GEOPHYSICAL RESEARCH LETTERS, VOL. 28, NO. 3, PAGES 435-438, FEBRUARY 1, 2001

Periodic Loading On A Creeping Fault: Implications For Tides

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Abstract. We study the effect of time varying normal and shear stress perturbations on a creeping fault and in which inertia is neglected. When interpreted in terms of earthquake triggering by earth tides, our results suggest that tidal triggering may exist but be very difficult to detect. We use a spring-block model with a rate-and state dependent friction law, loaded at constant velocity, and in which inertia is neglected. When a periodic stress is applied, a resonance exists which can destabilize sliding (i.e. earthquakes). However, when frictional parameters slightly vary along the fault, the observed phase lag between the response of the fault and the perturbating stresses close to resonance shows a broad and uncorrelated scattering.

It is now well accepted that faults are not subjected to spatially uniform stress. Correlations between Coulomb stress changes induced by previous earthquakes and aftershocks locations have been widely reported in the literature [*King et al.*, 1994]. These spatial variations in stress are accompanied by temporal changes of the state of stress on the fault. The latter may occur because of seismic waves generated by surrounding earthquakes [*Gomberg et al.*, 1998] and also by periodic (and non-transient) changes of stress. The most important periodic fluctuations of the stresses acting on a fault are due to earth tides (created by the gravitational attraction of the sun and the moon) and these variations are of order 0.001 to 0.004 MPa [*Vidale et al.*, 1998]. Water level changes in reservoirs also create periodic variations in stress but the time scales are of the order of a year (seasonal fluctuations).

Periodic loading in shear as well as in normal stress may highly influence failure. Considering periodic fluctuations of the normal stress in a spring-block model, *Perfettini* [2000] showed that such perturbations can destabilize sliding. In this case, the fault enters a stick-slip regime, and earthquake-like events are observed. This effect is enhanced close to the stability boundary, i.e. when the stiffness of the spring is close to the critical stiffness k_c, and when the period of the normal stress oscillations is close to a period T_c. Both k_c and T_c are defined further in the text. The model proposed by *Perfettini* [2000] is extended here to periodic perturbations of the normal as well as shear load, leading to qualitatively similar observations. These results suggest that tides may promote rupture providing that the effective stiffness k of the fault is close to k_c and that T_c is close to the period T of the external perturbations.

Paper number 2000GL011686.
0094-8276/01/2000GL011686$05.00

Perfettini [2000] showed that typical laboratory values of frictional parameters (i.e., a, b and L appearing in (1) and (2)) are compatible with T_c of the order of 12 hours, a value which is close to the diurnal tidal period.

The key problem is the detection of such a triggering effect among a population of faults. Following the work of *Dieterich* [1987] and *Dieterich* [1994], we consider a seismically active volume of the Earth. We assume this volume to contain a large number of faults, or patches, on which frictional properties slightly varying around a mean value. These patches could be part of the same major fault or be different isolated faults. Among them, some have started to (or will soon) nucleate an earthquake meaning that their effective stiffness is close to k_c. Assuming that a periodic stress of period T is applied to this volume, one may expect some of these patches to have frictional parameters compatible with $T = T_c$ and therefore to be good candidates for tidal triggering.

The evolution of friction on a fault is often described by rate-and state constitutive laws [*Dieterich*, 1979; *Ruina*, 1983], which were derived from laboratory experiments. These laws propose that friction τ is proportional to normal stress σ and depends on velocity as well as prior slip history: $\tau = \sigma\,\mu(\psi, V)$ where V is the sliding velocity and ψ is the so-called state variable which is introduced to describe the state of the sliding surface and its evolution with slip. The coefficient of friction μ is found to be described by [*Ruina*, 1983]:

$$\mu = \mu_0 + a\,\ln(V/V_*) + \psi \qquad (1)$$

where μ_0 and a are experimentally determined constants and V_* is a normalizing velocity. The variable ψ is proportional to the real area of contact associated with time-dependent creep [*Linker and Dieterich*, 1992]. For the evolution of the state variable, we use $d\psi/dt = F(\psi, V)$. *Ruina* [1983] proposed different expressions for F and in this work we will mainly use (herein called the Ruina, or slip, law):

$$F(\psi, V) = -(V/L)\,[\psi + b\,\ln(V/V_*)] \qquad (2)$$

where b and L are two constants determined experimentally. Laboratory experiments at variable normal stress [*Linker and Dieterich*, 1992; *Richardson and Marone*, 1998] shows that the slip law is usually in better agreement with the data than the commonly used ageing law. For a comparison between these two laws at variable normal stress, see *Perfettini* [2000].

