On estimation of flow magnitudes in planetary cores using magnetic observations

Chris Finlay[1] and Hagay Amit[2,3]

[1] ETH Zürich

[2] IPG, Paris

[3] LPG, Univ. Nantes

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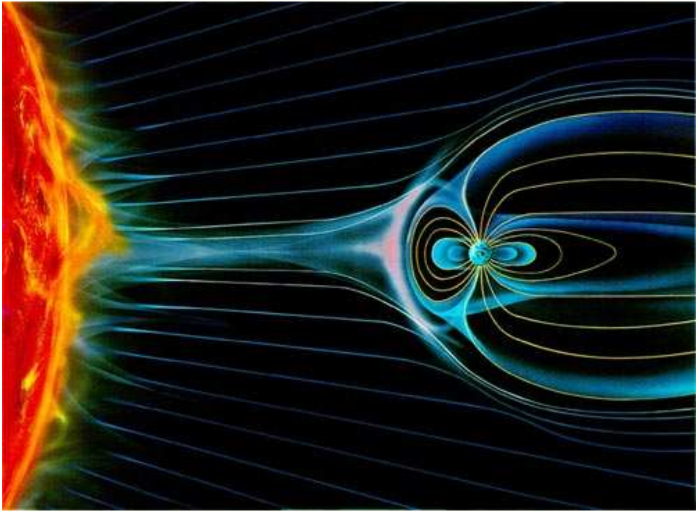
Talk Outline

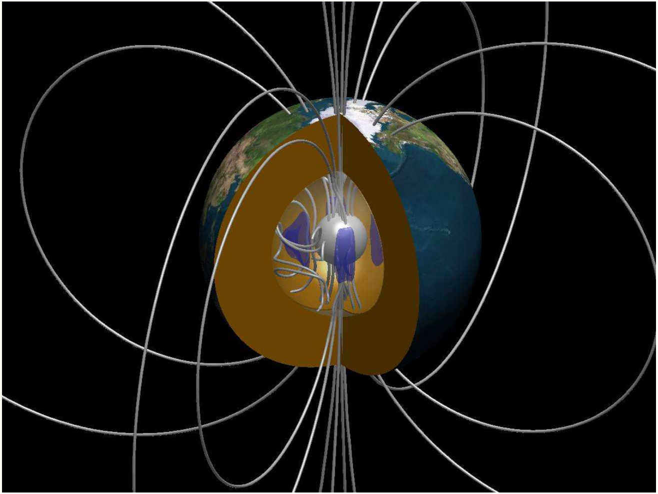
1. **Motivation**
2. Theory
3. Synthetic Tests
4. Global field models from magnetic observations
5. Flow magnitude in Earth's core
6. Discussion
7. Summary

Planetary Magnetic fields



- Mercury, Earth, Jupiter, Saturn, Uranus & Neptune possess strong internal magnetic fields that shield them against the solar wind.

Example: Dynamo located in Earth's core



(Image courtesy of Julien Aubert, IPG Paris)

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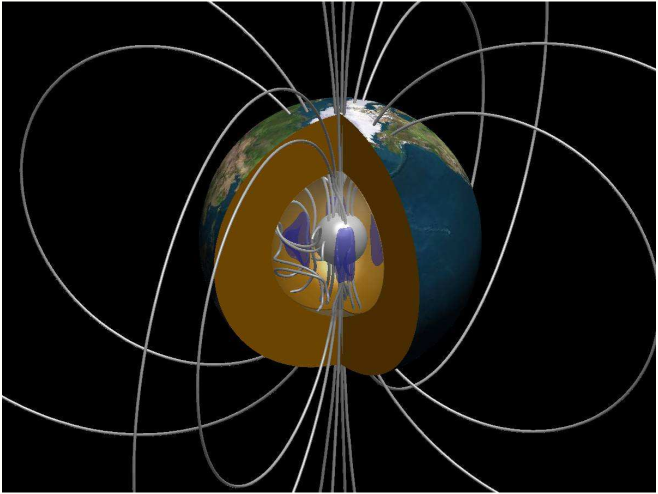

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- What type of motions occur in planetary cores to generate, sustain and cause the evolution of strong, global, magnetic fields?

Equations governing field generation and evolution

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- **Conservation of Momentum:**

$$\rho_0 \frac{\partial \mathbf{u}}{\partial t} + \rho_0 (\mathbf{u} \cdot \nabla)\mathbf{u} + 2\rho_0 \Omega(\widehat{\mathbf{z}} \times \mathbf{u}) = -\nabla P + \rho_0 \alpha g_0 T \frac{\mathbf{r}}{r_0} + \frac{1}{\mu_0}((\nabla \times \mathbf{B}) \times \mathbf{B}) + \rho_0 \nu \nabla^2 \mathbf{u}$$

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- **Heat transport:**

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \kappa \nabla^2 T$$

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  - Kinematics: e.g. Relative importance of advection vs diffusion

$$Rm = \frac{|\nabla\times(\mathbf{u}\times\mathbf{B})|}{|\eta\nabla^2\mathbf{B}|} = \frac{\mathcal{U}D}{\eta} \quad \text{Magnetic Reynolds Number}$$

- Need $Rm << 1$ for Frozen Flux Hypothesis

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- Or, if have magnetic observations of $\mathbf{B}$ and $\partial \mathbf{B}/\partial t$, then may obtain estimates of $\mathcal{U}$ via the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

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- **Is it possible to take another approach?**

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- Make simplifying assumptions & test using dynamo simulations.
- Utilize recent high quality satellite and observatory observations.
- Quantify range of plausible flow magnitudes.

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Simple theory of induction at core surface I

- Begin with the radial induction equation at the core surface

$$\frac{\partial B_r}{\partial t} + \mathbf{u_H} \cdot (\nabla_H B_r) + B_r(\nabla_H \cdot \mathbf{u_H}) = \eta \left[\frac{1}{r} \frac{\partial^2}{\partial r^2}(r^2 B_r) + \nabla_H^2 B_r \right]$$

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- Applying this operator, the RMS value of the radial secular variation at the core surface can then be written as

$$< \partial B_r / \partial t >=< \mathbf{u_H} \cdot (\nabla_H B_r) >$$

Simple theory of induction at core surface II

- From the definition of the scalar product

$$\cos\gamma = \frac{\mathbf{u_H}\cdot(\nabla_H B_r)}{|\mathbf{u_H}|\,|\nabla_H B_r|},$$

where γ is the angle between $\mathbf{u_H}$ and $\nabla_H B_r$.

