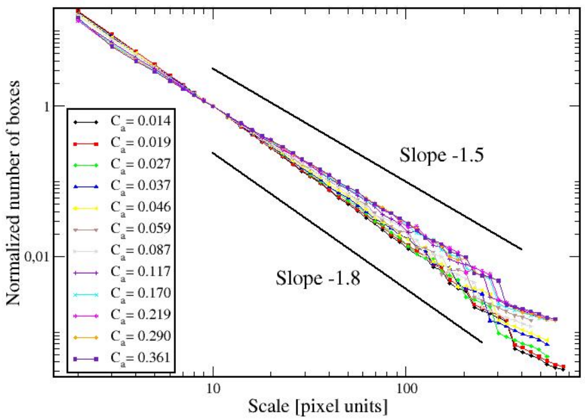
Finger width w_f



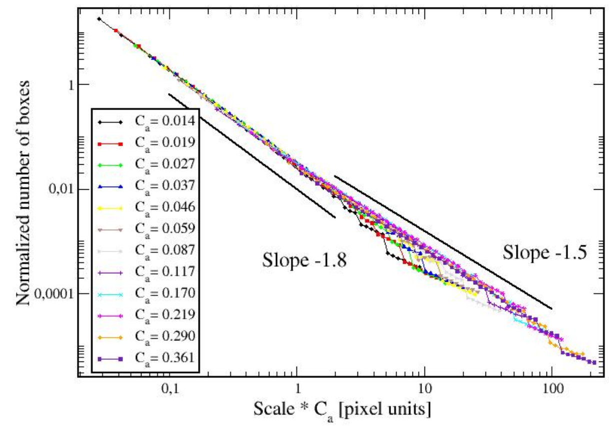


$\log_{10}(Ca) = -1.83, \quad -1.22, \quad -0.70$

Mass fractal dimension: Box counting



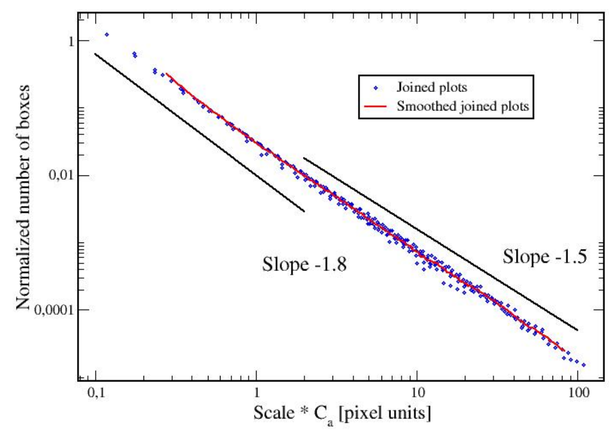
Collapse, scaled size $s \cdot Ca$



- D=1.8 corresponds to capillary fingering, or invasion percolation.

