Scale Dependence of Strength of the Earth's Crust

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Conceptual Summary (Size Matters)

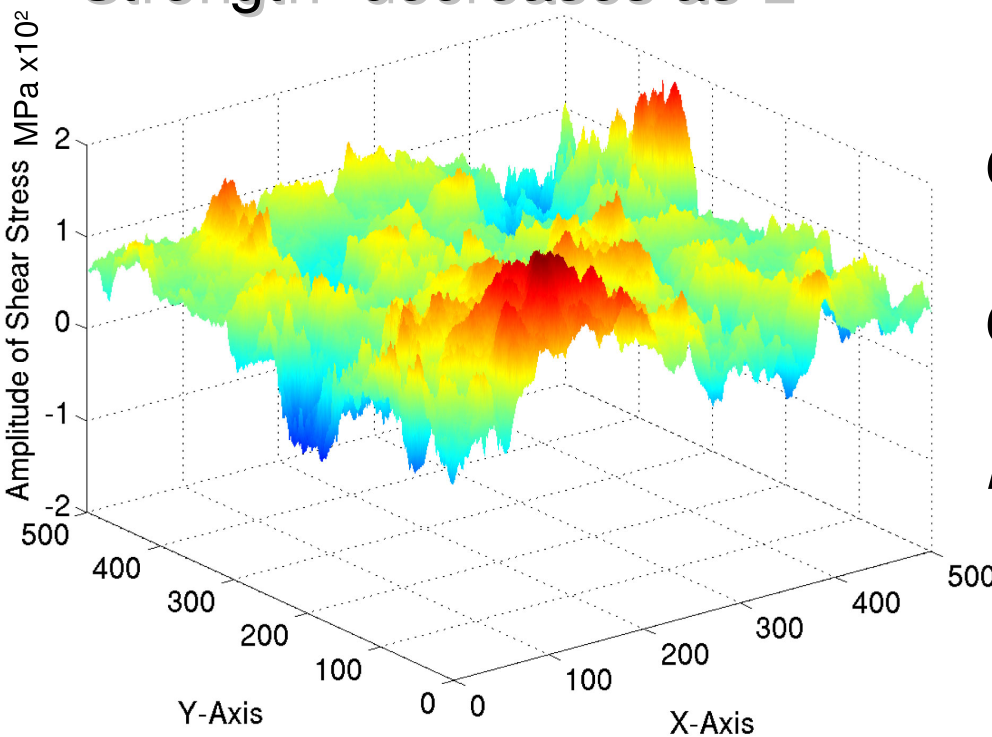
- Observed slip is spatially heterogeneous in earthquakes and inferred stress in the crust is strongly heterogeneous.
- Dynamic rupture produces heterogeneous slip when friction transitions dramatically between high static friction and low dynamic friction.
- Repeated heterogeneous ruptures result in a stress state that is described by a spatial power spectrum.
- I define “Strength” to be the spatial average of pre-stress within some region that fails.
- “Strength” can be determined directly from the spatial power spectrum of stress and it is a monotonically decreasing function of increasing length scale.

Fractal Shear Stress on a Plane

• When shear stress is measured on a plane, spatial variations similar to below explain many aspects of focal mechanisms.

• Amplitude of shear stress for a 2D cross-section, approximately 20 km x 20 km.

• “Strength” decreases as $L^{-1/3}$



$$\tilde{\sigma}(k) \quad k^{-0.8}$$

$\tilde{\sigma}$ is Fourier spectrum

k is wavenumber

Road Map for This Talk

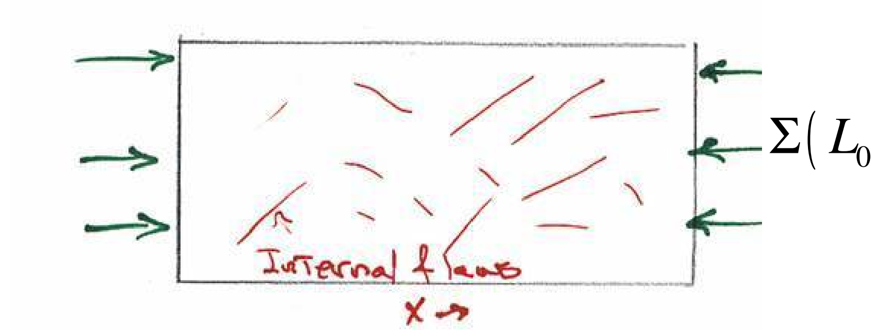
- Conclusions (already done)
- Definition of “strength” and some nasty, but important equations
- Origin of stress heterogeneity in the Earth
- A model of fractal stress
- Implications of fractal stress
- Conclusions

What is Strength?

- How much load can a solid sustain before it fails?
- For **ductile** materials (plastic), $< \ _{Y}$ where $_{Y}$ is apparently a material property with **units of stress**.
- **Brittle mode I** failure is not characterized by a yield stress, but instability of a flaw can be predicted by a fracture energy that seems to be a material property (**units of stress x length**).
- Failure of the Earth's crust is neither ductile nor is it mode I brittle failure. How to define "strength?"
- Most geologists seem to intend that strength is defined as a stress at which the crust yields. **$F_Y \div$ Area**
- We can define strength in such a way, but it means that strength depends on length scale. **(L), Area=L^2**

How can you define “strength” when stress is heterogeneous?

- In the lab, the measured (external) yield stress is the stress **averaged over the entire sample**.
- In the Earth, we estimate the size of the **stress in the Earth** and call it the strength.
- If stress inside the sample is uniform, then we can measure stress either way.



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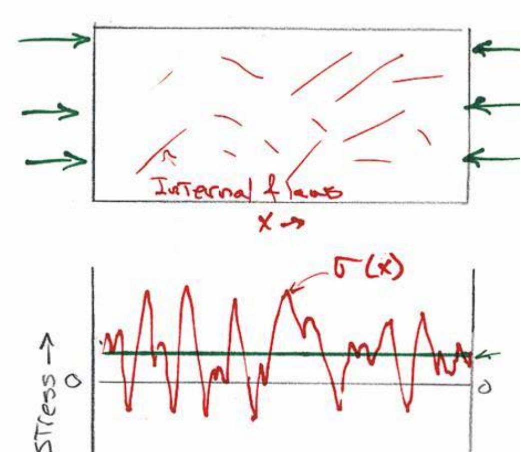
- In the lab, the external yield stress is measured = stress **averaged over the entire sample**.
- In the Earth, we estimate the size of the **stress in the Earth** and call it the strength.
- If stress inside the sample is uniform, then we can measure stress either way.
- But if stress is heterogeneous, then we mean that the **average stress** inside the sample has reached the **yield stress at that scale length.**



$\Sigma\left(L_0\right)$

$\Sigma\left(L_0\right)$

$$\sigma\left(x,y,z\right)=\Sigma\left(L_0\right)+\sigma^{Int}\left(x,y,z\right)$$

$$\overline{\overline{\sigma\left(\mathbf{x}\right)}}^{L} \quad \frac{1}{L^2}\,{}_{0}^{L}\,{}_{0}^{L}\,\sigma\left(x,y,z\right)dydz$$

$$\overline{\overline{\sigma^{Int}\left(\mathbf{x}\right)}}^{L_0}=0$$

$$\Sigma\left(L_0\right)=\left|\overline{\overline{\sigma\left(\mathbf{x}\right)}}^{L_0}\right|;\ \text{that is}\,\Sigma\left(L_0\right)>0$$

A Statistical Physics Definition of Strength

- At this point in time there is some (x)
- Earthquakes may occur at multiple locations and with multiple length scales
- The stress has developed through time, such that its **expected value** at length scale L is close to the strength (L).

A Statistical Physics Definition of Strength

- At this point in time there is some (x)
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$$\Sigma_{abs}\left(L\right) \quad E \left| \overline{\overline{\sigma\left(x, y, z\right)}}^{L} \right|$$

I can altenatively define **rms strength** as

$$\Sigma_{rms}^2\left(L\right) \quad E \quad \overline{\overline{\sigma\left(x, y, z\right)}}^{L} \quad ^2$$

- This definition provides some easier access to analytic analysis
- I will show later that it has very similar length scale behavior with $_{abs}$

- If (x) is a Gaussian stationary process,then

$$\Sigma_{rms}^{2}\left(L\right) \quad E \quad \overline{\overline{\sigma\left(x,y,z\right)}}^{L} \quad {}^{2}$$

$$= \; E \; \overline{\overline{\sigma\left(x,y,z\right)}}^{L} \quad {}^{2} \quad +Var \; \overline{\overline{\sigma\left(x,y,z\right)}}^{L}$$

$$=\left(\Sigma\left(L_0\right)\right)^2+Var \; \overline{\overline{\sigma^{Int}\left(x,y,z\right)}}^{L}$$

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$$=\left(\Sigma\left(L_{0}\right)\right)^{2}+Var \; \overline{\overline{\sigma^{Int}\left(x,y,z\right)}}^{L}$$

- To find the scale dependence of strength on length, we need to find

$$Var \; \overline{\overline{\sigma^{Int}\left(x,y,z\right)}}^{L}$$

- Use Parseval’s Theorem to conclude that

$$Var \quad \overline{\overline{\sigma^{Int}\left(x, y, z\right)}}^{L} \qquad \frac{1}{L_0^2} {}_{0}^{L_0} {}_{0}^{L_0} \left| \overline{\overline{\sigma\left(x, y, z\right)^{Int}}}^{L} \right|^2 dydz$$

$$= \frac{1}{L_0^2} {}_{L_0^{-1}} {}_{L_0^{-1}} \left| \overline{\overline{\tilde{\sigma}\left(k_x, k_y, k_z\right)^{Int}}}^{L} \right|^2 dk_y dk_z$$

- Use Parseval's Theorem to conclude that

$$Var \; \overline{\overline{\sigma^{Int}\left(x,y,z\right)}}^{L} = \frac{1}{L_0^2}\int_0^{L_0}\int_0^{L_0}\left|\overline{\overline{\sigma\left(x,y,z\right)^{Int}}}^{L}\right|^2 dydz$$

$$= \frac{1}{L_0^2}\int_{L_0^{-1}}\int_{L_0^{-1}}\left|\overline{\overline{\tilde{\sigma}\left(k_x,k_y,k_z\right)^{Int}}}^{L}\right|^2 dk_y dk_z$$

- After some mathematical manipulation, we can show that

$$Var \; \overline{\overline{\sigma^{Int}\left(x,y,z\right)}}^{L} = \frac{C}{L_0^2}\int_{L_0^{-1}}^{L^{-1}}\int_{L_0^{-1}}^{L^{-1}}\left|\tilde{\sigma}^{Int}\left(k_x,k_y,k_z\right)\right|^2 dk_y dk_z$$

- Therefore, for a stationary stochastic stress that is near failure at all length scales

$$\Sigma_{rms}^{2}\left(L\right)=\left(\Sigma\left(L_{0}\right)\right)^{2}+\frac{C}{L_{0}^{2}}\int_{L_{0}^{-1}}^{L^{-1}}\int_{L_{0}^{-1}}^{L^{-1}}\left|\tilde{\sigma}^{Int}\left(k_{x},k_{y},k_{z}\right)\right|^{2}dk_{y}dk_{z}$$

- That is, the shorter the length scale the larger the strength, since you are integrating to higher wavenumbers

- If the statistical variations in stress are assumed to be **isotropic,** then it can be written as a function of **radial wavenumber**

$$k_r = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$\Sigma_{rms}^2 \left(L \right) = \Sigma^2 \left(L_0 \right) + C \int_{L_0^{-1}}^{L^{-1}} \int_{0}^{2\pi} \left| \tilde{\sigma} \left(k_r \right) \right|^2 \left(k_r d\theta \right) dk_r$$

$$= \Sigma^2 \left(L_0 \right) + 2\pi C \int_{L_0^{-1}}^{L^{-1}} \left| \tilde{\sigma} \left(k_r \right) \right|^2 k_r dk_r$$

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$$= \Sigma^2\left(L_0\right) + 2\pi C \int_{L_0^{-1}}^{L^{-1}} \left|\tilde{\sigma}\left(k_r\right)\right|^2 k_r dk_r$$

- **A remarkably simple result!**

What if stress is a fractal?

- Fractal stress has a power spectrum defined by a power law and is the heterogeneous stress with the fewest parameters
- Assume that $\left|\tilde{\sigma}\left(k_r\right)\right| = Ck_r^{-\beta}$ then

$$\Sigma_{rms}\left(L\right) = \frac{C}{\sqrt{1-2\beta}}\frac{1}{L^{1-\beta}}, 0 \quad \beta < 1$$

How to Determine the spatial power spectrum of stress?

- Cannot directly measure distribution of stress at 10 km depth
- What if we knew fault failure physics and make a numerical simulation of the crust?

