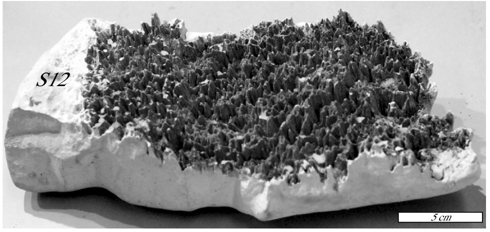

Stylolite formation process:

Surface Roughness Measurements and Process Modelization

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Outline

- Introduction and examples of stylolites
- Experimental techniques
- Results of analysis of stylolite roughness
- Simple model of stylolite formation
- Conclusion and Prospects

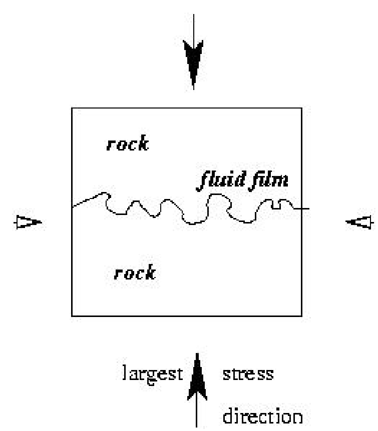
Introduction to stylolites

- Solid-solid interfaces found in some sedimentary rocks

- Roughly planar interfaces perpendicular to the main principal stress direction (so called anti-cracks): used as stress orientation markers by geologists

- Roughness of stylolites: appearance of printed lines on rock cuts, seen in many buildings.

- Formed under pressure-dissolution processes happening in shallow earth crust (depth km, during million years): fluid present during formation. Most important "plastic" deformation mechanism of Earth crust.

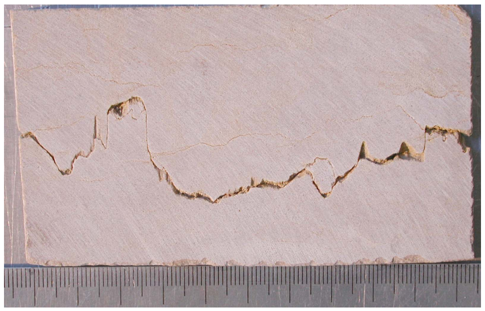


Examples of stylolites

Stylolite in marble wall used in building
Total view 10 cm
Courtesy: Duncan Heron, Duke University

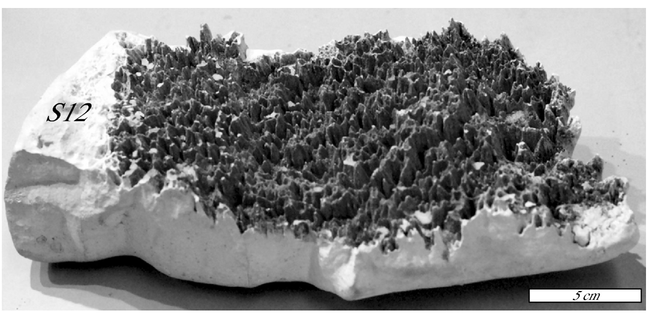
Micrograph of stylolite in Carbonate rock (dolomite)
Total view 1 cm
Courtesy: Dr. CM Yoo, SNU, Korea

opening a stylolite in limestones



Stylolite in cutted limestone
Total view 10 cm
F. Renard and J.P. Gratier, Grenoble.

After opening:

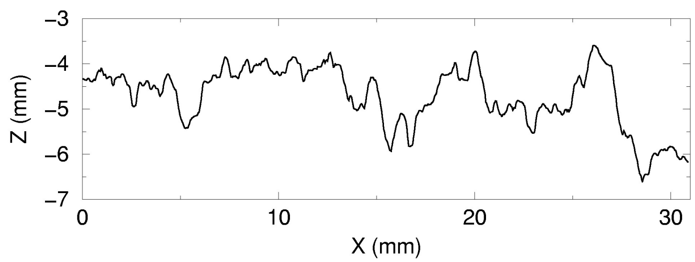


Interstitial clay allowed to open the stylolite
limestone from Vercors mountain, France.
F. Renard and J.P. Gratier, Grenoble.

Acquisition of surface profile

Profile $z(x,y)$ is obtained along the surface of a stylolite
Acquisition techniques:

- *Mechanical profilometer*
  needle in contact with surface, 1D profiles $z(x)$, precision around $\Delta x = 25\mu$m and $\Delta z = 7\mu$m.



