



# Advanced modelling of glacial isostatic adjustment: deviations from spherical symmetry in mantle material

Volker Klemann,  
Zdeněk Martinec<sup>1</sup>, Yoshiyuki Tanaka<sup>2</sup>, Erik Ivins<sup>3</sup>

Institut de Physique du Globe de Strasbourg

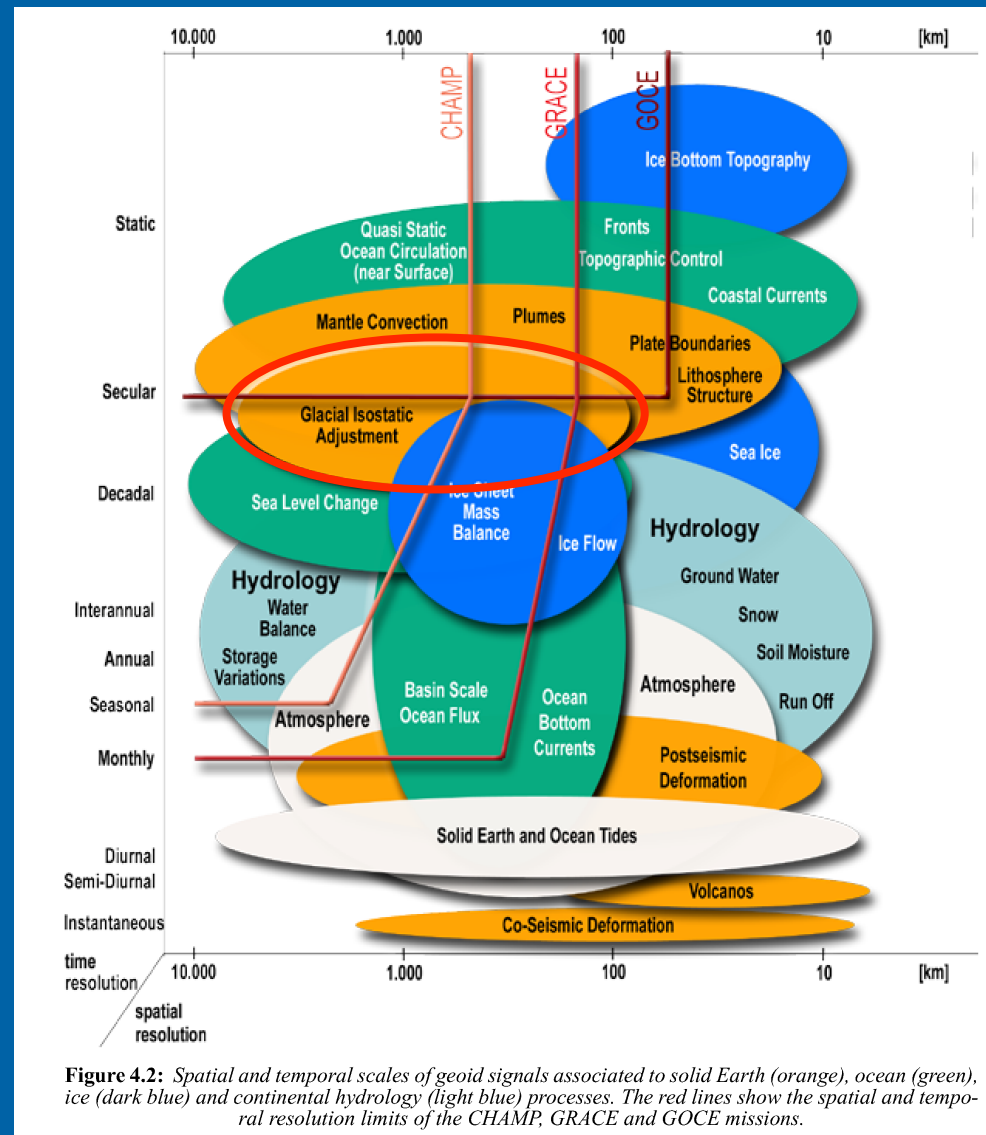
Sep 9, 2011 (Strasbourg)

GFZ German Research Centre for Geosciences, Potsdam  
[volkerk@gfz-potsdam.de](mailto:volkerk@gfz-potsdam.de)

<sup>1</sup>DIAS—Dublin, <sup>2</sup>Tokyo University, <sup>3</sup>JPL—Pasadena



# Where are we?



**Figure 4.2:** Spatial and temporal scales of geoid signals associated to solid Earth (orange), ocean (green), ice (dark blue) and continental hydrology (light blue) processes. The red lines show the spatial and temporal resolution limits of the CHAMP, GRACE and GOCE missions.



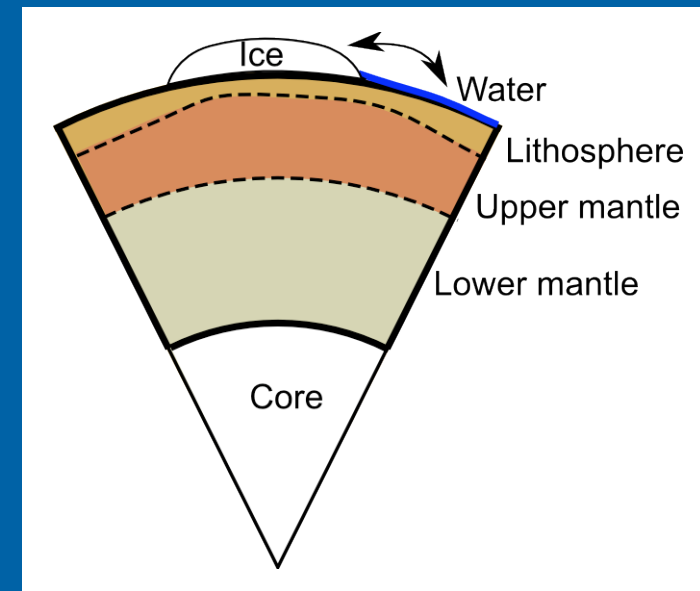
# Overview

1. The physical process of Glacial Isostatic Adjustment (GIA)
  - Last glacial cycle
  - Physical process of solid earth
  - Interplay of solid earth, ocean and ice sheets
2. Lateral heterogeneity
  - Variations of lithosphere thickness + plate boundaries
  - Consideration of non-linear rheologies (in prep.)
3. GIA induced geocenter motion
  - Influence of viscosity structure
  - Influence of glaciation history

# Features of GIA modelling



- PREM structure for shear modulus and density
- Viscosities:  $\eta_{UM}$  = lateral variable,  
 $\eta_{LM} = 1 \times 10^{22}$  Pa s
- Elastic lithosphere of variable thickness  
Predefined ice history (ICE5G)



- S-FE formulation (Martinec, GJI, 2000)
  - incompressible
  - self-gravitating
  - non-rotating
  - hydrostatically pre-stressed

- Uniqueness conditions
  - centre of mass
  - no surface net-rotation



# Numerical modeling

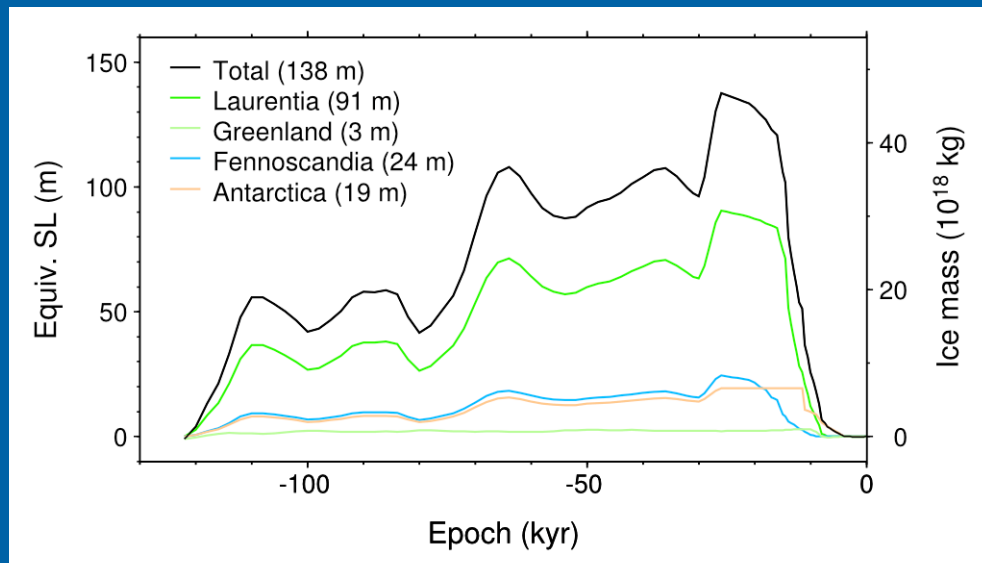
- Initial boundary-value problem of viscoelastic relaxation (Martinec, 2000)
  - Field equations of momentum
  - Linearly viscoelastic material (Maxwell-body)
  - Continuity equation (incompressibility)
  - Potential equation
- Boundary and interface conditions
  - Welded continuum of lithosphere and mantle
  - Buoyancy and free slip at CMB and surface
- Weak formulation for explicit time-differencing scheme

$$\delta \mathcal{E} \left( \mathbf{u}^{i+1}, \phi_1^{i+1}, \Pi^{i+1}, \delta \mathbf{u}, \delta \phi_1, \delta \Pi \right) = \delta \mathcal{F} \left( \delta \mathbf{u}, \delta \phi_1 \right)$$

