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Development of Roughness in Crack Propagation.

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Abstract. – We report on the statistical analysis of experimental measurements of the crack roughness for a fracture initiated from a straight notch in a granite sample. The crack surface is shown to develop a self-affine character over a correlation length ξ which depends on the distance to the notch y as a power law $\xi \propto y^{\alpha}$ characterized by a dynamic exponent $1/\alpha$ estimated to be 1.2 ± 0.15. The self-affine roughness exponent below the correlation length is found to be $0.80 \pm$ ± 0.05. The scaling is obtained from two maps of the surface, consisting each in more than 10^5 data points.

Starting from the pioneer work of Mandelbrot *et al.* [1], there has been numerous works focusing on a statistical characterization of the roughness of cracks (see ref. [2-4] for reviews). Scale invariance has been evidenced in a number of cases [1-20]. It is now firmly established that the topography of crack surfaces can be described as self-affine. Many different materials have been investigated, with different fracture behaviours from ductile to brittle, at very different scales from nanometre scale using atomic force or scanning tunnelling microscopy [5, 6], micrometre to centimetre scale using profilometry measurements on a variety of materials [7-9], image analysis techniques [1, 10-13] or other techniques [14], metre [15, 16] and up to kilometre scale [17] for geological faults.

The initial hope in measuring the surface topography was to relate this geometrical information to mechanical properties (such as toughness [1]) or to material characterization. However, some technical difficulties are related to the determination of the roughness exponent of a self-affine surface, and some often used tools—in particular those developed for isotropic fractals—may lead to biased measurements [21]. A critical analysis of the early published data, as well as more recent experimental data analysis, shows that in many cases the range of reported values for the roughness exponents was of extent comparable to real uncertainty. Subsequently, the roughness exponent was proposed to be universal [10, 7]. Although some measurements are in conflict with this conjecture, the amount of data which supports it is so large that the question is more to identify the parameters which eventually limit the validity of this law than to falsify it. In three dimensions, this exponent ζ amounts to $\zeta_{3d} \approx 0.80 \pm 0.05$ [7, 4], whereas for two-dimensional systems it assumes a lower value $\zeta_{2d} \approx \approx 0.66 \pm 0.05$ [9, 13, 20]. In addition, a comparable scaling has been extended to branched cracks. [22,23]

While experimental and numerical results seem to converge to the above results, there is at present a very poor understanding of this property in spite of its broad range of validity. In two dimensions, an analogy with perfect plasticity has been proposed which accounts for the value of the roughness exponent, $\zeta_{2d} = 2/3$, through a mapping [24] between the latter problem and the conformation of directed polymers in a random medium at zero temperature [25]. However, the three-dimensional generalization of this argument to minimal surfaces gives an exponent about 0.5 or less [26] which is much too small compared to the observed value. An interesting approach has been proposed recently by Bouchaud *et al.* [23]. The crack surface is considered as the trace of the crack front whose propagation is modelled by a non-linear Langevin equation. The latter, constructed on the basis of respecting all symmetries of the system, has to be identical to the one originally proposed by Ertaş and Kardar [27] for vortex line motion in superconductors. However, in spite of a rich phase diagram, most regions of the phase space give a roughness exponent which is somewhat smaller than the measured value. Moreover, in many cases, the surface roughness is not isotropic in the mean surface plane.

Many other models, in particular for interfacial growth, have been studied recently [4, 28] and a general scaling law for the development of the roughness has been established. Such a law is also relevant for both previously mentioned models: perfect plasticity and vortex lines. A selected set of articles focusing on this property has been recently collected and published [29]. The aim of this paper is to go beyond the characterization of fully developed roughness, and to analyse the development of the roughness from a border where a straight notch imposes a zero roughness. We will show that the fracture roughness obeys a scaling law comparable to the ones of the above-mentioned growth models.