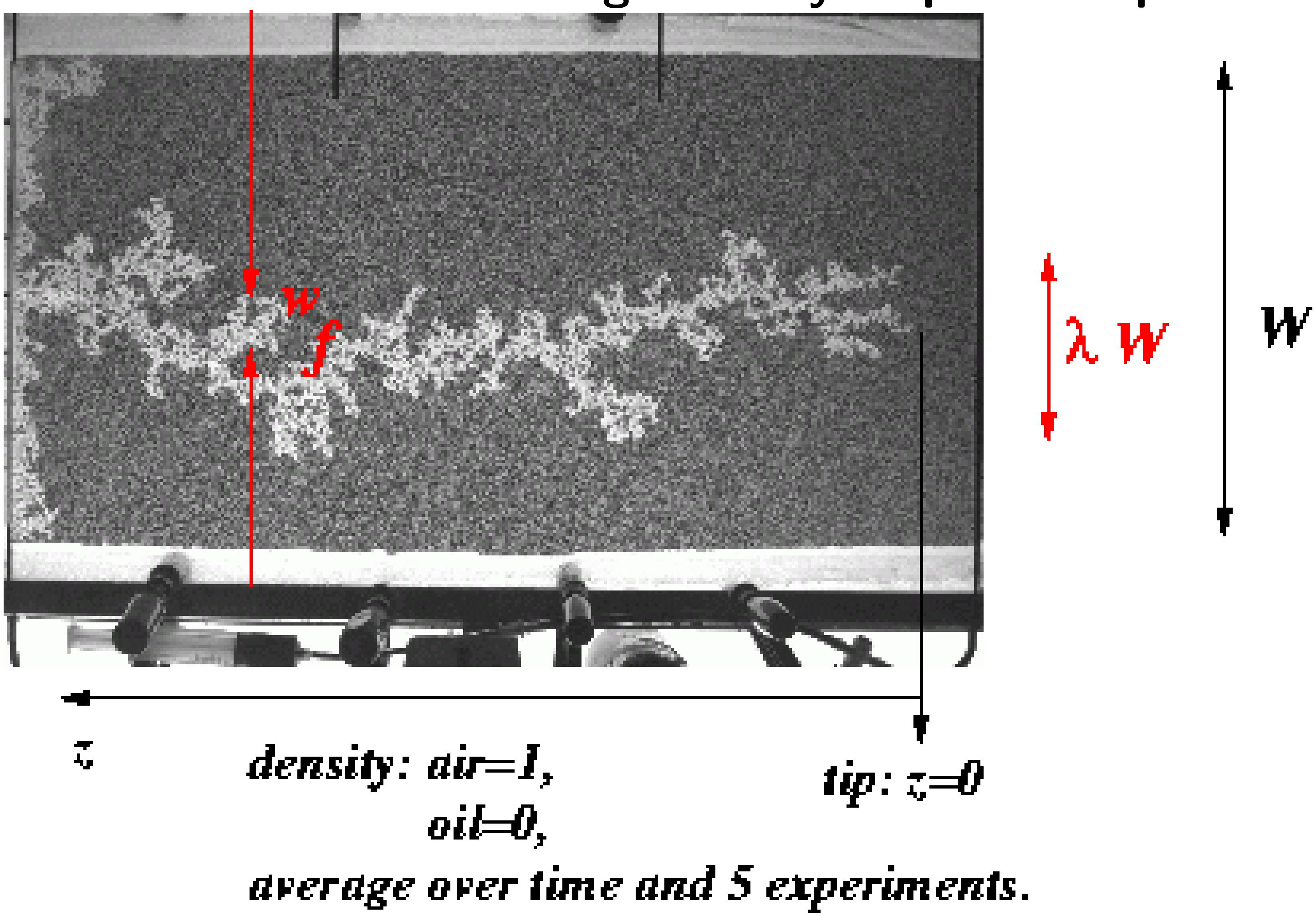
- D=1.5 is smaller than the Diffusion Limited Aggregation result, D=1.71. In the absence of granular material at high injection speed in Hele-Shaw cells, only a large scale exponent D=1.70 is found in experiments (Moore et al., Phys. Rev. E, 2002)