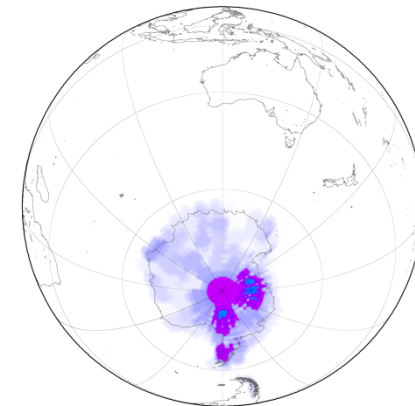
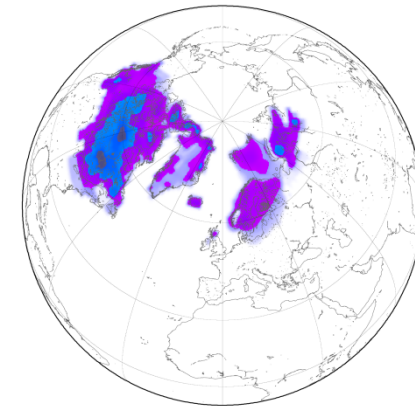


# Last glacial cycle

- Glaciated regions on northern hemisphere:
  - North America, Greenland
  - Fennoscandia
- On southern hemisphere
  - Antarctica



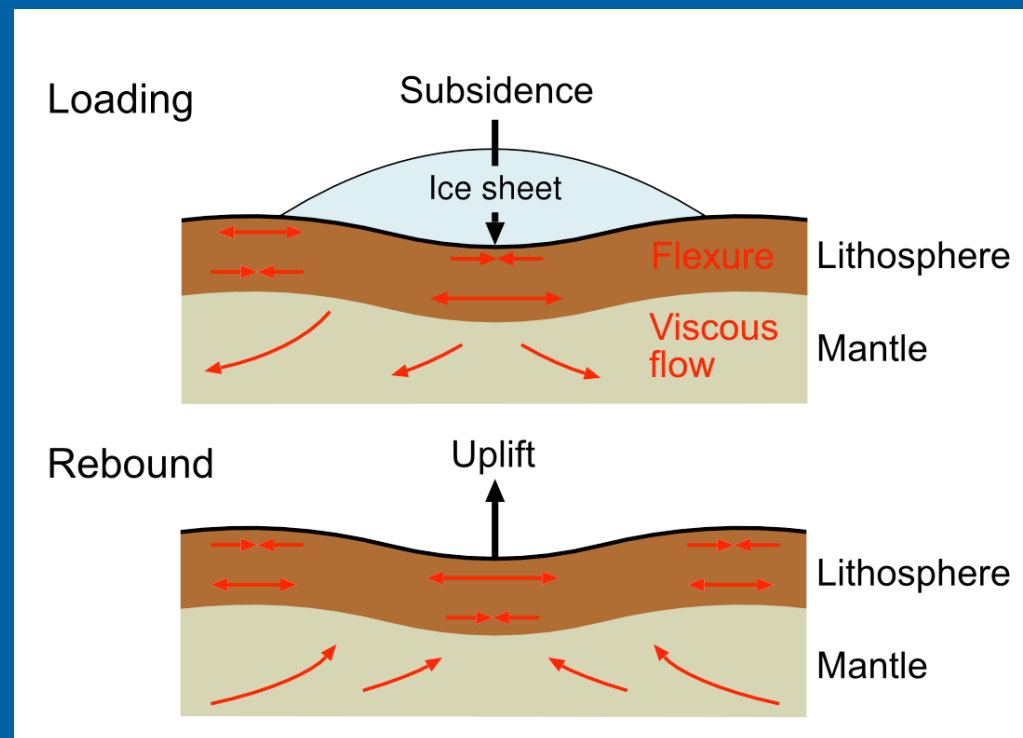
21 kyr b.p.



# Earth response to glacial loading (GIA)



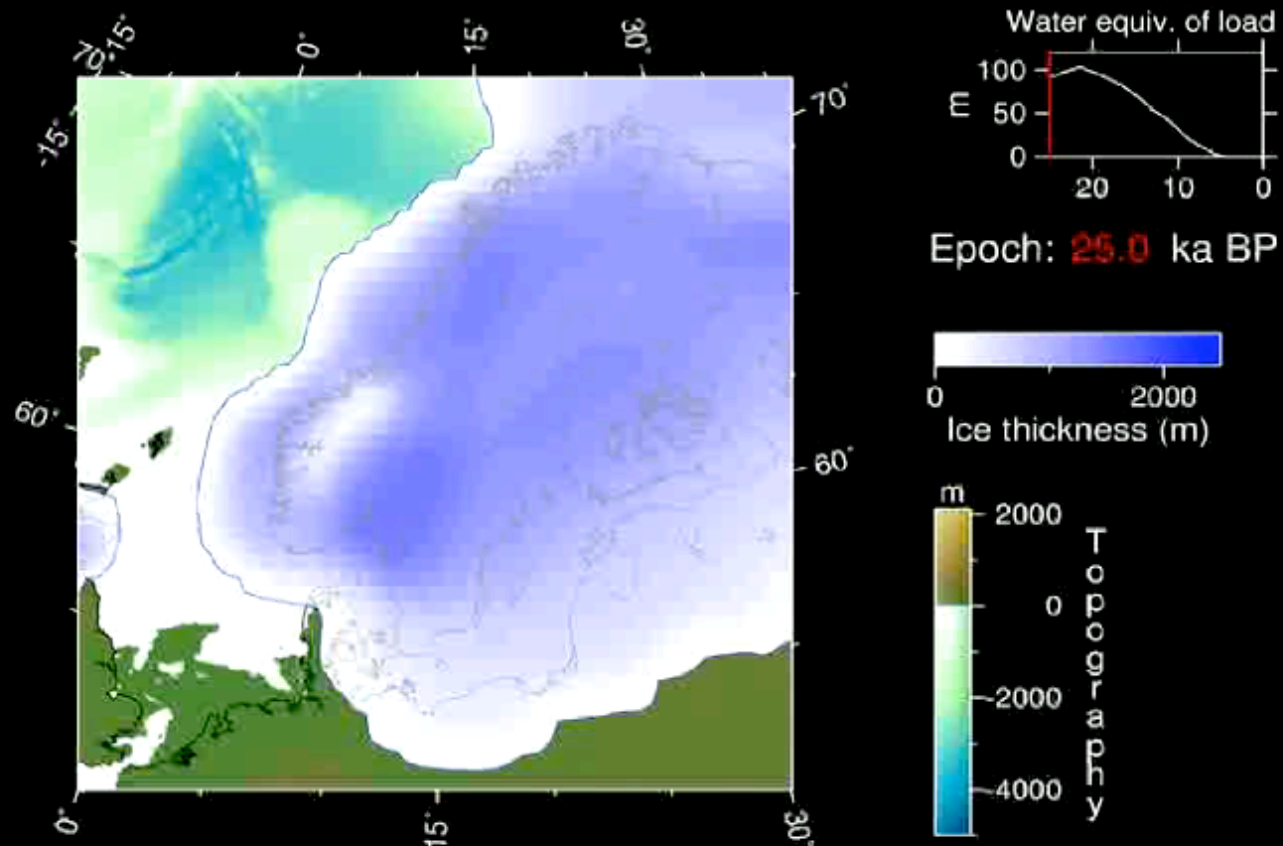
- Response to glacial loads
  - Extension:  $O$  (1000 km)
  - Thickness:  $O$  (1 km)
  - Period:  $O$  (100 kyr)
- Last glaciation terminated 8000 yr BP
- Present-day adjustment



# Interplay of solid earth, ocean and ice sheets



Variation of palaeotopography caused by GIA



© Volker Klemann, GFZ





# Features of earth models

- Standard models considered in GIA
  - Elastic lithosphere overlaying viscoelastic mantle
  - Materially incompressible
  - 1D stratification
  - Linear Maxwell viscoelasticity
  - Normal mode theory
- Non-standard models
  - Compressibility
  - Lateral viscosity variations
  - Stress-dependent viscosity

