Statistical Physics of Steady-State Two-Phase Flow in Porous Media

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Declining oil production, increasing energy consumption.

Paul Roberts: Arguably the most serious crisis ever to face industrial society. The End of Oil: On the Edge of a Perilous New World (2004)

Low-permeability carbonate rock: All fluid transport through fractures. But where are the fractures, and are there enough of them?

Tar sands of Alberta: Frozen goo.

Recovering stuck oil: EOR
Flooding Experiments

Instabilities: Transients

\[ M = 0.0001 \]

- \( Ca = \text{capillary number} = \frac{\text{viscous forces}}{\text{capillary forces}} \)
- \( M = \text{viscosity ratio} \)

\( Ca = 0.029 \)  \( Ca = 0.059 \)  \( Ca = 0.22 \)

(Måløy, Univ. Oslo)
Steady State Flow

What enters a RVE is statistically the same as what leaves it.

An RVE inside a reservoir will typically experience *steady-state conditions*, and not the transients in a flooding experiment.
Steady state

signifies that apart from statistical fluctuations, the system remains constant – or changes slowly on the scale of the fluctuations.

Hence, even if both fluids move and fluid clusters break up and merge, we will be dealing with a steady state as long as the averaged macroscopic parameters describing the flow remain constant (or change slowly on the time scale of the fluctuations).

A setup for studying steady-state flow in the laboratory:
Clusters in spatially homogeneous steady-state flow

Clusters have been artificially colored.

Tallakstad, Måløy

www.ntnu.no
Old wisdom:

- Equilibrium phenomena are easy
- Non-equilibrium phenomena are difficult

Steady state

Flooding

- Equilibrium thermodynamics is 150 years old
- Non-equilibrium thermodynamics is still being developed

Porous media:
- Flooding has been studied for a long time
- Steady-state flow has been largely neglected
A Numerical Model

Ca = 0.0046
M = 0.001

Ca = 0.0046
M = 100

Ca = 0.000046
M = 1

(Aker et al., 1998)
Dynamics at the pore level: How do the interfaces move?
Coalescence and Instabilities: Snap-off
Coalescence and Instabilities: Coalescence
Steady-state flow implemented on a torus

(Ramstad, 2006)

Biggest non-wetting cluster.

Flow Direction
Starting from a band of red fluid, the interfaces initially develop capillary fingering and stable displacement.

\[ Ca = 0.000032 \]
\[ M = 1 \]
Further on in the process:
Steady-state configuration is independent of initial preparation of system: It is a state.

(It is independent of initial conditions given the flow is fast enough for none of the fluids to get stuck due to capillary forces.)

Flow direction

Drainage and imbibition are now microscopic concepts.
Reconstructed pore networks

Pore Network from Berea Sandstone

Each pore is described by a number of geometric parameters.

Reconstruction by e.g. merging thin slices
Setting up steady-state flow in reconstructed network:

Must make the non-periodic structure periodic.
Making the three-dimensional network periodic in the flow direction:

(Ramstad, Øren, Hansen)
Evolution towards steady-state flow

\[ \text{Ca} = 0.015, \quad M = 1, \quad S = 0.5 \]

non-wetting saturation
Largest non-wetting clusters at different saturation levels in steady state

Critical saturation

Ca=0.015, M=1

S=0.59

S=0.65

S=0.67

S=0.71
Cluster size distribution corresponding to the saturations just shown.

At criticality

\[ N(s) \propto s^{-2.27 \pm 0.08} \]

Cluster size

Average over 20 samples
Relative density of largest cluster $P_\infty$ vs. saturation $S$.

Critical saturation vs. capillary number:

$$S_c = A + B \log Ca$$

Interesting consequences for radial flow
Parallel flow: Instabilities?
Kelvin-Helmholtz Shear Instability?

http://www.fluent.com/about/news/newsletters/04v13i2/a6.htm
Key: viscous fingering.
Non-wetting saturation profile normal to flow direction as function of time.
The derivative of the non-wetting saturation profile.

This defines a *mixing length scale* $\lambda$. 
Profile is stable and moves towards the non-wetting fluid: effectively an imbibition process.
Steady state in a reference frame moving with constant speed to the right.
Returning to the concept of a state.

A state is described by a small number of macroscopic parameters.

When the system is in such a state, it evolves through following a path in the space of all possible microscopic configurations such that the macroscopic averages remain constant. The equations that govern the path are the microscopic flow equations.

These equations are very hard to solve (e.g. numerically). But, they provide far too much information!

This information is needed to understand transients, but not to describe the dynamical steady state.

Is there a way to get the needed information to describe macroscopically the steady state without having to resort to complete overkill?
Sequence of configurations through time integration:

1 → 2 → 3 → 4

Δt

The order of the configurations has been randomized:

3 → 1 → 4 → 2

Δt

Does this randomization change the statistics?
If order plays no role: All steady-state properties will be completely described by the configurational probability distribution $W\{cf\}$ where $\{cf\}$ signifies the positions of all interfaces between the immiscible fluids in the porous medium.