Average collapsed



Large scale: width selection $\lambda = 0.4$

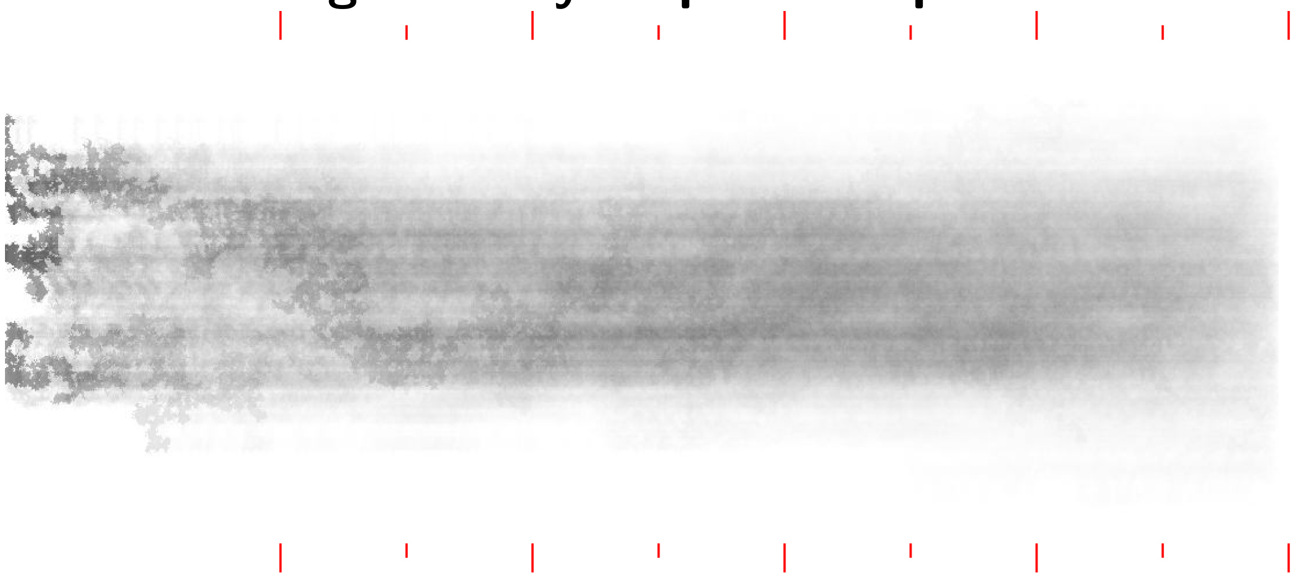
Definition of average density map over exps and time