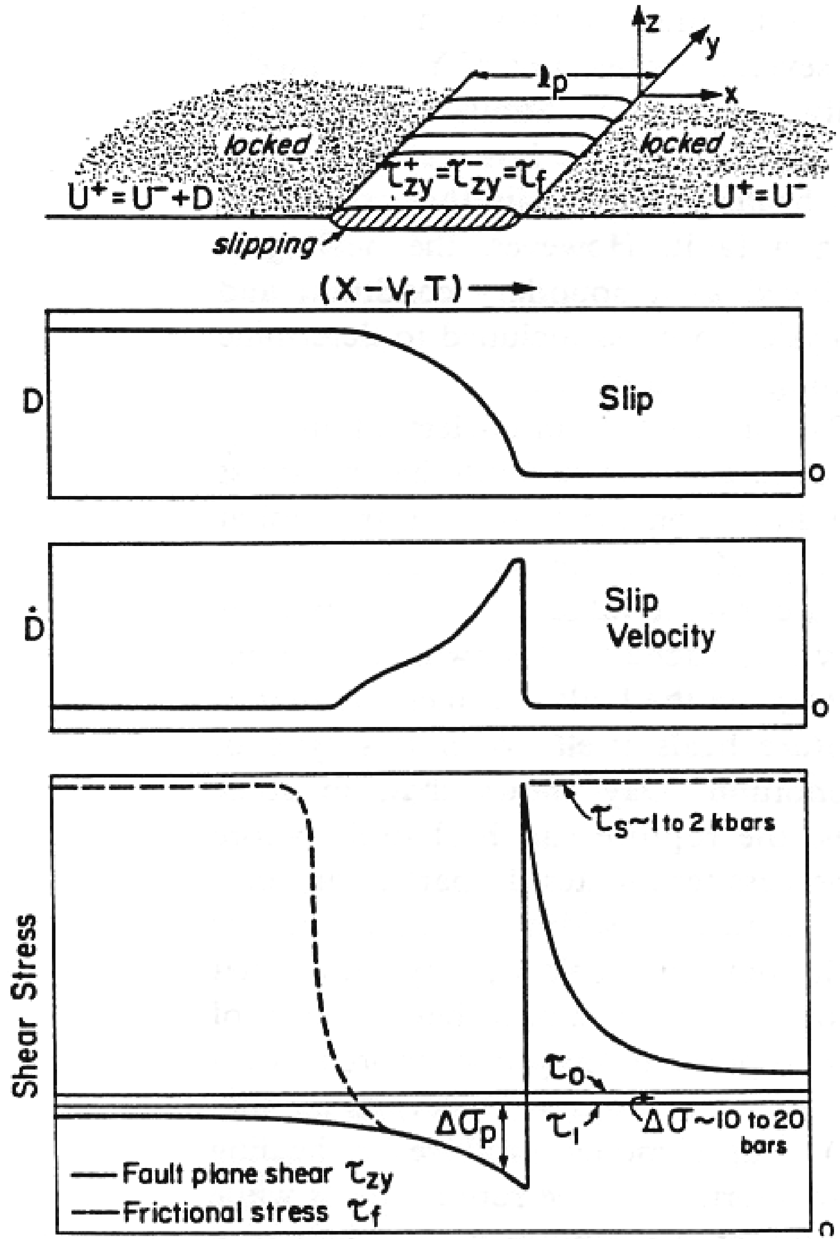
Many faults have very thin primary deformation zones, with no significant evidence of melting. Therefore the sliding friction must have been either very low, or the sliding must have been very slow.

Transmission line fault in Pacoima Canyon, S. Calif. 300 m strike-slip total offset.

How can the sliding friction be low?

- If sliding friction *was* high, it would drop after melting, but pseudo-tachylytes are uncommon.
- Cushion of steam (Sibson and later Rice)?
- Flash heating produces micron thick plasma (Rice suggested and Tullis has observed)?
- Slip along material interfaces causes dynamic normal stress variations on the same order as the shear stress, which can be very large at the crack front (Weertman; Andrews; Ben Zion).
- Several suggested models of low sliding friction have strong slip-velocity dependence.

Slip pulse rupture model … solitary waves of slip

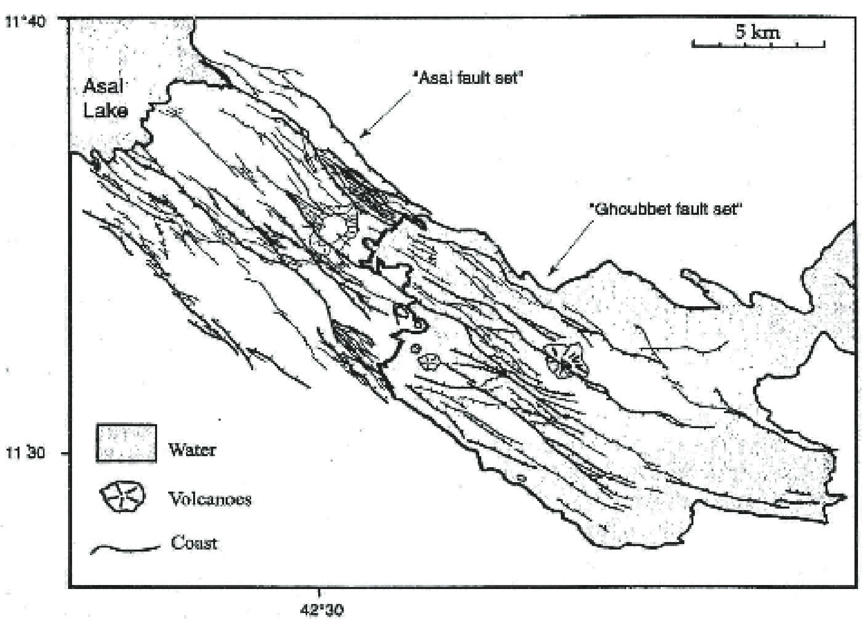


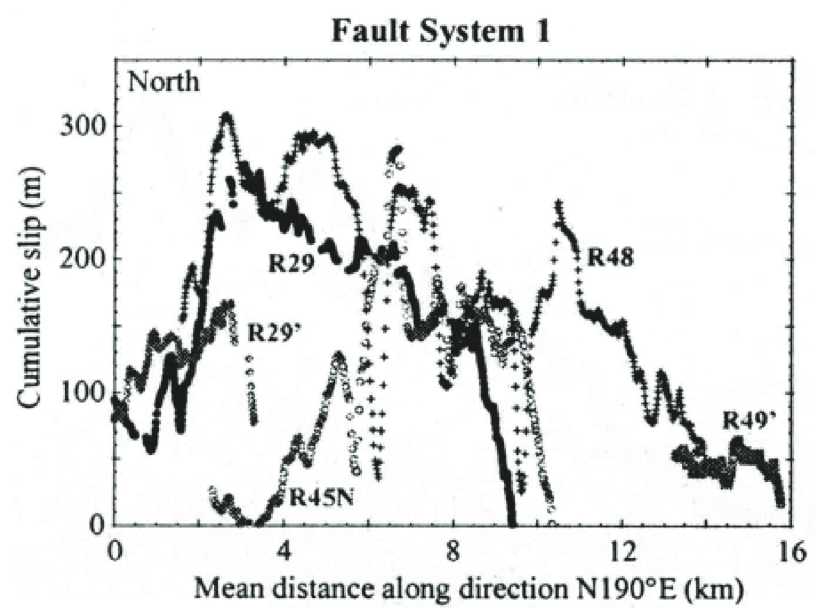
Slip pulse

- Slip pulse carries the information about slip vs length scaling.
- Particle-velocity-dependent friction produces slip pulses.
- Friction depends on slip velocity, but slip velocity depends on friction (highly nonlinear positive feedback system).
- The slip at any point depends on the distance between the rupture front and the healing front. Both velocities are unsteady, producing highly heterogeneous slip as a function of space.
- This nonlinear rupture freezes in short scale heterogeneities (large small-scale stresses)

Strain/Stress Heterogeneity from Slip vs. Length Data

- Implied local strain changes of 10^{-3} and stress changes of 100 MPa (McGill and Rubin, 1999)
- Mapped strain changes of the order $5x10^{-2}$ from altimetry data





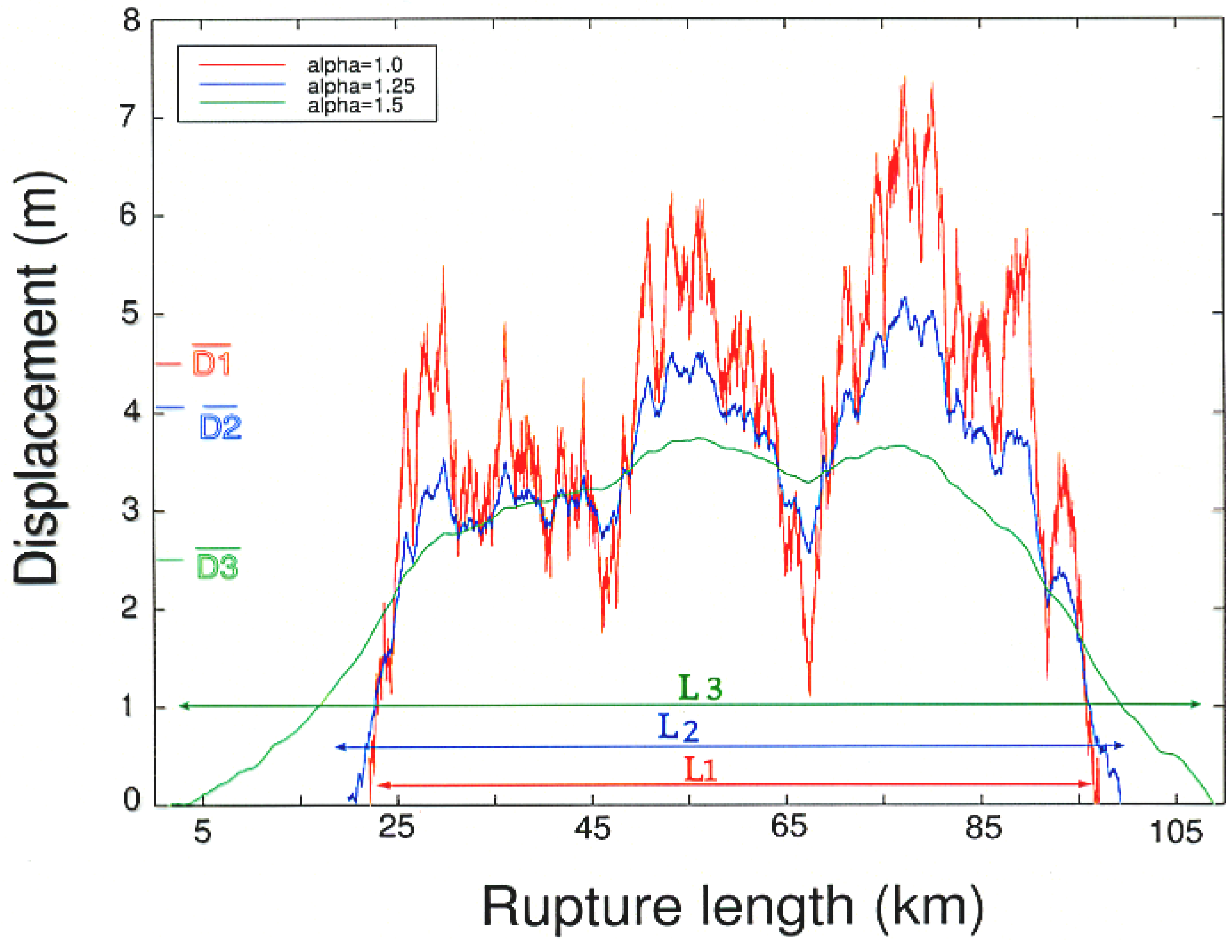
Can produce 1)power law frequency vs size and 2) slip vs. length scaling from 2 rules

1. Slip is fractal in space
2. Rupture is spatially contiguous

Of course, rupture dimensions are limited by the dimension of the fault

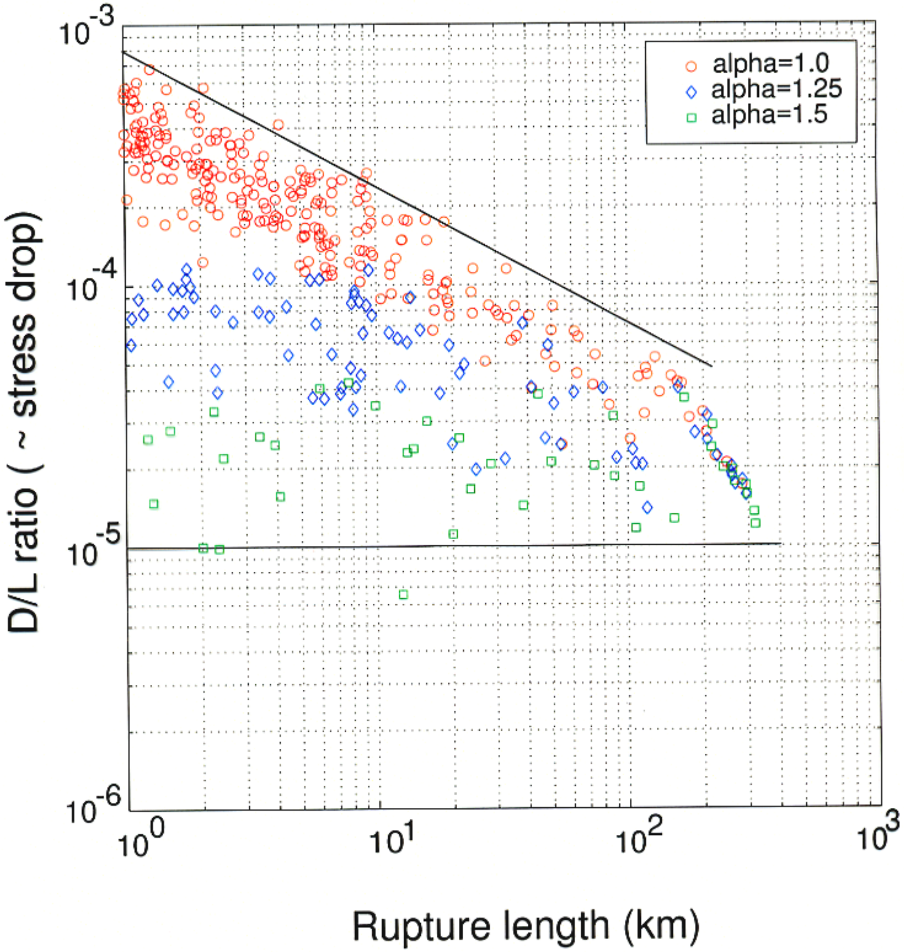
- Many fault networks seem to have fractal geometries