This property leads naturally to the introduction of a correlation length ξ which governs the largest scale over which correlations have developed parallel to the initial border. More precisely, below the scale ξ, the surface appears as self-affine with an exponent ζ. Above this scale, the height can be considered as uncorrelated, with the same statistical distribution. The observed scaling provides a power law relation between the correlation length and the distance to the notch.

The system on which the analysis is performed is a parallelipedic block of granite of size 25 cm × 25 cm × 12 cm. Two straight notches were first sawn on two opposite faces of the sample using a diamond saw. The one where the fracture is initiated is 2 mm deep with a triangular profile. The opposite one is shallow and its role is simply to direct the crack in the late stage of the crack propagation. The fracture has then been produced by the impact of a massive steel blade along the notch of initiation producing essentially a single crack. No fragments were observed to detach from the main crack. From direct observation of the crack surface it was not possible to distinguish the direction of propagation.

The analysis of the fracture roughness is based on the measurement of the surface topography $z(x, y)$ using a mechanical profilometer. The latter consists of a 50 μm radius stylus which probes the surface vertically (along the z-axis). A contact is detected by an induction cell mounted on the support of the stylus. Displacements along x and y were imposed by two high-precision step motors monitored by a PC. The accuracy and reproducibility of the apparatus was tested by measuring the same profile after a long series of intermediate displacements. We estimated the accuracy to be of 2 μm along x, 5 μm along y and 5 μm along z.

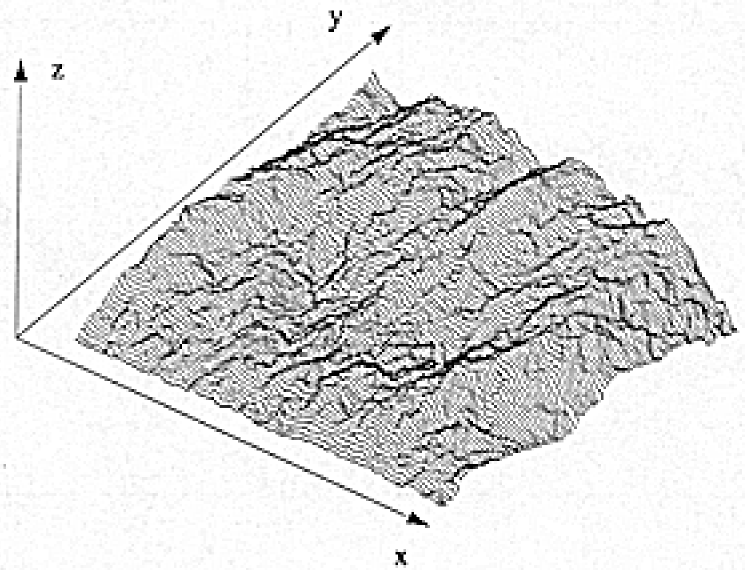

Fig. 1. – Map of the crack surface close to the notch. The notch was initially sawn along the x-axis close to $y = 0$, $z = 0$. As the distance to the notch y increases, the roughness develops.

Three maps of a hundred profiles parallel to the notch of initiation (parallel to x) were measured. One of them is shown in fig. 1. Two maps are built according to a regular grid. The spacing along the x-axis between the 1050 points is $\delta x = 50\ \mu\text{m}$. The distance between profiles (y-axis) is $y = 350\ \mu\text{m}$. So that a total of about 10^5 data points were collected to extract a map of the surface close to the notch. The total extent of the probed area was thus 5 cm × 4 cm, *i.e.* much smaller than the available area. The third map is sampled with a non-regular grid in order to focus on the area very close to the initiation of the fracture. The spacing follows an arithmetic series. The x step for this map is $\delta x = 80\ \mu\text{m}$ and the measured zone is 8 cm × 4 cm. For all maps the maximum distance of the profiles to the notch was chosen to be small enough so that the facing notch was not sensitive.

