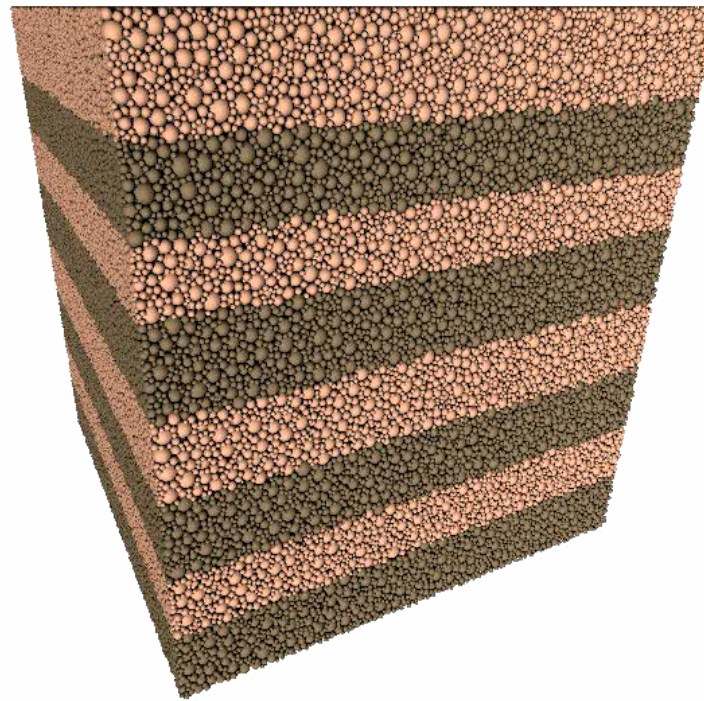


Numerical investigations on the mechanical response of fractured rocks



D. Weatherley

28 June 2012

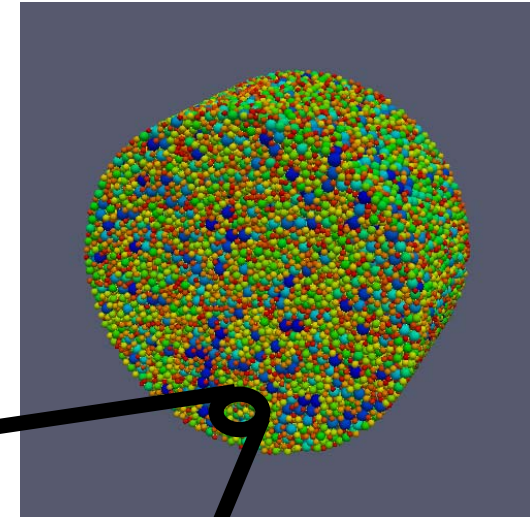
Outline

- Overview of Discrete Element Method
- Calibration of DEM rock fracture model
- Validation studies
- Method to model pre-existing fractures in DEM
- Mechanical response of pre-fractured rocks
- (Other DEM rock mechanics applications)

Discrete Element Method

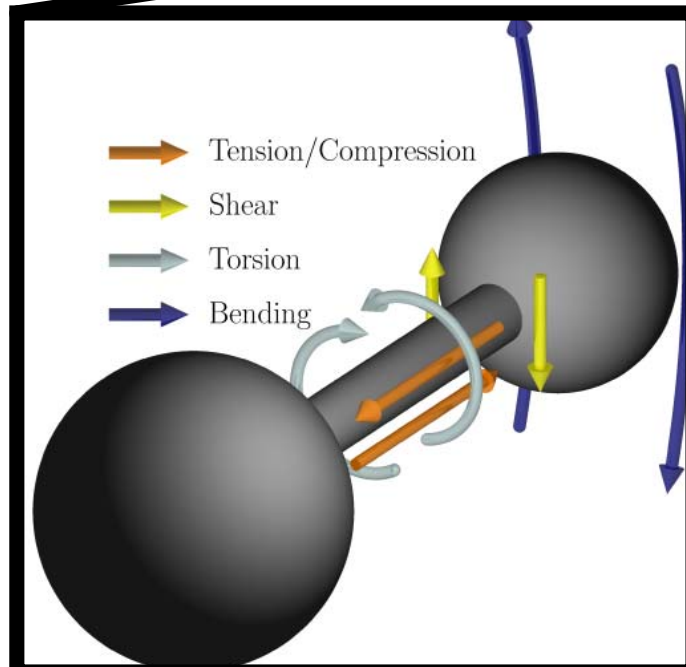


Materials may be represented as an assembly of indivisible spherical particles.



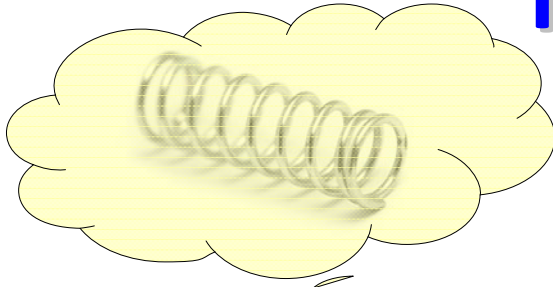
Having computed the net force acting on a particle, its future velocity and position may be calculated.
-- Newton's 2nd Law

-  Tension/Compression
-  Shear
-  Torsion
-  Bending



Particles undergo forces arising from interactions with nearby particles.

Bonded particle interactions for rock mechanics



Neighbouring particles connected by cylindrical linear elastic beams

Four model parameters govern micro-scale mechanical response:

- Elastic Modulus, E_b
- Poisson's ratio, ν_b
- Cohesive strength, C
- Internal friction angle, ϕ_b

Mohr-Coulomb failure criterion governs when bonds break

$$F_{ij}^N = K_{ij}^N \Delta U_{ij}^N \quad F_{ij}^S = K_{ij}^S \Delta U_{ij}^S$$

$$M_{ij}^B = K_{ij}^B \Delta \theta_{ij}^B \quad M_{ij}^T = K_{ij}^T \Delta \theta_{ij}^T$$

$$K_{ij}^N = \frac{\pi}{2} E_b (R_i + R_j)$$

$$K_{ij}^S = \frac{\pi E_b}{4(1 + \nu_b)} (R_i + R_j)$$

$$K_{ij}^B = \frac{\pi}{8} E_b (R_i + R_j)^3$$

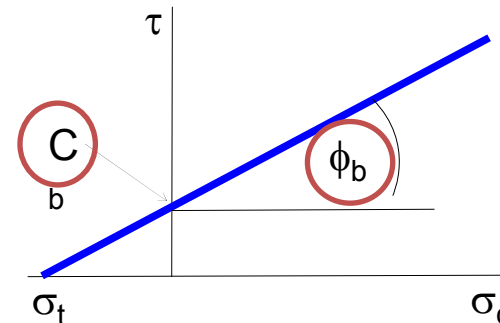
$$K_{ij}^T = \frac{\pi E_b}{8(1 + \nu_b)} (R_i + R_j)^3$$

Bonds break if:

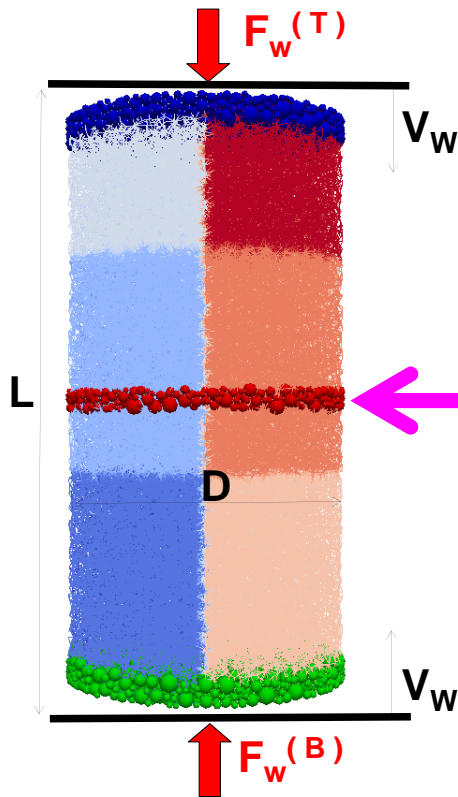
$$\tau^{ij} \geq C_b - \sigma_N^{ij} \tan \phi_b$$

$$\sigma_N^{ij} = \frac{F_{ij}^N}{A_{ij}} + \frac{|M_{ij}^B|}{I_{ij}} R_{ij}$$

$$\tau^{ij} = \frac{|F_{ij}^S|}{A_{ij}} + \frac{|M_{ij}^T|}{J_{ij}} R_{ij}$$



Calibration of model parameters



Numerical specimens are compressed between two planar walls moved at constant speed V_w