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- Substituting we find

$$<\partial B_r/\partial t> = <|\mathbf{u_H}|\,|\nabla_H B_r|\cos\gamma>$$

- But if $|\mathbf{u_H}|$, $|\nabla_H B_r|$ and $\cos\gamma$ are spatially uncorrelated then

$$\boxed{<\mathbf{u_H}> = \frac{<\partial B_r/\partial t>}{<\nabla_H B_r><\cos\gamma>}}$$

Simple theory of induction at core surface III

- For a spherical harmonic representation of the main field and secular variation this can be written in terms of the Lowes spectra as,

$$\boxed{< \mathbf{u_H} >= \frac{\sqrt{\sum_{l=1}^{\infty} \frac{(l+1)}{(2l+1)} S_l}}{\sqrt{\sum_{l=1}^{\infty} \frac{l(l+1)^2}{c^2(2l+1)} R_l} \; < \cos\gamma >}}.$$

where

$$R_l = (l+1)\left(\frac{a}{c}\right)^{2l+1} \sum_{m=0}^{l} (g_l^m)^2 + (h_l^m)^2$$

and

$$S_l = (l+1)\left(\frac{a}{c}\right)^{2l+1} \sum_{m=0}^{l} (\dot{g}_l^m)^2 + (\dot{h}_l^m)^2$$

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Suite of numerical dynamo models for tests

Case	Ra	Ek	Pm	R_m	Ro
$M1$	$3 \cdot 10^5$	10^{-3}	4	110	$2.75 \cdot 10^{-2}$
$M2$	$1.5 \cdot 10^6$	$3 \cdot 10^{-4}$	2	96	$1.44 \cdot 10^{-2}$
$M3$	$3 \cdot 10^6$	$3 \cdot 10^{-4}$	3	296	$2.96 \cdot 10^{-2}$
$M4$	$8 \cdot 10^6$	$2 \cdot 10^{-4}$	3	487	$3.25 \cdot 10^{-2}$
$M5$	$1.5 \cdot 10^7$	$1 \cdot 10^{-4}$	2	329	$1.65 \cdot 10^{-2}$
$M6$	$8 \cdot 10^6$	$1 \cdot 10^{-4}$	2	177	$8.85 \cdot 10^{-3}$
$M7$	$1.5 \cdot 10^7$	$1 \cdot 10^{-4}$	4	617	$1.54 \cdot 10^{-2}$
$M8$	$1.2 \cdot 10^8$	$3 \cdot 10^{-5}$	2.5	876	$1.05 \cdot 10^{-2}$
$M9$	$7.5 \cdot 10^6$	$2 \cdot 10^{-4}$	0.5	51	$2.04 \cdot 10^{-2}$
Earth	10^{20}	$3 \cdot 10^{-14}$	10^{-5}	$400 - 4000$	$10^{-6} - 10^{-5}$

- Calculated using pseudo-spectral dynamo code of Wicht (2002).
- $Pr = 1$ for all models considered.
- Models span at least order of magnitude in Ra, Ek and Pm.
- Models approach what is believed to be Earth-like Rm and Ro.

Results of tests: $< \cos\gamma >$ and $< \mathbf{u_H} >$

Case	$< \cos\gamma >$	$< \mathbf{u_H} >_{calc} / < \mathbf{u_H} >_{true}$
$M1$	0.633	0.849
$M2$	0.628	0.962
$M3$	0.661	0.902
$M4$	0.677	0.985
$M5$	0.667	0.989
$M6$	0.644	1.026
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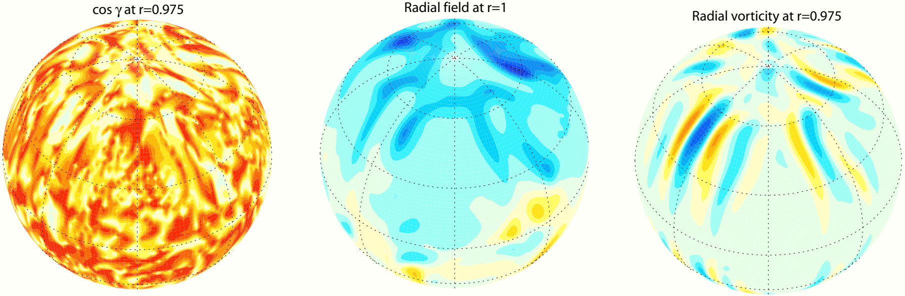
- Method typically retrieves true flow magnitude to within 10%.

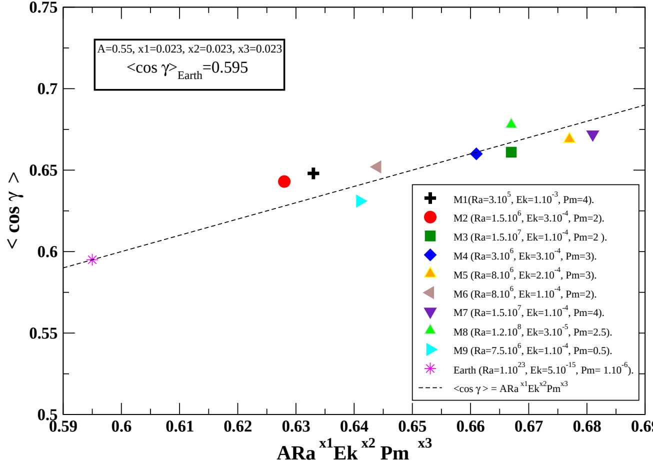
Spatial variations in $|\cos\gamma|$



(Snapshot from model M2)

- Bimodal distribution of $<\cos\gamma>$.
- Low $<\cos\gamma>$ e.g. at helical convection columns.
- High $<\cos\gamma>$ e.g. at drifting low latitude flux concentrations.
- No systematic correlation between $|\nabla_H B_r|$, $|u_H|$ and $\cos\gamma$.
- On average, $\gamma = 40 - 50^\circ$.

Variation of $<\cos\gamma>$ with control parameters



- Weak dependence on control parameters: $(Ra \cdot Ek \cdot Pm)^{0.023}$.
- For control parameters appropriate to Earth's core $< \cos\gamma > \sim 0.6$.

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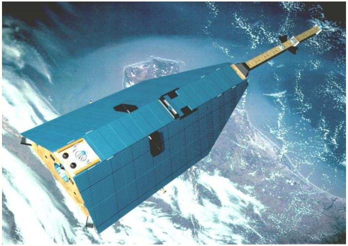
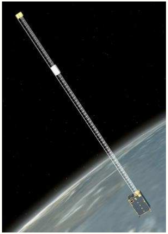

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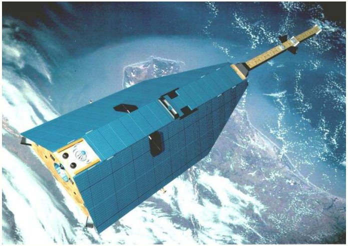

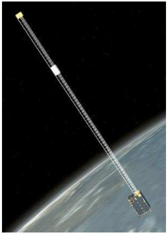

- How are these observations used to obtain core surface field models?

