

**Schmittbuhl, Hansen, and Batrouni Reply:** Alava and Zapperi (AZ) [1] question whether the fracture fronts we observe in [2] are self-affine. We use Family-Vicsek scaling to determine the two scaling exponents  $\alpha$  and  $z$ . AZ claim that this is not enough to determine whether the front is self-affine and they go on to point to the presence of overhangs as their evidence of fractality rather than self-affinity. In Fig. 1, we show details of *experimental* fracture fronts from Fig. 1 of [3]. There are significant overhangs, but these experimental fronts are self-affine. In fact, as long as one allows a damage cloud to develop, overhangs are unavoidable. Family-Vicsek scaling implies nonisotropic scaling: Scaling of the front width  $w$  with the width of the system  $L_x$  is different from that with the length of the system  $L_y$ . Such nonisotropic scaling is the defining property of self-affine surfaces.

We also point out that Ramanathan and Fisher [4] measured numerically  $\nu = 1.52 \pm 0.02$  using a very different fracture model, which by construction cannot produce fractal fracture lines. This is in complete agreement with our model, which gave  $\nu = 1.54$ . Their fracture fronts are also self-affine but with a very different roughness exponent.

AZ claim that the source of the  $L_x$  dependence of the damage length scale  $l_y$ , Eq. (9) in [2], comes from the rescaling of the model's elastic constants when changing  $L_x$ . However, this rescaling is necessary to ensure that the elastic *properties* remain unchanged when the system size is changed under uniform loading conditions. It is caused by the nonlocal nature of the problem introduced by the Green function  $\bar{G}_{i,j}$ , Eq. (4) in [2]. Under uniform loading condition, Eq. (5) in [2] reads  $u_i = u = \sum_j \bar{G}_{i,j} f_j = \sum_j \bar{G}_{i,j} b^2 \sigma$ . Here  $b$  is the lattice constant. If  $u$ , local deformation, and  $\sigma$ , local stress, are to be independent of size,  $L_x$  and  $L_y$ , we must have that

$$\sum_j \bar{G}_{i,j} = \text{constant}. \quad (1)$$

Since  $\bar{G}_{i,j} = \bar{G}_{i-j}$ , we may for aesthetic reasons write  $\sum_j \bar{G}_{i,j} = \text{constant} = \sum_{i,j} \bar{G}_{i,j} / (L_x \times L_y)$ , as there is no

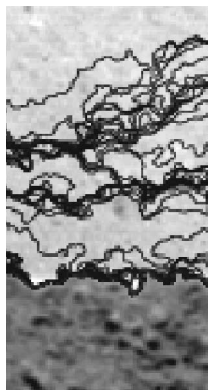


FIG. 1. Closeup of experimental fracture fronts from [3].

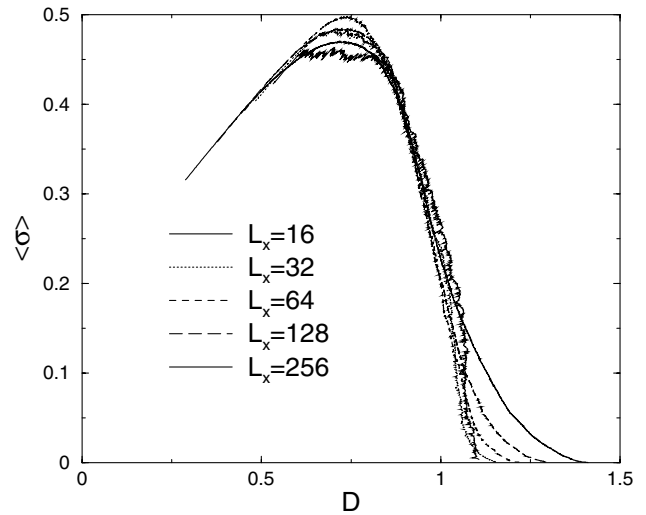


FIG. 2. Average stress  $\langle \sigma \rangle$  as a function of imposed displacement  $D$  from virgin system to complete failure. The rescaling of  $\bar{G}_{i,j}$ , Eq. (1) and of the threshold distribution ensures that the curves collapse for different system lengths  $L_x$  and fixed  $L_y = 128$ .

dependency on the index  $i$  in Eq. (1). This was the way we chose to present it in [2]. We point out again that both indices in the double sum  $\sum_{i,j}$  run over  $L_x \times L_y$  sites. We demonstrate the correctness of the rescaling in Fig. 2, where we show the collapse of the loading curves obtained for different system sizes after using Eq. (1). Only when the elastic properties have been rescaled as just described, may one proceed to use finite-size scaling as done in [2].

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