$Ca = 0.060$

Large scale: width selection $\lambda = 0.4$

Average density map over experiments and time



$Ca = 0.058$

Large scale: width selection $\lambda = 0.4$

Contour at half-maximum average density

$Ca = 0.058$

Large scale: width selection $\lambda = 0.4$

Comparison with Saffman-Taylor finger generic shape

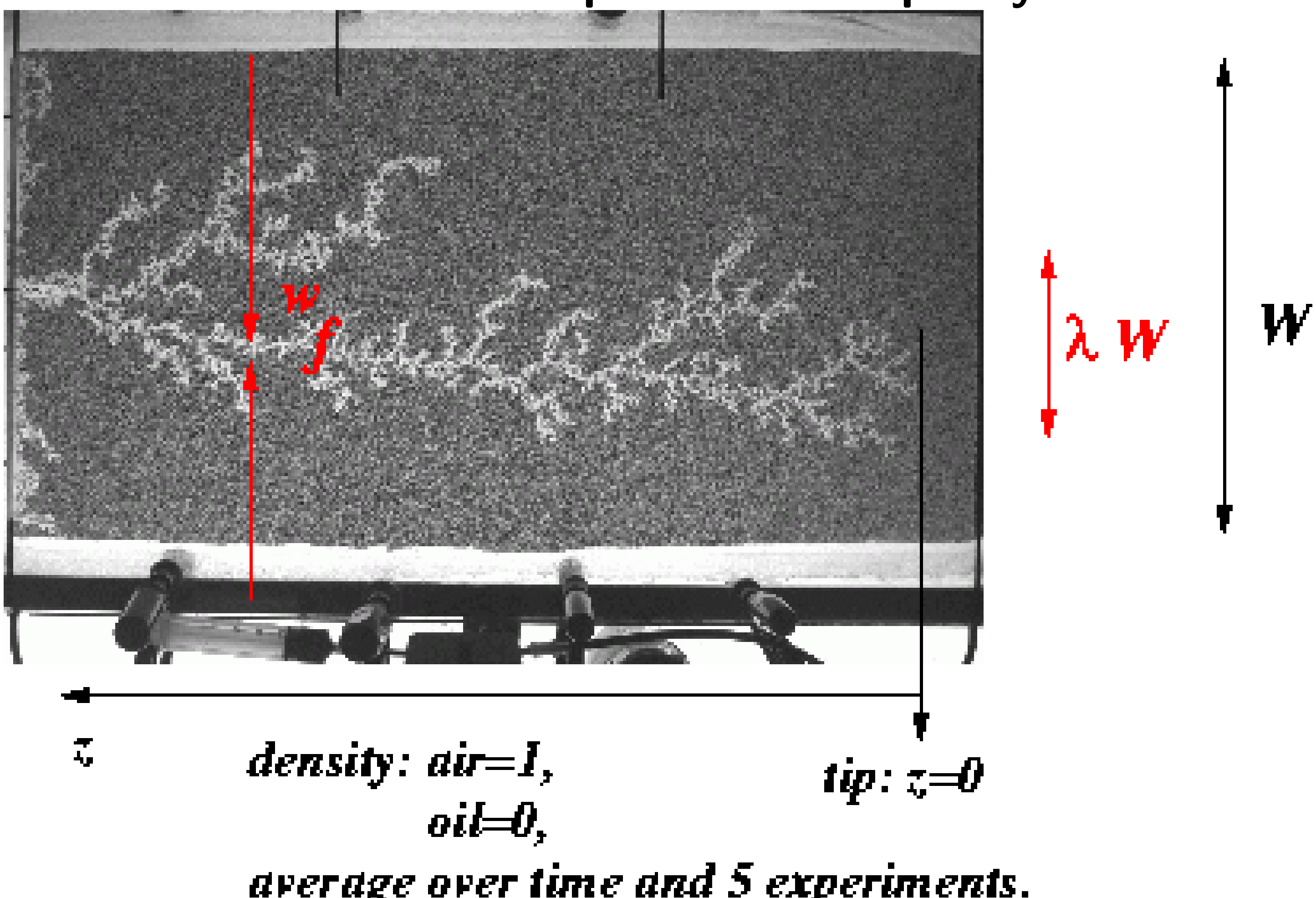
Some similarities with Saffman-Taylor finger shape, selected width $0.35 < \lambda < 0.45$

(conjecture of Arneodo et al, 1996: for DLA, average half-density profile = SF finger, $\lambda = 0.62$ Somfai et al 2003: confirmation for selected width, difference in the detailed head shape.

Similar conclusion for present experiment, selected width: $0.35 < \lambda < 0.45$