Time-dependent field modelling

- Deterministically correct data for magnetospheric and crustal magnetic field; then model as potential field with internal source,

$$V(r,\theta,\phi,t) = a\sum_{l=1}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+1} g_l^m(t)\,Y_l^m(\theta,\phi),$$

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- Find model that minimizes following cost function at core surface,

$$\Theta = [\mathbf{d} - \mathbf{f}(\mathbf{m})]^T \mathbf{C}_e^{-1} [\mathbf{d} - \mathbf{f}(\mathbf{m})] + \mathcal{R}(\mathbf{m})$$

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- $\mathcal{R}(\mathbf{m})$ is a norm measuring spatial + temporal complexity at CMB.

A core suface regularized model for 21st century

- Preliminary model:
  - CHAMP, Ørsted and SAC-C: xCHAOS datatset of N. Olsen.
  - Data span 2000-2008
  - Quiet-time, night-side, vector data only < 60 deg geomag. lat.
  - Sub-sample to get quietest data on equal area grid, reset every 0.2yrs
  - Regularization: entropy norm in space and a curvature norm in time.
  - L1 norm measure of misfit (IRLS).
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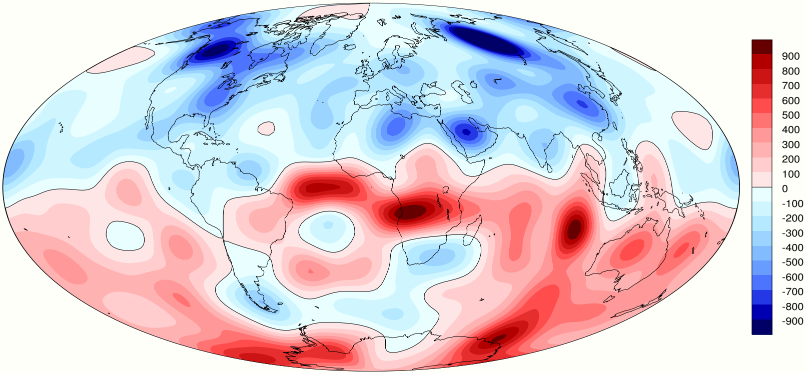
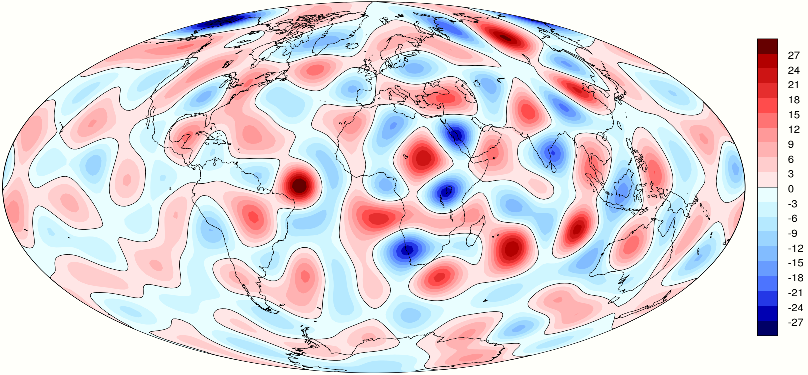
- In next section results with the preliminary model are presented; tests with other models (e.g. GRIMM, xCHAOS) give similar results.

Radial field at core surface in 2004

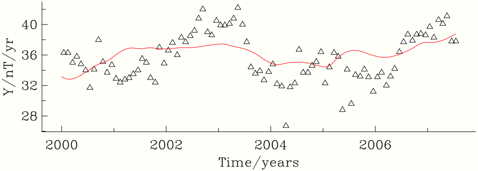


Secular variation at core surface in 2004



Fit to observatory data

- NGK



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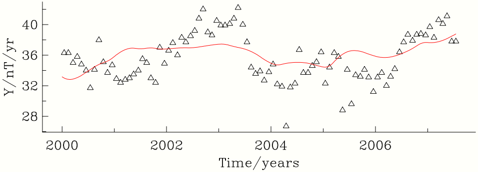

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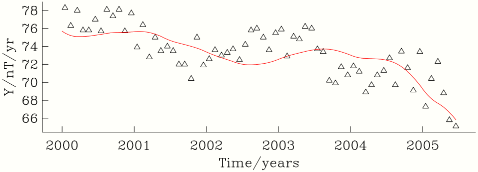

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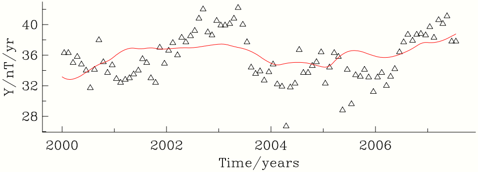

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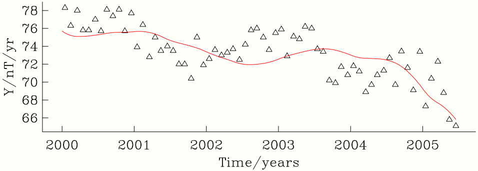

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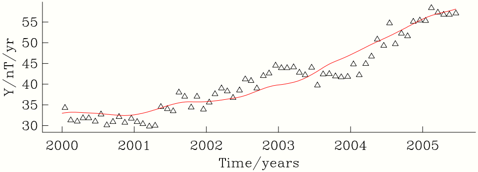
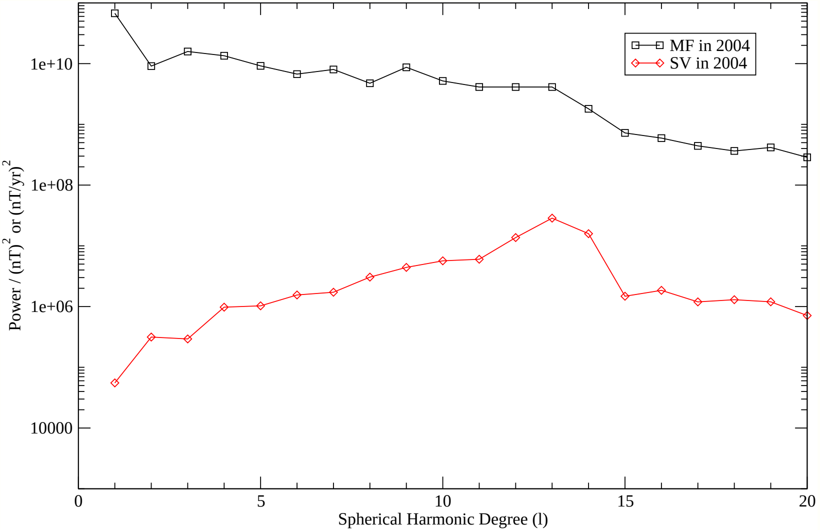