We model a fault or a patch as a rigid block in uniform contact connected to a spring of stiffness k which represents the surrounding elastic media. The load point is moving at constant velocity V_0 to simulate tectonic loading. We neglect the effect of inertia (this assumption will be discussed below). When rate-and state frictional effects are considered, it has been shown that for constant normal stress σ_0 (see for

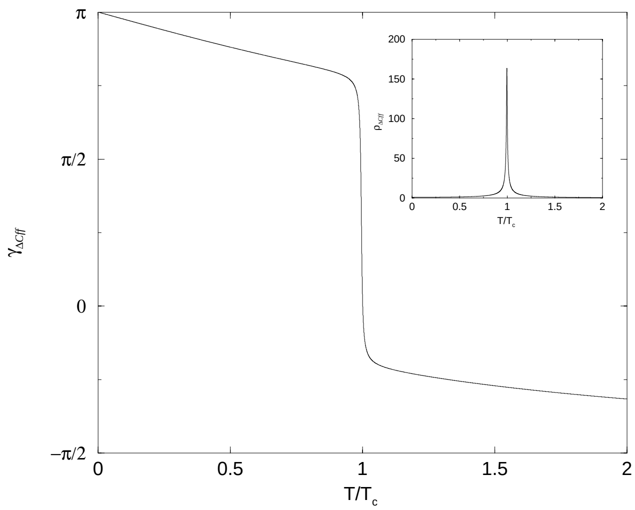

Figure 1. Phase lag $\gamma_{\Delta Cff}$ between Coulomb stress change ΔCff and external perturbation as a function of the period of the forcing. Parameters are $a = 0.005$, $b - a = 0.003$, $V_0 = 10^{-9}$ m.s^{-1}, $k = 1.01\, k_c$, $\bar{\tau} = 0$ and $\bar{\sigma} = 10^{-4}\,\sigma_0$. In the upper left corner is displayed the normalized modulus of ΔCff for the same set of parameters. The presence of a sharp peak illustrates the resonance phenomenon.

example [*Rice and Ruina*, 1983]) the slider is stable when the stiffness exceeds a a critical stiffness $k_c = \sigma_0\,(b-a)/L$ and is unstable otherwise. At the critical stiffness $k = k_c$, the spring-slider system shows stable oscillations with period $T_c = 2\,\pi\,\sqrt{a/(b-a)}\,(L/V_0)$. Stable sliding corresponds to creeping faults while unstable sliding, often called stick-slip behavior, is related to seismically active faults.

In our model, it is crucial to estimate accurately the stiffness of a fault. For two elastic half-spaces and neglecting dynamical effects, shear stress is related to slip discontinuity by $k = G\,\Sigma/\lambda$, with $\Sigma = \pi/(1-\nu)$ for in-plane slip and $\Sigma = \pi$ for anti-plane slip [*Cochard and Rice*, 1997], where G is the elastic shear modulus, ν the Poisson ration, and λ the wavelength. For a slipping patch of length h, the stiffness k in an elastic continuum is $k(h) = G\,\Sigma/h$. Using a similar model, *Dieterich* [1987] described the nucleation process as the growth of a patch of zero initial length (infinite stiffness), embedded in a homogeneous medium with 'adjoining portions of the fault locked prior to the onset of instability'. This *quasi-static* nucleation stops when the stiffness of the fault reaches the critical stiffness k_c. That occurs when the size of the patch reaches h_* defined by $k(h_*) = k_c$ leading to $h_* = G\,\Sigma/k_c$ [*Dieterich*, 1986; *Rice*, 1993]. Then the *dynamic* nucleation starts, leading to a dynamic event or earthquake. Our model applies to a quasi-static stable regime and our results may be applied either to creeping faults or to patches where *quasi-static* nucleation is underway (before *dynamic* nucleation).