# Laterally variable earth models

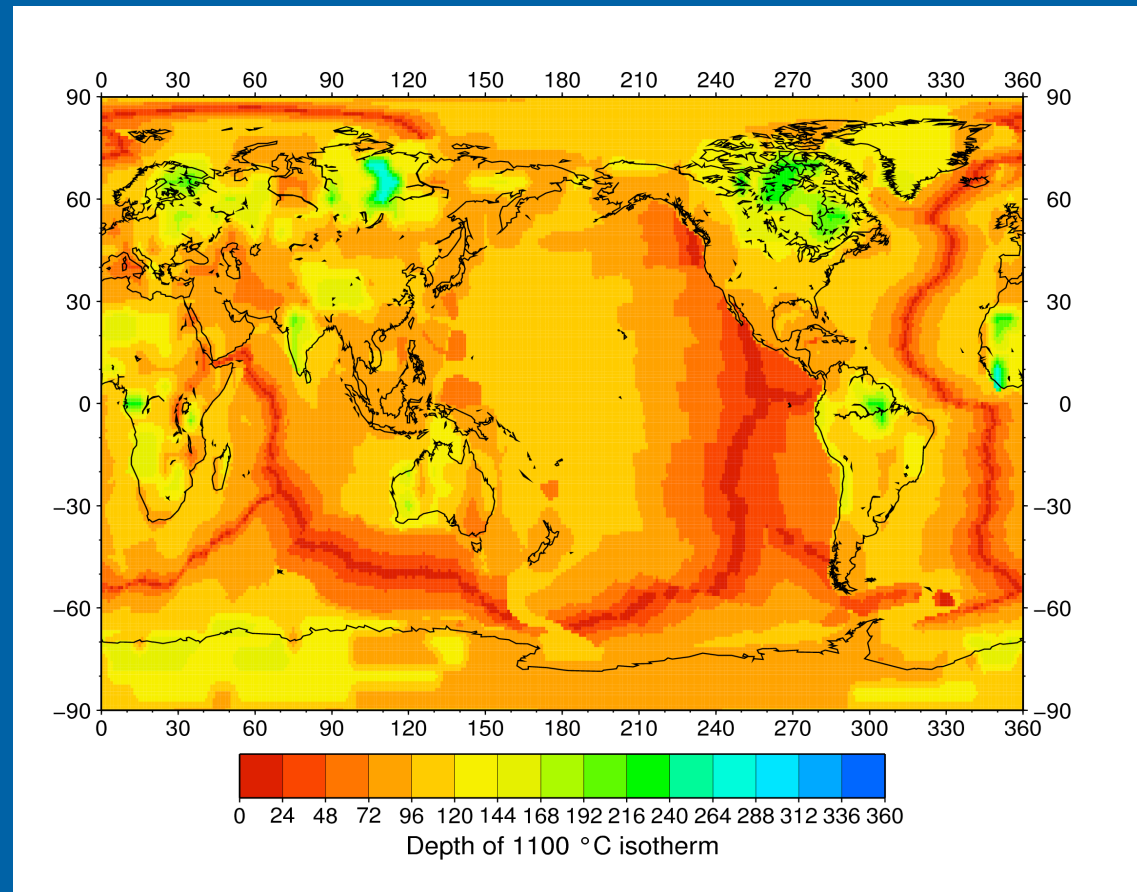


- Lateral variations of lithosphere thickness
  - Influence of GIA on plate motions
- Regionally varying response to glacial loading
  - Differences between Laurentide, Fennoscandia
  - Response in regions with strong tectonic features like Alaska, Iceland, Patagonia, Antarctic Pen.
- Lateral variations of mantle viscosity
  - Dichotomy between W- and E Antarctica

# Thickness of elastic lithosphere



- Depth defined by characteristic isotherm (1100 °C)
- Mosaic
  - Continental lithosphere from thermal data (Artemieva, Tectonophysics, 2006)
  - Oceanic lithosphere from ocean floor ages (Müller et al., JGR, 1997)

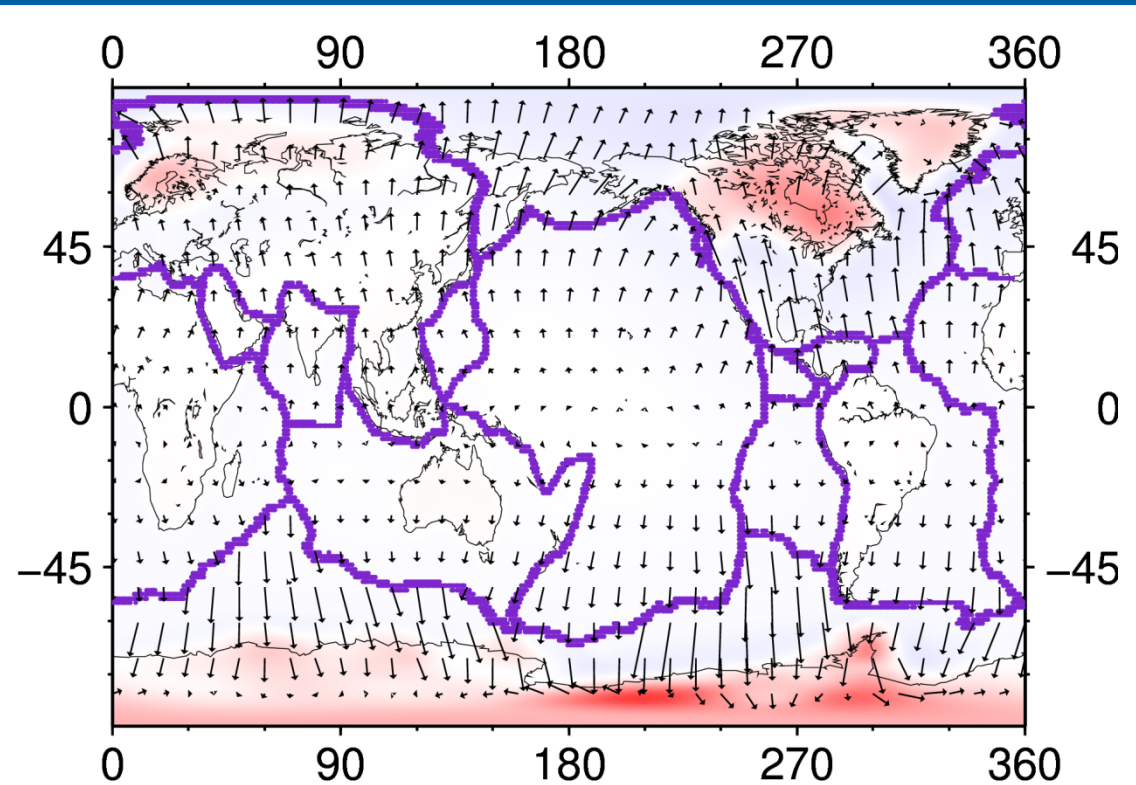




# Induced surface motion

## 3D earth structure

- plate boundaries defined as 200 km wide low-viscosity intervals
- variable thickness of elastic lithosphere
- 1D mantle
- ICE3G history
- fixed coastlines



Present-day velocities:

→ 2.5 mm/a



-20 -10 0 10 20

Horizontal comp.

Vertical component (mm/a)



# Cross section I

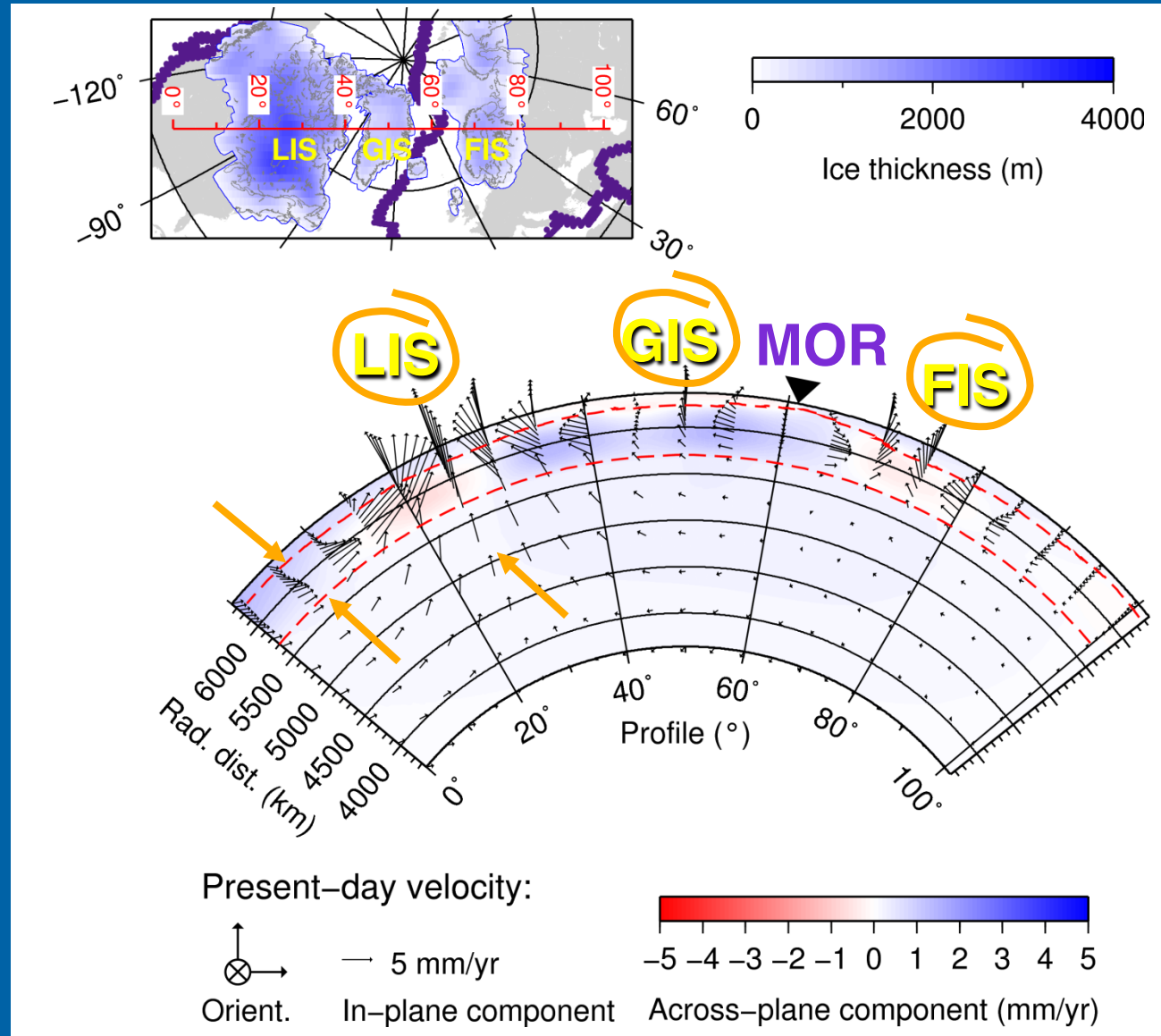
Mid Ocean Ridge  
(MOR)

Ice sheets:

Laurentide (LIS)

Greenland (GIS)

Fennoscandia (FIS)