A configuration is fully described by the position of all interfaces.

This leads to the possibility to exchange time integration by more efficient methods.
Metropolis Monte Carlo Sampling

Configurational probability

\[ W\{cf\} \]

Old configuration \( \rightarrow \) Test configuration

\( \{cf_{\text{old}}\} \) \( \rightarrow \) \( \{cf_{\text{test}}\} \)

Chosen by random change of old configuration.

Draw a random number \( r \in [0,1] \).

If \( W\{cf_{\text{old}}\}/W\{cf_{\text{test}}\} > r \): Reject test configuration.

If \( W\{cf_{\text{old}}\}/W\{cf_{\text{test}}\} \leq r \): Accept test configuration.
We need to know what $W_{\text{cf}}$ is?

Assumption:

All configurations with same energy are equally probable.

But, what is energy?

Abstract

The properties of granular materials can be well defined, that is be a branch of physics, but conventional statistical mechanics is inadequate to handle what amounts to the physics of disordered packings of hard-core particles, either static or driven by external forces. A new approach that employs statistical-mechanical concepts is offered for the description of such systems. An analysis of the stress field in static granular packings is given within the framework of this approach. There are more conventional systems such as polymer glasses which have a rather similar statistical physics to granular media, and some speculative ideas are offered which are a real departure from conventional glass theories.
Statistical mechanics of steady-state flow

Microscopic energy function:

\[ E\{\text{cf}\} = (1/2) \min \sum_{<ij>} (p_i - p_j - p_{ij}^c)^2/\mu_{\text{eff}}^{ij} \]

Pressure field solves the Kirchhoff equations.

Effective viscosity

Capillary pressure

Temperature \(1/T = d\Sigma/dE\)

Boltzmann distribution

Configurational entropy \(\Sigma = \log N(E)\)

Basic assumption

(All configurations with same energy are equally probable.)

\[ W\{\text{cf}\} = W(E\{\text{cf}\}) \]
All of thermodynamics now follows.

Partition function:

\[ Z = \sum \{cf\} \ W\{cf\} \]

Average energy:

\[ E = \sum \{cf\} \ E\{cf\} \ W\{cf\} / Z \]

\[ E = E(\Sigma, P) \]

\[ (\partial E/\partial P)_\Sigma = - Q \]

\[ (\partial E/\partial \Sigma)_P = T \]

Thermodynamic variables:

Pressure P
Total flow rate Q
Saturation S
Fractional flow f
Energy E
Capillary pressure \(P^c\)
Entropy \(\Sigma\)
Temperature T
Viscosity ratio M
Free energy \(G = -T \log Z\)

...
Numerical model: *Controlling P (or Q) and saturation S.*

Tallakstad, Måløy, Univ. Oslo: *Controlling P (or Q) and fractional flow f.*

Monte Carlo: 
\[ S = S(T, P) \]
\[ f = f(T, P) \]
0th Law of Thermodynamics:

System A in equilibrium with system B, System B in equilibrium with system C  
⇒ System A in equilibrium with system C.

⇒ existence of parameters describing the systems which are equal when they are in equilibrium with each other: Temperature T.
Temperature:

Boltzmann weight:

\[ W_{\{cf\}} = \exp[-(1/2T) \min \Sigma_{<ij>} (p_i - p_j - p_{ij}^c)^2/\mu_{ij}^{eff}] \]

\[ W_{\{cf\}} = W(P, p^c, T) \]

Viscous pressures are all proportional to pressure \( P \)

Rescaling:

\[ W(P, p^c, T) = W(P/T^{1/2}, p^c/T^{1/2}, 1) \]

Setting \( T \propto P^2 \)

Equation of state

\[ W(P, p^c, T) = W(1, 1/Ca, 1) \]

Only dynamical parameter is Capillary number (besides \( M \)).
1st Law of Thermodynamics

\[ dE = \delta Q - \delta W \]

- Reversible work related to change in flow rate \( Q; P \, dQ \).
- Irreversible work on the microscale related to creating new interfaces in the pores.

2nd Law of Thermodynamics

In an isolated system, a process can occur only if it increases the entropy of the system.

The steady state is at maximum entropy.
3rd Law of Thermodynamics

\[ T \to 0 \implies \Sigma \to 0 \]

The zero-temperature state, i.e., when \( P = 0 \), the state with the least number of interfaces is the optimal one. This is when the two fluids are fully separated. This state has zero entropy.
A thermodynamic equation (Not yet a complete story)

Fractional flow as a function of saturation $S$.

Pressure difference as a function of saturation $S$.

Keeping total flow rate $Q$ constant.