Diametric strain is measured by recording the displacement of particles in a ring encircling the sample
Axial stress is measured from the net particle-wall forces acting on each wall:

$$\sigma_a = \frac{2(F_W^{(T)} - F_W^{(B)})}{\pi D^2}$$

Axial strain is measured from the positions of the two walls:

$$\varepsilon_a = 1 - \frac{P_W^{(T)} - P_W^{(B)}}{L}$$

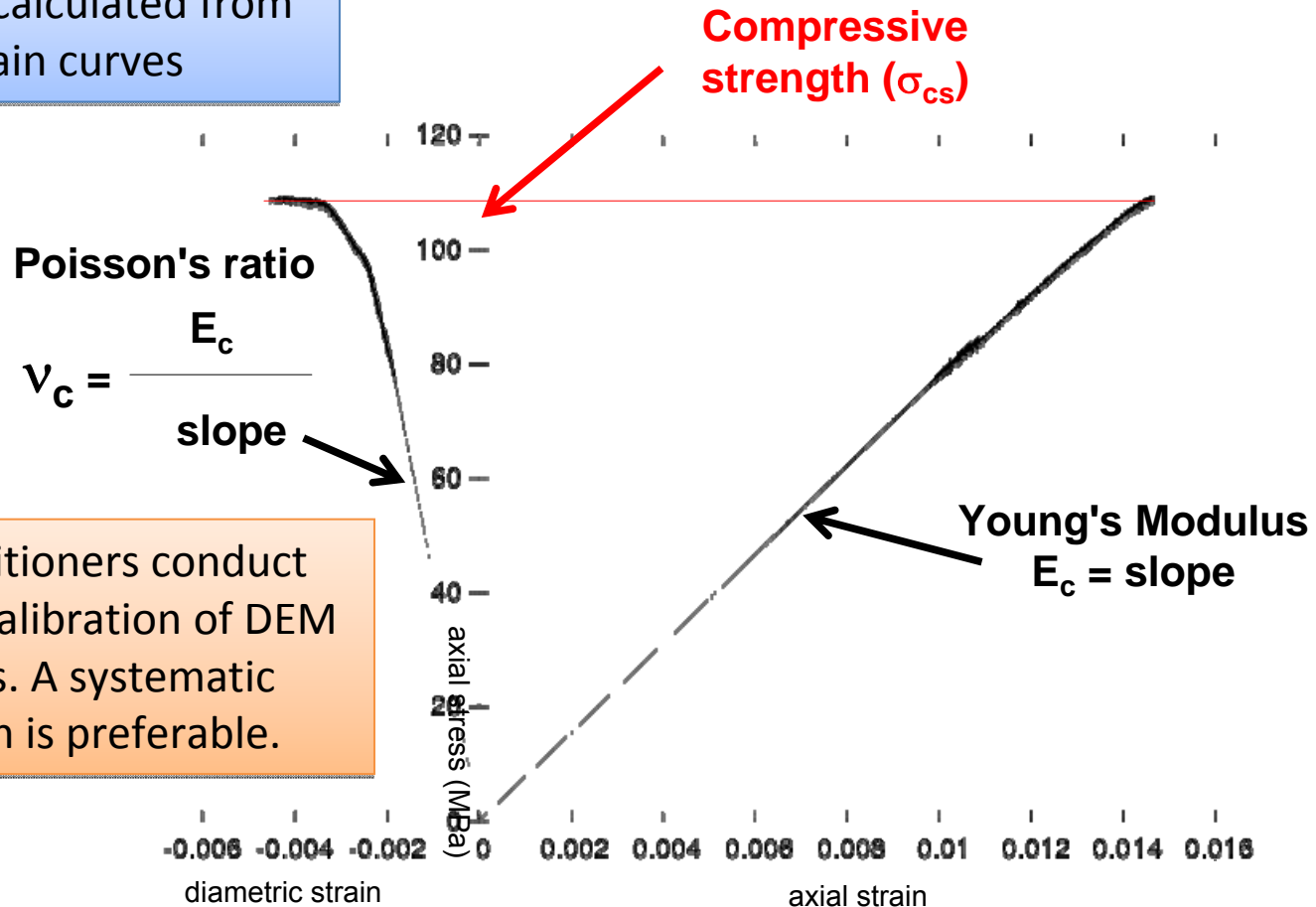
Model parameters to calibrate:

- Elastic modulus (E_b)
- Poisson's Ratio (ν_b)
- Cohesive strength (C_b)
- Internal friction angle (ϕ_b)



Calibration of model parameters

Material properties of numerical specimens are calculated from stress-strain curves



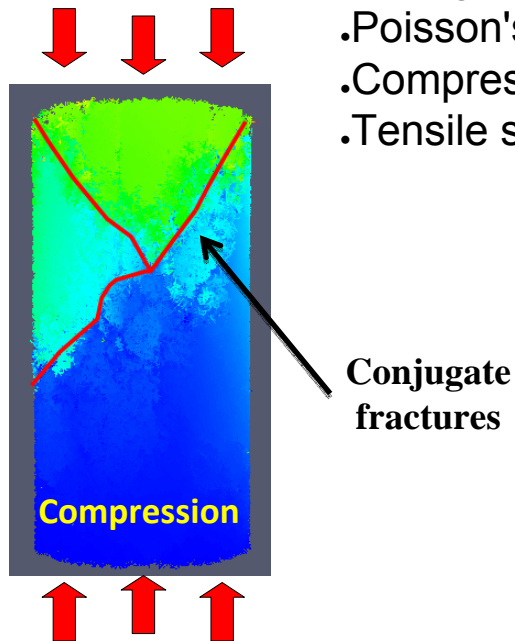
Most DEM practitioners conduct “trial-and-error” calibration of DEM rock specimens. A systematic parametrisation is preferable.

Systematic calibration procedure

1) A series of control experiments are conducted in which each of the **four model parameters are varied** independently

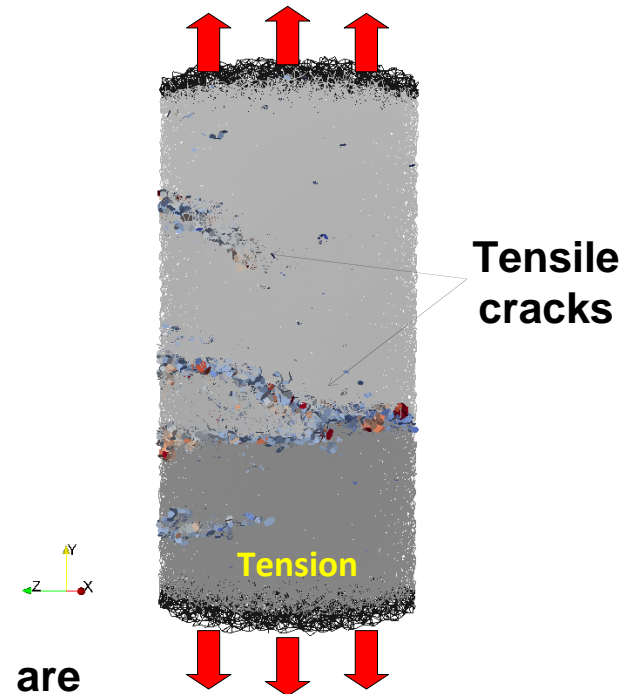
2) For each set of model parameters, both **Uniaxial Compression and Direct Tension tests** are conducted to measure macroscopic mechanical properties:

- .Young's modulus (comp./tension)
- .Poisson's ratio (comp./tension)
- .Compressive strength
- .Tensile strength

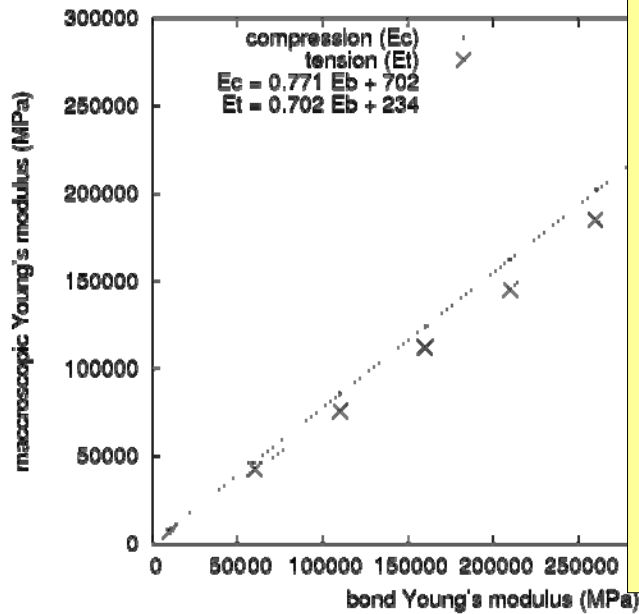


3) **Empirical relationships are obtained** relating mechanical properties to the four model parameters

4) Statistical variability of mechanical properties is investigated by repeating the experiments using specimens with **differing particle packing arrangements**



Young's modulus and Poisson's ratio



Calibration Relationships:

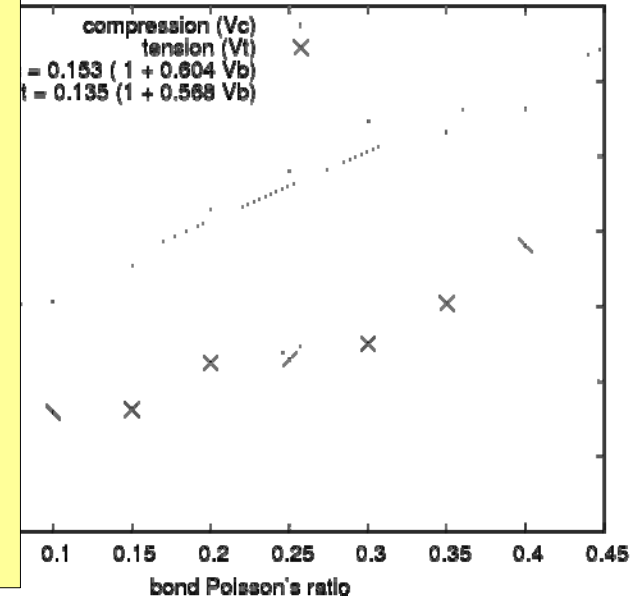
$$E_c \sim \kappa_1 E_b \left(1 - \frac{\nu_b}{4}\right)$$

Geometry dependent

$$\nu_c \sim \kappa_2 (1 + 0.6\nu_b)$$

constant?