- *Laser triangulation*
  higher acquisition speed allows for surface map $h(x,y)$ with precision around $\Delta x = 7$ to 50μm and $\Delta z = 2\mu$m. Comparison with mechanical setup: check that optical variations are height variations.

Analysis of surface roughness

Determination of **self affine** character of stylolite surfaces:

Statistical invariance under anisotropic rescaling

$$x \to \lambda x, y \to \lambda y, z \to \lambda^\zeta z,$$

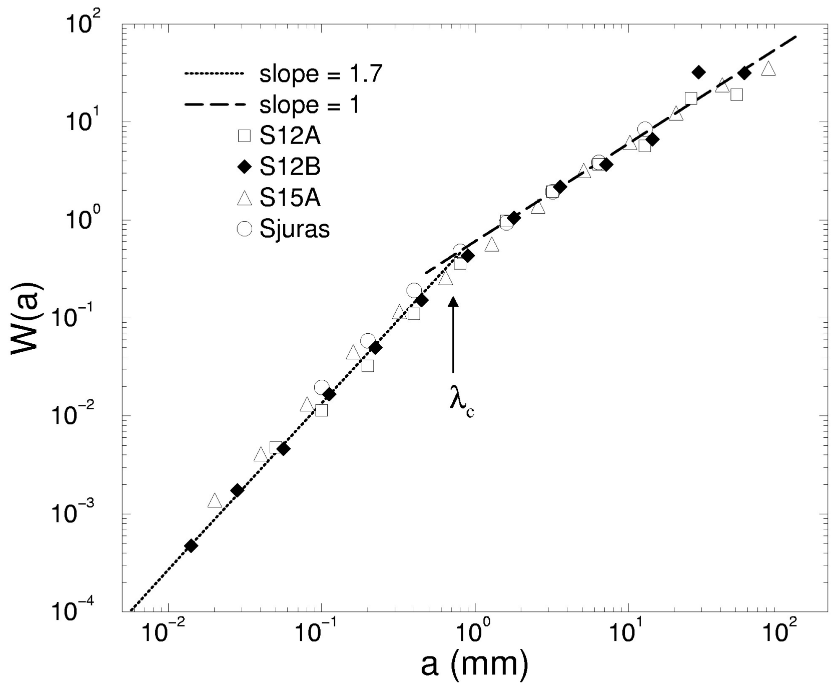
with a roughness (Hurst) exponent ζ.

Equivalence: average z-width determined on scale l along (x, y) plane scales as

$$w(l) \propto l^\zeta.$$

Use of Wavelet power spectrum

Using 1D wavelets, the average power spectrum is expected as $W(L) \propto L^{0.5+\zeta}$. (Simonsen, Hansen & Nes, Phys Rev E 1998).



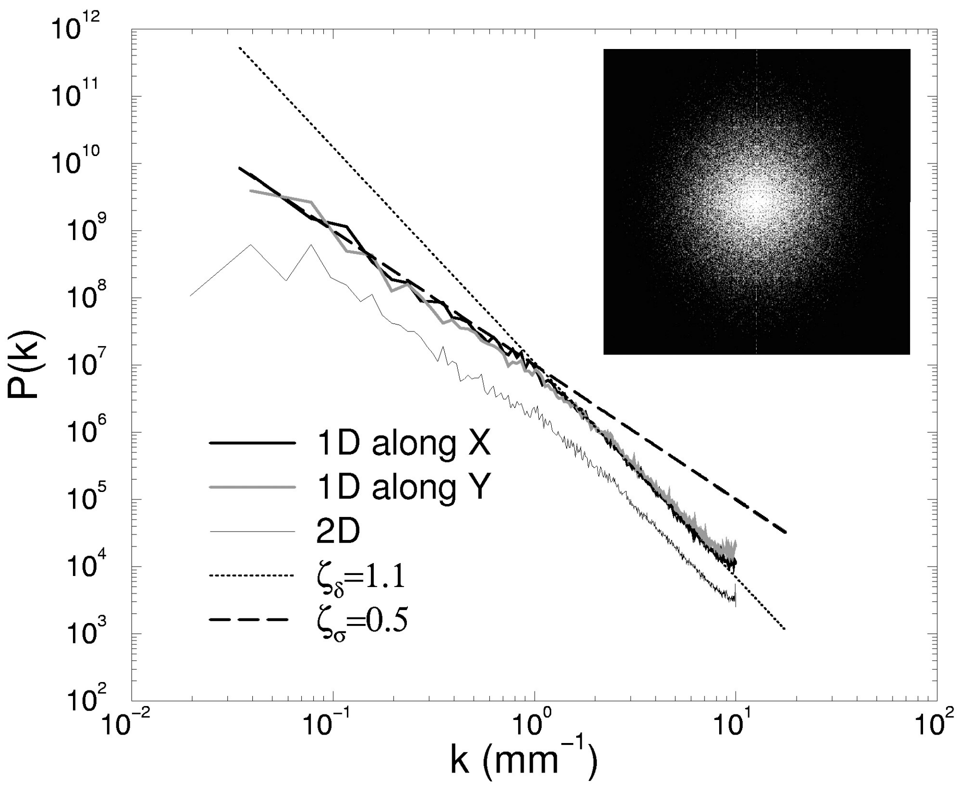
Average wavelet spectrum of 1D topographic profile of 4 stylolites: stylolites are self affine surface with two different regimes:

- *small scale:* $\zeta_2 \simeq 1.2$
- *large scale:* $\zeta_1 \simeq 0.5$

Cross-over scale: $L_c \sim 1mm$

Fourier power spectrum technique

Second technique to determine these roughness exponents:
Fourier power spectrum, which behaves in d dimensions for a self affine surface as $P(k) \propto k^{-(2\zeta+d)}$ (Barabasi & Stanley, Fractal concepts in surface growth, 1995)

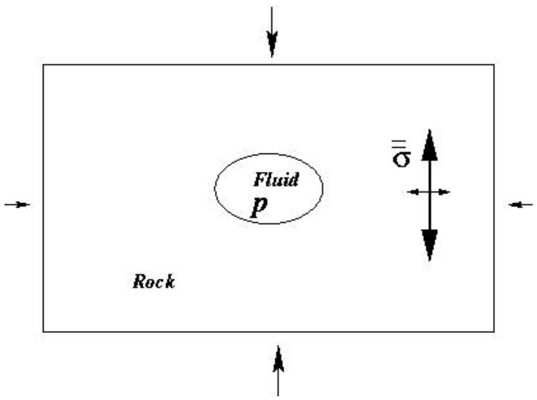


One and 2D analysis lead to

- *small scale:* $\zeta_2 \simeq 1.1$
- *large scale:* $\zeta_1 \simeq 0.5$

Considerations about the origin of this morphology

Initially, trapped fluid pockets burried in sedimentary rock



- Rock sustains an anisotropic stress, in contrast with the fluid
- Mechanical equilibrium at the solid-fluid interface

Departure from initial chemical equilibrium under stress (stress dissolution): potential energy difference goes mainly as (Kassner et al, Phys Rev E 2001)

$$\Delta\mu = \Omega(u_e + \gamma\kappa)$$

where $\Delta\mu$: potential energy difference for calcite between solid and fluid, u_e: elastic energy per unit volume in the solid, γ: surface energy, κ: curvature, and Ω: molar volume. u_e grows with σ.

Relationship with dissolution speed

With slow processes and homogeneous calcite concentration in the fluid, the dissolution speed goes as

$$\begin{aligned} v_n &= (k\Omega/RT)\Delta\mu \\ \Delta\mu &= \Omega(u_e + \gamma\kappa) \\ u_e^0 &= \alpha p_0^2/E \end{aligned}$$

with v_n: dissolution speed, k: surface reaction rate,
$p_0 = -Tr(\boldsymbol{\sigma}^0)/3$, E: rock's Young modulus, $\alpha \sim 1$.