Given equal surface areas, islands with rougher topography have higher average elevations



$\tilde{D}(k) \sim k^{-\alpha}$, where k wavenumber

From J. Liu and T. Heaton



Earthquakes compatible with $1.25 < \alpha < 1.5$

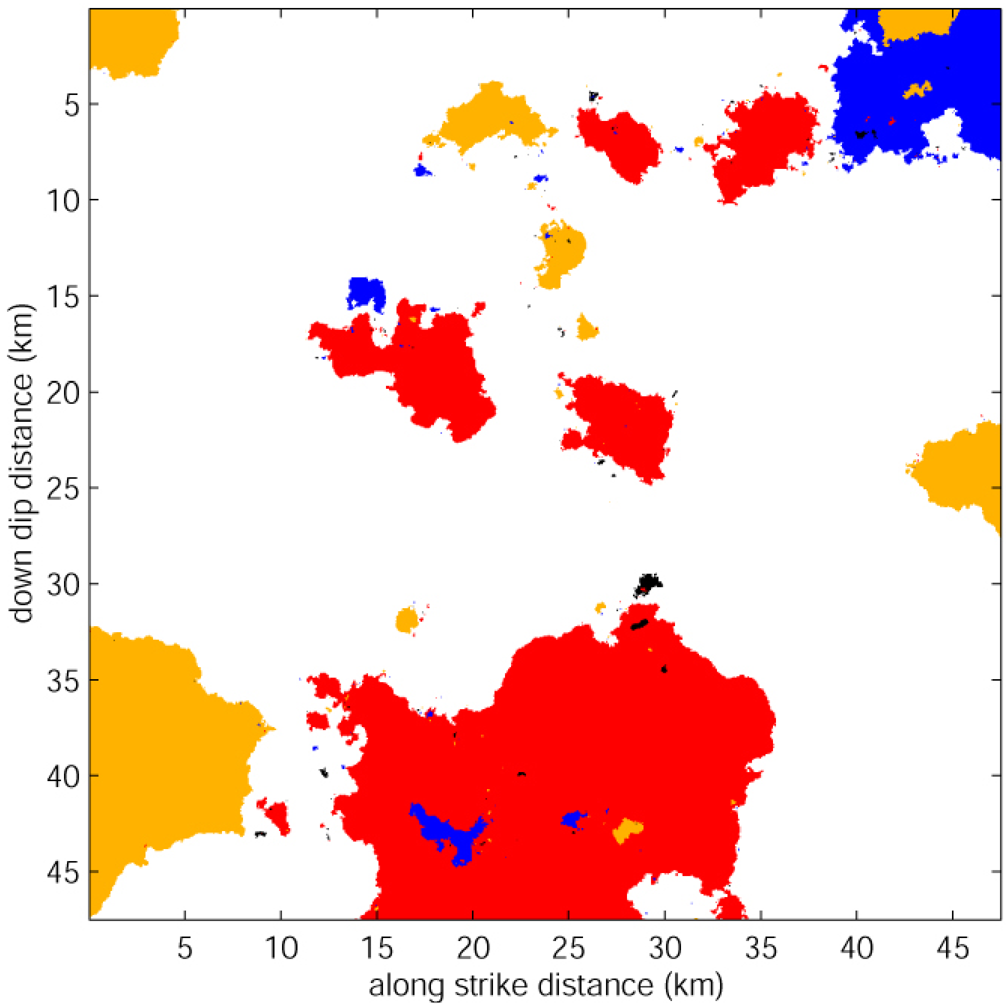
When extended to 2-d, $\alpha = 1.4$ gives G-R relation $\log N = a - M$

$\Delta \mu \left(\bar{D}/L \right) \quad \sigma$

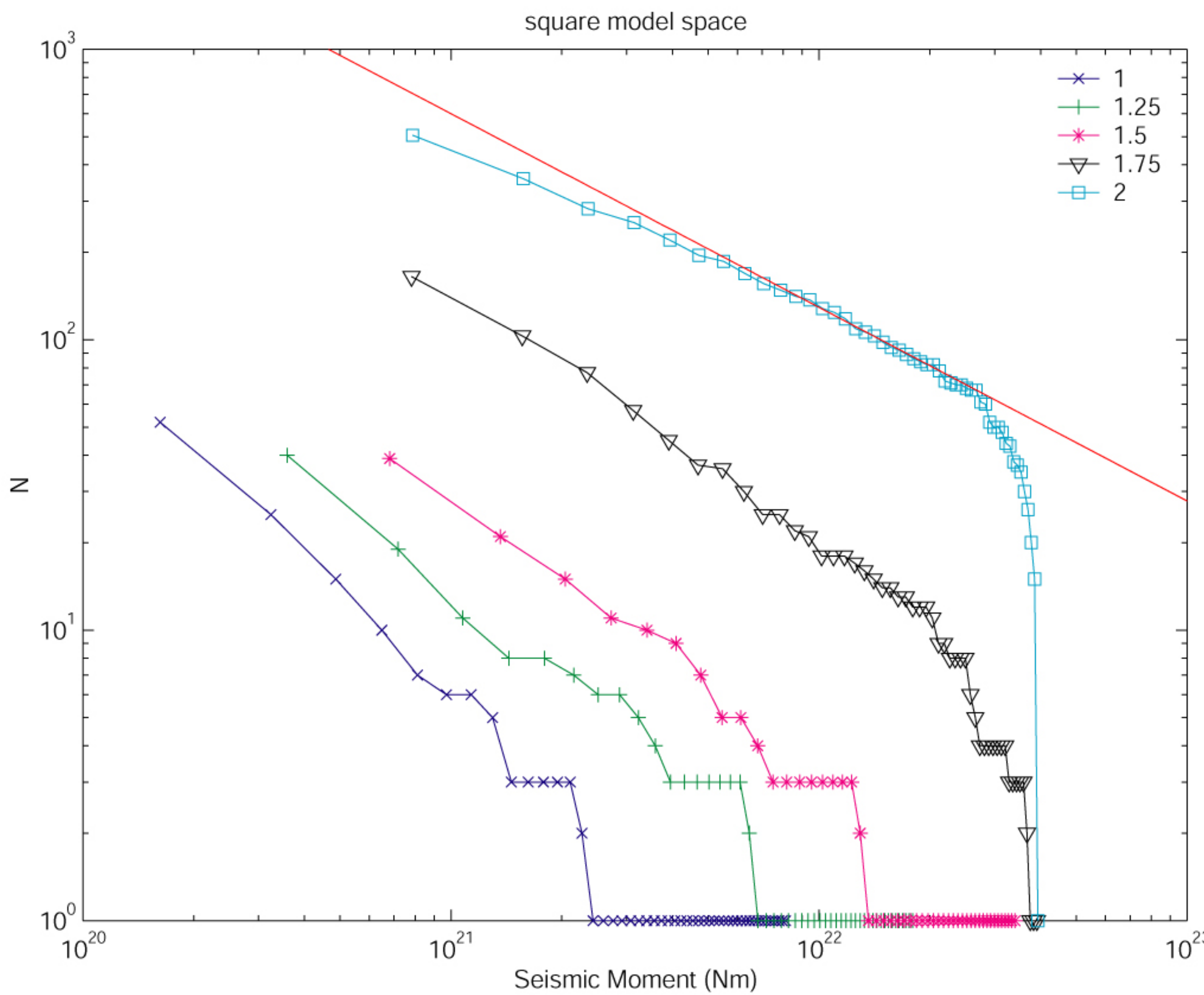
$\Delta \quad \sigma$ is a stochastic parameter

The more heterogeneous the slip, the higher the stress drop.

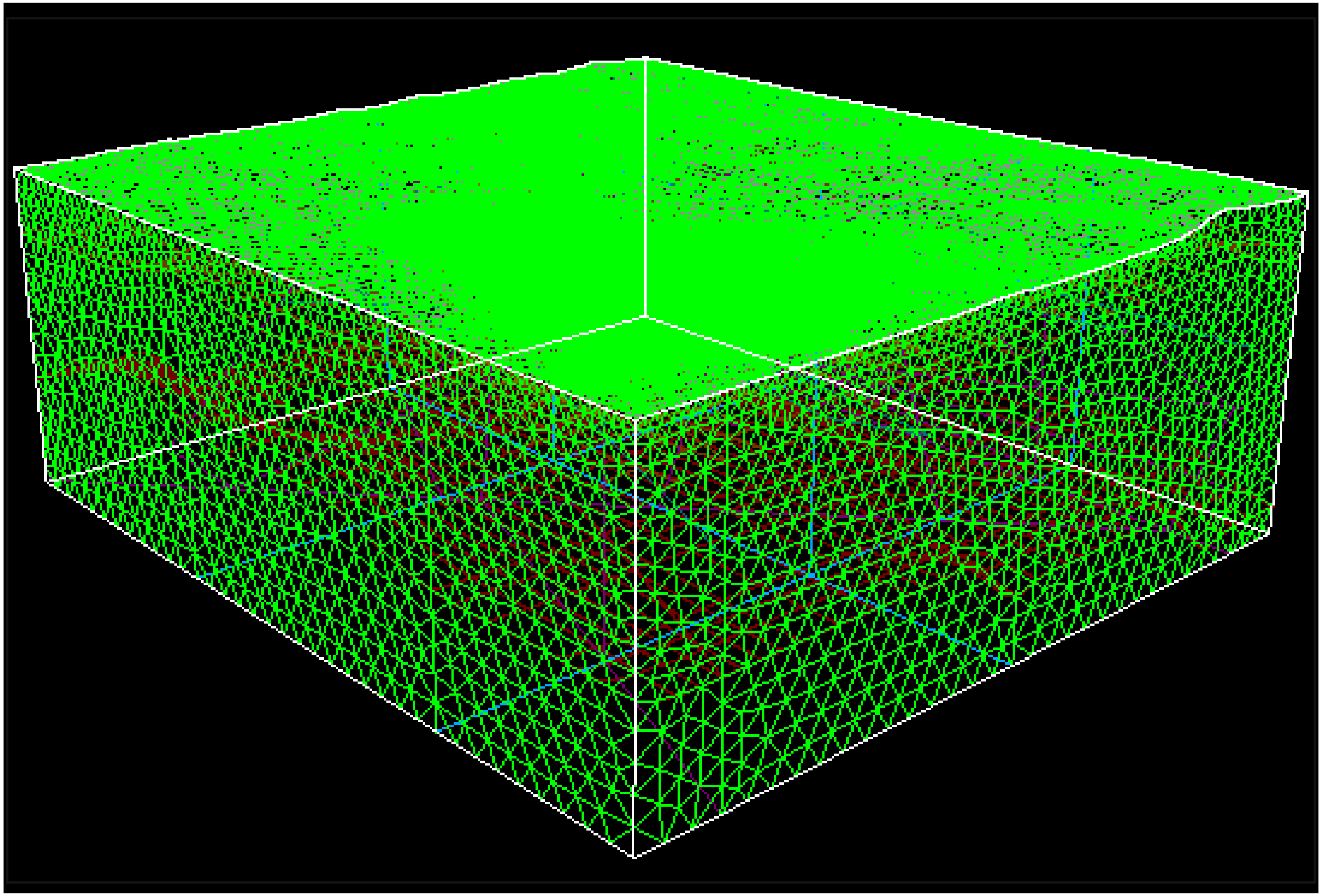
Map view of fractal islands (Dicaprio)



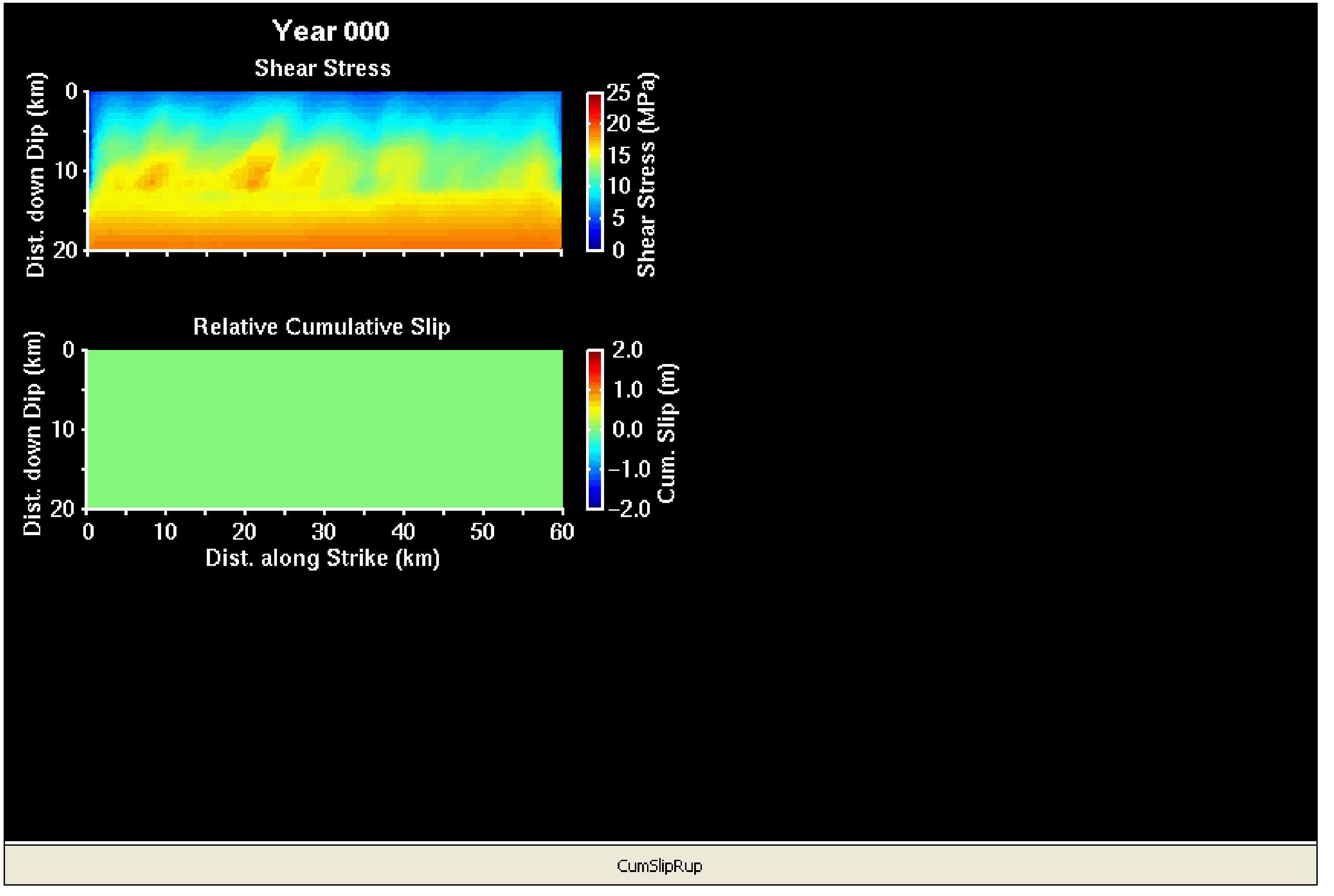
Frequency/magnitude for 2-d fractal islands (Dicaprio)