$$\frac{E_c}{E_t} = \frac{\nu_c}{\nu_t} \sim \kappa_3$$



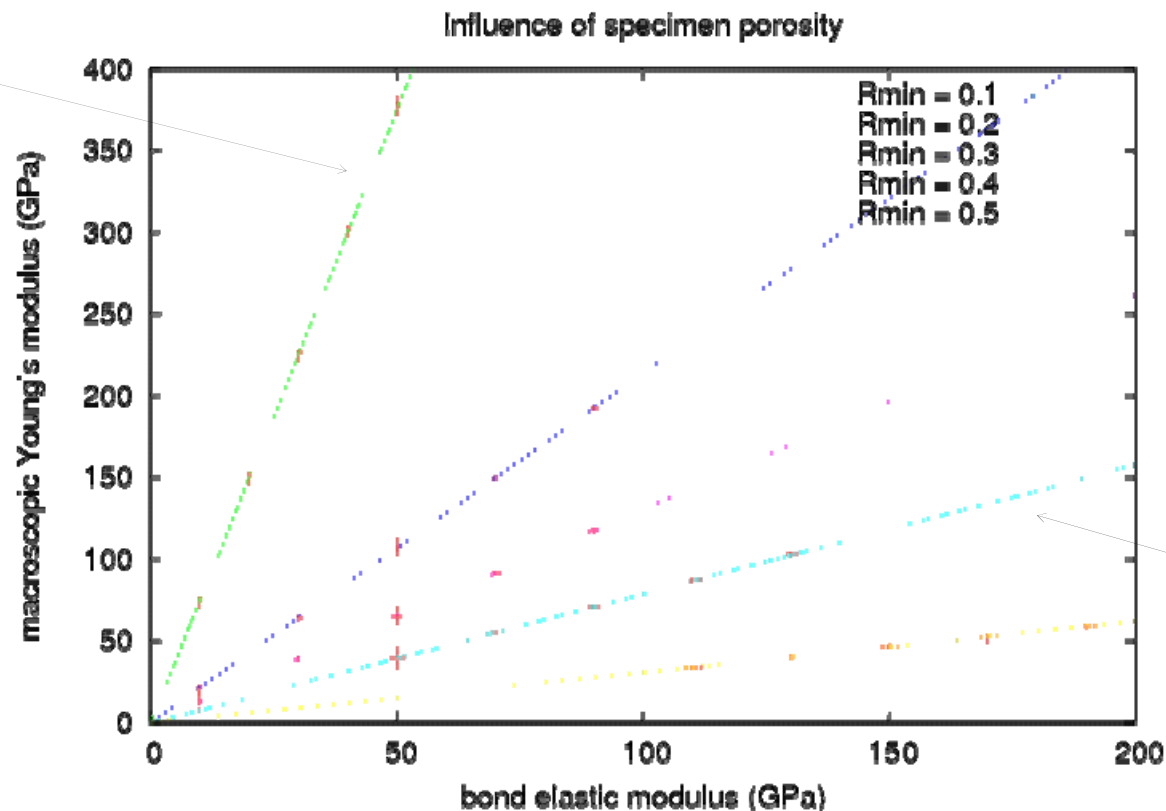
Both the compressive and tensile Young's moduli **increase linearly** as the bond modulus is increased.

The compressive Poisson's ratio is **10% larger** than the tensile ratio. Both **increase linearly** as the bond Poisson's ratio increases.

A similar approach yields calibration relationships for peak strength and tensile:compressive strength ratio as functions of the bond cohesion and bond friction angle.

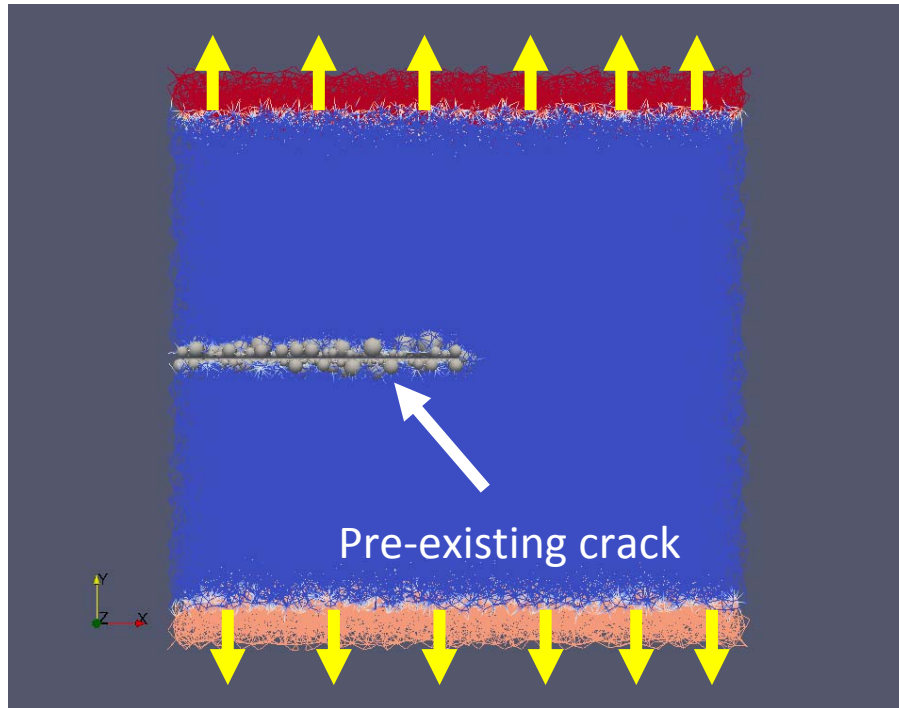
Geometrical “tuning” parameters

Decreasing the artificial porosity of specimens (by decreasing the minimum particle radius) increases the ratio of macroscopic Young's modulus to bond elastic modulus (likewise for σ_{cs} vs. C_b)



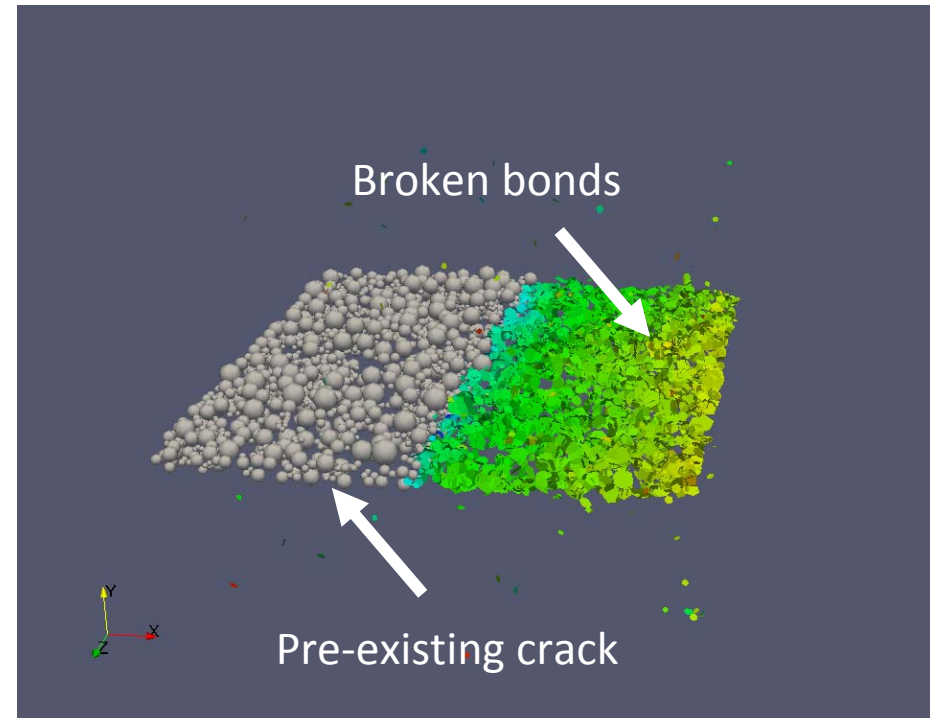
More detailed investigations have indicated that the statistical properties of the bond network govern the ratio between macroscopic and microscopic elastic moduli

Validation: Mode I crack propagation



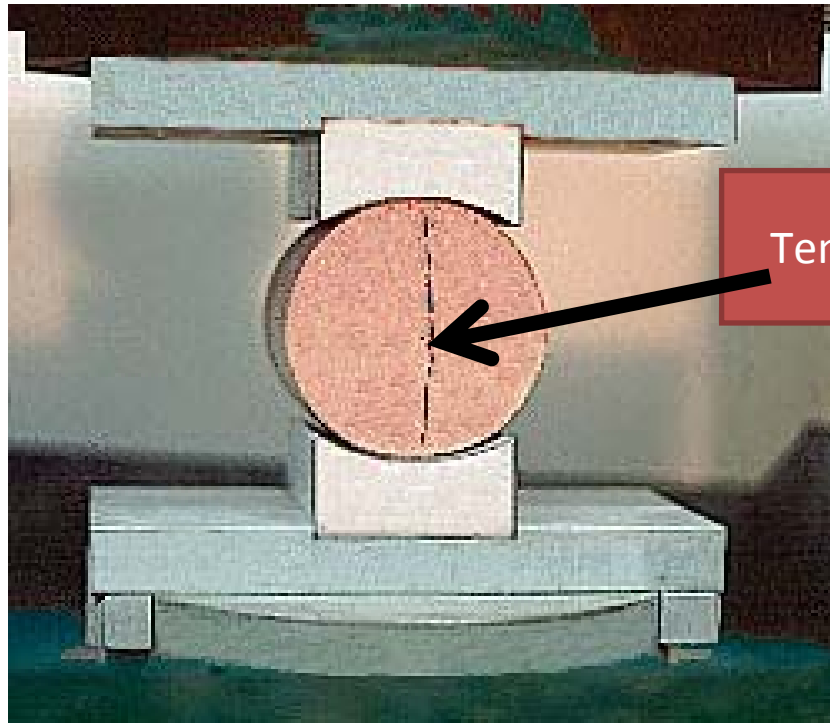
A cube of intact rock with a single pre-existing crack is extended at constant rate

Locations of broken bonds are consistent with predictions of Griffith crack theory: quasi-static propagation of a tensile crack parallel to the initial crack