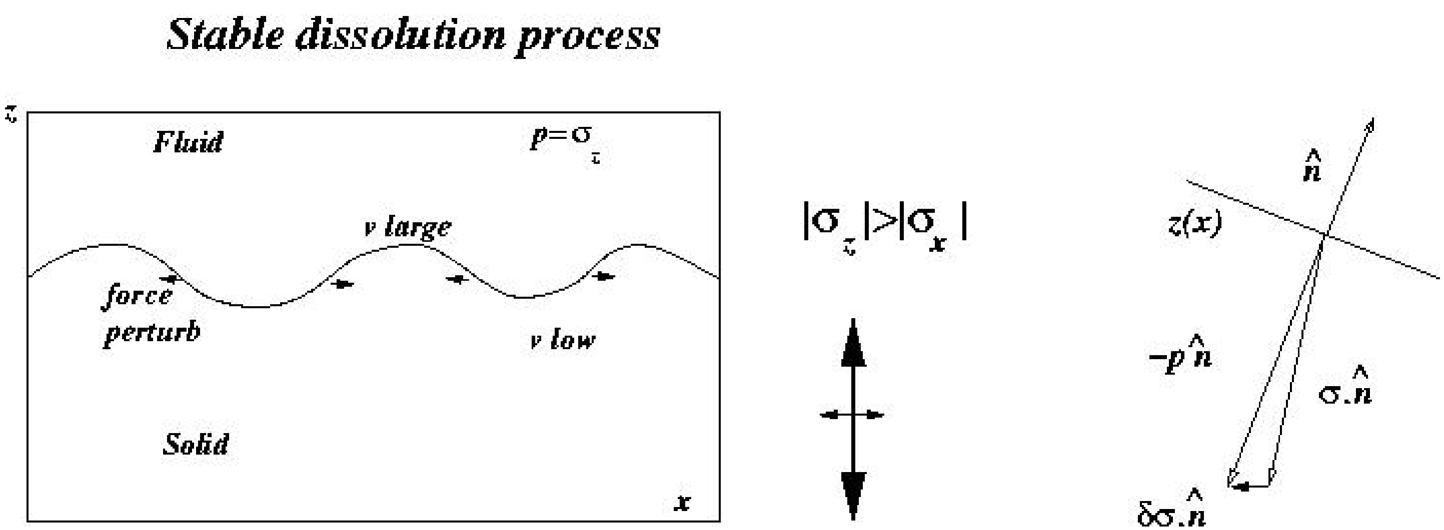
Average dissolution speed for a typical calcite water interface:

$$v_n^0 \simeq 8 \cdot 10^{-6} m/year$$

Stability of a rock-water interface

Consideration of the problem for single rock-fluid interface, first without any disorder.

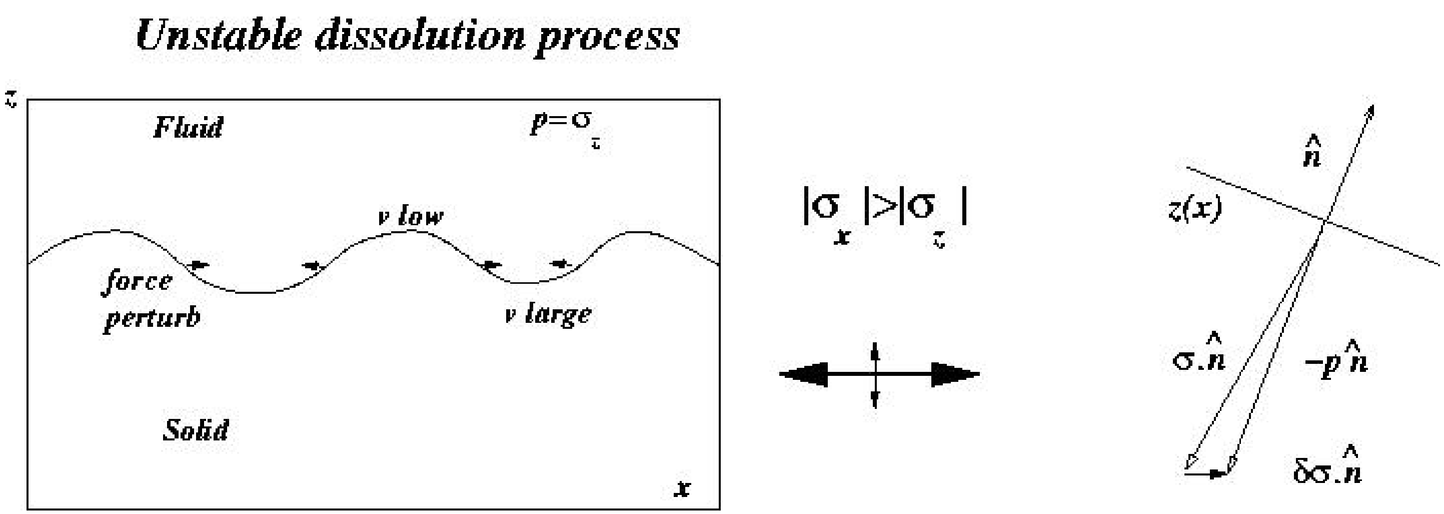
Surfaces normal to the largest stress axis: stable case



Any shape departure from a plane results in slowing down the dissolution grooves and accelerating the crests left behind: stable situation. Dissolution plane propagates at homogeneous constant speed v_n^0.