In order to study the influence of periodic loading, we add to the loading stress as well as the normal stress a small periodic component. With this condition, the equations of motions are [*Perfettini et al.*, 2000]:

$$\left.\begin{array}{c} \dot{u} = V_0 - V,\ \dot{\psi} = F(\psi, V) \\ k\,u + \Delta\tau(t) = \sigma(t)\,\mu(\psi, V) \end{array}\right\} \qquad (3)$$

where $u(t)$ is the relative displacement of the slider compared to the loading point and $\sigma(t) = \sigma_0 + \Delta\sigma(t)$. $\Delta\tau(t)$ and $\Delta\sigma(t)$ are periodic perturbations in the loading shear or normal stress respectively. We assume that the system is in steady-state (all the derivatives in (3) are set to zero) prior to perturbations and that $\bar{\tau} = |\Delta\tau(t)|$ is much smaller that the initial steady-state stress τ_0 (the same hold for the normal stress, i.e. $\bar{\sigma} = |\Delta\sigma(t)| \ll \sigma_0$). As in *Perfettini* [2000], we solve the linearized equations of the problem with solutions of the form $x = x_0 + \mathrm{Im}[x_1\,\exp(i\,\omega\,t)]$ where x is either relative displacement u, state variable ψ, friction τ or velocity V.

To quantify the influence of the perturbing stresses (normal as well as shear) on rupture, we introduce a Coulomb failure function Cff defined as:

$$Cff(t) = \tau(t) - \mu_{ss}\,\sigma(t) - \tau_c \qquad (4)$$

where τ_c is the cohesion stress (considered constant). μ_{ss} is the coefficient of friction in steady-state given by (1) for $V = V_0$ and $\psi = \psi_{ss}$, where ψ_{ss} is defined by $F(\psi_{ss}, V_0) = 0$. The variation of Cff named ΔCff is:

$$\Delta Cff = [k\,u_1 + \Delta\tau(t)] - \mu_{ss}\,\Delta\sigma(t) \qquad (5)$$

We introduce the dimensionless frequency $q = \omega\,L/V_0 = 2\,\pi\,L/(V_0\,T)$. At $T = T_c$, we get $q_c = \sqrt{(b-a)/a}$. Using the linearized version of (3) together with (5), we find that the amplitude of Coulomb fluctuations is:

$$\left.\begin{array}{c} |\Delta Cff| = |\frac{\hat{q}\,(\bar{\sigma}\,\mu_{ss} - \bar{\tau})}{\hat{k}}| \times \\ \sqrt{\frac{\hat{q}^2 + q_c^2}{(1-\hat{q}^2/\hat{k})^2 + q^2\,(1-1/\hat{k})^2}} \end{array}\right\} \qquad (6)$$

where we have introduced $\hat{q} = q/q_c$ and $\hat{k} = k/k_c$. For periodic perturbations of the normal stress, *Perfettini* [2000] showed that the slider-block system exhibits a resonance phenomenon for stiffness close but still greater than the critical stiffness k_c. As the period of perturbations T varies, the change in frictional stress exhibits a peak response. Figure 1 shows the phase lag $\gamma_{\Delta Cff}$ and modulus of ΔCff as a function of the normalized period of excitation for a stiffness of

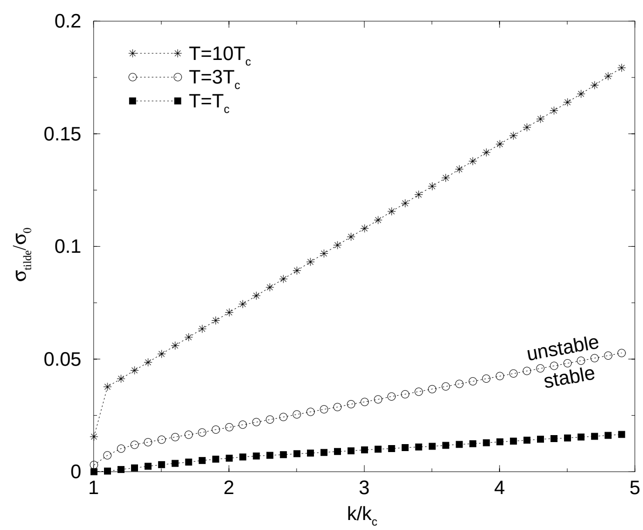

Figure 2. Stability curves of the quasi-static solutions as a function of stiffness for T=1, 3 and 10 T_c. Parameters are identical as in Figure 1. This figure shows the high sensitivity of stability to the period of the forcing term.