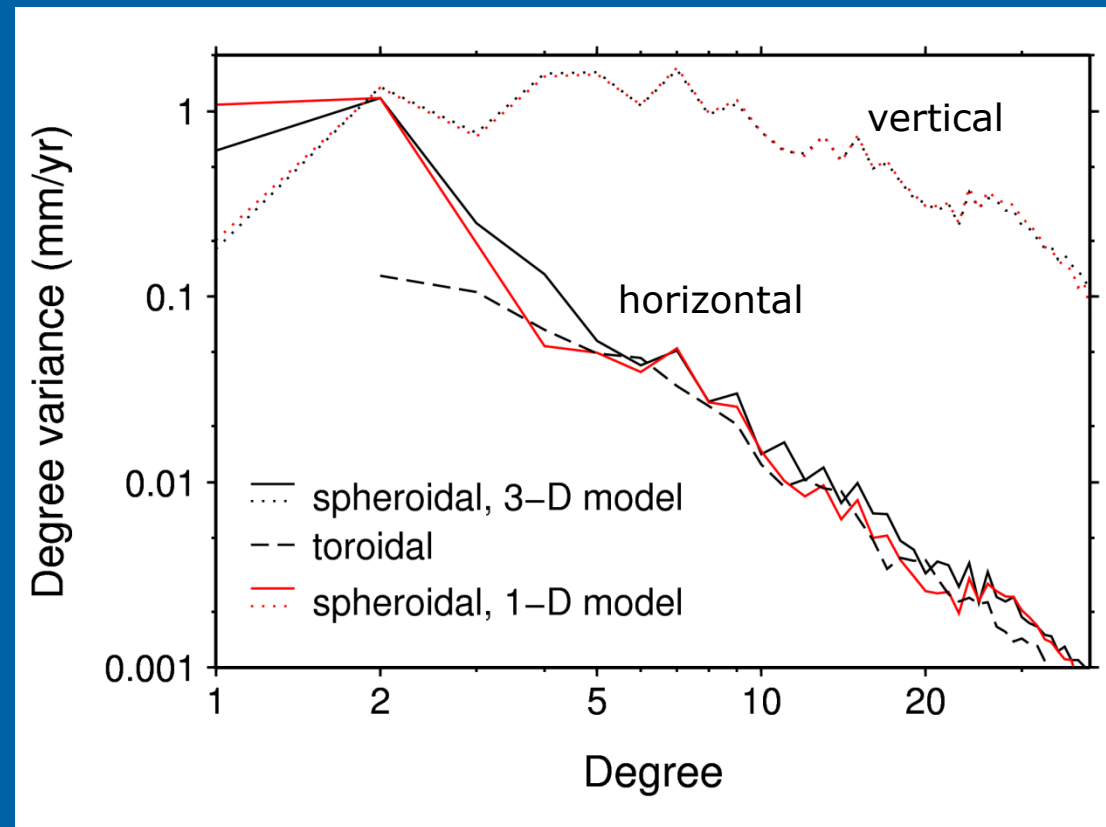


# Degree variances of surface motion



## Influence of plates

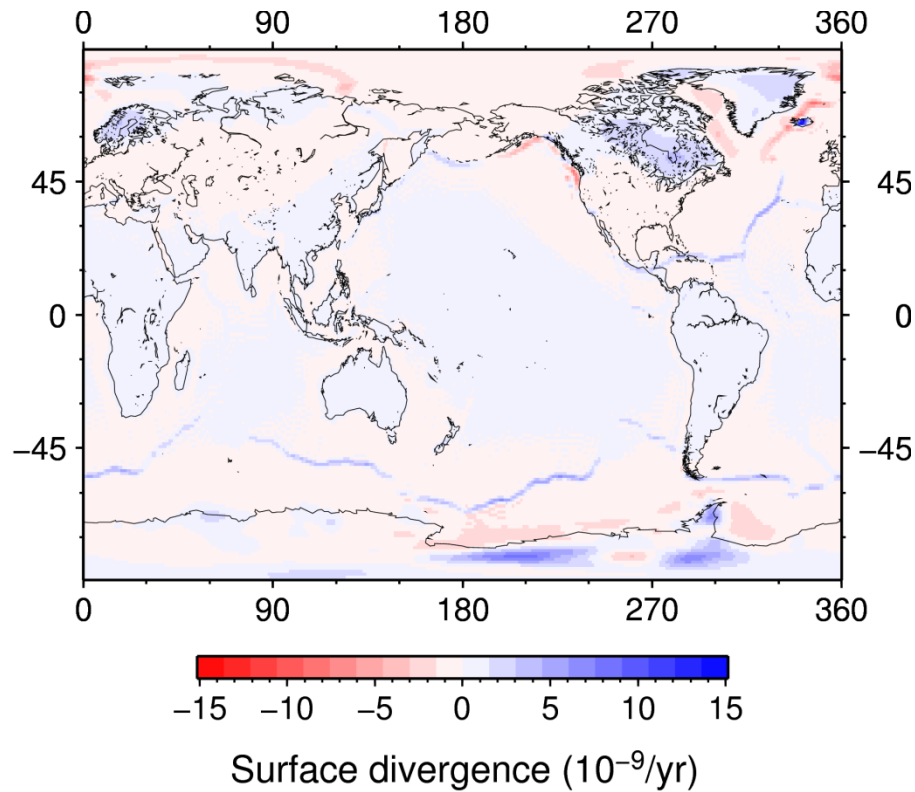
- Equipartitioning of spheroidal and toroidal component of GIA induced horizontal motions for  $j > 3$
- Equipartitioning appears in plate motions driven by convective flow (e.g. Čadek & Ricard, EPSL, 1992)
- Toroidal motion vanishes for  $j = 1$  due to uniqueness condition of no surface net-rotation



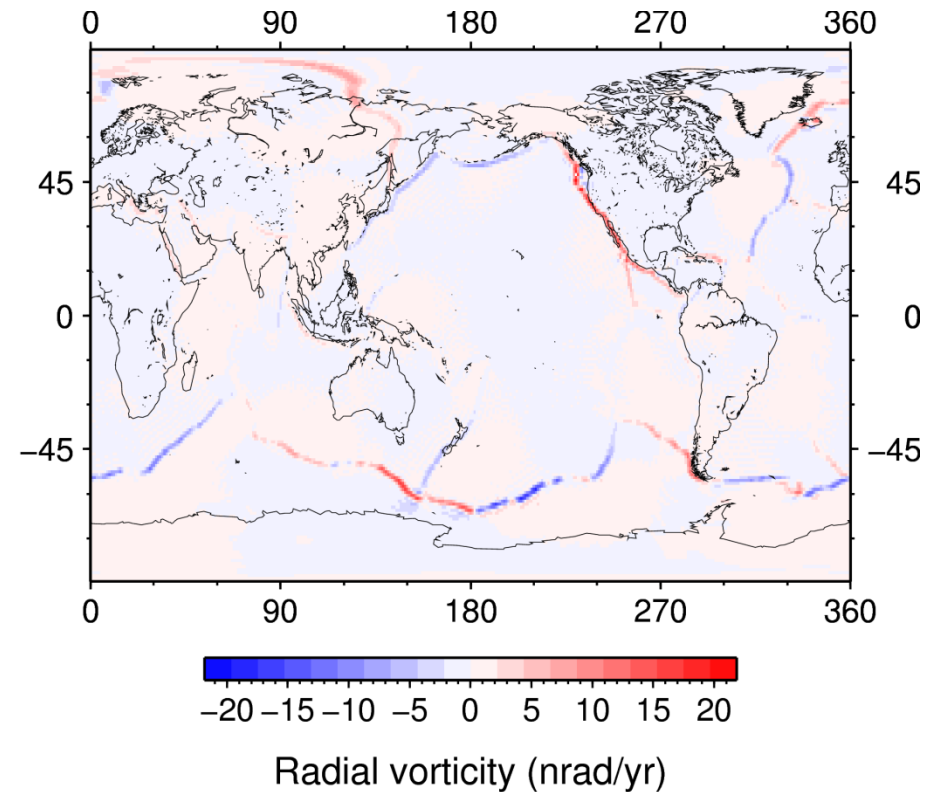
Klemann et al. 2008, J. Geodyn.



# Divergence and vorticity



GM 2008 Apr 9 12:06:40



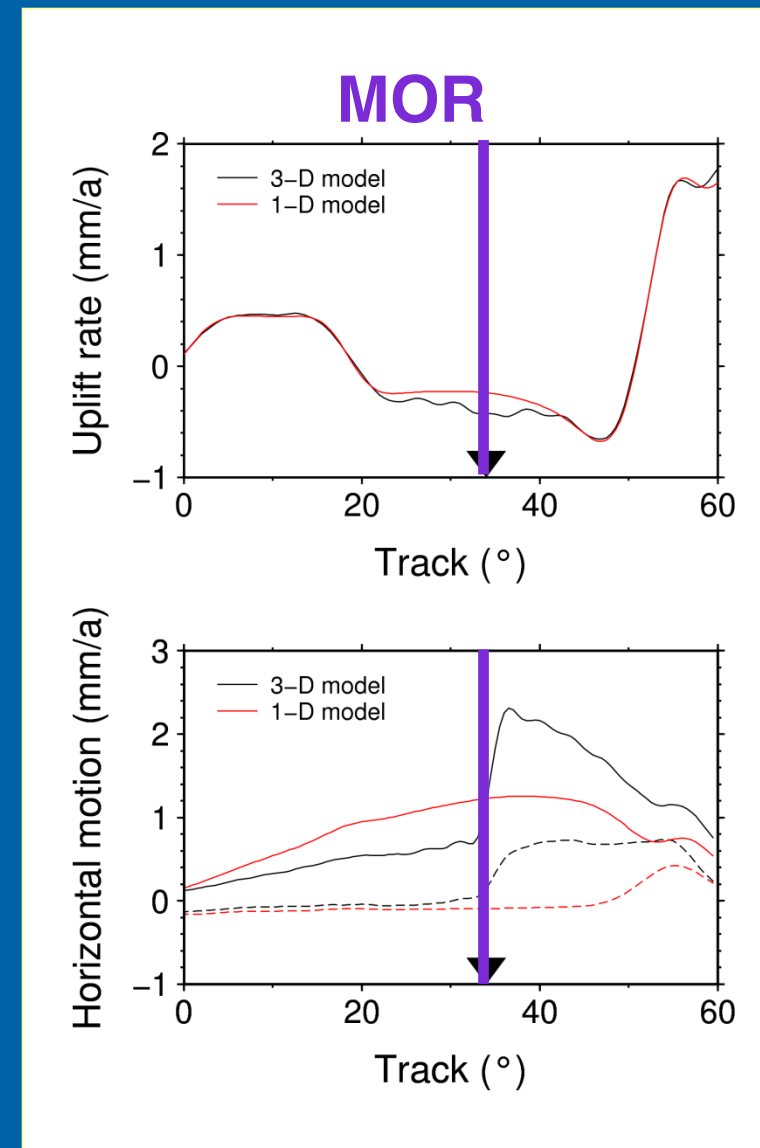
GM 2008 Apr 9 12:06:48

Klemann, et al., 2008, JGdyn

# Influence of mid ocean ridge



- Vertical displacement
  - small variation
- Horizontal displacement
  - velocity jump of  $> 1$  mm/yr across the MOR
- But, this is only small perturbation to the observed sea-floor spreading of 7 cm/a in this region