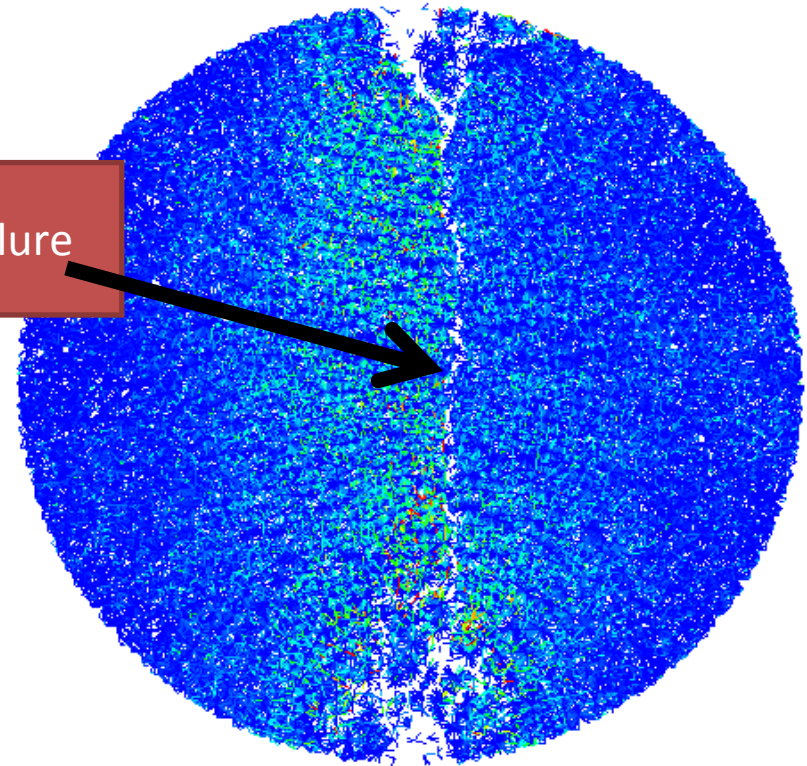


Validation: In-direct tension tests

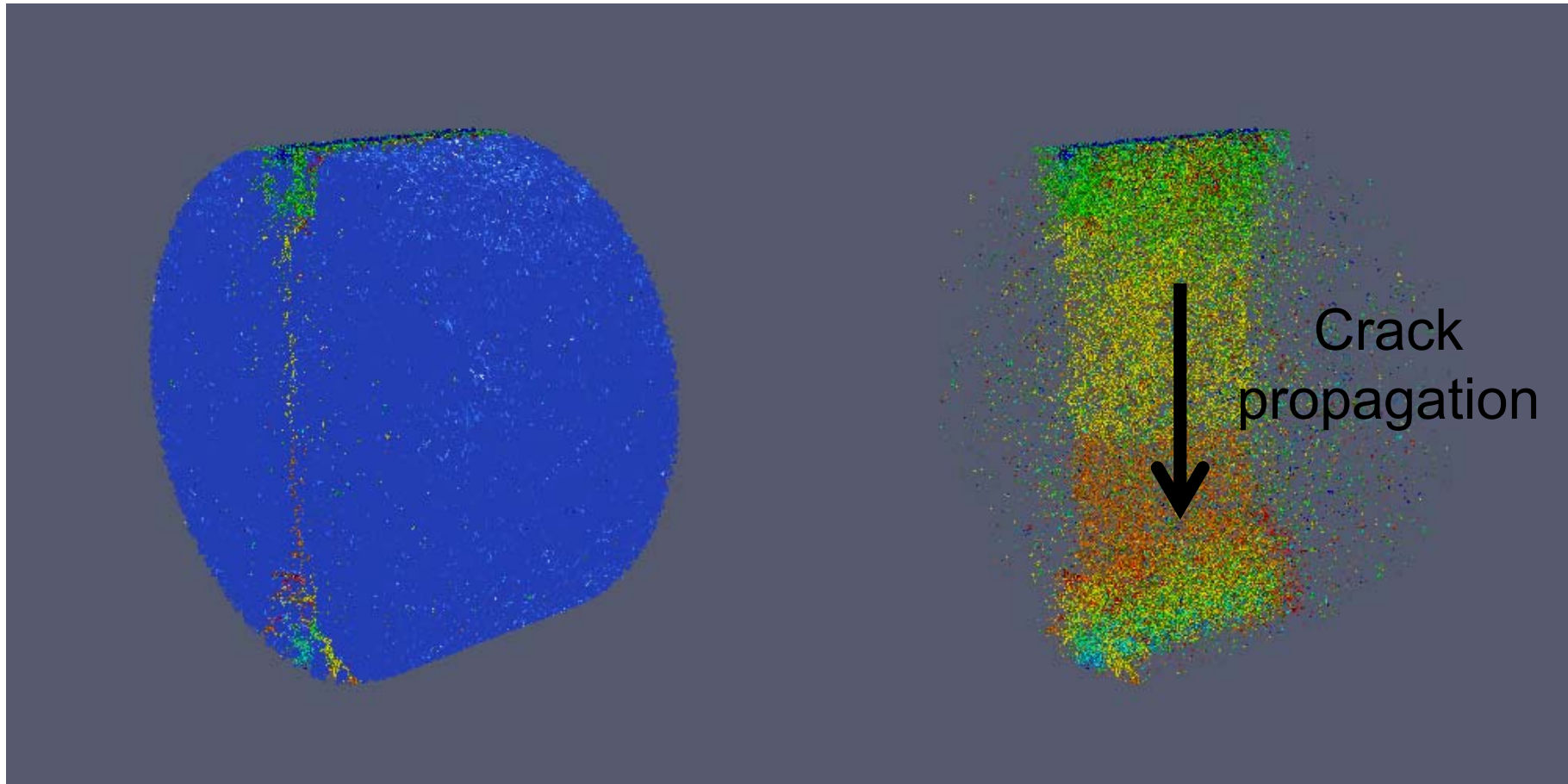
Having calibrated the DEM bonded particle model for rock, one must demonstrate it accurately predicts brittle failure under different geometrical and loading conditions.



Tensile Failure

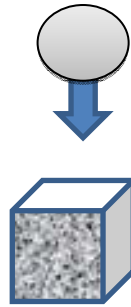


Validation: In-direct tension tests

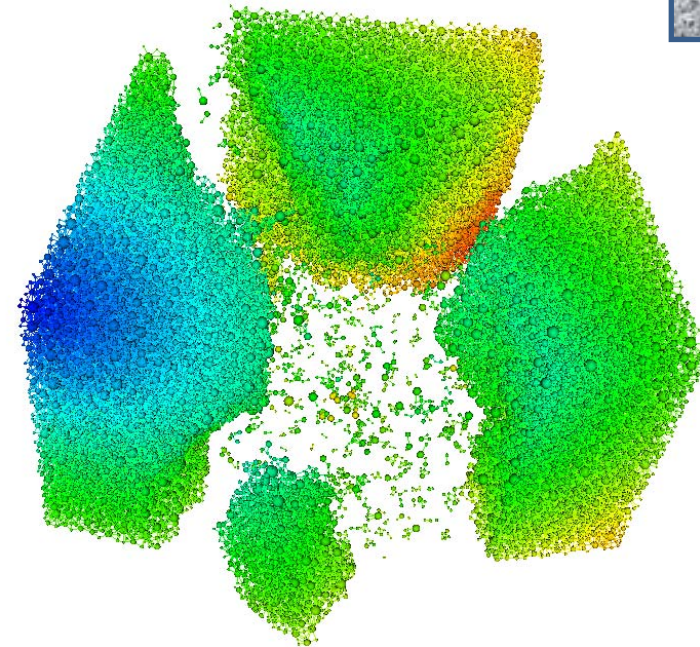


Crack propagation speed in Brazilian tests is approximately 850m/s, as expected from Griffith Crack Theory

Validation: Drop weight tests



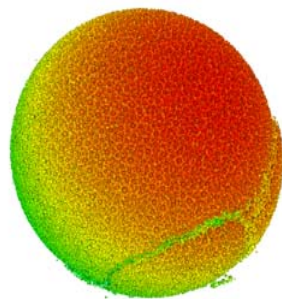
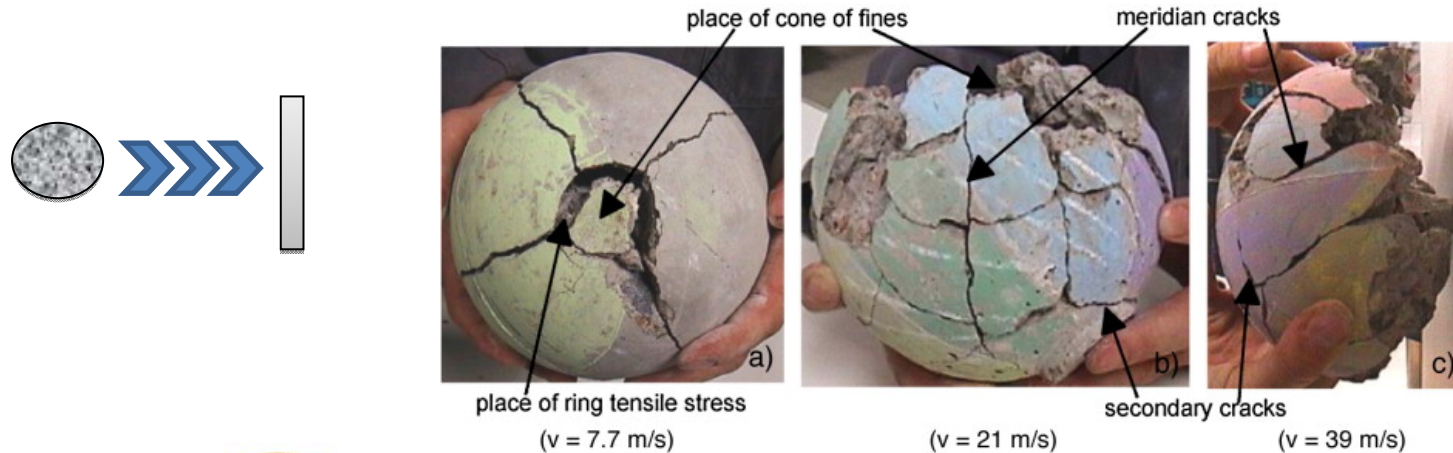
Courtesy: R. Chandramohan, JKMRC.



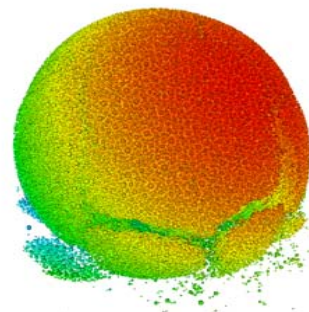
A cubic specimen is impacted with a steel ball dropped from variable heights. The DEM model reproduces the fracture patterns observed.