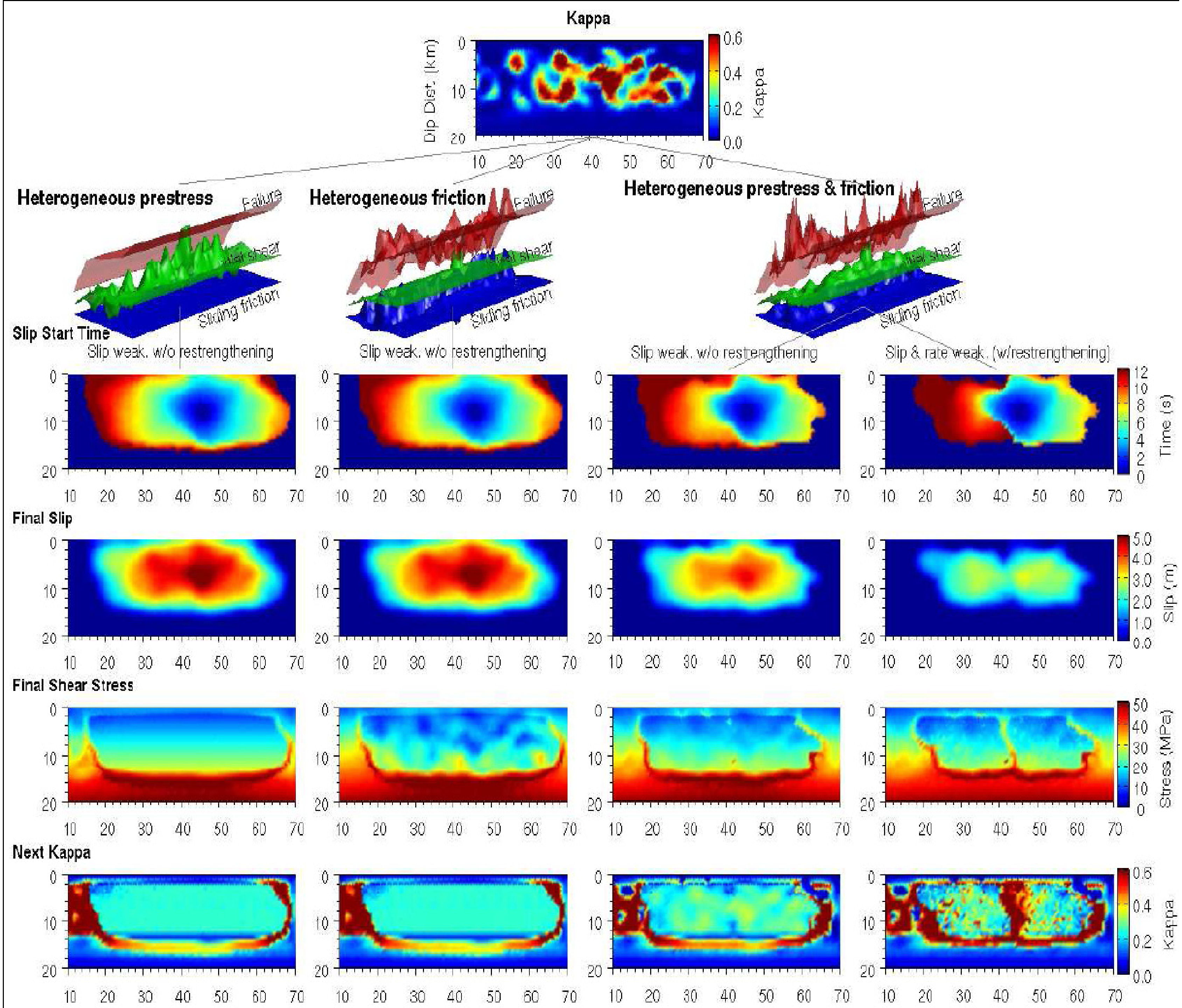
Finite element mesh created by Aagaard. A Frictional plane cuts the middle the mesh. Opposite sides of the mesh are horizontally displaced at 5 cm/yr. Spontaneous dynamic rupture is calculated for many events.



240 years of simulated strike-slip earthquakes



- Time is broken into 12 intervals of 20 years each.
- Earthquakes that occur in each 20-interval are shown.
- Some intervals contain more than 1 earthquake.
- Lithostatic pressure is included



Kappa
Dip Dist. (km)
Heterogeneous prestress
Heterogeneous friction
Heterogeneous prestress & friction
Failure
Initial shear
Sliding friction
Slip Start Time
Slip weak. w/o restrengthening
Slip & rate weak. (w/restrengthening)
Time (s)
Final Slip
Slip (m)
Final Shear Stress
Stress (MPa)
Next Kappa
Kappa

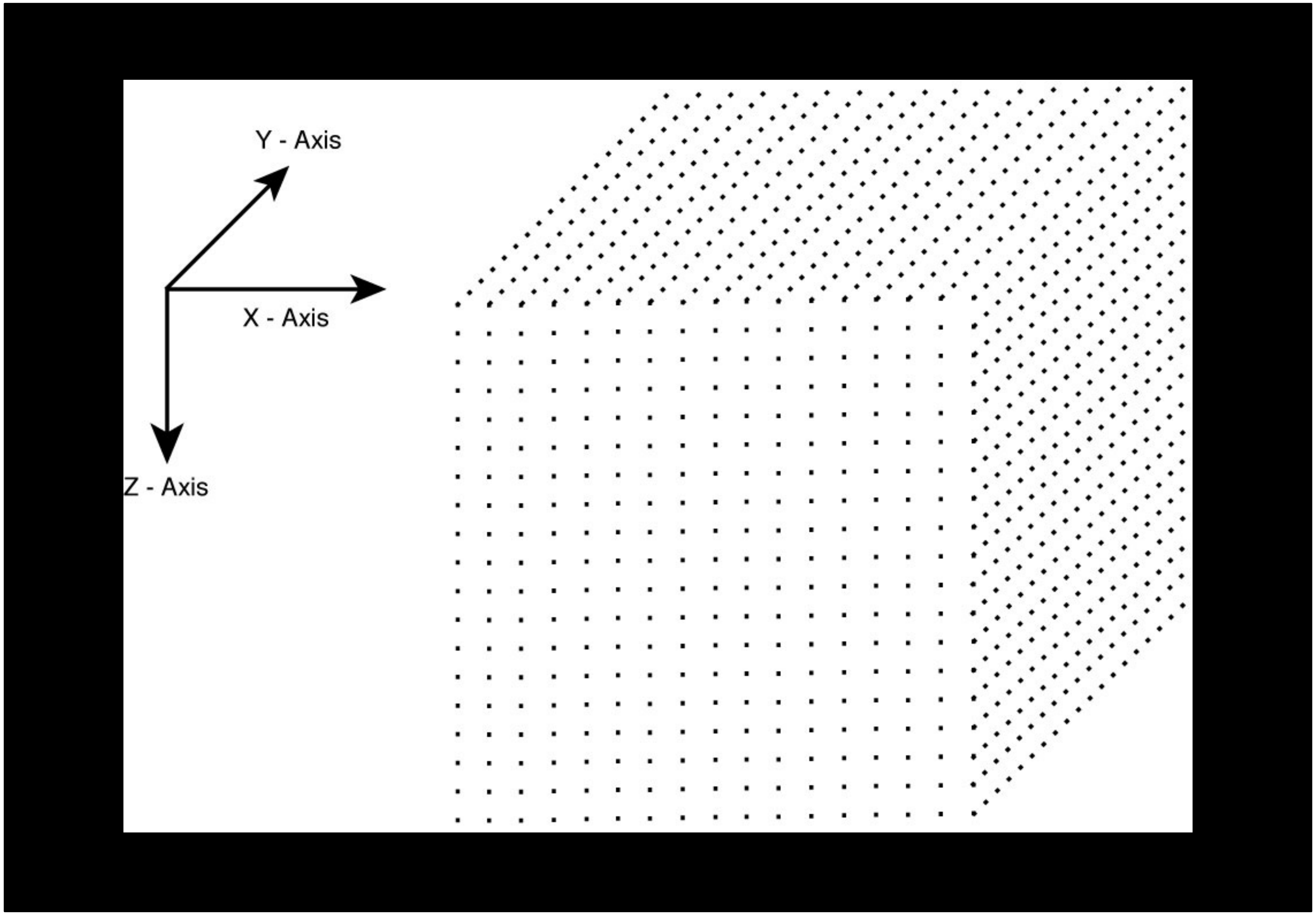
Dynamic rupture simulation is nice, but …

- It’s a computer killer!
- Strong velocity weakening requires very large grid and small time steps … the problem loses its length scales and becomes ill posed
- Makes lot of small events … hard to predict when it will make a large one
- Real earth has complex fault systems
- We are still a long way from realistic simulations
- **SPECULATION … Strong velocity dependence leads to fractal slip … there are no length scales.**

- Undeterred … we guess the stress distribution
- Deborah Smith and I have been constructing a models of stochastic stress in a 3-d grid
- We make a spatial and temporal model of the stress tensor in the crust and predict attributes of seismicity catalogs

Example 3D Grid

We Generate Stress and Compute Time to Failure at Each Point



Our Interseismic Stress Equation

$$\boldsymbol{\sigma}(\mathbf{x},t) = \boldsymbol{\sigma}_B$$

$\boldsymbol{\sigma}_B$ The background stress. The spatially and temporally averaged stress tensor. In other words, this is the stress left over when all time and space variations are subtracted. It is approximately what stress inversions attempt to solve for.

Our Interseismic Stress Equation

$$\boldsymbol{\sigma}\left(\mathbf{x},t\right)=\boldsymbol{\sigma}_{B}+\dot{\boldsymbol{\sigma}}_{T}t$$

$\boldsymbol{\sigma}_{B}$ The background stress. The spatially and temporally averaged stress tensor. In other words, this is the stress left over when all time and space variations are subtracted. It is approximately what stress inversions attempt to solve for.

$\dot{\boldsymbol{\sigma}}_{T}t$ The temporally varying stress due to plate tectonics. This term drives points toward failure. It can be derived from GPS velocity fields. It may have a different orientation than σ_B.

Our Interseismic Stress Equation

$$\boldsymbol{\sigma}\left(\mathbf{x},t\right)=\boldsymbol{\sigma}_B+\dot{\boldsymbol{\sigma}}_T t+\boldsymbol{\sigma}_H\left(\mathbf{x}\right)$$

$\boldsymbol{\sigma}_B$ The background stress. The spatially and temporally averaged stress tensor. In other words, this is the stress left over when all time and space variations are subtracted. It is approximately what stress inversions attempt to solve for.

$\dot{\boldsymbol{\sigma}}_T t$ The temporally varying stress due to plate tectonics. This term drives points toward failure. It can be derived from GPS velocity fields. It may have a different orientation than σ_B.

$\boldsymbol{\sigma}_H\left(\mathbf{x}\right)$ Our spatially varying stress due to all of the stress changes caused by local inelastic deformations such as faulting, compaction, fluids, thermal stresses. By definition, spatial average is zero.

Defining Heterogeneity Ratio

$$HR = \frac{\sqrt{Mean \quad I'_2\left(\boldsymbol{\sigma}_H\left(\mathbf{x}\right)\right)}}{\sqrt{I'_2\left(\boldsymbol{\sigma}_B\right)}}$$

$\sqrt{I'_2}$ is a scalar invariant

and is the size of the

maximum shear stress

Filtering These Six Quantities of the Stress Tensor

For Each Scalar, Principal Stress, σ_1, σ_2, σ_3

- Start with random Gaussian noise with a mean of zero.
- Apply a 3D spatial filter, that spatially smoothes the noise.

It produces α spectral fall-off of 1D cross-sections.