# GIA induced plate motion

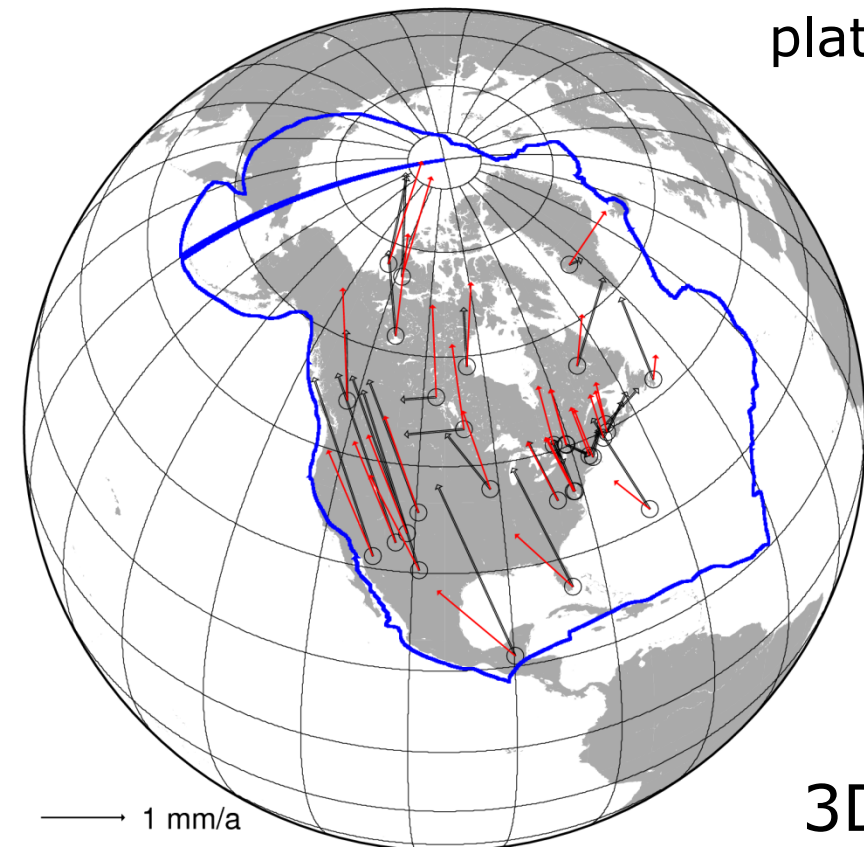
Motion of NA plate determined from ITRF 2005 (Altamimi et al., JGR, 2007) and **corrections due to GIA**

Model	Lon.(°E)	Lat. (°N)	$\Omega$ (°/Myr)
ITRF-2005	-87.4 $\pm 0.6$	-4.3 $\pm 0.9$	0.192 $\pm 0.002$
1-D	+1.1	-0.0	+0.002
3-D	+2.4	+3.6	+0.008

- GIA induced motion
  - < 10 % of observed plate motion
  - above accuracy of determined plate motion

Lon = 136.5 °E. Lat=-39.8 °N,  $\Omega$  = 0.015 °/Myr

North American  
plate





# Summary

- Lateral variations in lithosphere structure are present
- The applied method of spectral—finite elements allows us to consider lateral viscosity variations in a quite efficient way
- Separating the lithospheric plates by visous zones:
  - Toroidal surface motion of similar amplitude like spheroidal horizontal component is induced
  - Velocity field becomes discontinuous across plate boundaries
- GIA induced rotation of continental plates
  - Induced rates modify observed rates at the level of accuracy
  - GIA induced volocity fields are largely modified by 3d structure
- Future plans
  - Consideration of non-linear rheology. I/O is prepared (Thank you Gabi !)



# Non-linearity

- Considering experimentally inferred rheological data in GIA modelling
- Power-law creep in lithosphere and upper mantle
- Largest effect of non-linearity appears where large shear stresses are present.
  - During the deglaciation phase
  - At the load margins
  - Where strong lateral variations in earth structure are present
- Composite rheology

$$\dot{\epsilon} = \left( \frac{1}{2\eta_{\text{diff}}} + A\sigma^{n-1} \right) \sigma + \frac{1}{2\mu} \dot{\sigma}$$



# Geocentre motion

- Back to 1d modelling
- Convention
  - $u^{GC} := u^{CF} - u^{CM}$
- GIA contribution
  - Influence of viscosity structure on present day motion
  - Influence of glaciation history



# Why 1d is sufficient

$$\mathbf{u}_{gc} := \mathbf{u}_{cf} - \mathbf{u}_{cm}$$

or in components

$$u_{gc}^x = -\frac{1}{2} \sqrt{\frac{2}{\pi}} \operatorname{Re}\{U_{11} + 2V_{11} + 3F_{11}/g_0\}$$

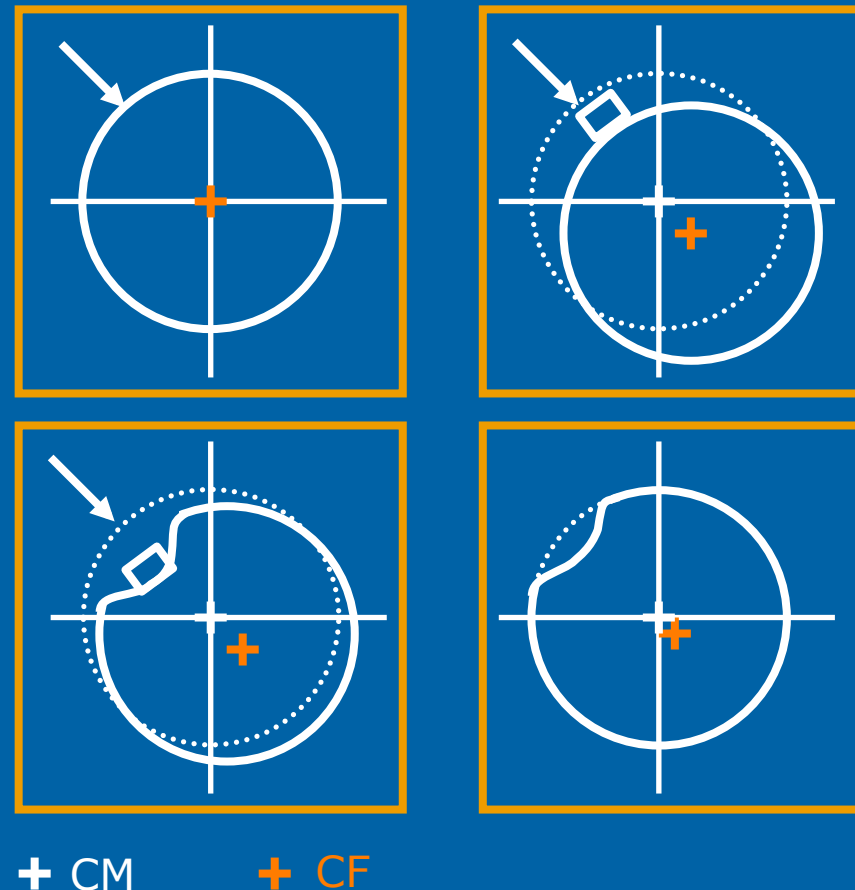
$$u_{gc}^y = \frac{1}{2} \sqrt{\frac{2}{\pi}} \operatorname{Im}\{U_{11} + 2V_{11} + 3F_{11}/g_0\}$$

$$u_{gc}^z = \frac{1}{2} \sqrt{\frac{1}{\pi}} (U_{10} + 2V_{10} + 3F_{10}/g_0)$$

# CM and CF motion due to GIA



- Surface loading
  - CM towards load
  - GC in opposite direction
- Viscoelastic compensation
  - downward displacement
  - CM away from load
  - CF away from load
- After deglaciation
  - CM first away from load area than moves towards load centre
  - CF towards load area