Validation: Ballistic impact tests

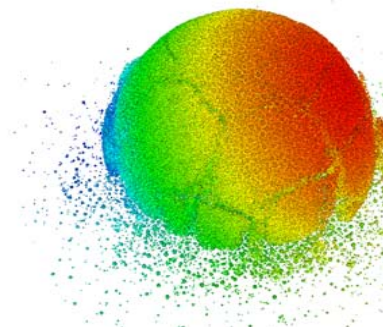
Khanal et al. (2004) Int. J. Min. Proc., 86 p.104



$V=14\text{m/s}$



$V=28\text{m/s}$



$V=56\text{m/s}$

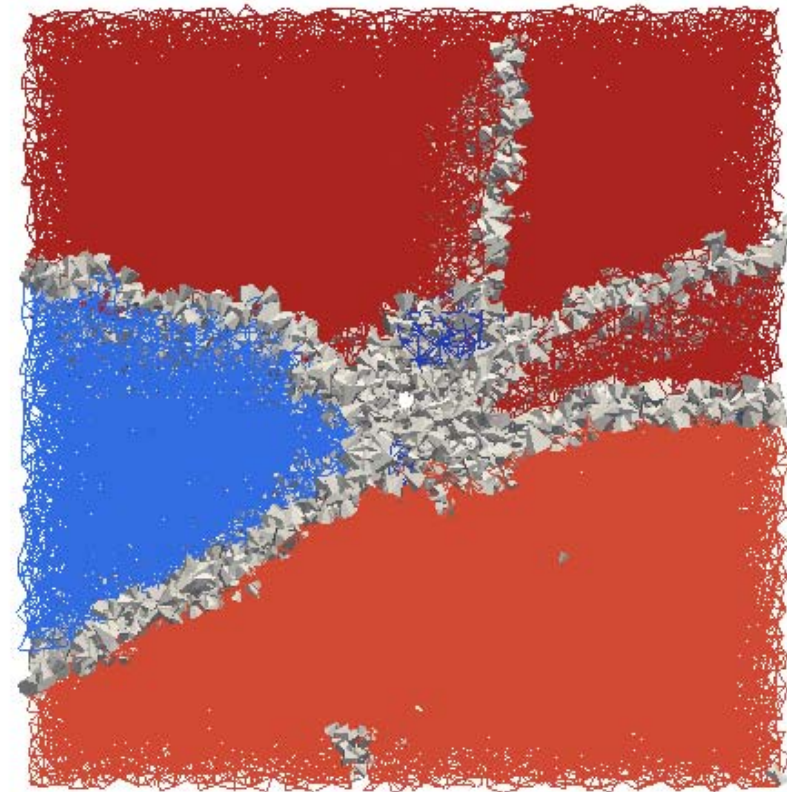
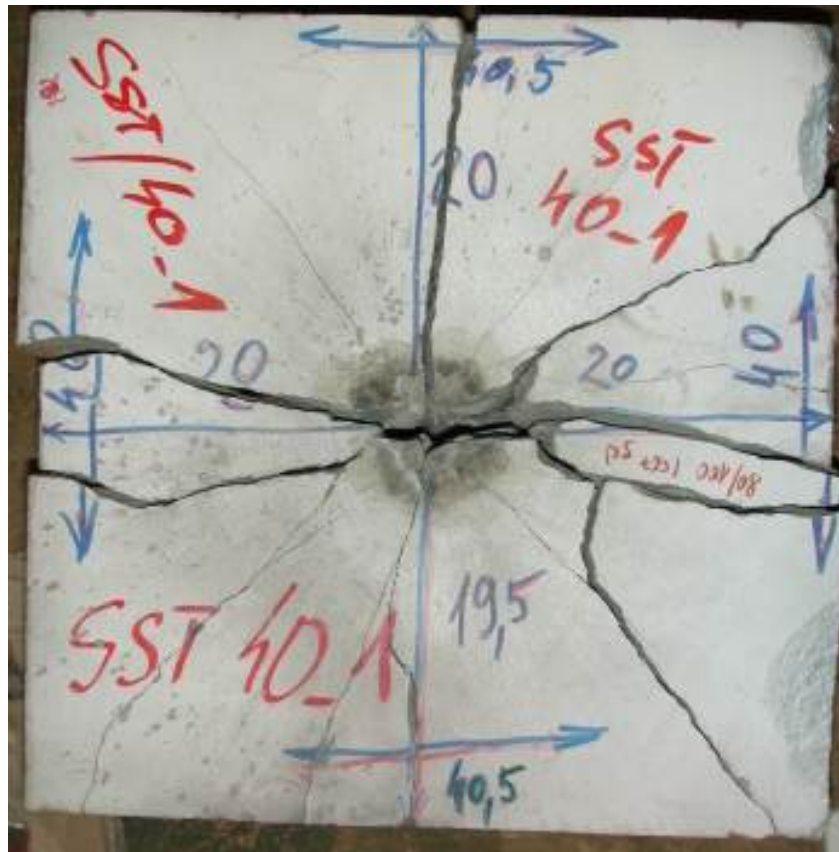


$V=70\text{m/s}$

Simulations of ballistic impact of a spherical specimen result in meridian cracks consistent with laboratory observations

Validation: Blast damage

Moser et al. (2003), Leoben, Austria.



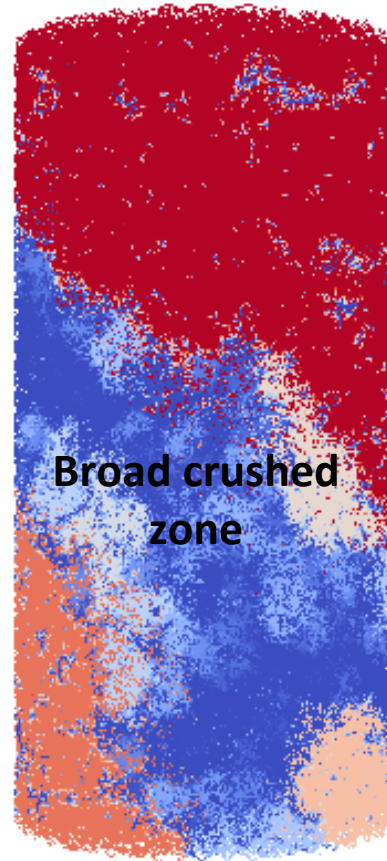
Detonation within a hole drilled centrally in a cubic specimen results in a central crushed zone with tensile cracks radiated outwards.

What about shear failure?

All of the validation results presented thus far are dominated by tensile failure of the DEM rock specimen

Uniaxial compression of cylindrical DEM rock specimens have realistic stress-strain curves however the failure patterns are not consistent with laboratory observations of shear failure.

Localisation of shear failure does not occur in the DEM rock model. Hypothesise that this is due to a lack of internal micro-structure.

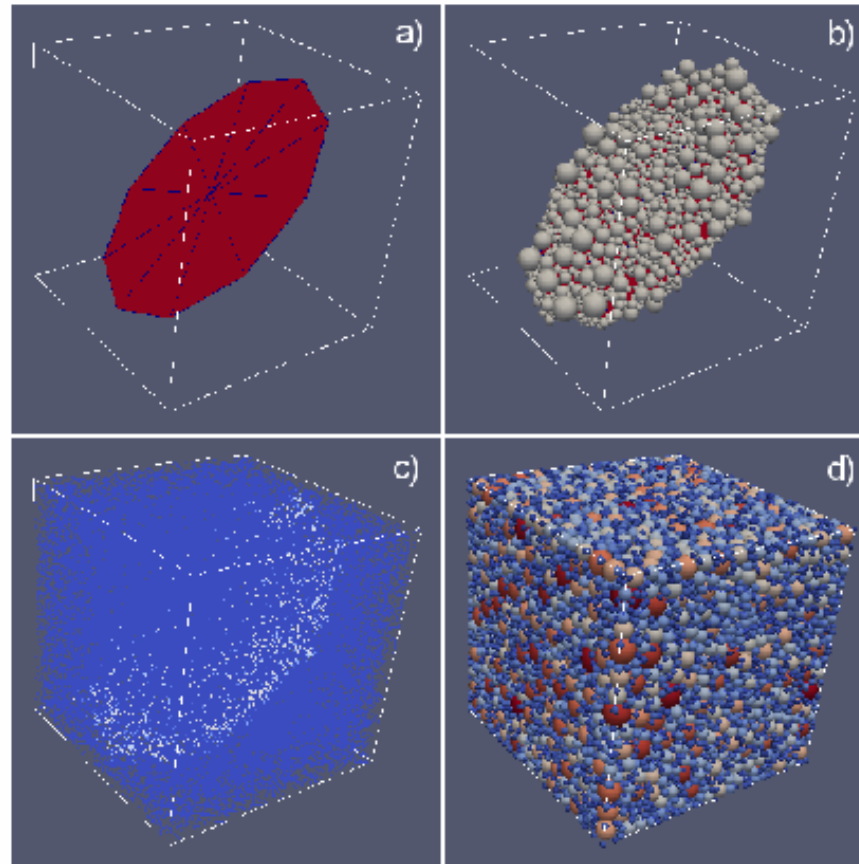


Fragments produced are delineated by different colours.



Locations of broken bonds

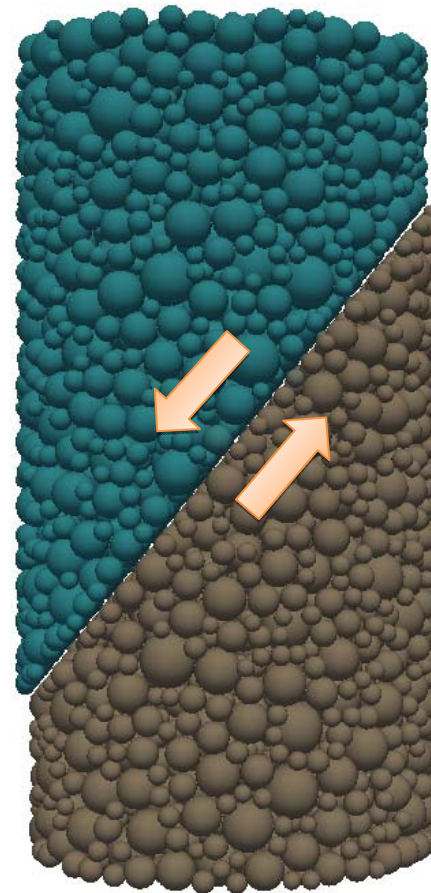
Modelling pre-fractured rocks



- a) A pre-existing crack is modeled as a triangulated surface
- b) Particles are packed either side of the crack(s)
- c) The surrounding volume is filled with particles
- d) Adjacent particles interact via brittle-elastic beam interactions

Validation: Stable sliding

A single through-going crack bisects a cylindrical rock specimen.