Spectra at core surface



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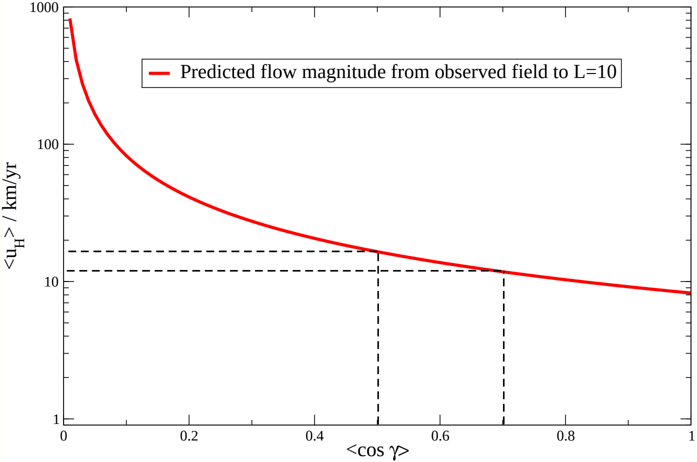
Variation of $<\mathbf{u_H}>$ estimate with $<\cos\gamma>$



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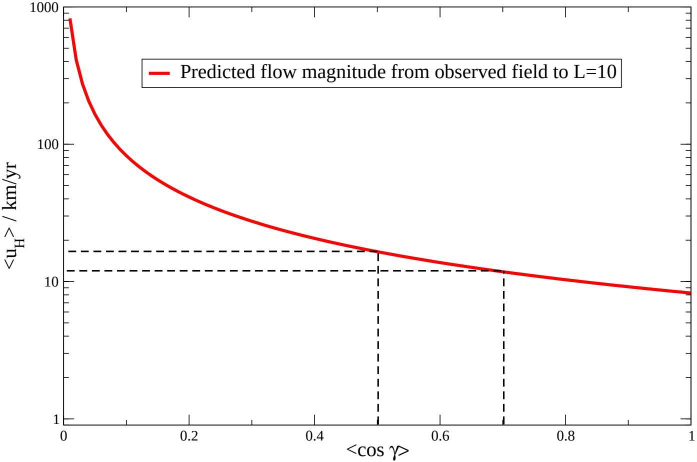
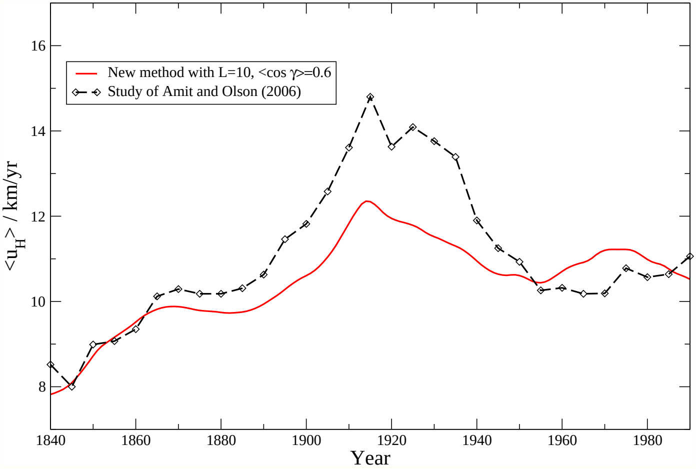


- $< \mathbf{u_H} >= 13.6$km/yr for range of $< \cos\gamma >= 0.6$.
- $< \mathbf{u_H} >= 10 - 17$km/yr for range of $< \cos\gamma >= 0.5 - 0.7$.

Temporal variation $<u_H>$ ($L = 10$,$<\cos\gamma>$=0.6)



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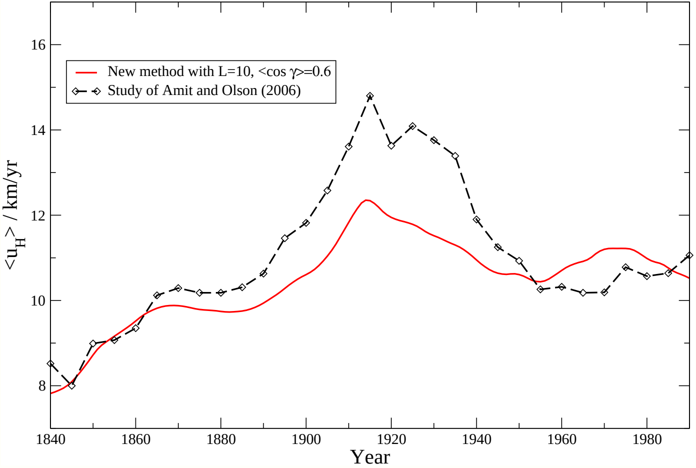

- For 1840-1990 (*gufm1*) obtain similar variations to previous studies.
- Can use method to monitor variations in magnitude of core flow.

Variation of $<u_H>$ with truncation degree L

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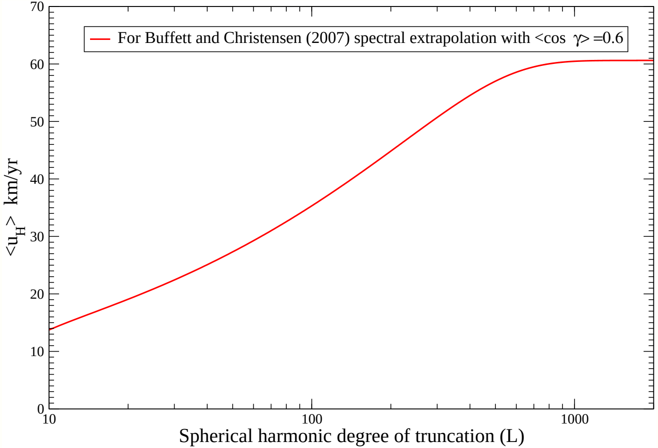