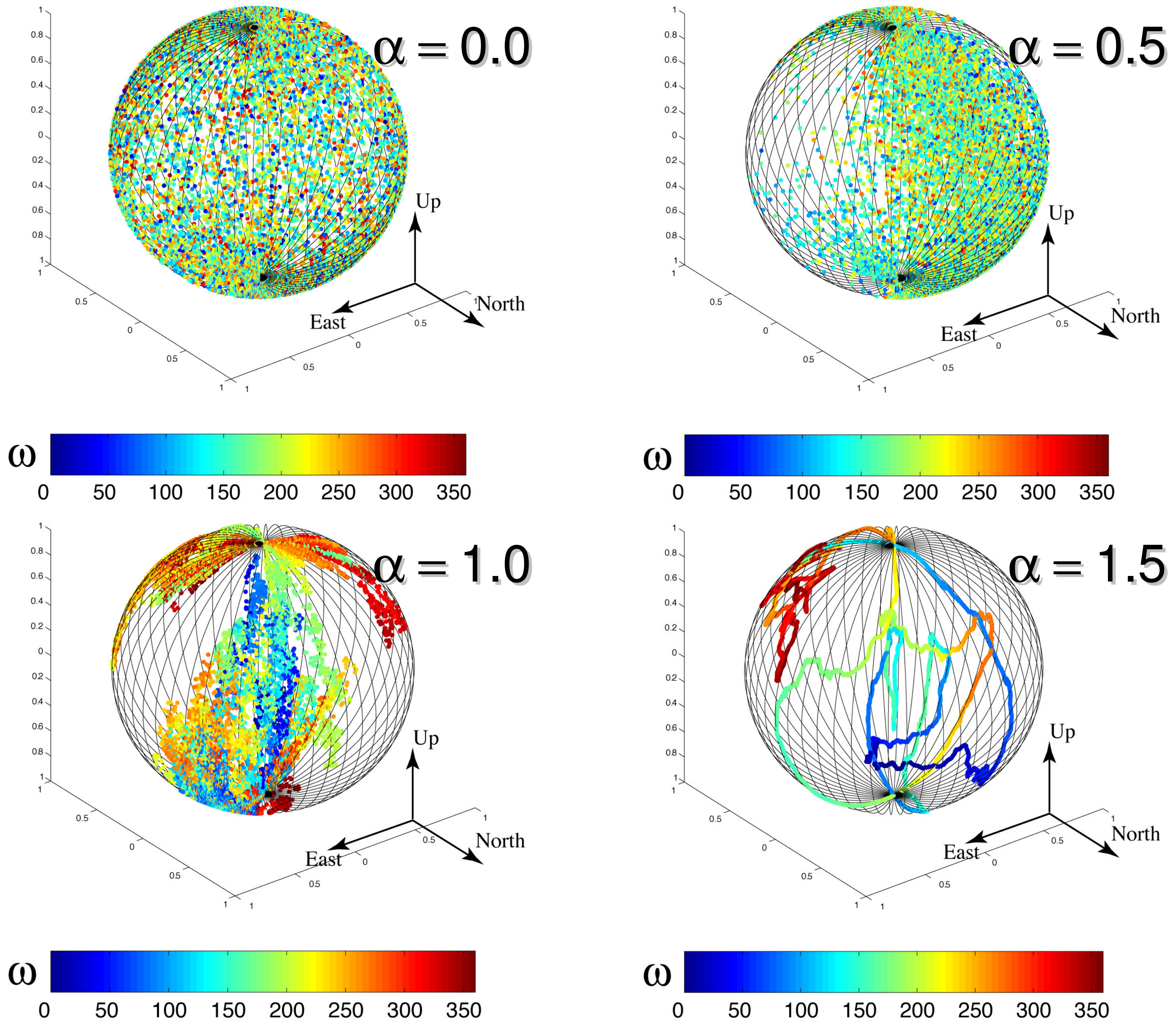
- Normalize.

For Three Orientation Angles, (ω, [θ, ϕ])

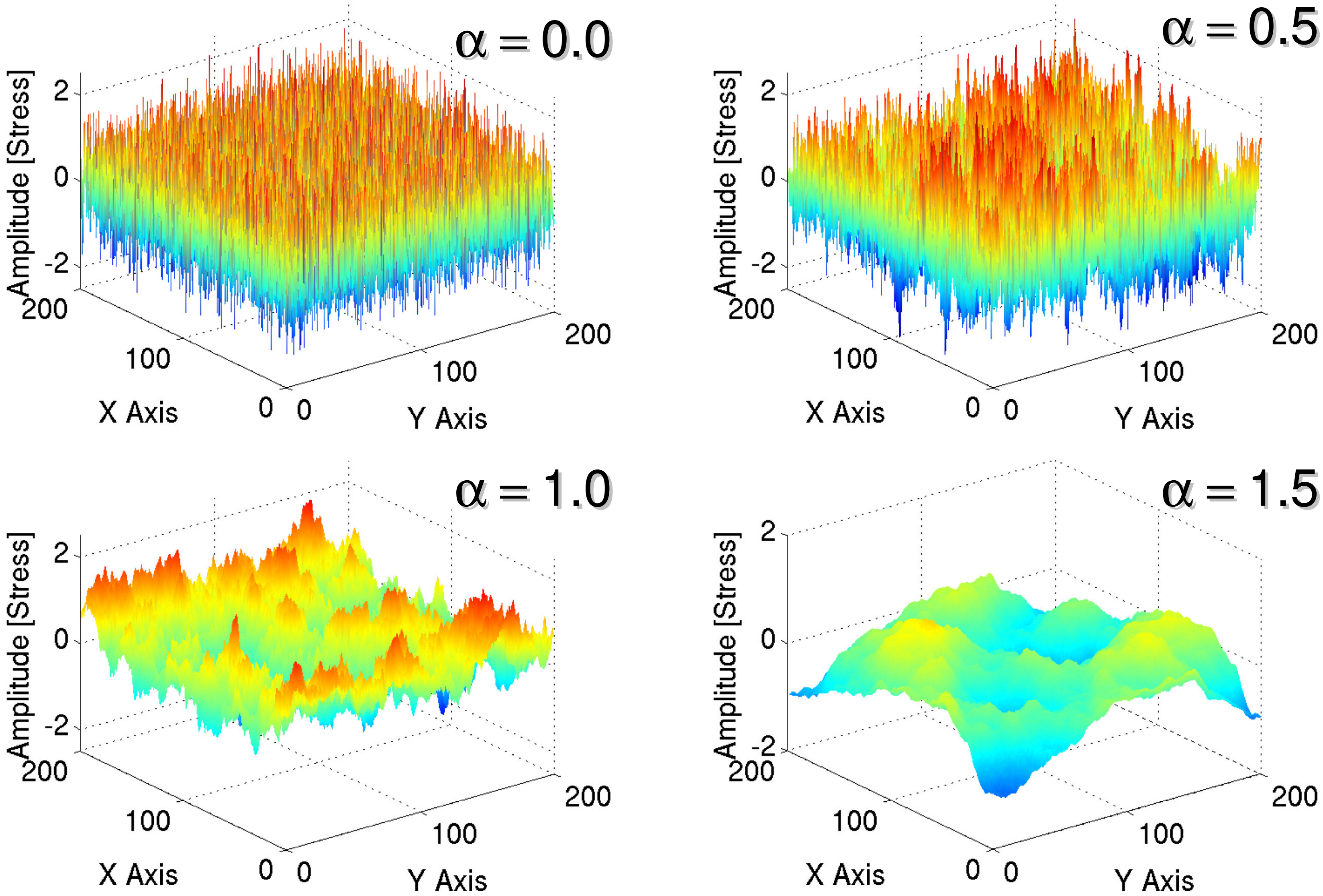
- Start with completely random orientations, using quaternions.
- Apply 3D spatial filter.
- Normalize so that:

$$0° \le \omega \le 360°, \text{ mean } \omega = 180°$$
$$0° \le \theta \le 180°, \text{ mean } \theta = 90°$$
$$0° \le \phi \le 360°, \text{ mean } \phi = 180°.$$

Filtered (ω, [θ, φ]) for 1D Lines



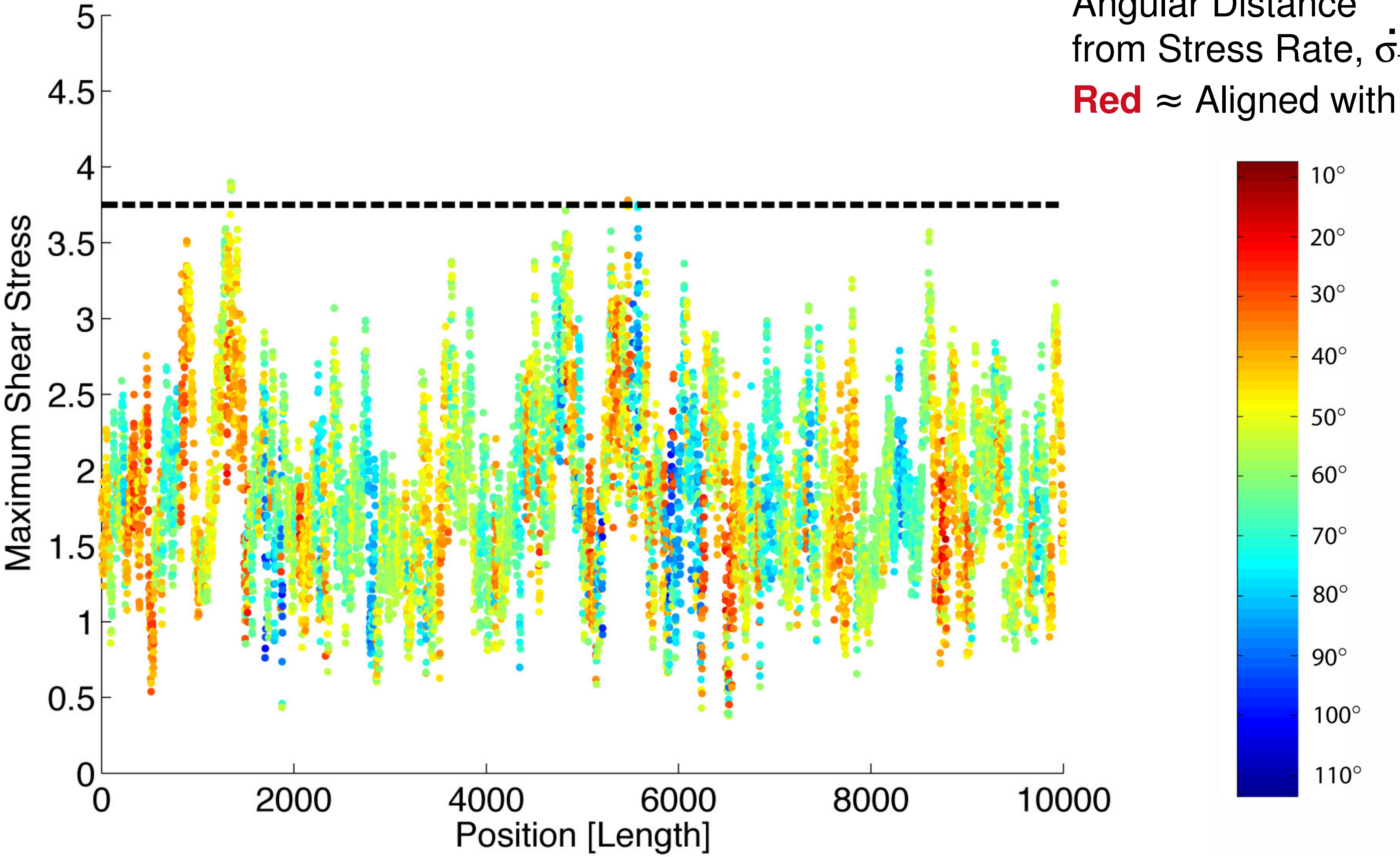
2D Cross-Sections of 3D Filtered Principal Stresses



Time = 1.00 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

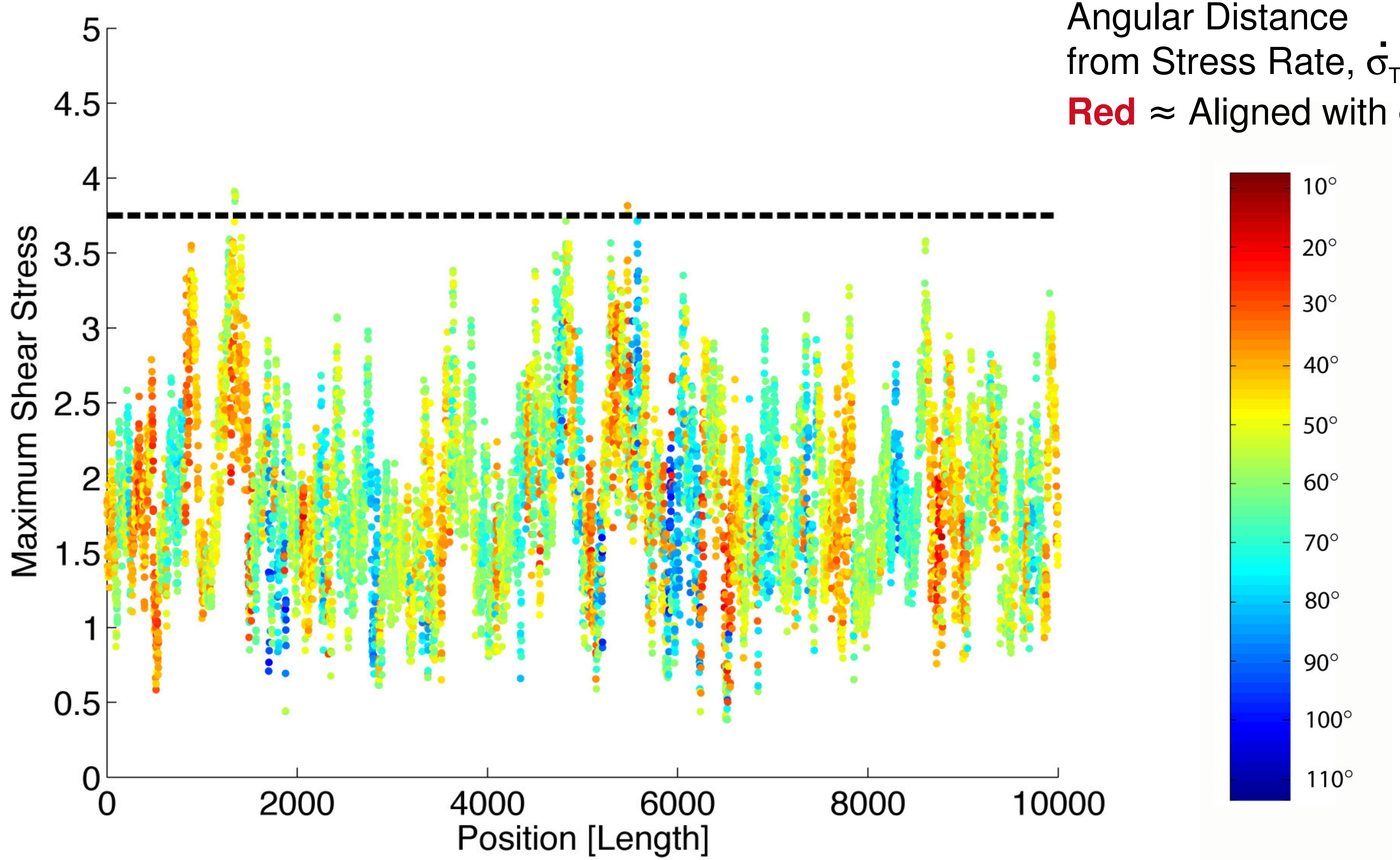
Red $\approx$ Aligned with $\dot{\sigma}_T$.