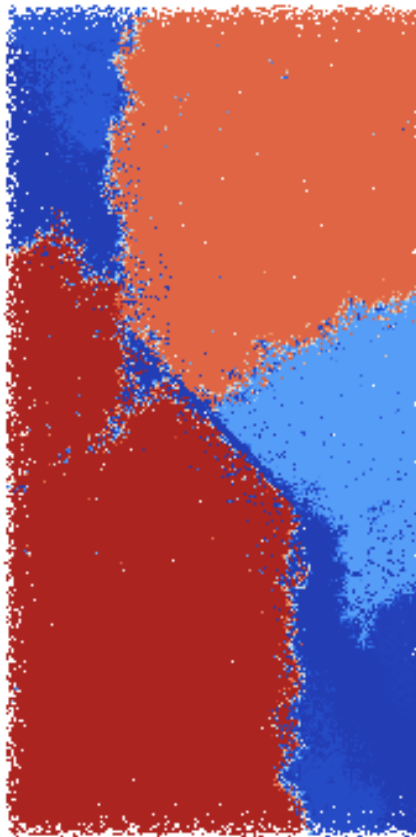


lateral displacement (mm)

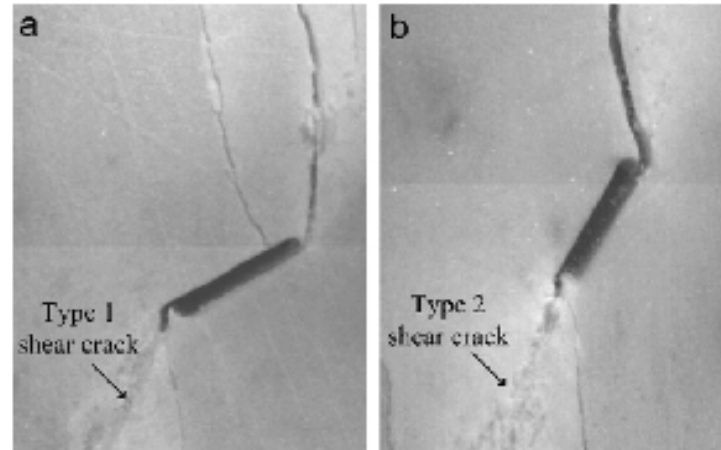


Frictional sliding along the crack is achieved under uniaxial compression of the specimen, as expected.

Validation: Wing cracks

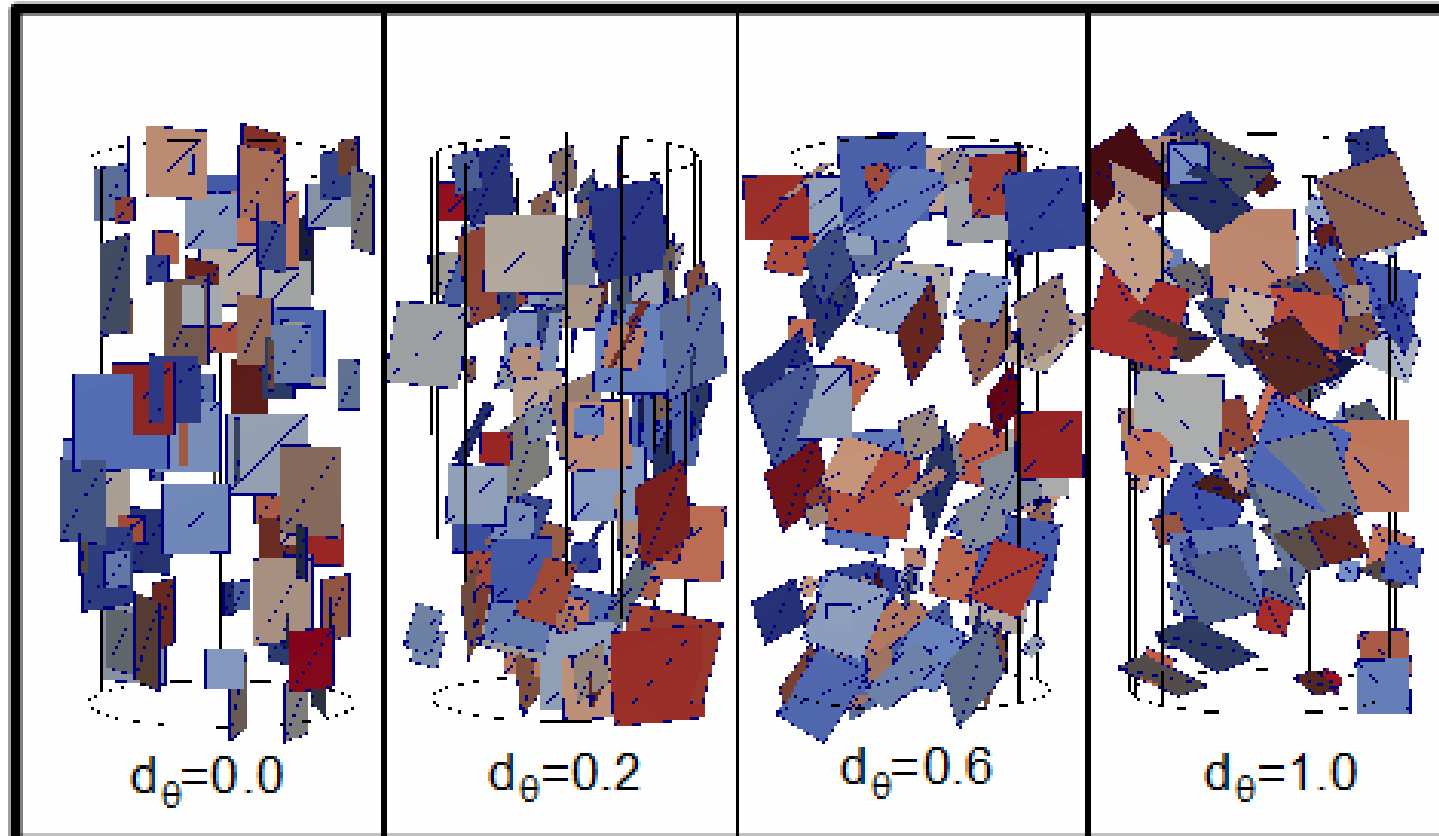


Fragment Mass (g)



Uniaxial compression of a sample containing a single penny-shaped crack results in formation of wing-cracks qualitatively similar to theoretical predictions and laboratory observations.