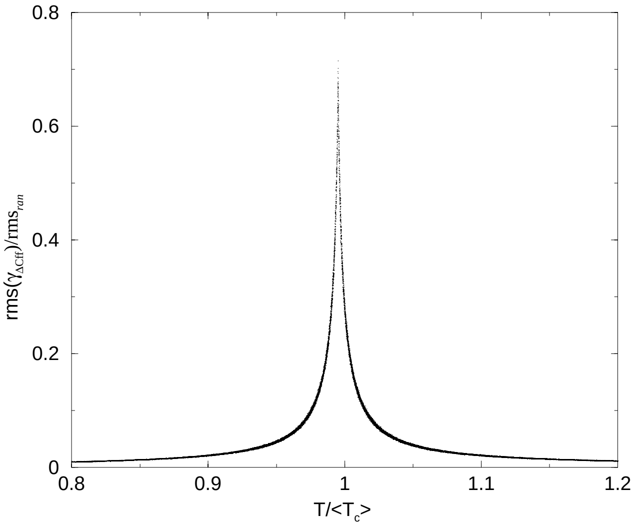

Figure 3. Normalized rms of the phase angle $\gamma_{\Delta Cff}$ as a function of normalized period $T/<T_c>$ using $n = 1000$ points. The difference $b-a$ was varied according to a Gaussian probability distribution with 0.003 mean and a rms of 1 % of this mean. Stiffness k was set to $1.01\,k_c$ (in order to have a significant response of the Coulomb stress fluctuations $|\Delta Cff|$). L is equal to $2\,\mu$m and a to 0.005, values consistent with laboratory measurements. V_0 was assigned to 10^{-9} m.s^{-1} which is the order of magnitude of plate velocities. The average normal stress σ_0 was set to 100 MPa, which is an average value of the lithostatic pressure at a 4 km depth and was weakly varied with amplitude $\bar{\sigma} = 10^{-4}\,\sigma_0$. For periods close to $<T_c>$, the curve exhibits a sharp peak.

$k = 1.01\,k_c$, a normal stress perturbation $\bar{\sigma} = 10^{-4}\,\sigma_0$ and no perturbation of shear stress ($\bar{\tau} = 0$). A resonance is observed for $T = T_c$ and the phase lag exhibits a step around this period. It is of interest to note that for $q = q_c$ and $k = k_c$, the denominator of (6) is nil, and the response of the system becomes unbounded. In that case, the quasi-static approximation ceases to be valid and the description of the system should be extended and account for inertial effects. If the sliding patch was in the late stage of *quasi-static* nucleation ($h \simeq h_*$ or equivalently $k \simeq k_c$), one may expect this resonance phenomenon to be significant and enhance the likehood of failure.

The stability of the quasi-static solutions is influenced by the amplitude of external perturbations and is studied numerically. We limit our presentation of stability results to perturbations of the normal stress but similar quantitative conclusions are reached if both variations (shear and normal stresses) are accounted for. The quasi-static oscillatory solutions derived earlier are numerically stable when perturbations are small enough. If the system is initialized to such solutions, the observed response is purely sinusoidal with a period equal to the period of the perturbation, but with a magnitude that strongly depends on the period of the perturbation. As $\bar{\sigma}$ is increased, the non-linear terms neglected in the analytical derivation become important, and can lead in some cases (high enough $\bar{\sigma}$) to instability which ensues by an infinite velocity (inertia is not considered in this model and therefore nothing prevents velocity from becoming unbounded) and more generally, to a divergence of all the variables describing the system. For this reason, a divergence of V leads to an infinite value of ΔCff. In the numerical runs, instability is supposed to occur when the sliding velocity V exceeds by a factor of 2 the maximum velocity inferred from our quasi-static derivation. Figure 2 displays three stability curves delimiting the stable (below) and unstable (above) domain in the ($\bar{\sigma}/\sigma_0, k/k_c$) plane for three values of the period of perturbation T (T =1, 3 and 10 T_c). The stability boundary is shown to be highly sensitive to normal stress variations.