Time = 2.00 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

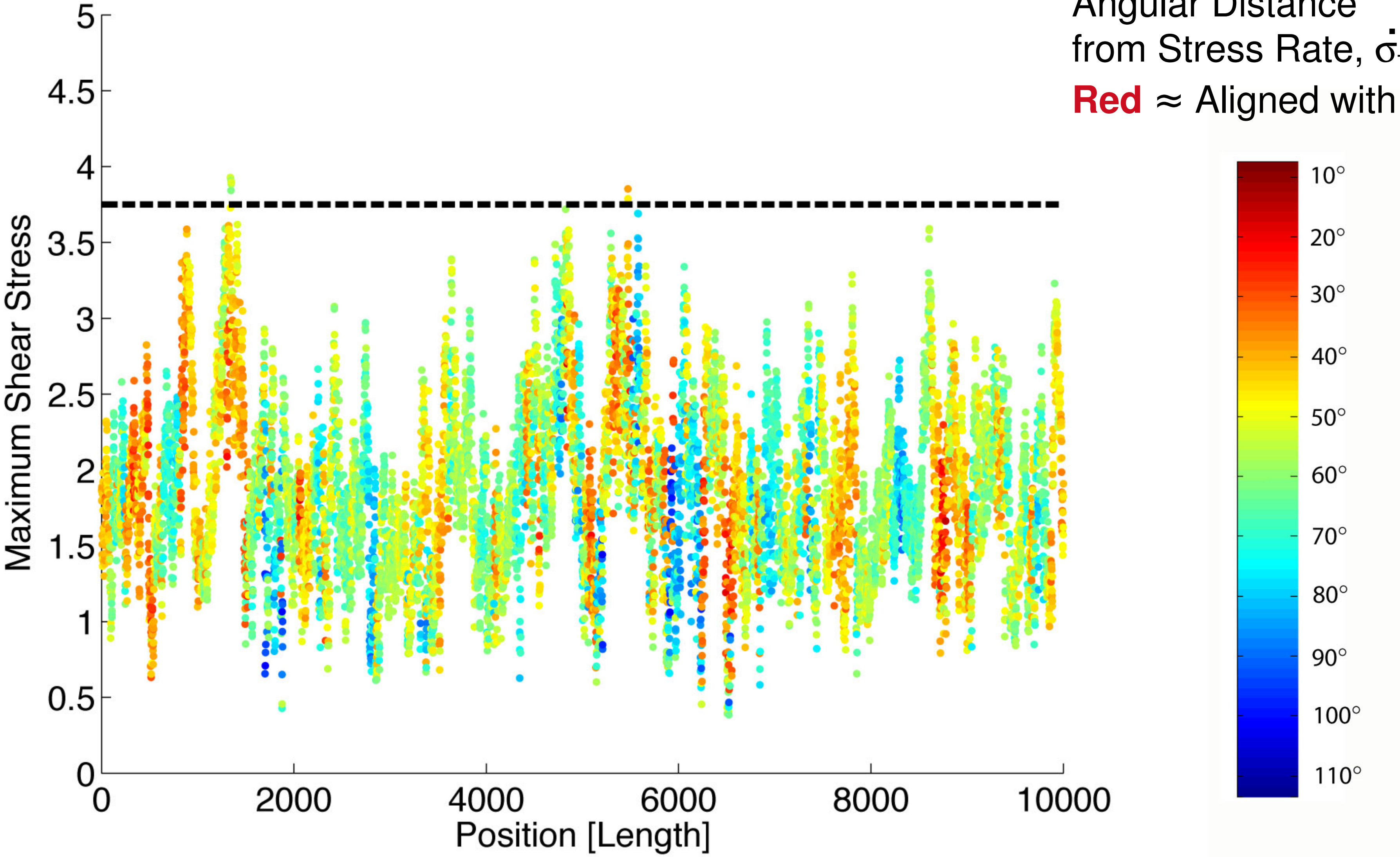
Red ≈ Aligned with $\dot{\sigma}_T$.



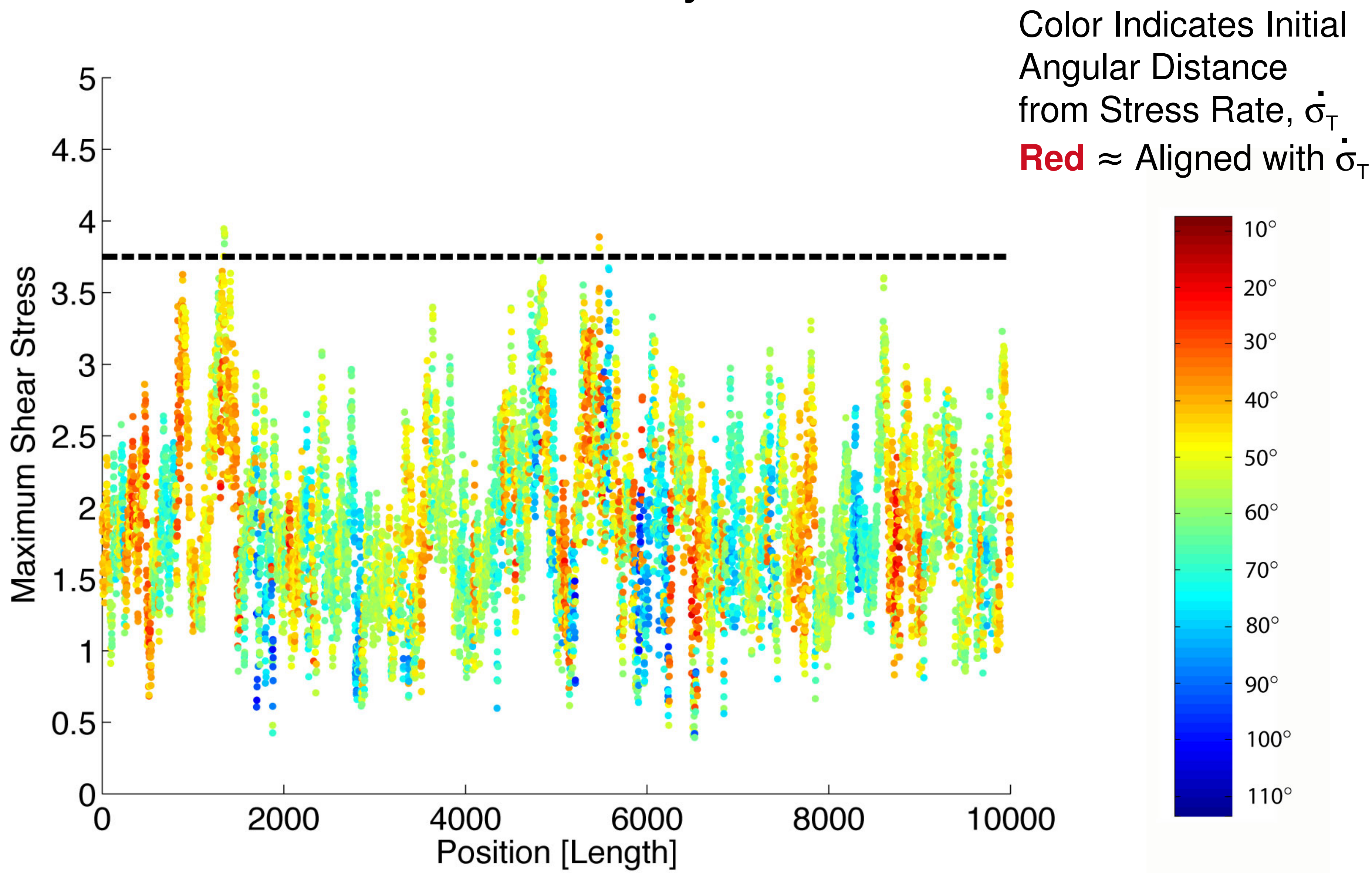
Time = 3.00 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

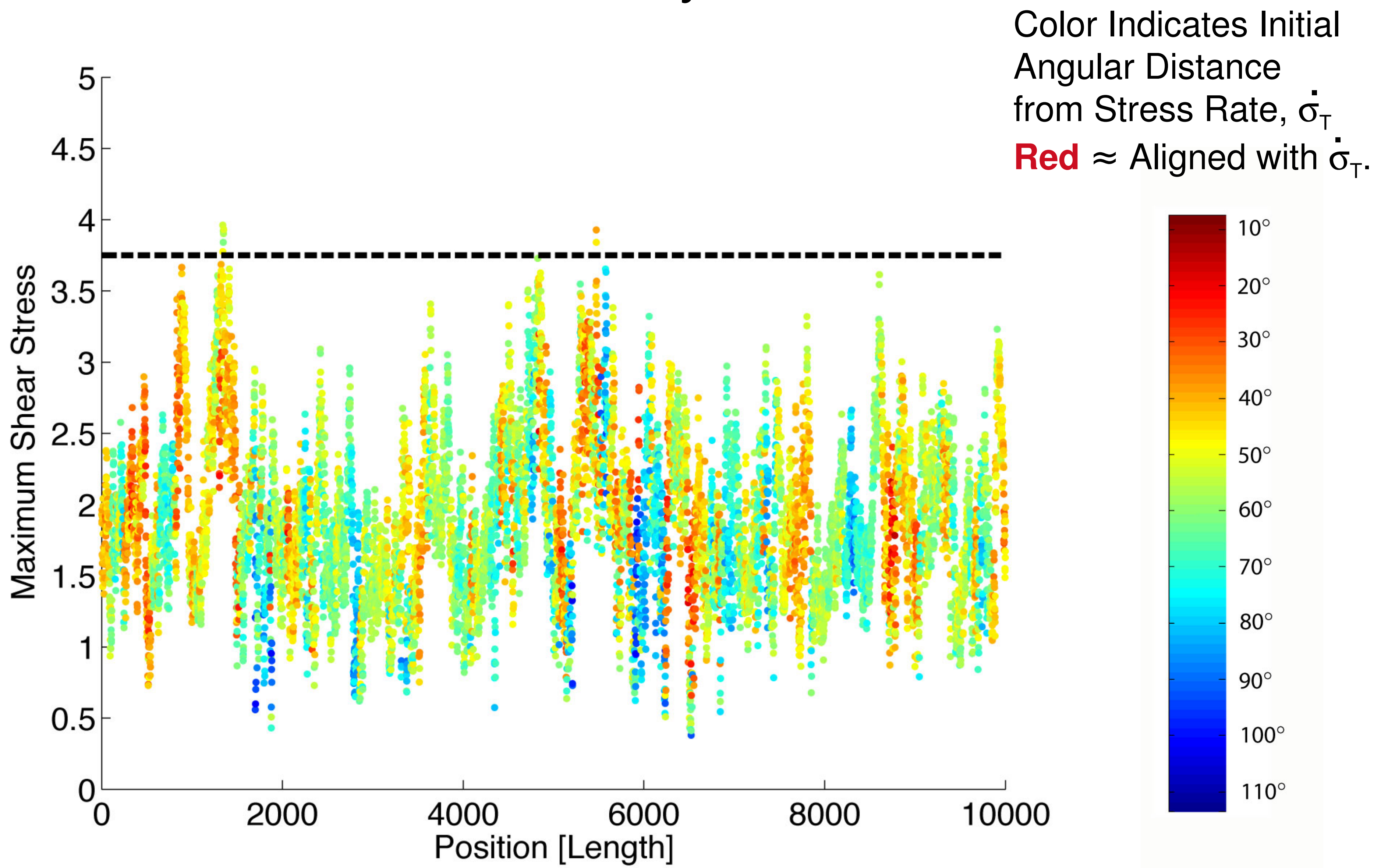
Red ≈ Aligned with $\dot{\sigma}_T$.