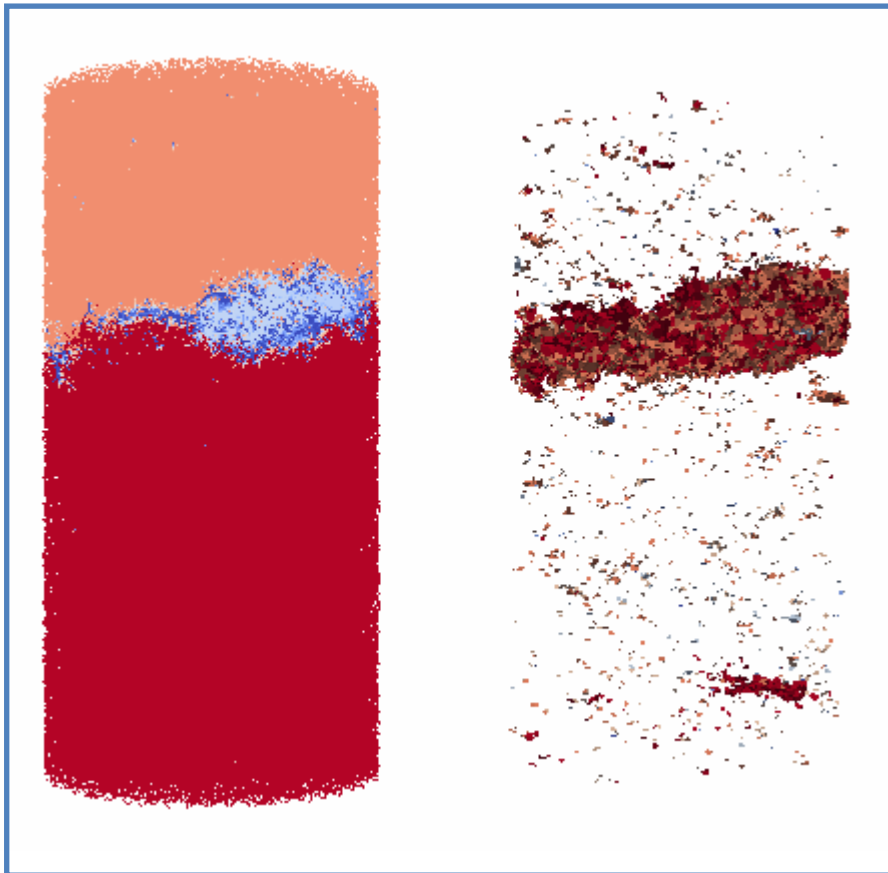
Fracture distributions and orientations



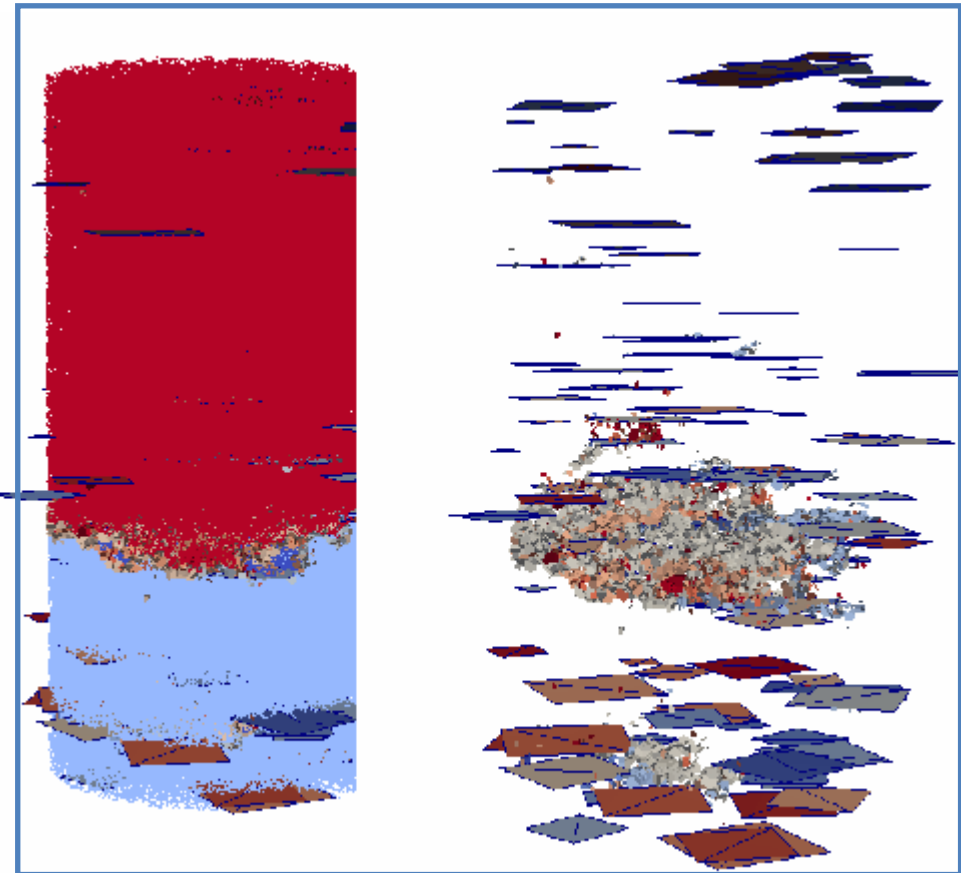
Cylindrical specimens are populated with randomly located planar cracks. The size-distribution is either uniform or fractal. A parameter known as the **deviation factor** (d_θ) governs crack orientations.

Failure under tension

PRISTINE SPECIMEN



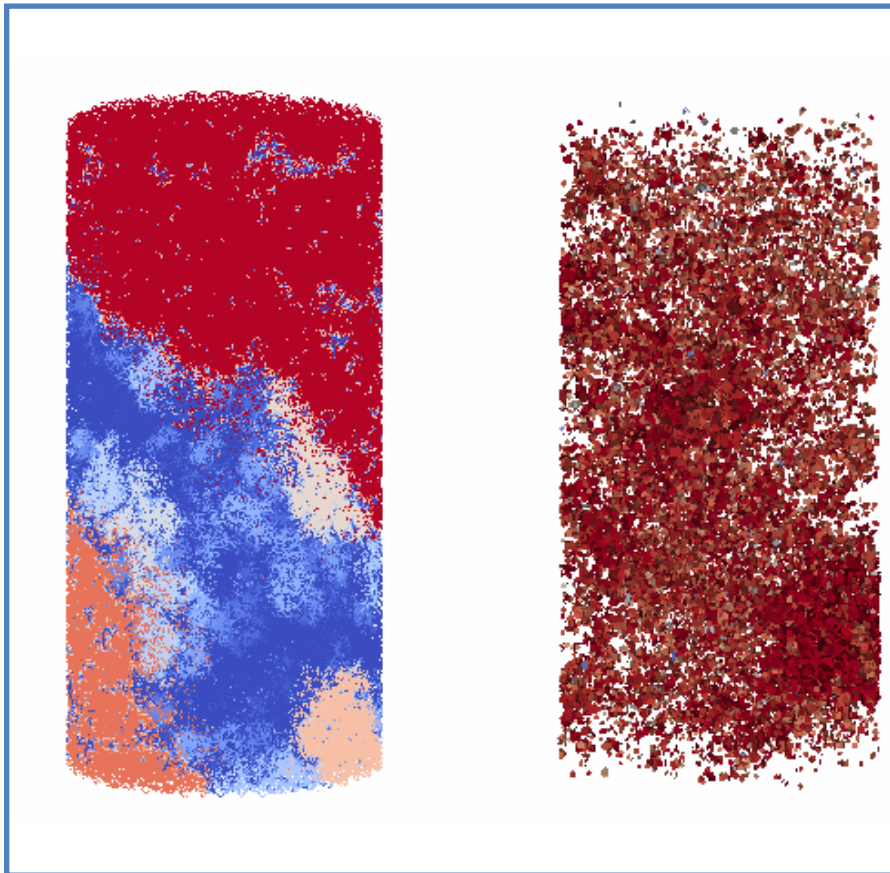
PRE-FRACTURED SPECIMEN



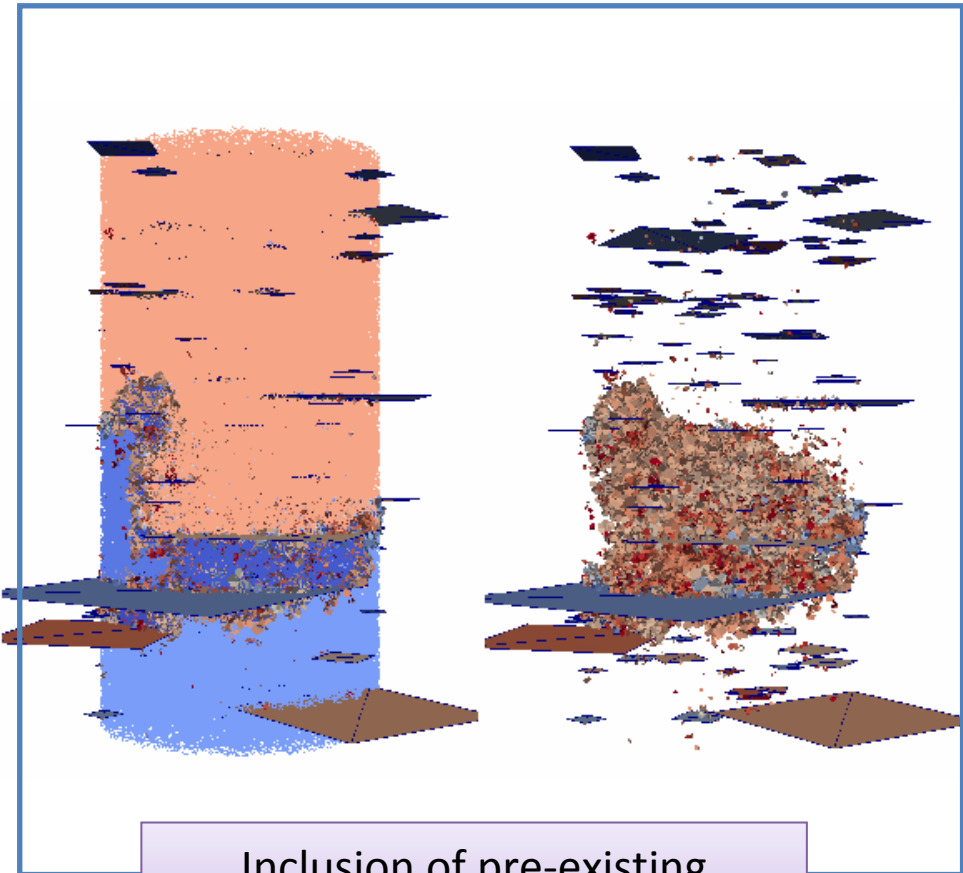
Tensile failure is somewhat more localised with the inclusion of pre-existing fractures.

Failure under compression

PRISTINE SPECIMEN



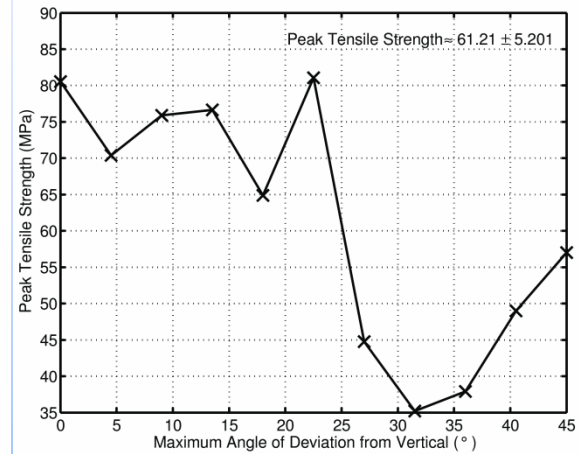
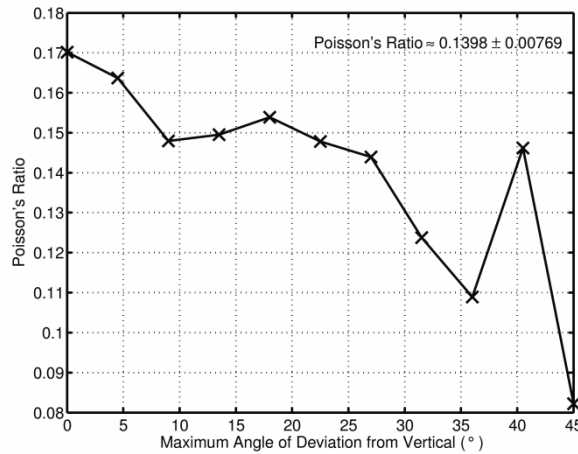
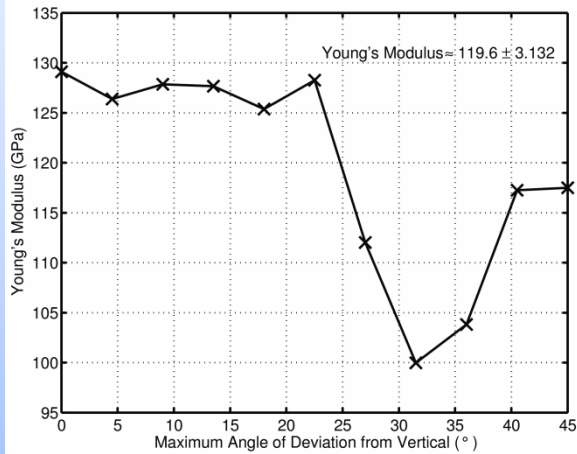
PRE-FRACTURED SPECIMEN