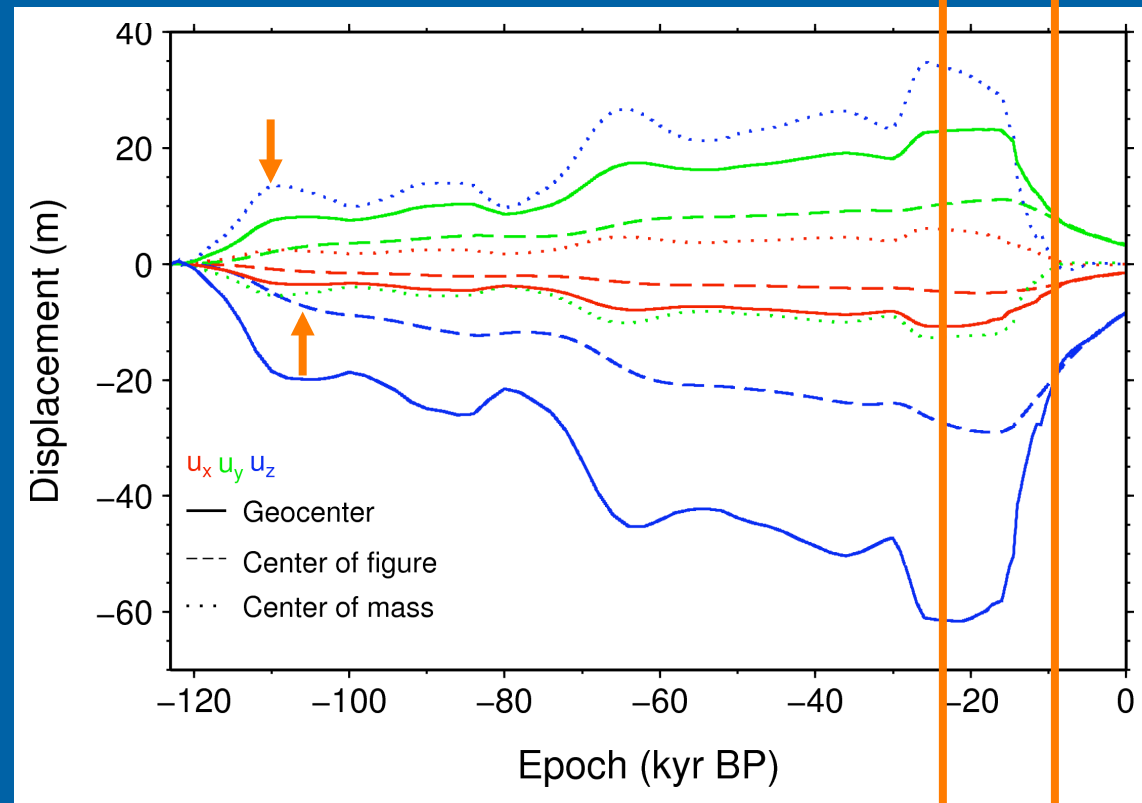


# GC, CM and CF motion

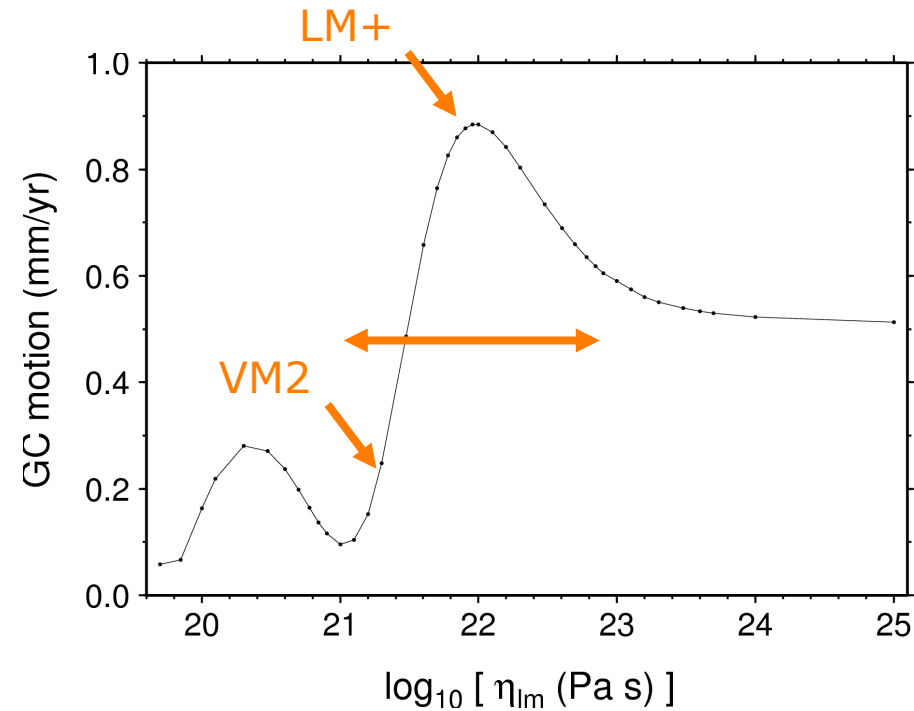
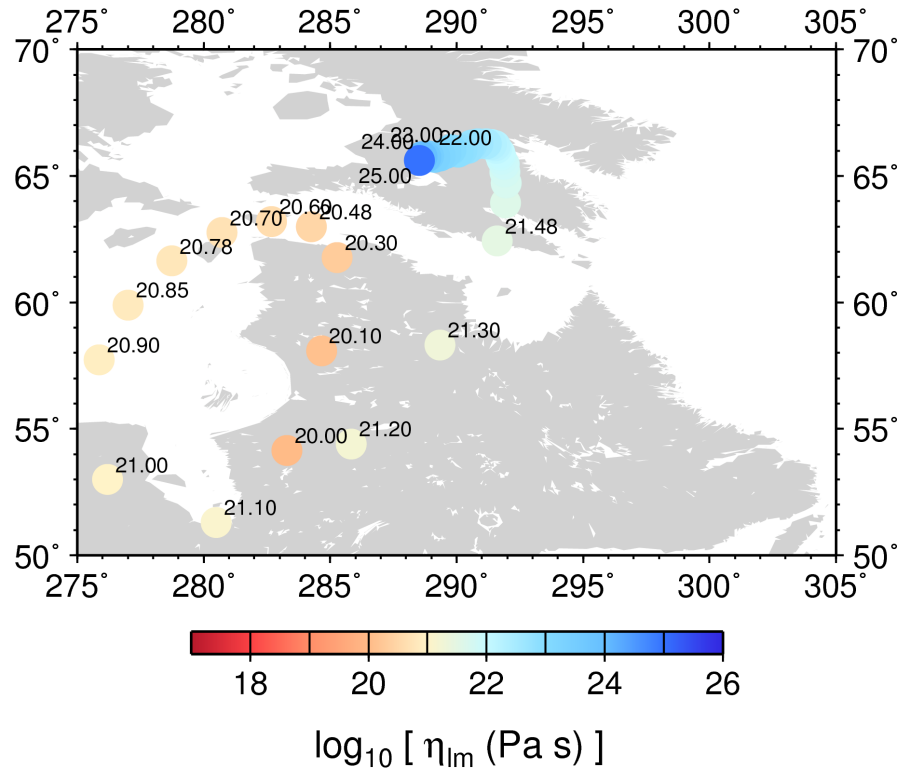
$$\int_V \rho \mathbf{u} dV = 0$$

- Evolution of motions in the CE realization during last glacial cycle

- CF is delayed and opposite to CM
- amplitude of GC is largest during LGM and reaches 70 m
- after deglaciation CM is negligible and GC is dominated by the delayed CF



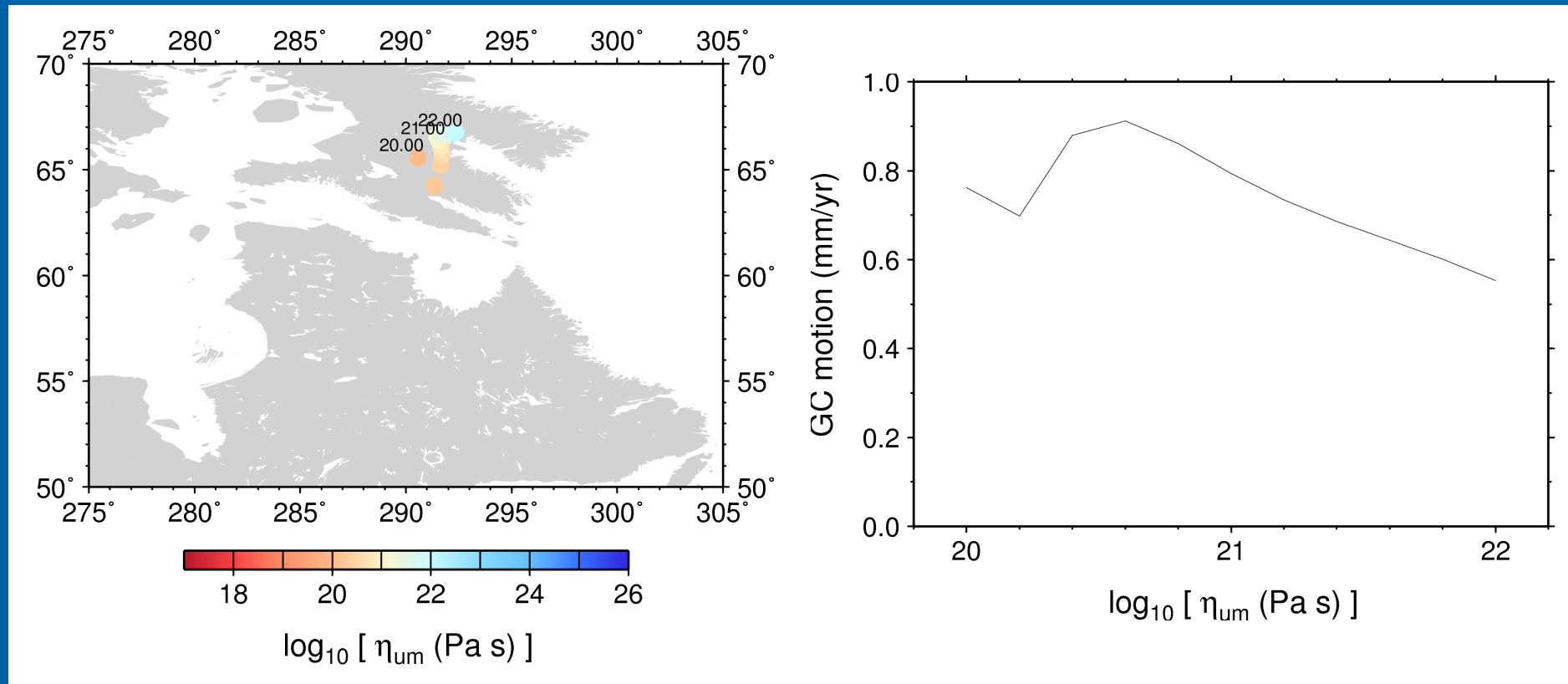
# Influence of lower-mantle viscosity on GC motion