Large scale: width selection $\lambda = 0.4$

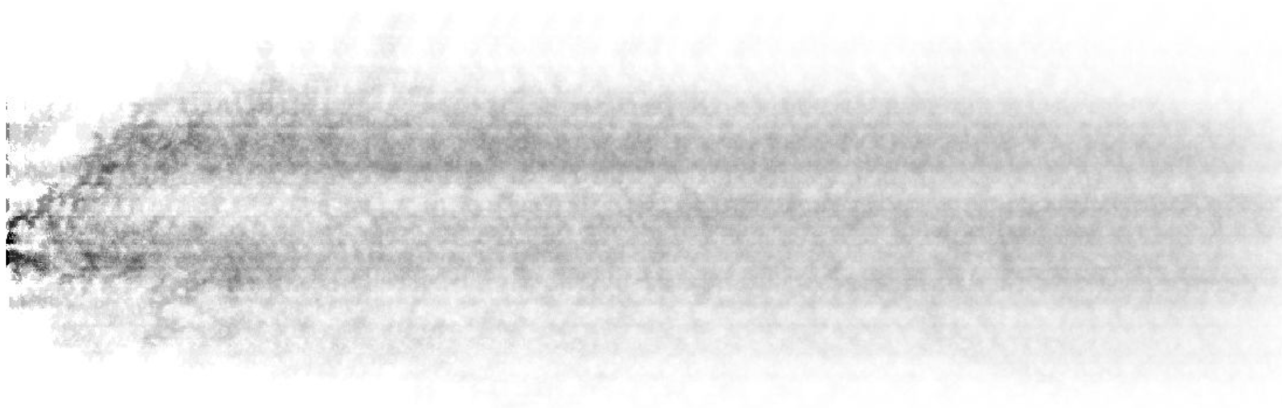
Result independent of capillary number



$Ca = 0.20$

Large scale: width selection $\lambda = 0.4$

Average density map over experiments and time



$Ca = 0.22$

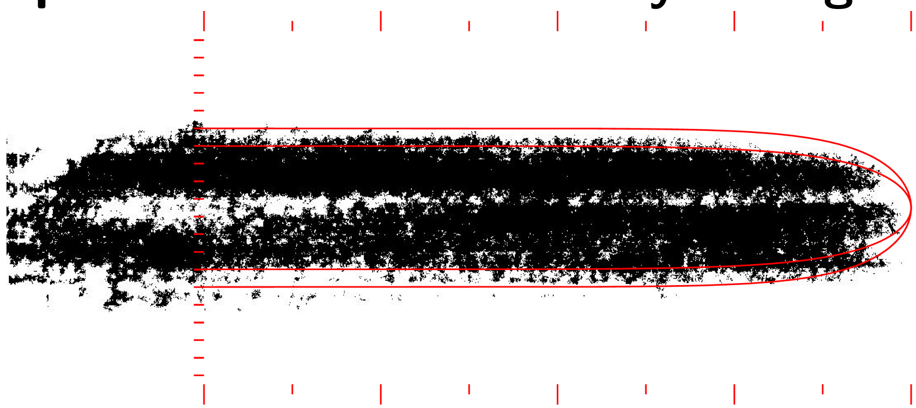
Large scale: width selection $\lambda = 0.4$

Contour at half-maximum average density

$Ca = 0.22$

Large scale: width selection $\lambda = 0.4$

Comparison with Saffman-Taylor finger generic shape

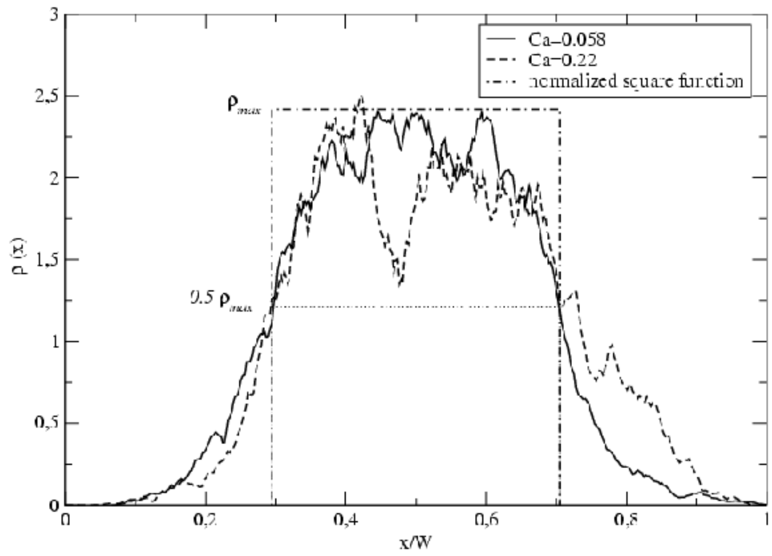


$Ca = 0.22$

w_f depends on Ca, not the width selected $0.35 < \lambda < 0.45$ as long as $Ca > 0.03$

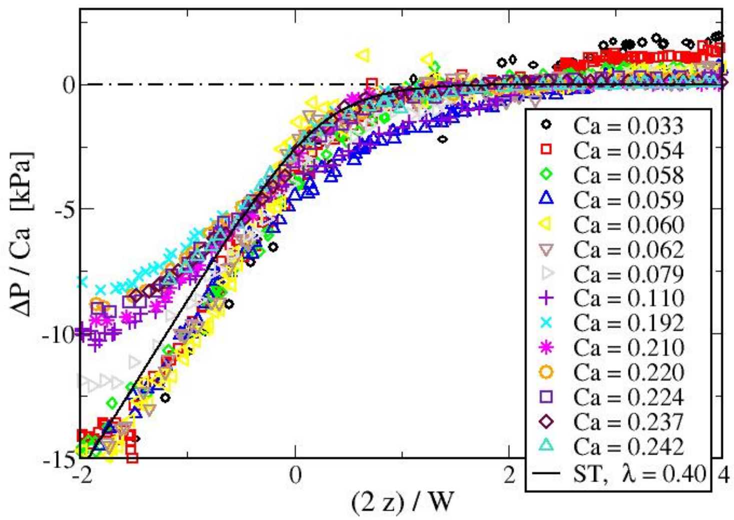
Normalized density along transverse-to-flow direction