Instability of a perpendicular rock-water interface

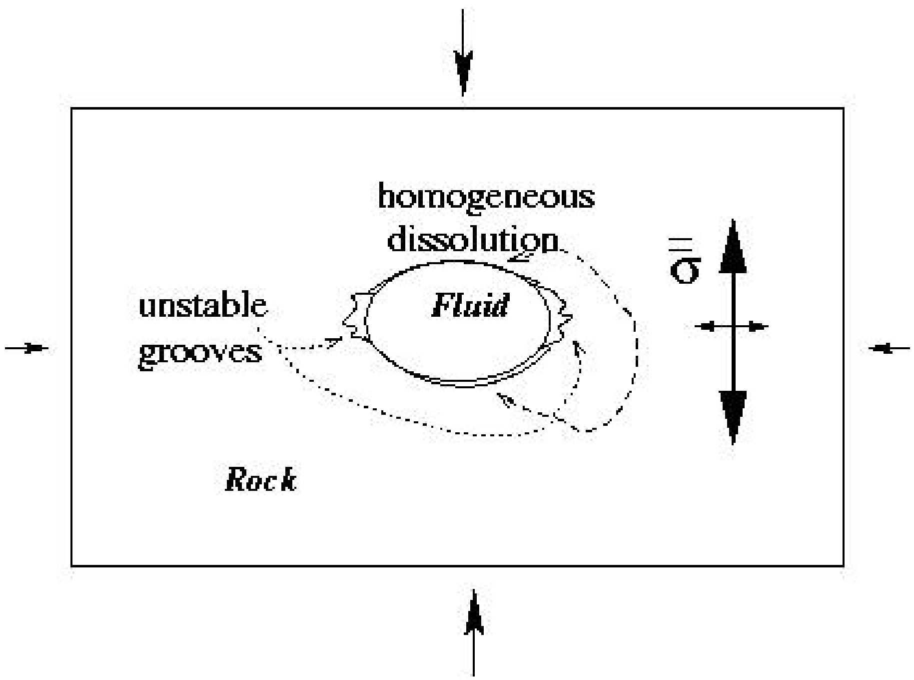
Surfaces tangential to the largest stress axis: unstable case



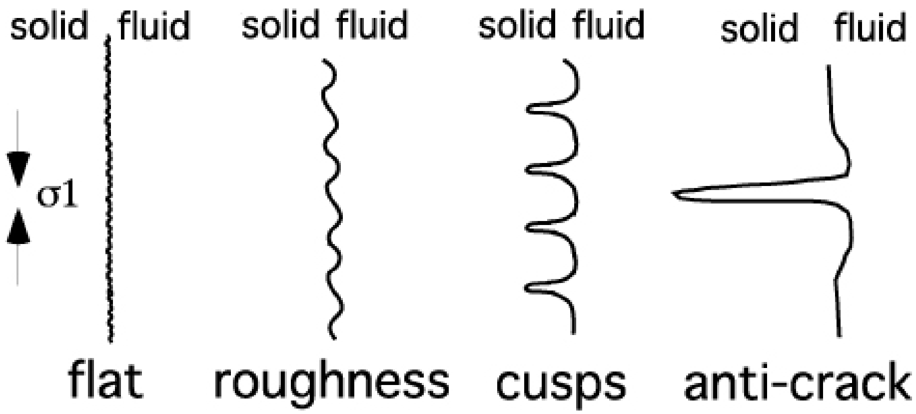
Any shape departure from a plane results in accelerating the dissolution grooves and slowing down the crests left behind: unstable situation. Small scales are stabilized by surface tension, large ones are destabilized by elastic interactions: Azaro-Tiller-Griensfeld (ATG) instability (Kassner et al 2001), where a certain characteristic wavelength grows fastest. Any dissolution plane develops penetrating grooves of this wavelength in addition to the average homogeneous dissolution.