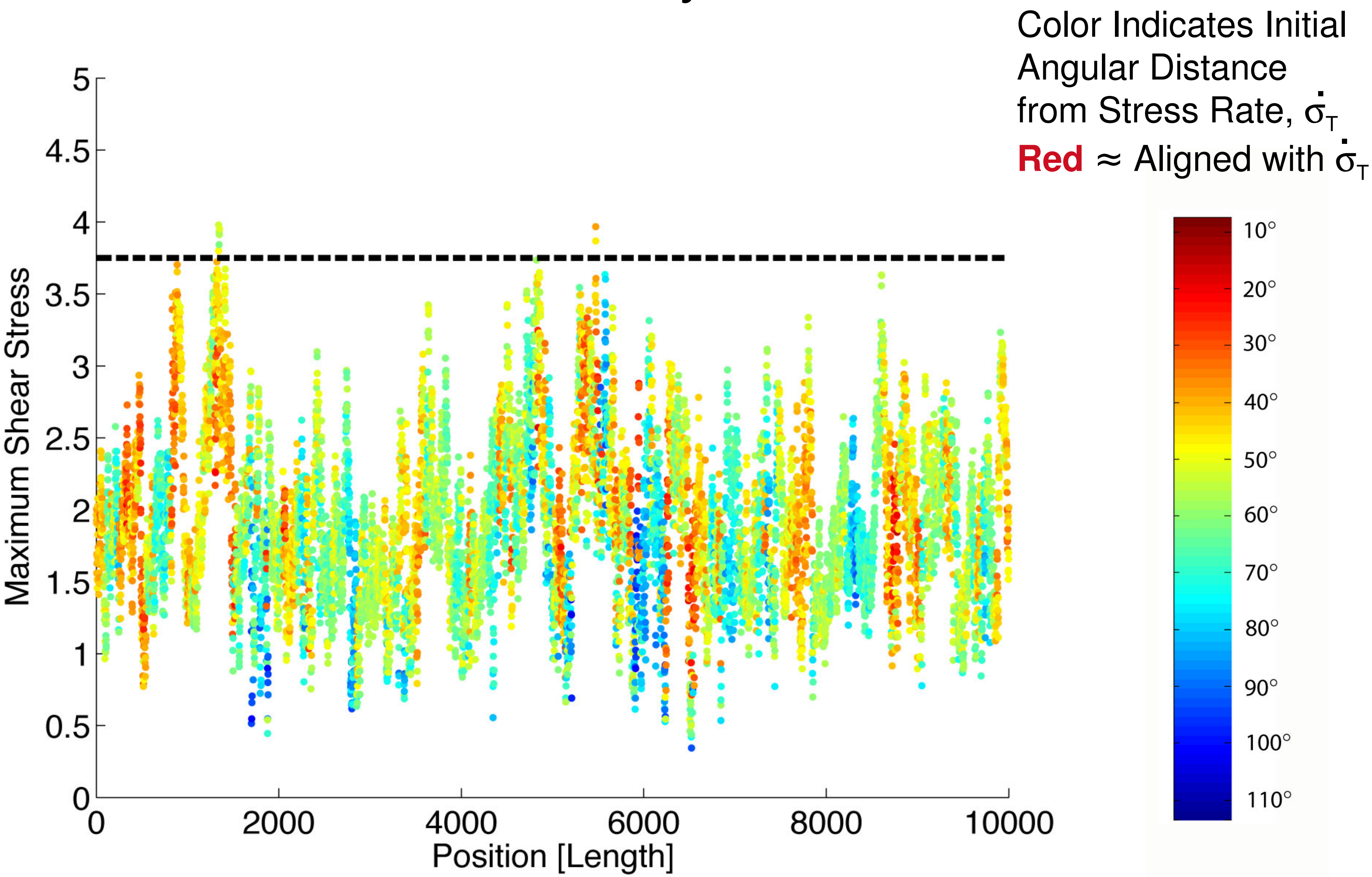
Time = 4.00 years



Time = 5.00 years



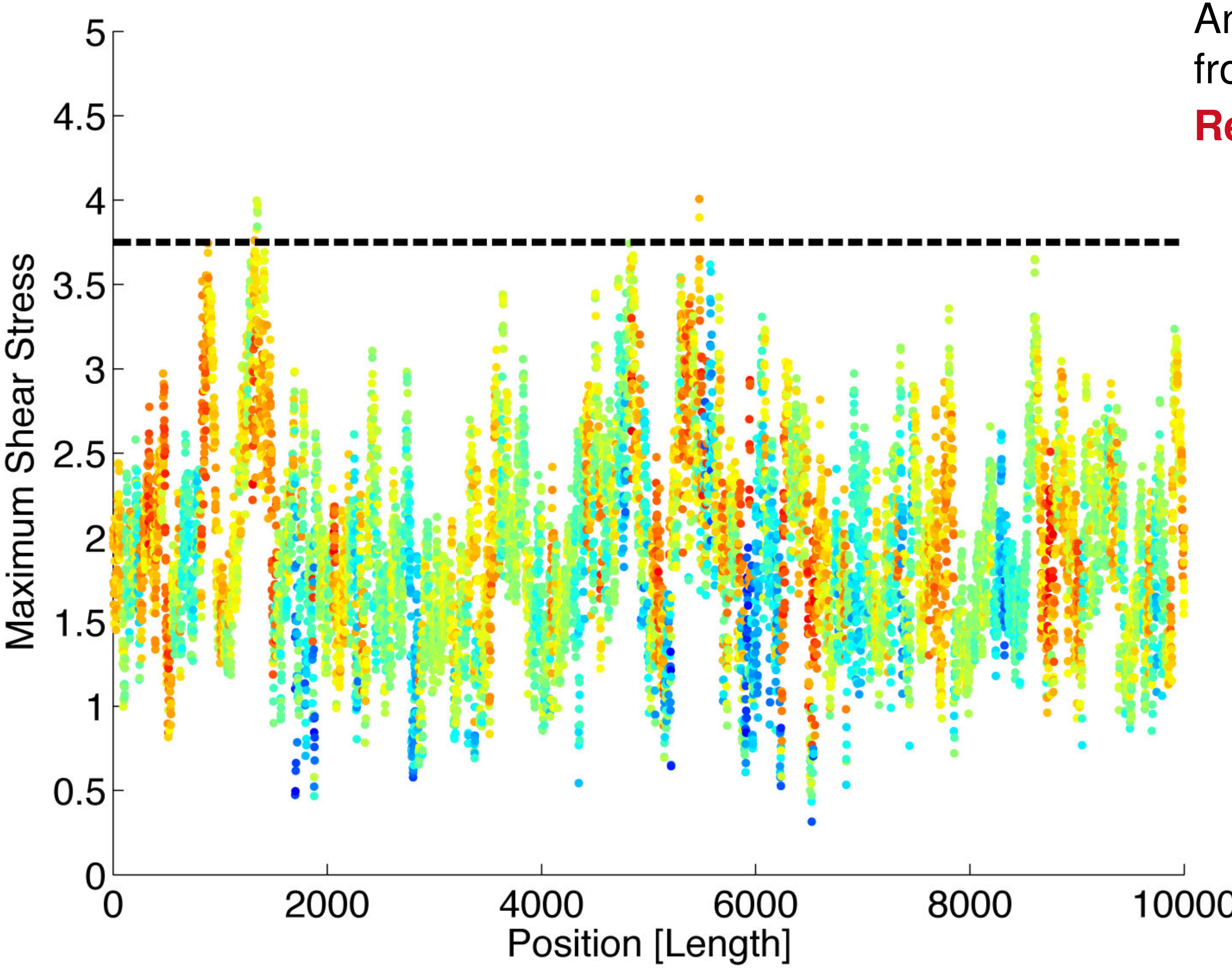
Time = 6.00 years

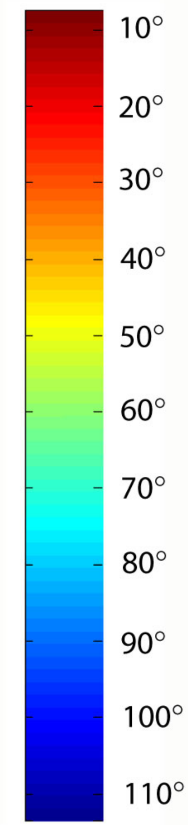


Time = 7.00 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

Red $\approx$ Aligned with $\dot{\sigma}_T$.

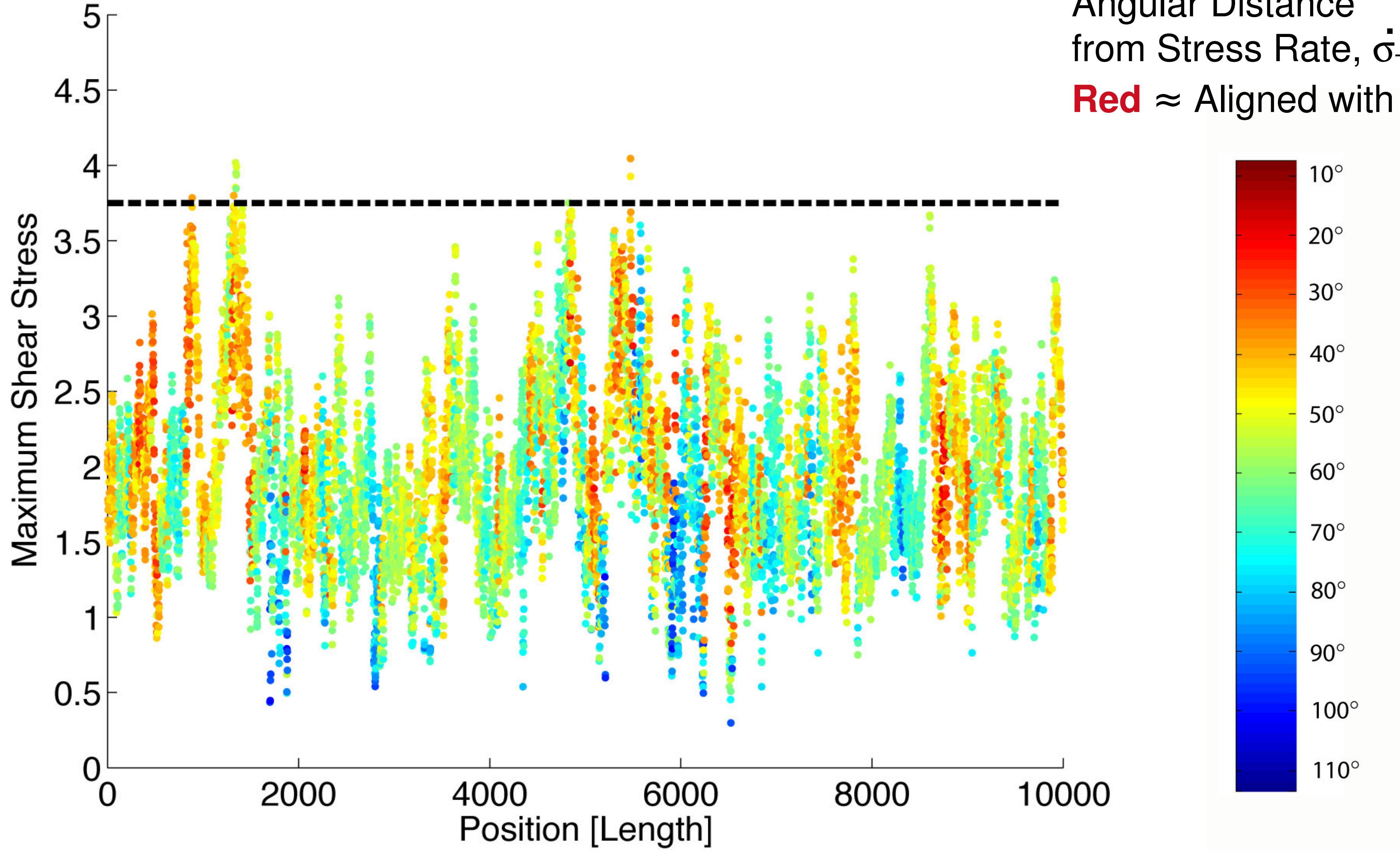


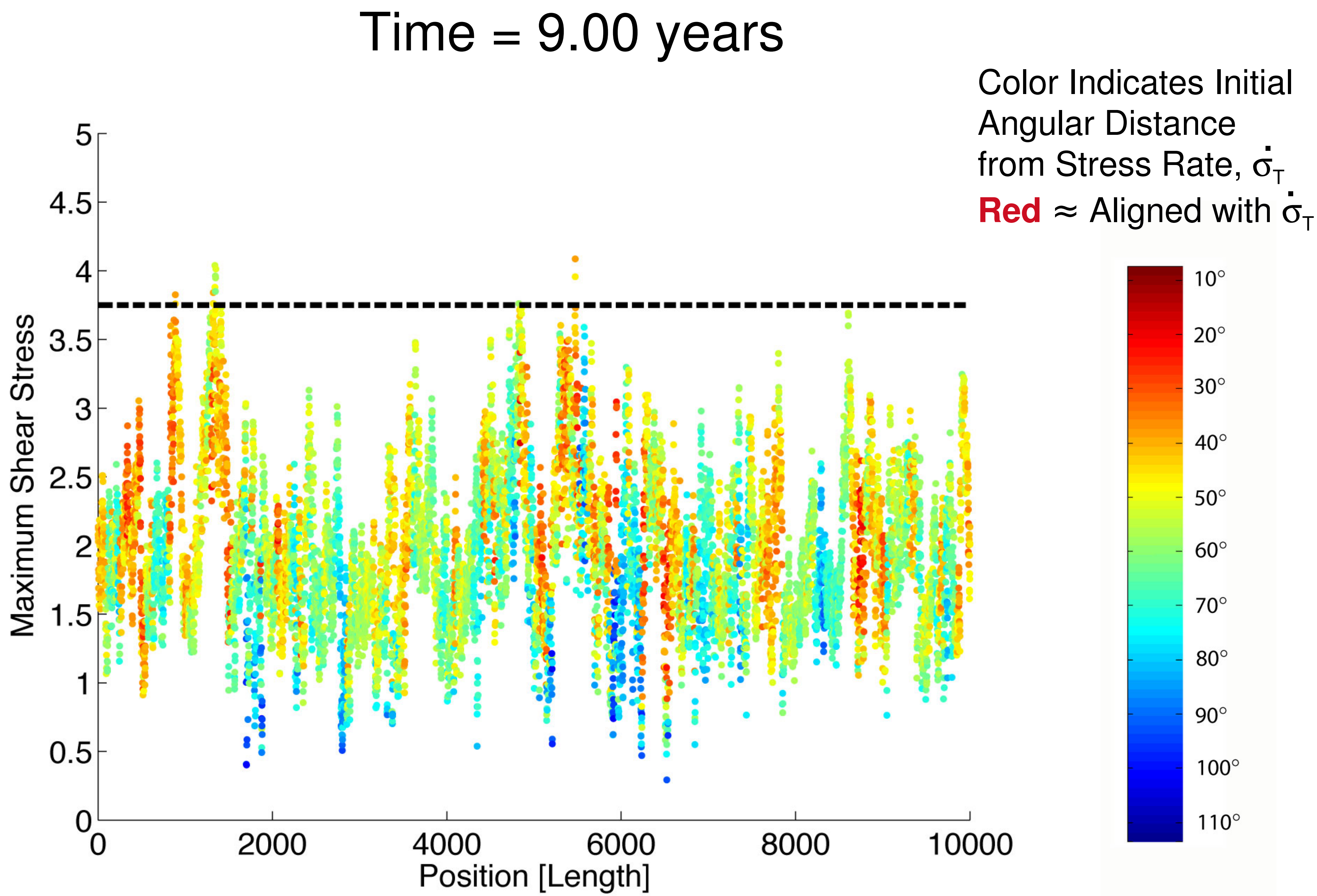


Time = 8.00 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

Red ≈ Aligned with $\dot{\sigma}_T$.