(Knudsen and Hansen, 2002, 2004)
Fig. 1 - The figure shows that the relation between pressure and the derivative of the fractional is of the form in eq. (2). For these curves the viscosity ratio $M = 1$. Simulations with four different capillary numbers are presented for 2D, and two different capillary numbers for 3D. The nonwetting saturation $S_{nw}$ serves as parameter for the curves, and the intervals in which the data is plotted are as follows: (2D) $Ca = 3.2 \times 10^{-2}$: $S_{nw} \in (0.14,0.93)$, $Ca = 1.0 \times 10^{-2}$: $S_{nw} \in (0.14,0.83)$, $Ca = 3.2 \times 10^{-3}$: $S_{nw} \in (0.14,0.83)$, $Ca = 1.0 \times 10^{-3}$: $S_{nw} \in (0.20,0.73)$; (3D) $Ca = 1.0 \times 10^{-2}$: $S_{nw} \in (0.10,0.90)$, $Ca = 1.0 \times 10^{-3}$: $S_{nw} \in (0.20,0.75)$. Valid for all $M$. 

$$P = A \frac{dF}{dS}Q + B$$

At constant $Q$
Reminder: Buckley-Leverett equation

\[- \left( \frac{dF}{dS} \right) \left( \frac{\partial S}{\partial x} \right) = \left( \frac{A \phi}{Q} \right) \left( \frac{\partial S}{\partial t} \right)\]

Derived by assuming conservation of Saturation in a one-dimensional tube. The equation

\[P = A \frac{dF}{dS} + B\]

cannot be gotten from this equation nor from the fractional Darcy equations.

This is a new equation.
One more observation

\[ P = CF^2 + DF + E \]

dF/dS has a maximum for some F.

Fig. 2 – The figure shows fitted curves to \( P(F_{nw}) \) according to eq. (3). The results are from the same series of simulations as the curves in 2D in fig. 1, only showing results for two lower capillary numbers. The shown region of \( F_{nw} \) is the one where the fit is good. For these capillary numbers this region is large, for the two higher values of \( \text{Ca} \) shown in fig. 1, the region of good fit is \( \text{Ca} = 3.2 \times 10^{-2} ; F_{nw} \in (0.15, 0.45) \), \( \text{Ca} = 1.0 \times 10^{-2} ; F_{nw} \in (0.10, 0.50) \). The data is from five realizations of the porous network giving slightly shifted curves as we can see. The scattering of the data becomes larger for smaller capillary numbers due to increased hysteresis and history effects.
Experimental results from the literature

This is the analytical form of the fractional flow vs. saturation curve.

This corresponds to having an analytical form for the relative permeability curves.

Fig. 6 – Data from the literature: Sharma and Yen (SY) [21], Peters and Khataniar (PK) [7], Braun and Blackwell (BB) [5, 22]. The water saturation is also the wetting saturation $S_w$. Different normalizations of $S_w$ in the respective papers decide the placement of the curves. The SY curves are typical samples of the set of functional forms that are employed when discussing how various physical parameters influence fractional flow. Regarding PK and BB curves, see fig. 7. All the curves are fitted to the solution of eq. (1) as is given in eq. (4).
More Phase Transitions in the Steady State  (2D system)

(Single-phase flow of wetting fluid)

(Single-phase flow non-wetting fluid)

Fig. 2. The stepwise simulation of two phase transitions. In both cases $Ca = 1.00 \times 10^{-3}$ and $M = 10$. The saturation is changed in steps up and down past the transition. The system is run for 7 s (physical time) at each step. (a) Starting at single-phase wetting flow, increasing the saturation above the transition to two-phase flow, and then lowering the saturation back to the initial level, the system shows hysteresis. (b) Starting in the two-phase region and increasing saturation to single-phase nonwetting flow, and decreasing again, no hysteresis appears.

(Knudsen and Hansen, 2006)
Three-dimensional parameter Space:

- Saturation S
- Viscosity Ratio M
- Capillary Number Ca

Both fluids move

Only one fluid moves

Fig. 3. The phase diagram for six selected values of Ca. The x-marks are the transition points from the simulations. The uncertainty in each point is not marked, but it is substantial. The scattering of points in parameter space gives an indication of the uncertainty. Phase boundaries are indicated by solid lines, see Section 4. The phase diagrams are divided into three regions, counting from the left hand side: single-phase wetting flow, two-phase flow, and single-phase nonwetting flow.

Fig. 4. The phase diagram for three selected values of M. The simulations were performed at constant saturation, but with varying Ca. The simulated points indicate the dynamical phase boundaries. In the lower left part of the diagrams, there is single-phase wetting flow; in the middle upper part, two-phase flow; and in the lower right part, single-phase nonwetting flow.
Summary:

1. Steady-state flow is a state.
2. Cluster distribution shows a critical point that depends on the flow rate.
3. Cross flow generates a stable foam layer moving with constant speed orthogonal to the average flow direction.
4. \[ E = \frac{1}{2} \min \Sigma_{<ij>} (p_i - p_j - p_{ij}^c)^2/\mu_{ij}^{\text{eff}} \]
5. \[ W\{\text{cf}\} = W(E\{\text{cf}\}). \]
7. \[ T = P^2/\mu^{\text{eff}}. \]
8. \[ P = A \left( \frac{dF}{dS} \right)_Q + B \] leads to expressions for relative permeability.