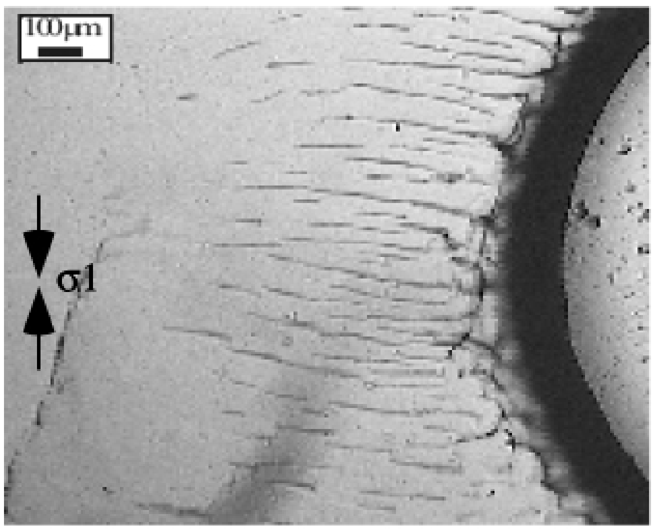
Consequence on initial evolution of trapped fluid pocket

A trapped fluid should slightly dissolve (or recrystallize if concentration gets high enough) the rock along the surfaces normal to the principal stress axis, and penetrate in grooves of characteristic wavelength the rock in the directions along the weakest stress: development of anti-cracks (Koehn et al, American Journal of Science 2003).