We follow the formalism applied to growth models [29] and define the roughness $\sigma(y, \Delta)$ over a window size Δ along x as

$$\sigma(y, \Delta)^2 = \left\langle \frac{1}{\Delta} \sum_{i=1}^{\Delta} z(x_i, y)^2 - \left(\frac{1}{\Delta} \sum_{i=1}^{\Delta} z(x_i, y) \right)^2 \right\rangle_j , \tag{1}$$

where the brackets $\langle \ldots \rangle$ stand for an average over the position of the window j. The expected behaviour is

$$\sigma(y, \Delta) = \Delta^{\zeta} \varphi(y/\Delta^{1/\alpha}), \tag{2}$$

where the scaling function assumes the following behaviours:

$$\begin{cases} \varphi(x) \sim x^{\zeta\alpha}, & \text{for } x \ll 1, \\ \varphi(x) \sim \text{const}, & \text{for } x \gg 1. \end{cases} \tag{3}$$

Thus at a large distance from the border, the roughness is only dependent on the window size as $\sigma \propto \Delta^{\zeta}$. This reveals a self-affine character of the crack surface with a roughness exponent ζ.

On the opposite, for a small distance y, the roughness no longer increases when the window size exceeds a correlation length $\xi \propto y^{\alpha}$. Therefore, for large window sizes the

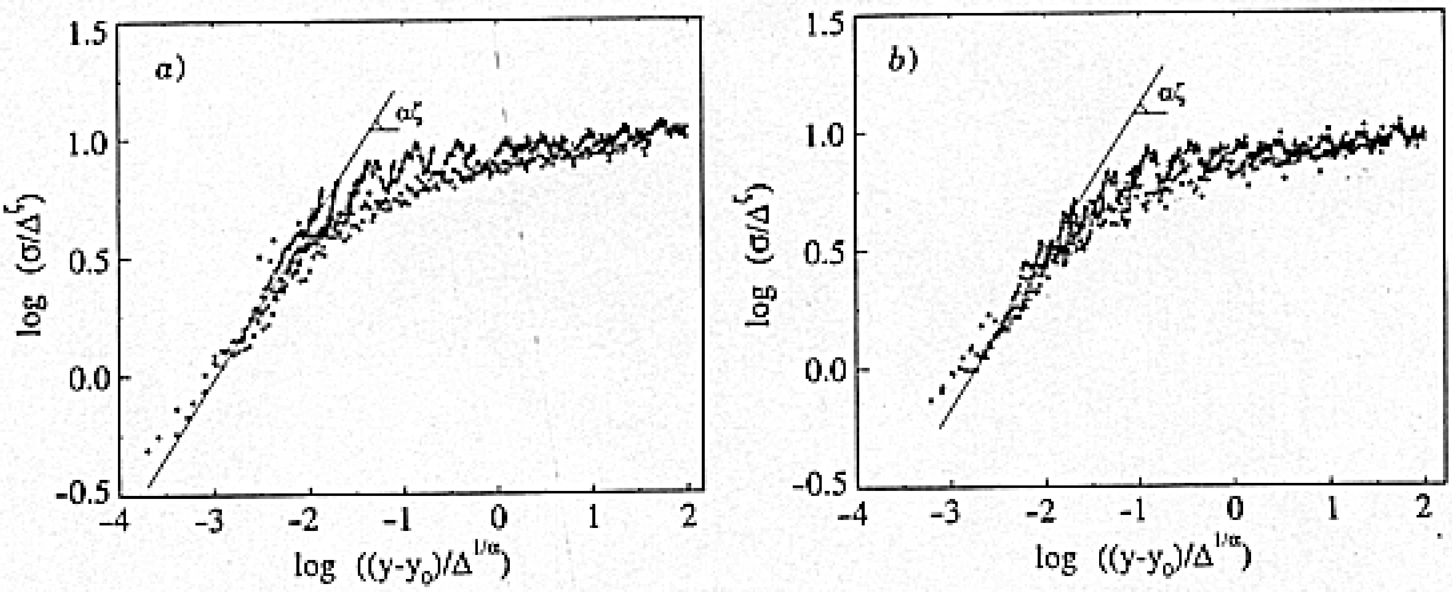

Fig. 2. – In fig. *a*) is plotted the data collapse of the roughness $\sigma(y, \Delta)/\Delta^{\zeta}$ over a window of size Δ along the x-axis *vs.* the rescaled border distance $y/\Delta^{1/\alpha}$ for various window sizes Δ. Figure *b*) is the same as *a*) for a different map.