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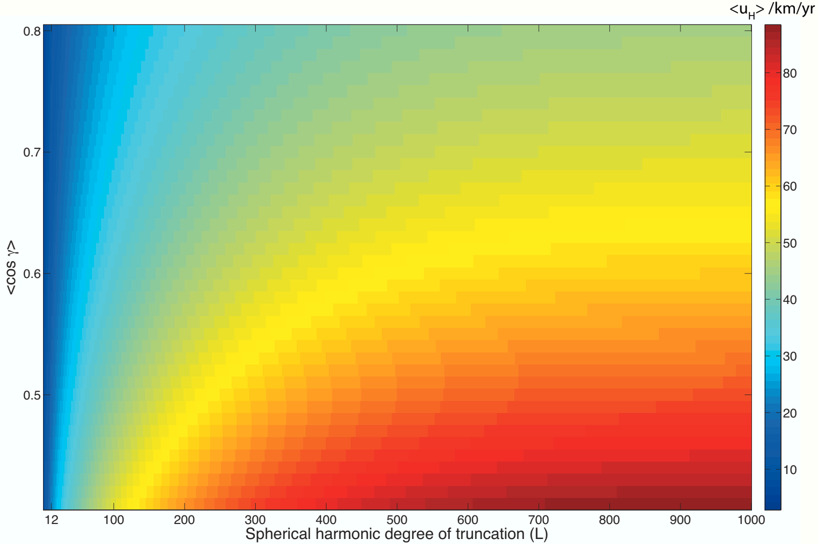

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- Estimated upper limit: $< \mathbf{u_H} >\sim 70$ km/yr

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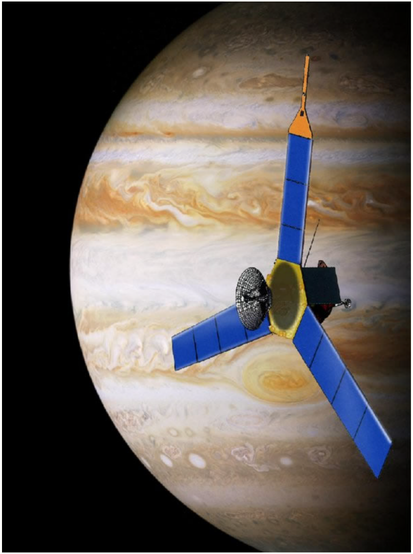
- **Can we apply the method to other planets?**

Application to other planets: Jupiter



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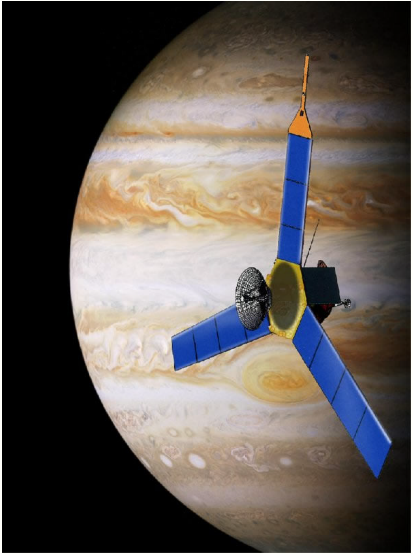

- JUNO (NASA), orbiter mission to arrive in 2016.

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- Bepi Colombo (ESA): two orbiters to reach Mercury in 2019.

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- Method could in future be applied to other planets.

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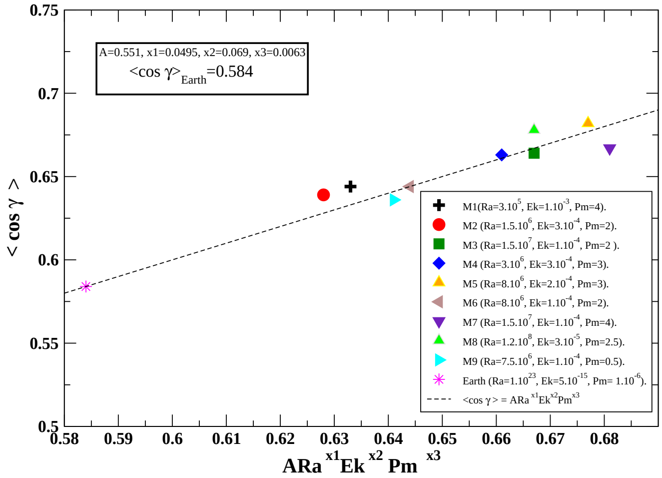
3 parameter scaling law



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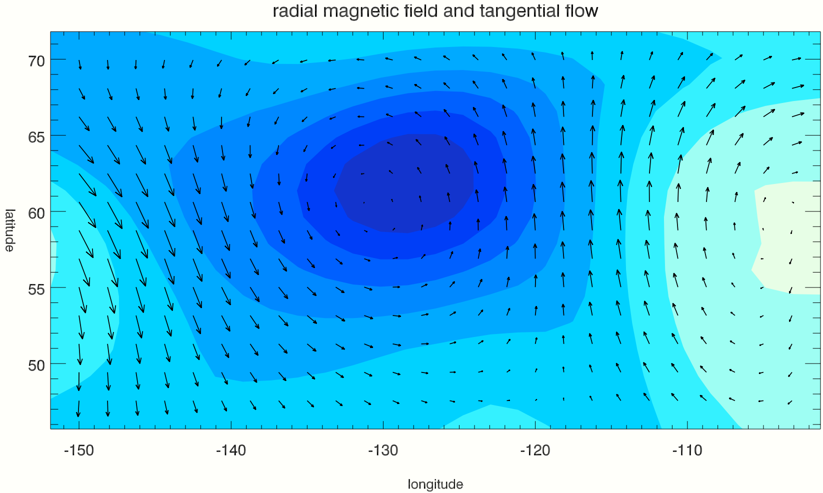
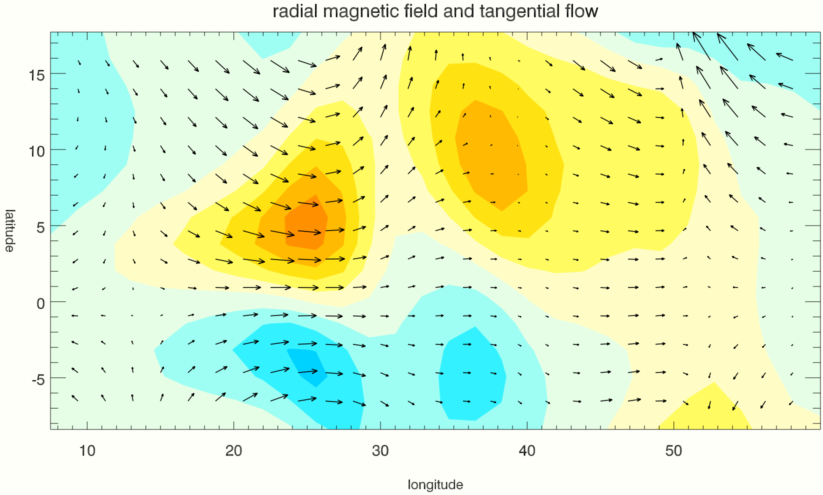
- Simulations have $Ra \cdot Ek \cdot Pm$ comparable to planetary cores.