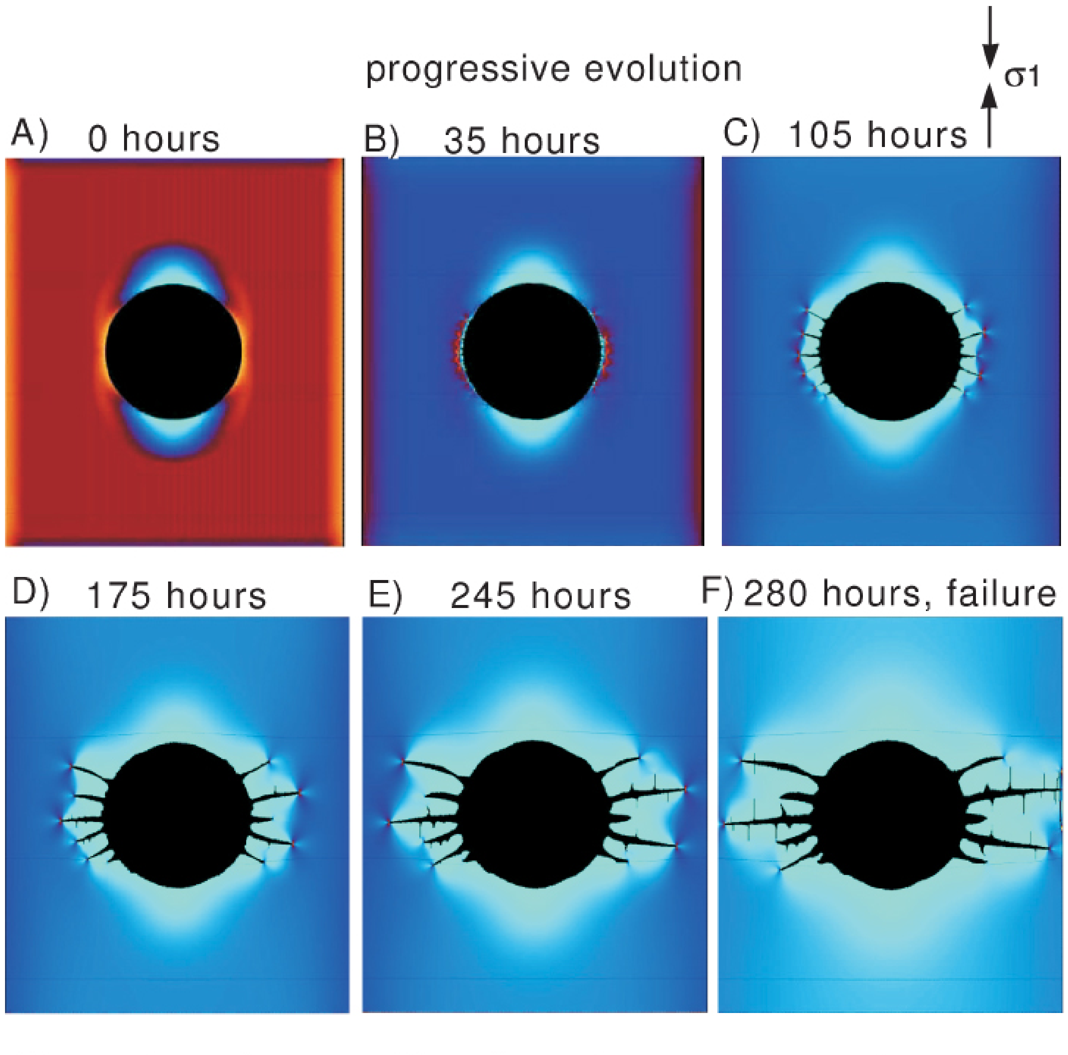


Comparison with other experiments and simulations





den Brok et al, Geoph. Res. Lett. 2001: stress dissolution of a salt crystal with trapped water.



Koehn et al, American Journal of Science 2003: simulations of similar processes, using a discrete model for elasticity coupled with pressure-dissolution laws

Quantification

A mathematical analysis of the initial roughness development confirms this intuitive analysis:

$z(x)$: solid-fluid interface, limit $|\partial_x z| \ll 1$

Stress in the solid: $\boldsymbol{\sigma} = \boldsymbol{\sigma}^0 + \boldsymbol{\sigma}^1$, stress imposed at infinity plus perturbation due to non planar interface.

$\sigma_s = |\sigma^0_{zz}| - |\sigma^0_{xx}| > 0$ for the stable case, < 0 otherwise.

Mechanical equilibrium: $\sigma^0_{zz} = p$, force perturbation on solid $\boldsymbol{\sigma}^1 \cdot \hat{n} = \sigma_s \partial_x z \hat{x}$.

Using Elastostatic's Green tensor, and integrating along the third direction (plane strain, translational invariance), comes for
$u_e = [(1+\nu)\sigma_{ij}\sigma_{ij} - \nu\sigma_{kk}\sigma_{ll}]/4E = u^0_e + u^1_e$:

$$u_e^0 = \alpha p_0^2/E \quad (1)$$

$$u_e^1 = -\beta(p_0\sigma_s/E)\int dy(\partial_y z)/(x-y) \quad (2)$$

with an average solid pressure $p_0 = -(2\sigma^0_{xx} + \sigma^0_{zz})/3$, and two dimensionless positive constants $\alpha = [9(1-2\nu) + 2(1+\nu)\sigma_s^2/p_0^2]/12$ and $\beta = \nu(1-2\nu)/\pi$, where E is an effective Young's modulus, and ν the Poisson coefficient.

Surface dynamic equation (dimensionless):

$$\partial_t z(x,t) = v_0 + \partial_{xx} z - \frac{\ell}{L^*}\int dy \frac{\partial_y z}{x-y} \quad (3)$$

where $L^* = \gamma E/(\beta p_0 \sigma_s)$, ℓ: unit length, $\tau = \ell^2 RT/(\gamma k \Omega^2)$: time unit.

An elementary bump disappears for $\sigma_s > 0$, or grows for $\sigma_s < 0$

Where are then the stylolites? Rocks are disordered

Assume a small disorder in the implied quantities (e.g. Young Modulus), quenched in the material properties of the rock: heterogeneity associated with micrometric grains, typically $\ell = 10\mu$m. Interface normal to largest stress direction (stabilizing elastic interactions)

By perturbation to first order, in the ref frame of the homogeneously moving average front, $z' = z - v_0 t$, the surface growth equation becomes

$$\partial_t z'(x,t) = \partial_{xx} z - \frac{\ell}{L^*} \int dy \frac{\partial_y z}{x-y} + \eta(x, z(x)) \tag{4}$$

with a quenched random term
$\eta(x, z'(x)) = [\alpha \ell p_0 / (\beta L^* \sigma_s)] \cdot [(\delta E/E) + (\delta k/k) - (\delta\alpha/\alpha)]$

Two first terms are stabilizing, only the quenched disorder destabilizes the interface

Small scale limit of the equations

for scales $l \ll L^*$, elastic interactions can be neglected and this reduces to a Laplacian regime:

$$\partial_t z'(x,t) = \partial_{xx} z' + \eta(x, z'(x))$$

modified Edwards Wilkinson equation (Proc Roy Soc A 1982) with a quenched noise.