roughness of the crack varies with y as

$$\sigma \propto y^{\zeta\alpha} . \tag{4}$$

In order to check the validity of the scaling form (2), we plotted $\sigma(y, \Delta)/\Delta^{\zeta}$ *vs.* $y/\Delta^{1/\alpha}$ as shown in fig. 2*a*) and *b*) for two different maps. Due to the difficulty of measuring profiles extremely close to the notch, and because of the roughness of the notch itself, we introduced as a free parameter the origin y_0 of the y-axis. However, this parameter was only varied in a small interval $\pm 3\,\delta x$ which was checked to be consistent with the experimental set-up. For clarity only a small subset of data is shown on each graph. In order to obtain a plot similar to fig. 2, we have to impose trial values of the exponents α and ζ. The latter exponent $\zeta = 0.80 \pm 0.05$ was measured independently of profilometry measurements—not reported here—performed at large distances from the boundaries. The α exponent as well as the offset y_0 were estimated by looking for the best data collapse. We also checked that treating ζ as an unknown parameter yields the same results.

Once the two exponents and the offset are fixed, as in fig. 2, the data collapse on the scaling function φ. We can thus check the scaling behaviour (3) self-consistently by fitting the φ evolution on both sides of the crossover point. We have drawn a power law of exponent $\alpha\zeta$ as a guide to the eye for small window sizes. For large arguments, the scaling function should behave like a constant. Both behaviours appear to be consistent with the data using the determined exponents.

This data collapse procedure was also tested blindly on a well-documented model which is known to display a similar property, the free-energy landscape of directed polymers (DP) (along y) attached to a straight line at $y = 0$. A similar amount of data was generated (100 profiles of 1024 points each), and α and ζ were measured using the same data collapse procedure. We measured $\alpha = 0.65$ and $\zeta = 0.45$, values which are consistent with the known exponents $\alpha = 2/3$ and $\zeta = 1/2$. A similar scatter of data was observed in the DP problem and the crack roughness.

We do not address here the question of the origin of this dynamical scaling. It is indeed possible that the velocity at which the crack front advances also affects the development of the crack roughness. For homogeneous materials such as PMMA or glass it has been reported [30, 31] that the surface structure depends on the crack velocity. Since the crack velocity depends on the crack location in the first acceleration phase, it cannot be excluded that the dynamical exponent we measure here combines a roughness-velocity dependence with the acceleration of the crack front. An analysis along these lines has been very recently proposed by Bouchaud and Navéos [32]. Nevertheless, in the two limit cases where the crack velocity can be considered as constant, *i.e.* in a quasi-static crack propagation, or in a very fast fracture where the crack velocity saturates to a constant value, one expects that the roughness depends on the distance to the notch. Therefore, presumably in the most general case a combination of the crack position and the crack velocity should govern the geometrical structure of the fracture surface.

From the analysis of a crack surface starting from a straight notch, we have obtained for the first time a scaling law for describing the growth of roughness of the crack surface which indicates the existence of a correlation length with growth as a power law of the distance to the notch. The dynamic exponent which governs this growth is $1/\alpha \approx 1.2 \pm 0.15$.

This new result constitutes an additional property which should be quantitatively recovered in models which attempt to describe the crack propagation, *e.g.* through a Langevin approach. In the approach proposed by Bouchaud *et al.* [23] the development of roughness is qualitatively similar to the experimentally determined scaling form, but the value of the dynamic exponent is large (3/2) compared to our result. It would be very interesting to test the above results on other materials, other scales, and other fracture modes so as to probe its universality which is expected to be comparable to that of the ζ exponent, and to investigate the role of the crack velocity.

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