The easiest way to measure and detect the possible tidal triggering is to study the phase lag between earthquakes occurrence and tidal loading peaks. *Tsuruoka* [1995] studied this feature on a catalog of 998 earthquakes with magnitude greater than 6. He observed no correlation for strike-slip and thrust-type earthquakes occurrence but a correlation was noticed for normal-fault type earthquakes. Including a larger number of events (13,042 earthquakes), *Vidale* [1998] discussed tidal triggering by resolving the normal, shear and Coulomb stress on earthquake fault planes of his catalog. They found a weak (2 %) correlation which was not considered as significant.

In order to test phase shift sensitivity to frictional parameters, the difference $b-a$ was varied using a Gaussian probability with a mean of $<b-a>= 0.003$ and a root mean square (rms) of 1 %. We define $\mathrm{rms}_{ran} = \pi/\sqrt{3}$ which corresponds to the rms of an uniform probability density distributed over an interval of $2\,\pi$. Figure 3 shows the rms of the phase angle of the Coulomb failure function, $\mathrm{rms}(\gamma_{\Delta Cff})$ normalized by rms_{ran} as a function of the normalized period $T/<T_c>$ of the perturbating stress. A value of 1 means that the variable is randomly distributed. The rms is negligeable as long as T is far from $<T_c>$. For $T \simeq <T_c>$, the rms is maximum and can lead to values higher than 0.7 rms_{ran}. This feature is due to the jump of the phase lag around T_c mentioned earlier. Dispersion of the modulus of ΔCff also exhibits a sharp peak around $<T_c>$. For the set of parameters used in figure 3, we find $\mathrm{rms}(|\Delta Cff|) \simeq 3 < |\Delta Cff| >$ while $\mathrm{rms}(\gamma_{\Delta Cff}) \simeq 23 < \gamma_{\Delta Cff} >$.

Using an initial population of patches leading to a constant rate of seismicity, each of them obeying equation (3) with $\bar{\sigma} = 0$, *Dieterich* [1987] studied the variation of the seismicity due to periodic perturbations of the loading stress. He found that the maximum change R_a of instability rates over the loading cycle, verify $R_a = 2\,\bar{\tau}/a$ and is independent of the period of the perturbation. This last result is in contradiction with the recent work of *Lockner* [1999] who studied on rock samples the influence of periodic fluctuations of the load (normal as well as shear) on the occurrence of stick-slip events. Their results show that triggering depends on the amplitude according to the results of *Dieterich* [1987] but also on the frequency of perturbations of the loading stresses. This is in agreement with our model which predict a high sensitivity of the triggering effect to the period of the external perturbations of stresses.

The absence of observed tidal triggering is consistent with our results: For periods close to T_c and stiffnesses close to k_c, fluctuations in the frictional parameters of only 1 % can lead to a high dispersion of the observed phase angle. If a global phase angle was measured for such a statistical population of patches (by measuring the time delay between earthquake occurrence and the previous peak in tidal stress), the uncertainties (or dispersion) would make the results unreliable. An equivalent conclusion would be reached if one was to look at successive events on the same fault (as in the characteristic earthquake point of view) since there is no reason for frictional properties on a real fault not to vary slightly with time.

In conclusion, the influence of normal and shear stress perturbations on a creeping patch have been studied using a spring-slider model subjected to rate-and state- dependent friction laws. As in *Perfettini* [2000], a resonance exists as the period of the perturbations T is varied. At critical stiffness k_c and for $T = T_c$, the model predicts an unbounded response of the slider. This suggests that nucleating faults (k close to k_c), when submitted to periodic perturbations in stresses applied at periods close to T_c, may be brought towards failure. Examination of a Coulomb failure function confirms this conclusion. However, for such faults, variations in the frictional parameters (here fluctuations of the difference $b-a$) lead to high variability in the observed phase angle. Our description may explain the absence of observed correlation between earthquakes occurrence and tides even though tidal triggering of earthquakes is possible.

Acknowledgments. We are grateful to Jim Rice and Raúl Madariaga for their generous help.

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(Received April 13, 2000; revised September 15, 2000; accepted September 28, 2000.)