- Variation in direction of GC motion  $\sim 2000$  km
- Velocity of motion varies by almost one magnitude
- largest sensitivity between  $10^{21}$  and  $10^{23}$  Pa s

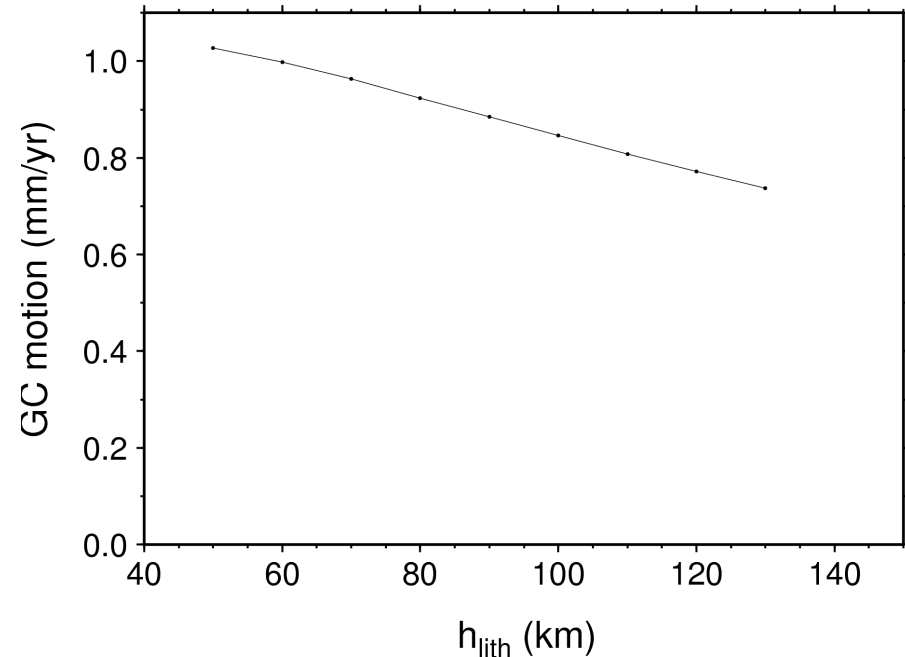
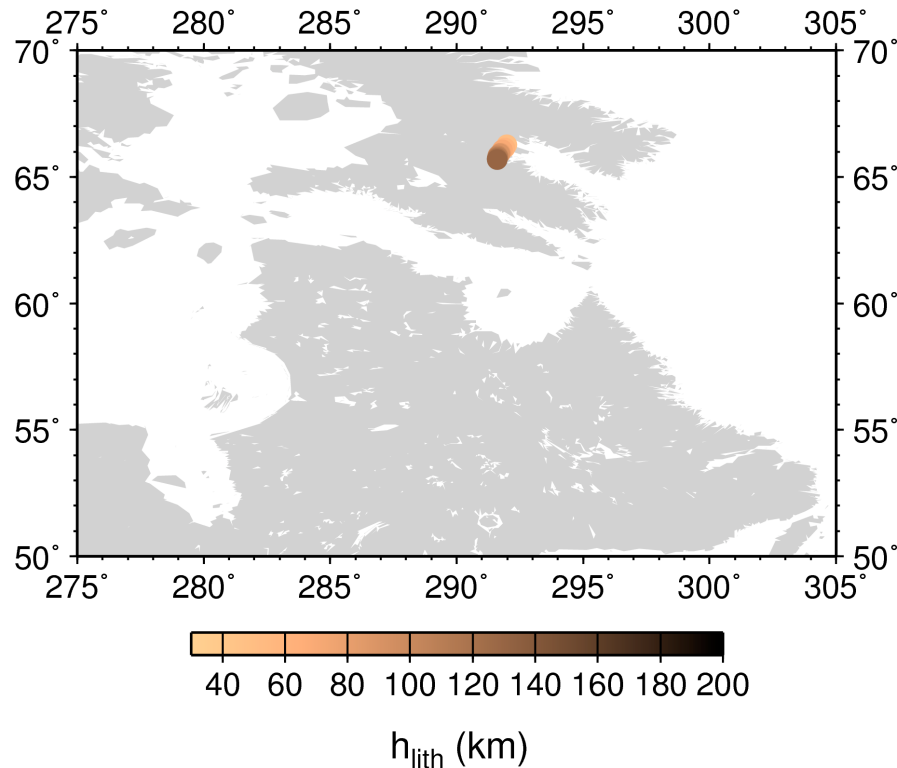


# Influence of upper-mantle viscosity on GC motion



- Influence on direction of motion is much smaller
- At  $3 \times 10^{20}$  Pa s amplitude is largest and decreases linearly on logarithmic scale

# Influence of lithosphere thickness on GC motion

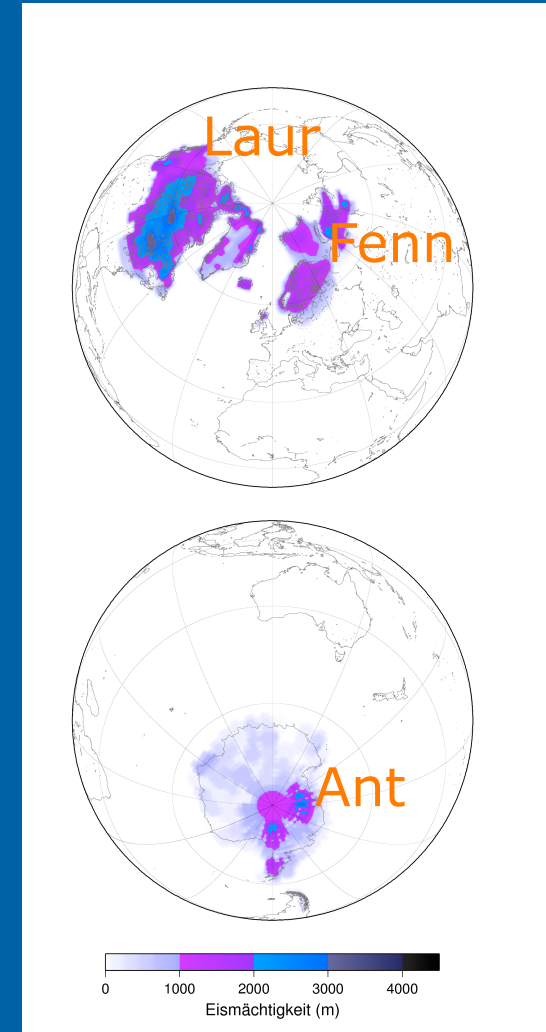


- Influence of lithosphere thickness on direction is negligible
- Velocity decreases linearly with lithosphere thickness

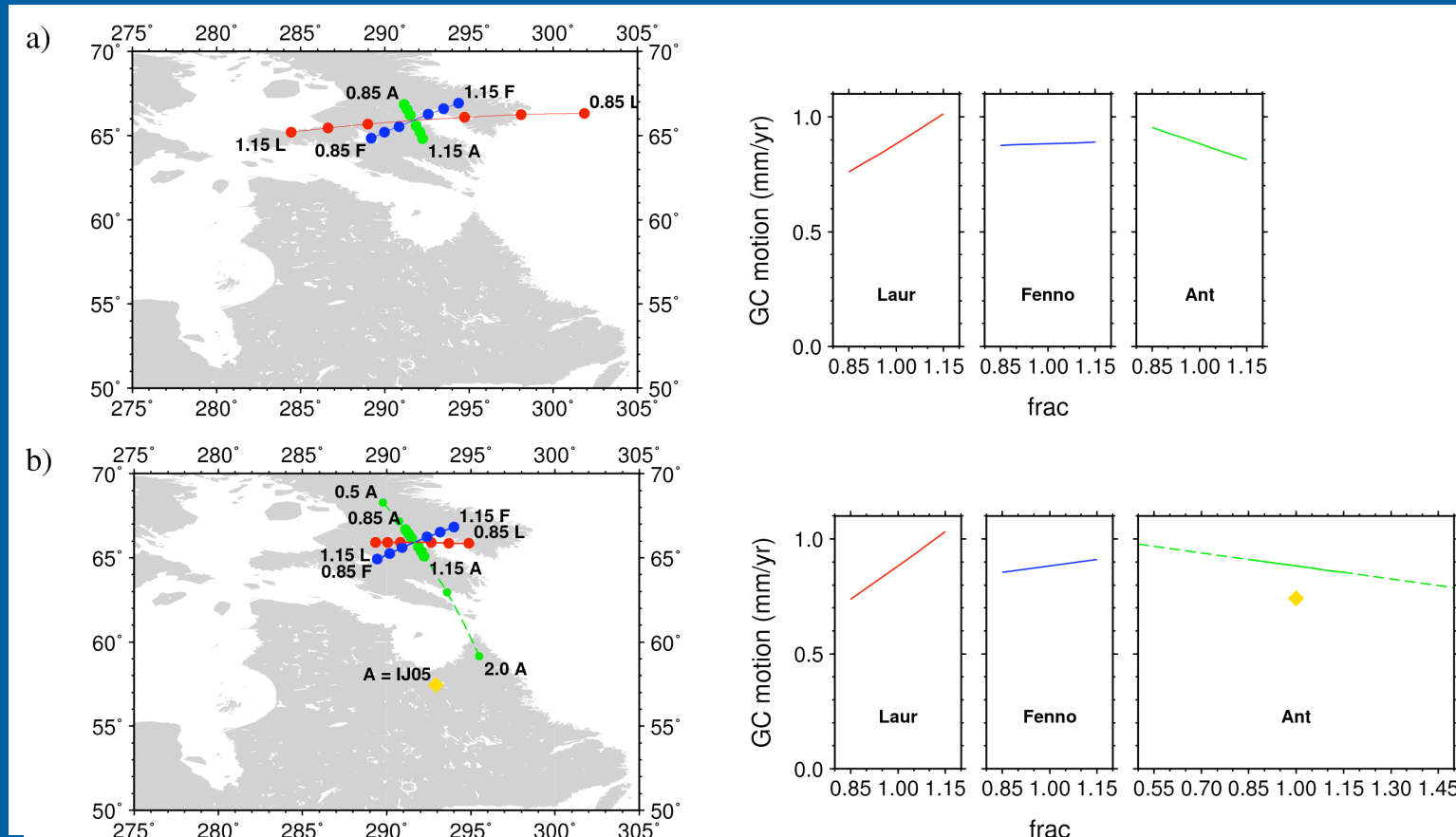
# Influence of glaciation history on GC motion