Near high latitude columnar convection rolls in M2



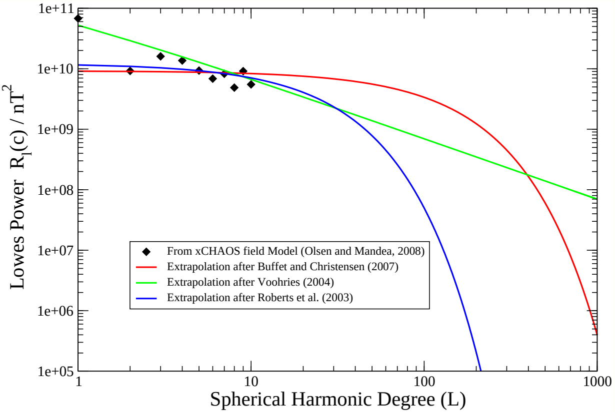
- Flow often tends to be approximately aligned with contours of B_r.

Rapidly drifting flux features at lower latitudes



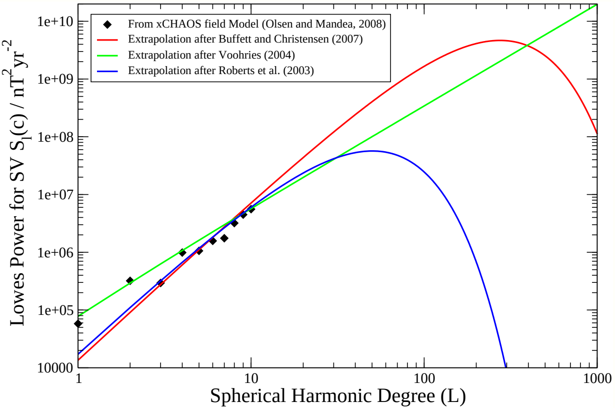
- Flow often tends to be across with contours of B_r.

Extrapolation of main field spectrum



- Explored 3 possible empirical extrapolations of main field spectra.
- RED: $R_l(c) = \bar{R}\chi^l$ (Buffett & Christensen, 2007).
- GREEN: $R_l(c) = K(l + 1/2)/l(l + 1)$ (Voohries, 2004)
- BLUE: $R_l(c) = Aexp(-Bl)$ (Roberts et al., 2003)

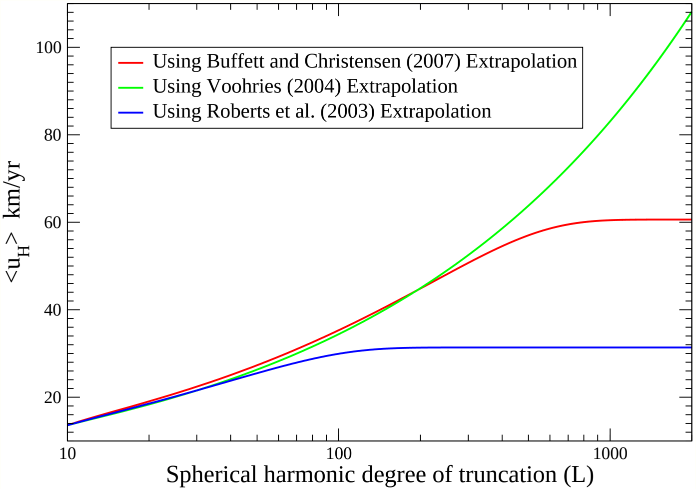
Extrapolation of secular variation spectrum



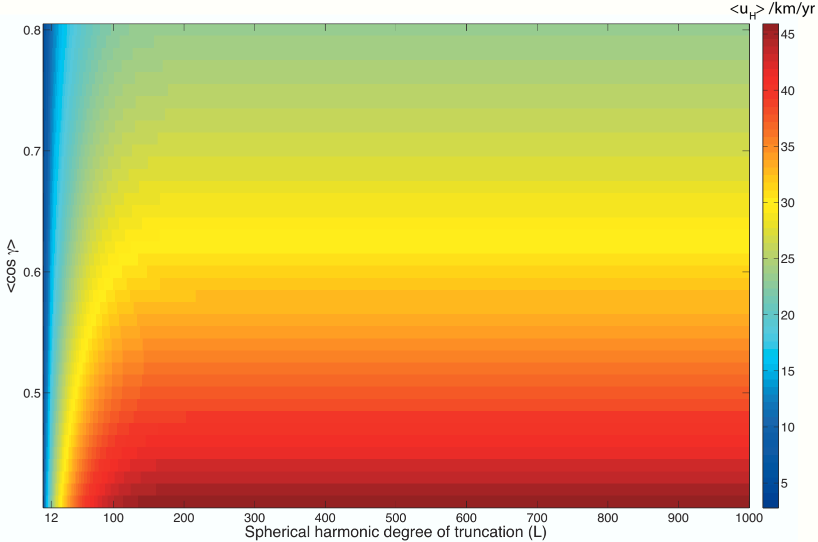
- SV spectra obtained by assuming relation between MF and SV at large scales continue to hold out to the dissipation scale.
-

$$\tau_L = \sqrt{\frac{R_L(c)}{S_L(c)}} = CL^{-D}.$$

Variation of $<u_H>$ with truncation degree L

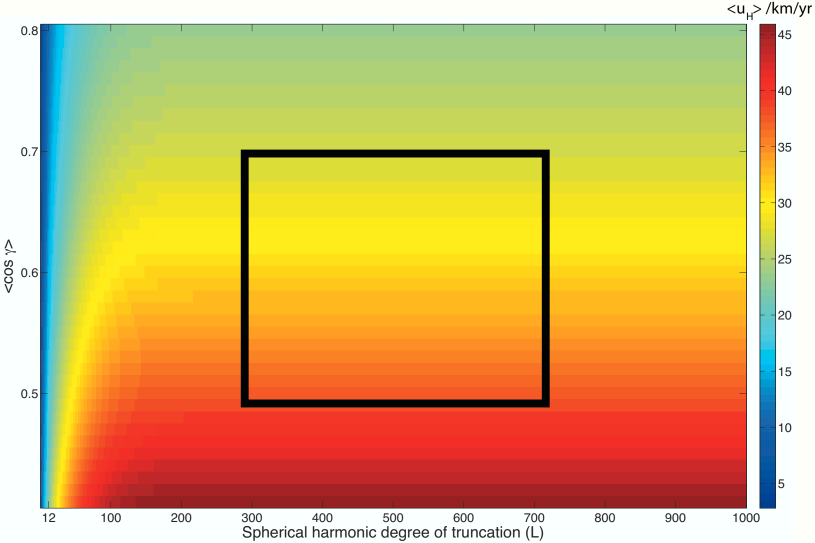


Variation of $\langle u_H \rangle$ with $\langle \cos\gamma \rangle$ and L



For MF extrapolation:

Variation of $\langle u_H \rangle$ with $\langle \cos\gamma \rangle$ and L



For MF extrapolation: $R_l(c) = Aexp(-Bl)$ (Roberts et al., 2003)

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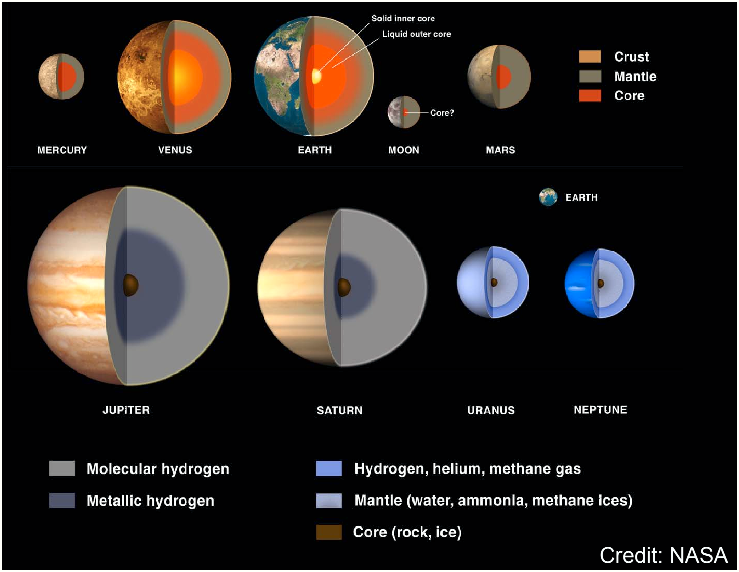
- Scaling law for $< \cos\gamma >$ from suite of numerical dynamos.
- Sufficient magnetic observations to model MF and SV spectra.
- Estimate of magnetic dissipation scale required.
- Range NOT formal limits, but provide quantified estimate.

Planetary composition



3.1 Field Modelling Methodology

- Assuming the region between the satellite observations and the CMB is to a first approximation an insulator then we write,

$$\mathbf{B} = -\nabla V$$

- Then since the magnetic field is divergent free, V satisfies Laplace's equation and it can be written as a sum of spherical harmonics:

$$V(r,\theta,\phi,t) = a\sum_{l=1}^{L}\left(\frac{a}{r}\right)^{l+1}\sum_{m}^{l}[g_l^m(t)\cos m\phi + h_l^m(t)\sin m\phi]P_l^m(\theta)$$

- We choose to expand the Gauss coefficient in a basis of cubic B-splines,

$$g_l^m(t) = \sum_{n=1}^{T} g_l^{mn} B_n(t)$$

- A knot spacing of 0.2yrs and maximum SH degree L=24 are employed.

- A well-converged CMB field solution is then sought by mimimising the cost function:

$$\Theta = [\boldsymbol{d} - \boldsymbol{f}(\boldsymbol{m})]^T \boldsymbol{C_e}^{-1} \boldsymbol{W_f} [\boldsymbol{d} - \boldsymbol{f}(\boldsymbol{m})] + \mathcal{R}(\boldsymbol{m})$$

- The weight matrix is iteratively updated (IRWLS) to implement an L1 misfit measure.

3.2 Specification of regularization norms

- The regularization added to the cost function can be expressed as:

$$\mathcal{R}(\boldsymbol{m}) \quad = \quad \frac{\lambda_S}{(t_e - t_s)} \int_{t_S}^{t_e} \int_{\mathrm{CMB}} R_S(\boldsymbol{m}) \; d\Omega dt \quad + \quad \frac{\lambda_T}{(t_e - t_s)} \int_{t_S}^{t_e} \int_{\mathrm{CMB}} R_T(\boldsymbol{m}) \; d\Omega dt$$

- The regularization added to the cost function are chosen to take the form:

Model QQ: $R_S(\mathbf{m}) = B_r^2$ $\qquad R_T(\mathbf{m}) = \left(\frac{\partial^2 B_r}{\partial t^2} \right)^2$

Model EQ: $R_S(\mathbf{m}) = -4d \cdot S(B_r)$ $\qquad R_T(\mathbf{m}) = \left(\frac{\partial^2 B_r}{\partial t^2} \right)^2$

where $S(x) = \psi{-}2d{-}x\, ln \left(\frac{\psi + x}{2d} \right)$ with $\psi = \sqrt{x^2 + 4d^2}$ and $d =$ 'default magnitude of x'

- Regularization in space needed to: (i) Ensure convergence at CMB
  (ii) Choose minimum norm soln
  (Shure, Parker and Backus, 1982)

- Regularization in time necessary to ensure that time evolution at CMB is smooth and that SV and SA are well converged.

3.3 Why use maximum entropy methods?

• Classical quadratic regularization tends to over-smooth images by assigning low probability to models with sharp contrast.

• Ed Jaynes (1957) set out the rationale for using a maximum entropy method to assign probabilities in the lack of other information:

> “ the maximum-entropy estimate ... is the least biased estimate possible on the given information; i.e. it is maximally non-committal with regard to missing information”

Jaynes (1957), Physical Review, Vol 106, pp 620-630.

• The maximum entropy method has been applied with great success in diverse areas e.g. **astronomy, image processing and medical tomography.**

• It was introduced to geomagnetism by Jackson (2003) we implement it using the method of Gillet et al., (2007).