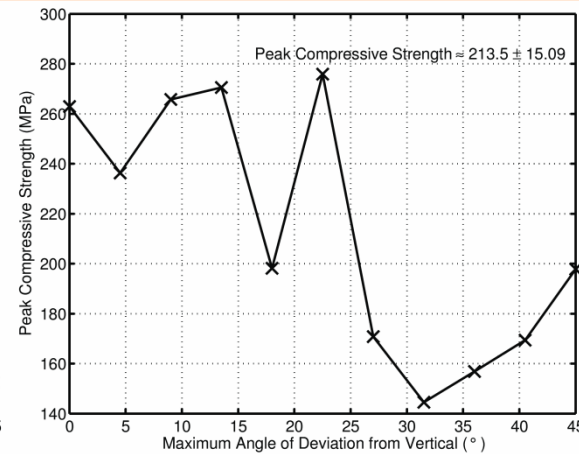
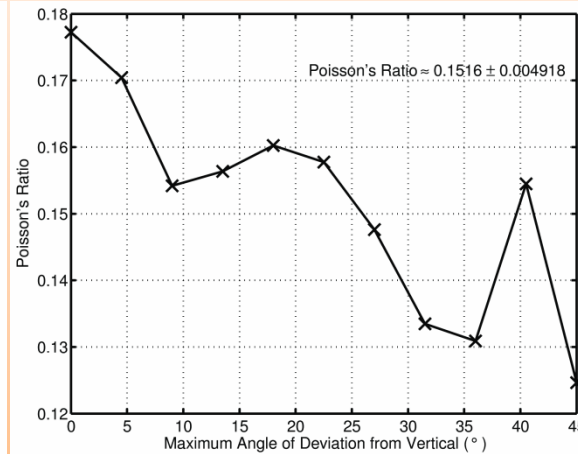
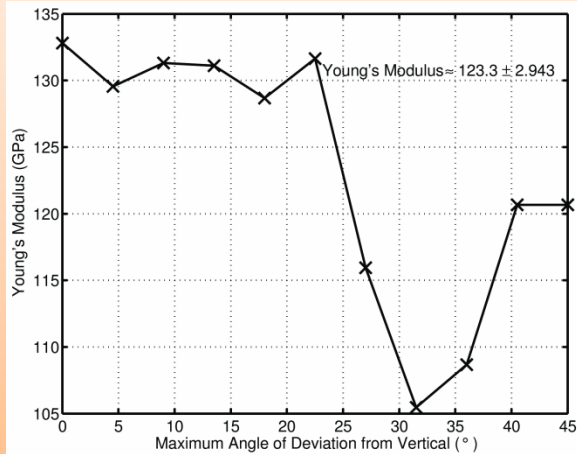
Inclusion of pre-existing fractures localises shear failure.

Mechanical response vs. orientation

TENSION



COMPRESSION

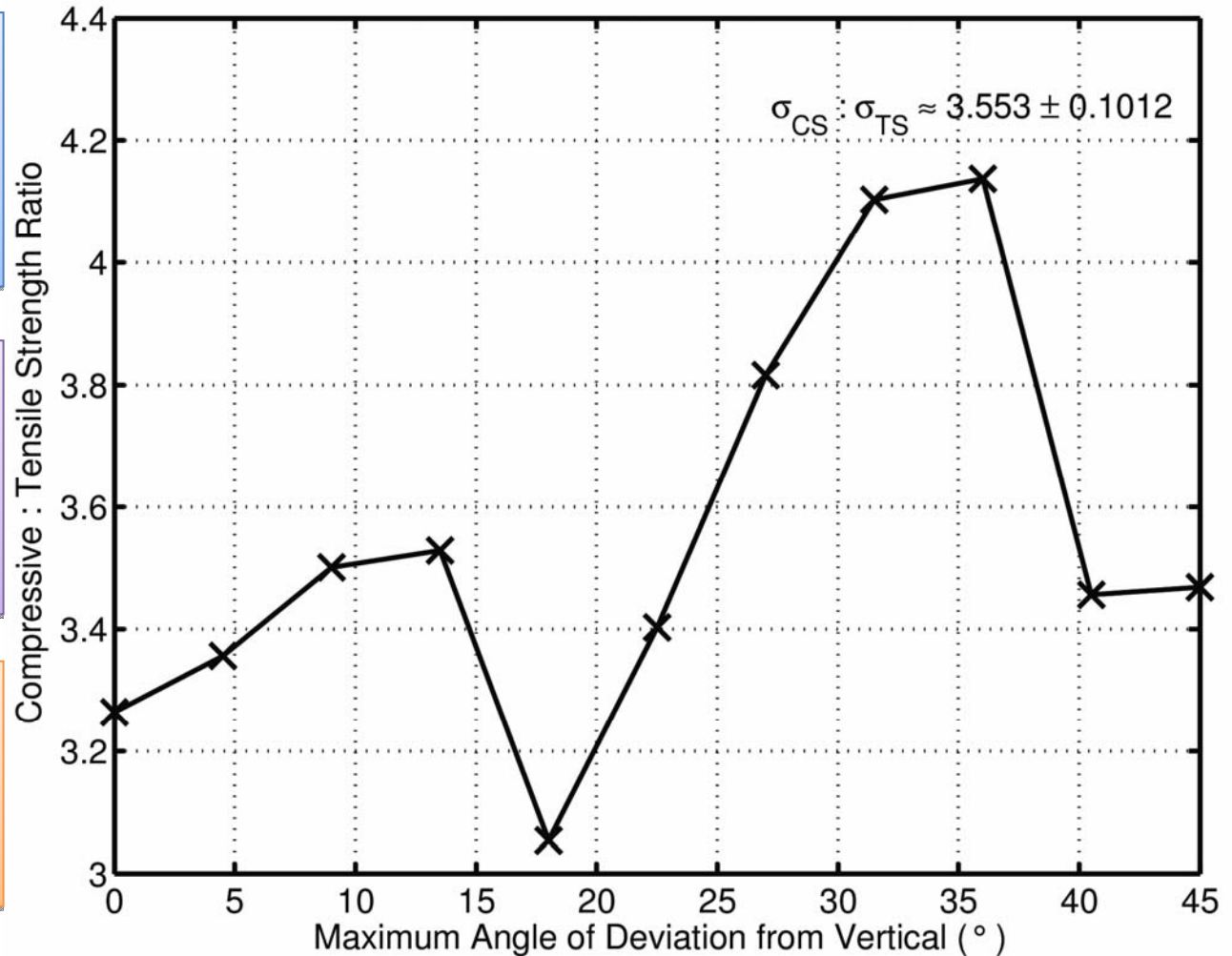


Ratio of compressive to tensile strengths

Rocks typically have compressive strength between 10-50 times larger than tensile strength

Even for pre-fractured DEM specimens, the compressive strength is too small relative to tensile strength

Or perhaps the tensile strength is too large relative to compressive strength?



Conclusions

The Discrete Element Method is a valuable tool to gain insight on the fracture mechanics of brittle-elastic solids, particularly for conditions resulting in tensile failures

Systematic calibration exercises provide some understanding of how micro-mechanical properties “scale-up” topologically to give rise to macroscopic mechanical properties

A simple technique has been developed to incorporate pre-existing fractures into DEM rock specimens that results in qualitatively different fracturing patterns than “pristine” DEM rock specimens.

Some aspects of the mechanical response of DEM rock specimens fail to match laboratory measurements, particularly the Poisson’s ratio and ratio of compressive to tensile strengths

Computational Caving Science

TIME

1dec

1year

1day

1sec

1cm

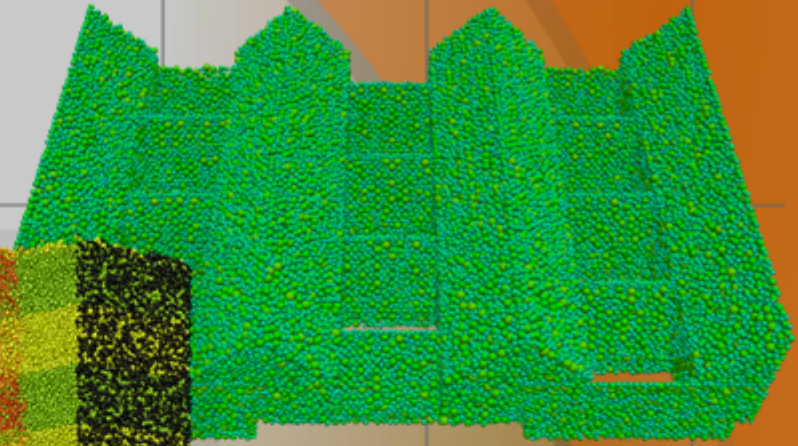
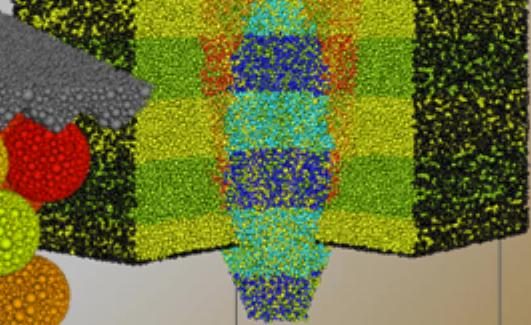
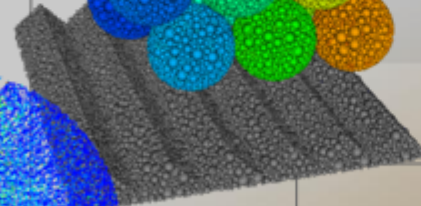
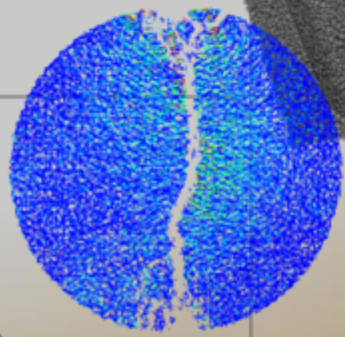
10cm

1m

10m

100m

SPACE



Questions?