- Main areas of Pleistocene glaciation are
  - Laurentide
  - Fennoscandia
  - Antarctica
- Considered earth models
  1. LM+ ( $10^{22}$  Pa s)
  2. VM2 ( $4 \times 10^{21}$  Pa s)
- Experiments to analyse sensitivity
  - a) Variation of load thickness where total mass of ice is conserved
  - b) Variation of load thickness without conservation of ice mass



# Influence of glaciation history on model LM+

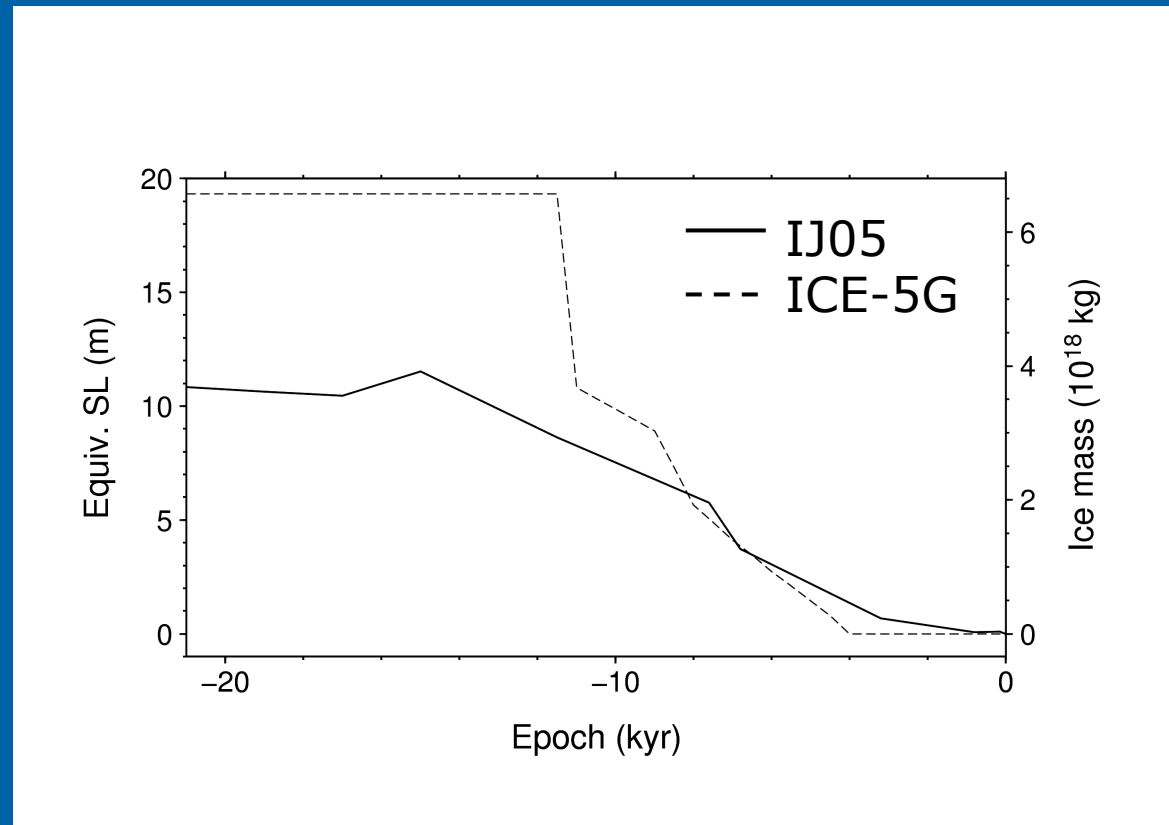


- Variation of Laurentide shows largest influence
- When total mass is conserved variations are larger
- Replacing Ant by IJ05 has strong influence

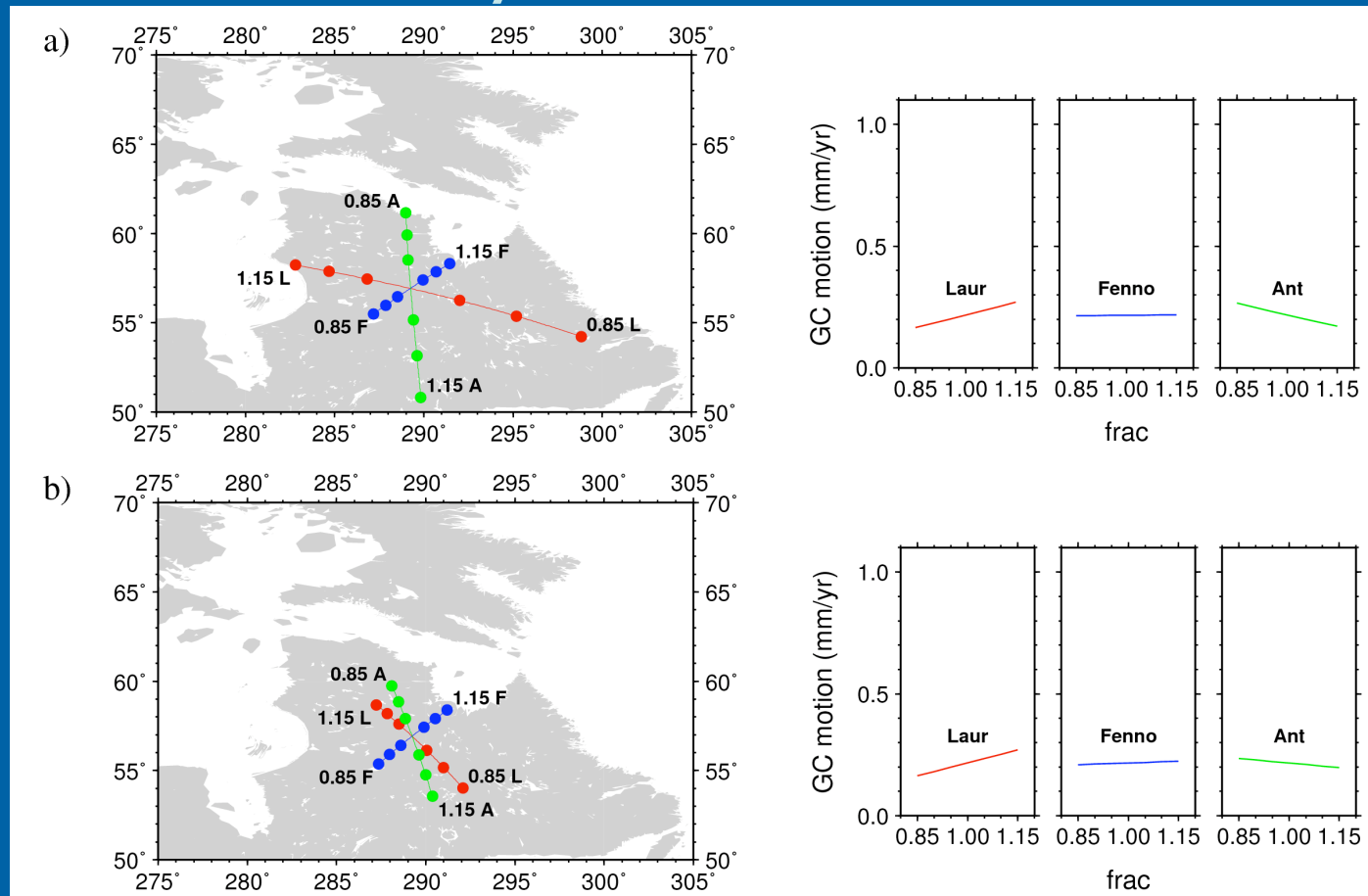
# Deglaciation history of Antarctica



- IJ05 (Ivins & James, 2005)
- Half the amount of ice than ICE-5G
- < 8 kyr, mass of IJ05 is larger than ICE-5G



# Influence of glaciation history on model VM2



- Pattern is similar
- Amplitudes are reduced due to the much smaller lower mantle viscosity



# Summary

- Predicted contribution of GIA to present time GC motion
  - is dominated by relaxation process
- Direction of motion is robust
  - towards western part of Hudson Bay
  - amplitude of  $\sim 0.1$  to 1 mm/yr
- Influence of mantle-viscosity structure
  - relaxation process in lower mantle is dominating
  - increase of viscosity from  $10^{21}$  to  $10^{22}$  Pa s increases geocenter motion from  $\sim 0.1$  to 1 mm/yr
- Influence of glaciation history
  - Laurentide ice sheet is dominant
  - Sensitivity to termination of Antarctic deglaciation

Klemann & Martinec (2009, Tectonophysics)



V Klemann,  
D Wolf  
GIA and solid  
earth modelling

# Thank you for your attention