3.3 Why use maximum entropy methods?

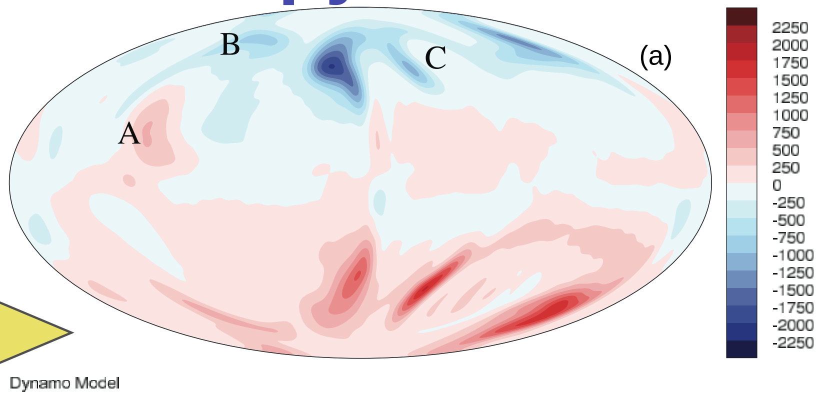
Image Processing
(a) Original
(b) With Gaussian noise
(c) With 50% loss
(Gull & Skilling, 1984)

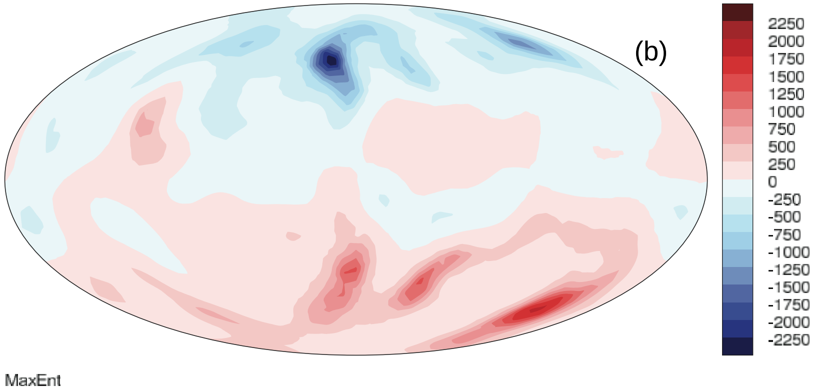
CMB Field Modelling
(a) Original (dynamo)
+ crustal field
(b) Maxent retrieval
(c) Quadratic retrieval
(Jackson et al., 2007)

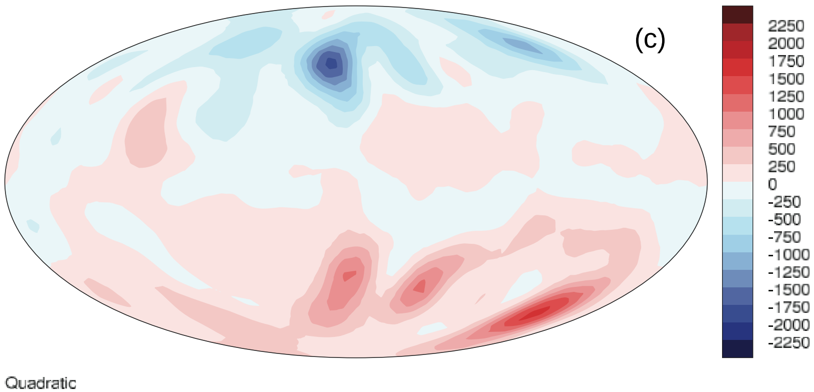
(a)


(b)

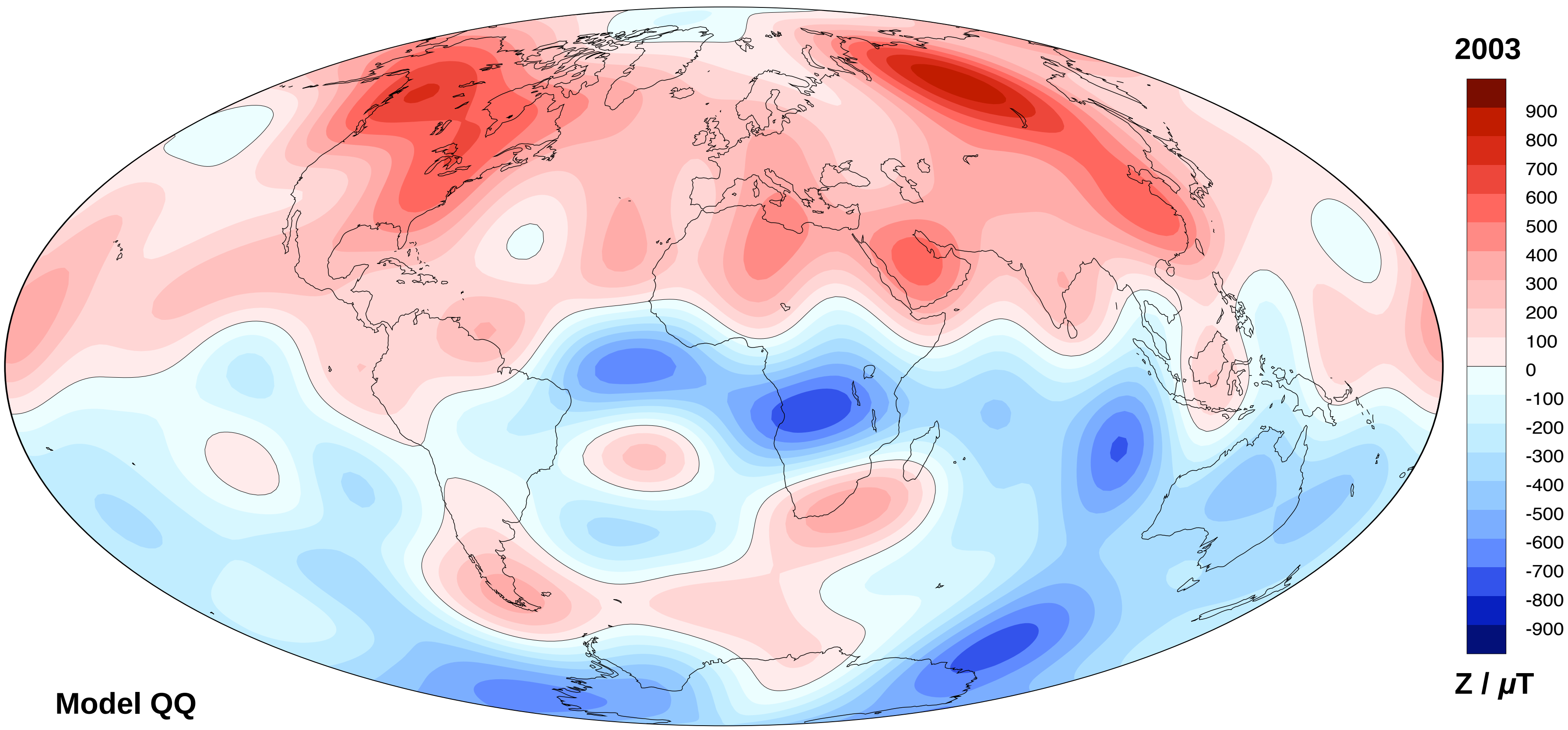

(c)





4.3 CMB field: Z component



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