This leads to self-affine surfaces of roughness $\zeta \sim 1.2$ (Roux and Hansen, J Physique I, 1994): agreement with experiments ($\zeta \sim 1.1$)

Large scale limit of the equations

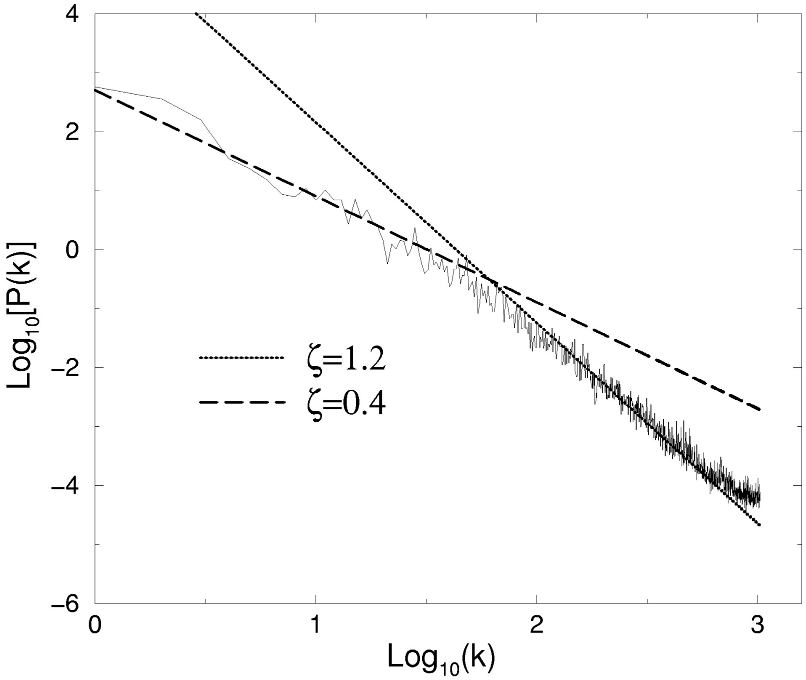
for scales $l \gg L^*$, surface tension can be neglected and this reduces to a mechanical regime:

$$\partial_t z'(x,t) = -\frac{\ell}{L^*}\int dy \frac{\partial_y z}{x-y} + \eta(x, z'(x))$$

Known model for elastic line on a disordered pinning landscape, or for mode I fracture front in a disordered solid.

This leads to self affine surfaces of roughness $\zeta \simeq 0.4$ (Tanguy, Gounelle and Roux, Phys Rev E 1998)

Simulation of the dynamic equation with both stabilizing terms and quenched noise



The Fourier power spectrum displays the expected small and large scale self-affine characteristics (1D discrete simulation: J. Schmittbuhl)

Quantification of the prefactors and geological relevance

In addition to these mappings, the characteristic units are known as function of the rock properties.

The cross over scale $L^* = \gamma E/(\beta p_0 \sigma_s)$ is function of the pressure during formation, through p_0 and σ_s.

Determining the cross over L^* at lab allows to determine such stress value during formation, and consequently depth of the rock during stylolite formation. Assuming as an order of magnitude $p_0 \sim \sigma_s$ and characteristics values for limestone elastic properties and water calcite reaction rates, $L^* \sim 1mm$ leads to a typical depth of 1 km.

Stylolites can thus be considered as **fossils of the stress magnitude**.

In addition, the dynamic behavior of these models is known, and the prefactor associated with the dynamics can be evaluated through the above from rock material properties. Estimate of time to saturation at observation scale is around a few hundred years,: stylolite roughness is always at saturation value for a geologist.

Conclusions and perspectives

- Stylolites are self affine surfaces, with two different self affine characteristics:
- large scale roughness exponent $\zeta \sim 0.5$
- small scale roughness exponents $\zeta \sim 1.1$
- Model: Solid fluid interface, normal to largest stress direction
- maps onto EW with quenched noise at small scale,
- or onto problem elastic line on disordered substrate at large scale
- cross over scale allows for an estimate of the stress and depth during formation: stress fossils
- further experiments and models of stylolite formation, to test this argument, should include quenched disorder in the solids.