$\rho(x) \propto n(x)$, normalized density (direction perp. to previous plots), cumulated on all z at W or more from tip or inlet, at $Ca = 0.058$ and 0.22:



Relationship between Saffman-Taylor $\lambda = 0.4$, and poroviscous flow through lateral pressure field

Comparison of the pressure scaled with Ca, measured in experiments at three positions (17$\%$ of lateral extent from side boundary), and calculated in Saffman-Taylor problem with finger of width $\lambda = 0.4$.

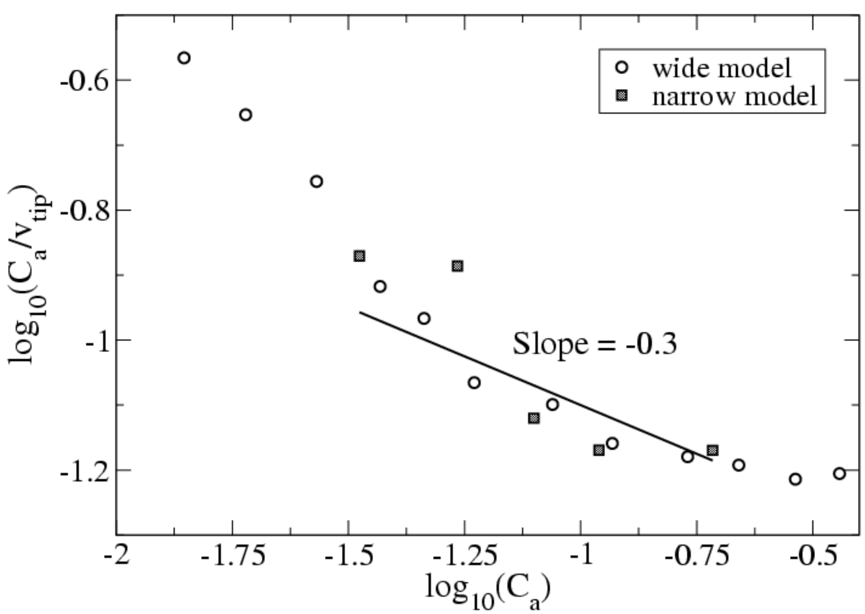


v_{tip} revisited and w_f

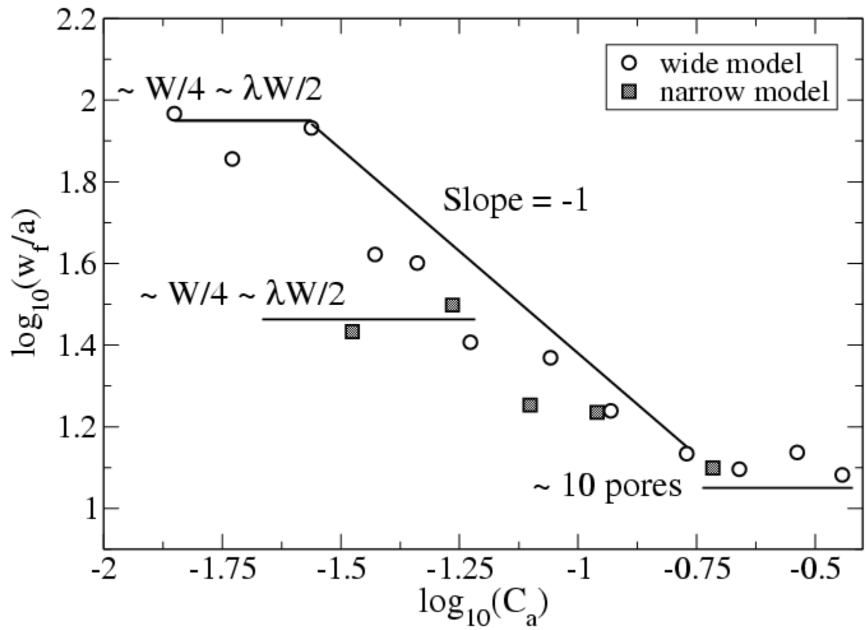
- The characteristic finger width w_f is measured as the average intersection length of cuts perpendicular to the flow direction with trapped clusters removed

- w_f is the cutoff scale between capillary and viscous fingering

Finger tip speed v_{tip}



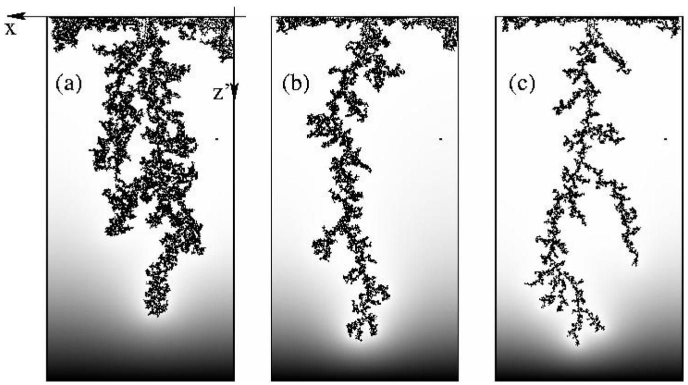
Finger width w_f



Selection of λ and $D = 1.5$

Laplace problem: $\nabla^2 P = 0$

Boundary condition ahead: $\nabla P \propto Ca$



Boundary condition along the invasion cluster: influence of the capillary threshold lower cutoff Laplacian Growth: $v \propto \nabla P$

DLA: Growth proba $\propto \nabla P$

η-DBM: Growth proba $\propto (\nabla P)^2$

Here, interface speed along a given cluster, averaged over possible values of the next throat threshold:

$$v(x,z) = \frac{2\kappa}{\mu} \frac{(P_0 - P(x,z) - P_t(x,z))}{a} \ . \tag{1}$$

$N(P_t(x,z))$ capillary pressure distribution. Assume a flat capillary pressure distribution with lower limit P_t^{min}, upper limit P_t^{max} and width W_c. Expectational value of the interface velocity (average value over the capillary threshold distribution)

$$\begin{aligned} \langle v(x,z) \rangle &= \int_{P_t^{\text{min}}}^{P_0 - P(x,z)} \frac{2\kappa}{a\mu} \left(P_0 - P(x,z) - P_t(x,z)\right) && (2) \\ & \cdot He\left(P_0 - P(x,z) - P_t(x,z)\right) dP_t / W_c && (3) \\ &= \frac{\kappa}{a\mu W_c} \left(P_0 - P(x,z) - P_t^{\text{min}}\right)^2 \quad . && (4) \end{aligned}$$

Analogy to Dielectric Breakdown Model, $\eta = 2$.

Model	Fractal dimension	Width selected
DLA, DBM $\eta = 1$	D=1.71	$\lambda = 0.62$
	(Somfai 2003)	(Somfai 2003, Arneodo 1996)
viscous fingering	D=2 , D=1.70	$\lambda = 0.50 - 0.60$
(empty Hele Shaw)	(slow, ST)(fast)	(Moore-Swinney2003)
DBM $\eta = 2$	D=1.4-1.5	$\lambda < 0.5$
	(Pietronero, Mathiesen 2003)	(Somfai 2003)
viscous fingering	D=1.5-1.6	$\lambda = 0.40$
in porous medium		