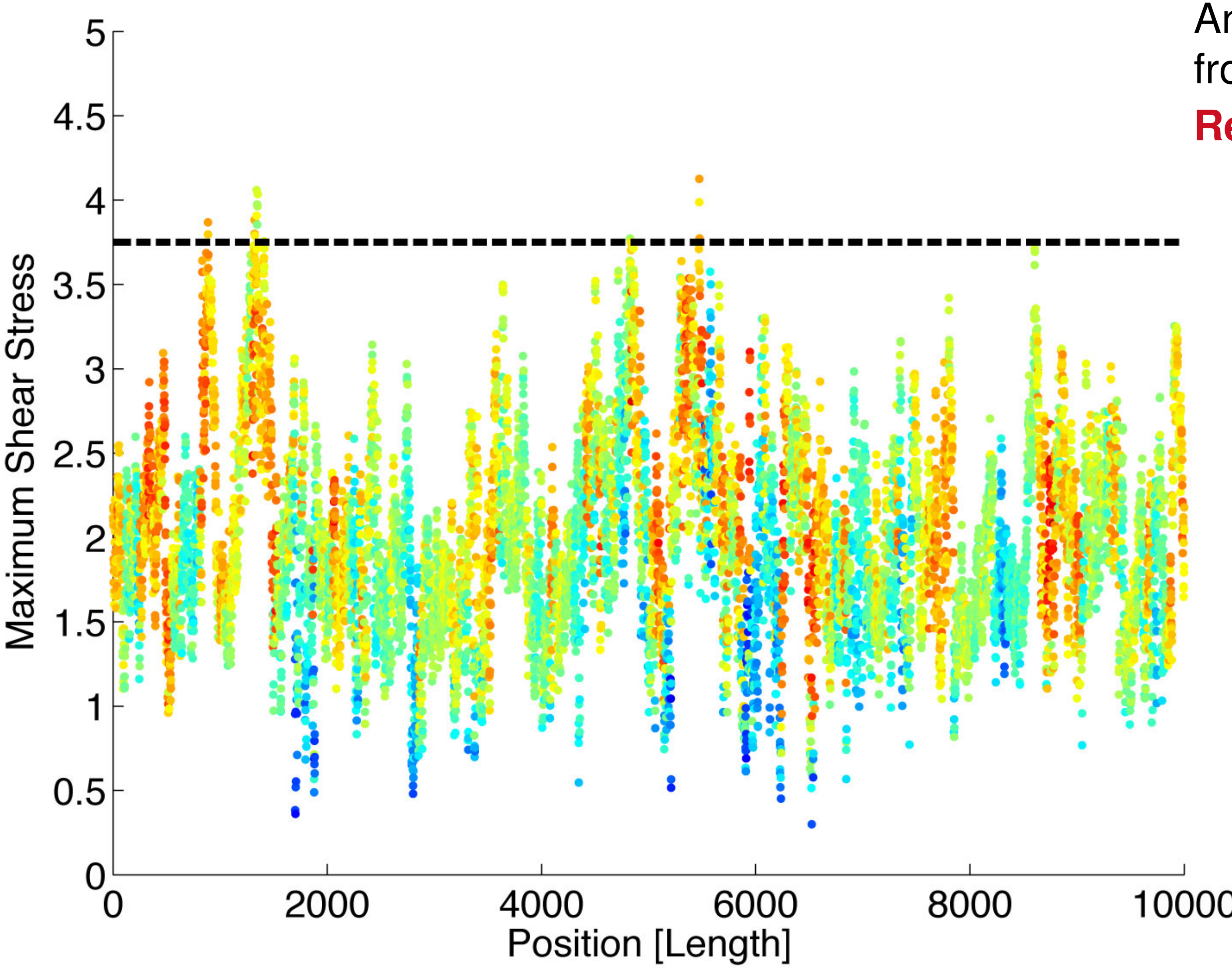


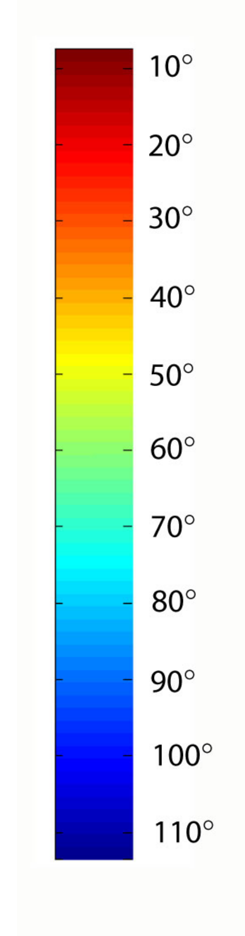

Time = 9.00 years
Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$
Red ≈ Aligned with $\dot{\sigma}_T$.
Maximum Shear Stress
Position [Length]
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
2000
4000
6000
8000
10000
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°

Time = 10.0 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

Red $\approx$ Aligned with $\dot{\sigma}_T$.

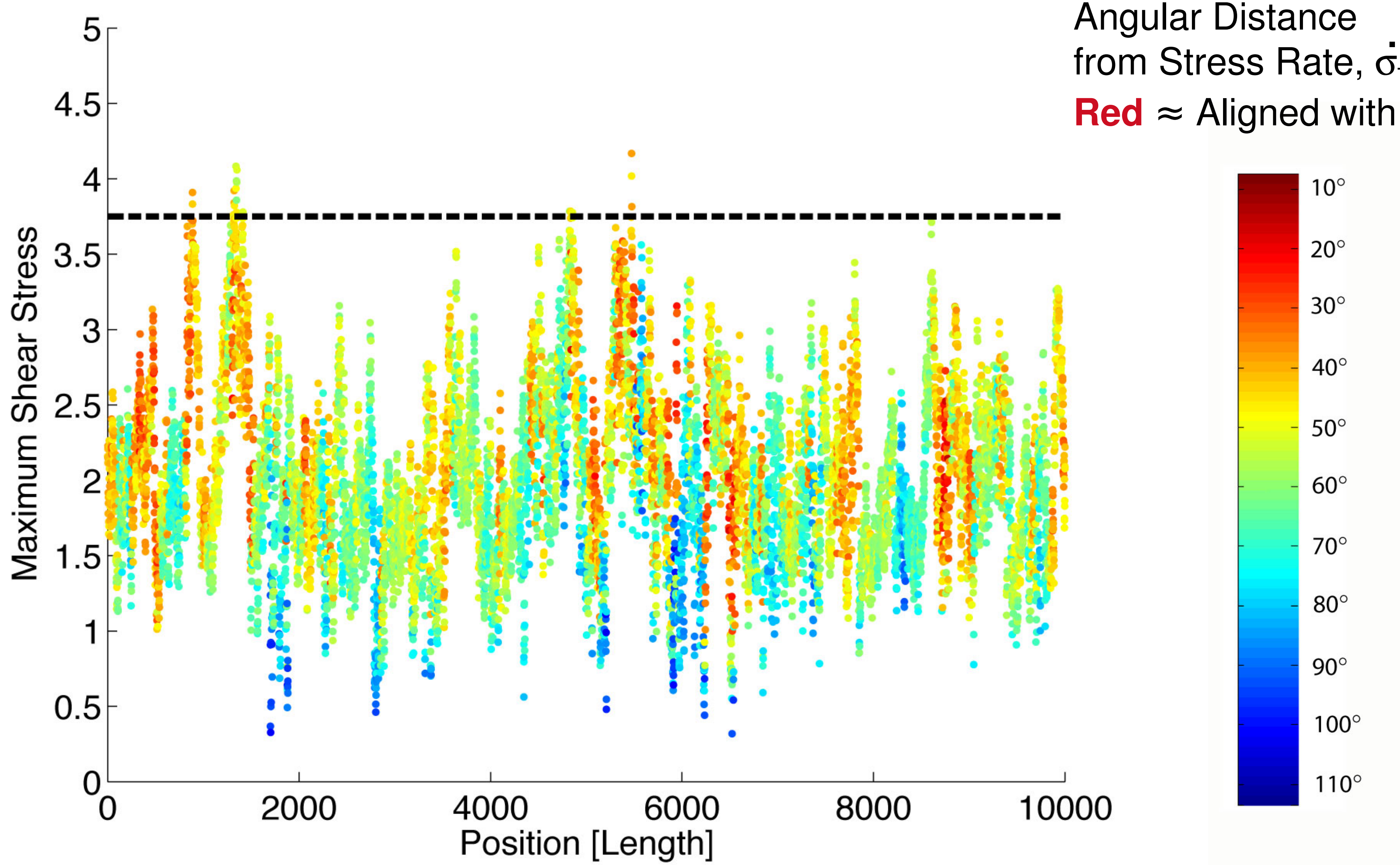




Time = 11.0 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

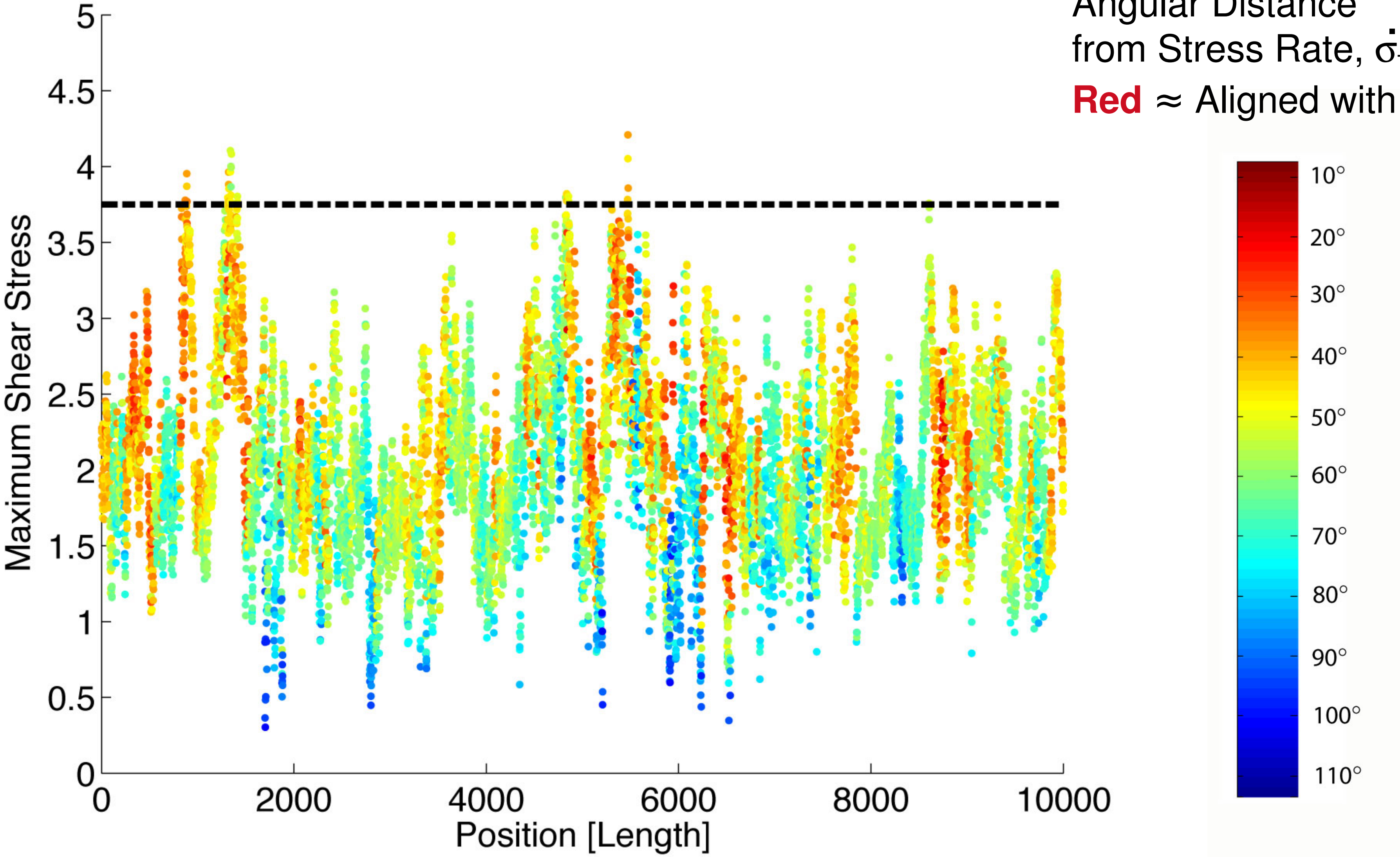
Red ≈ Aligned with $\dot{\sigma}_T$.



Time = 12.0 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

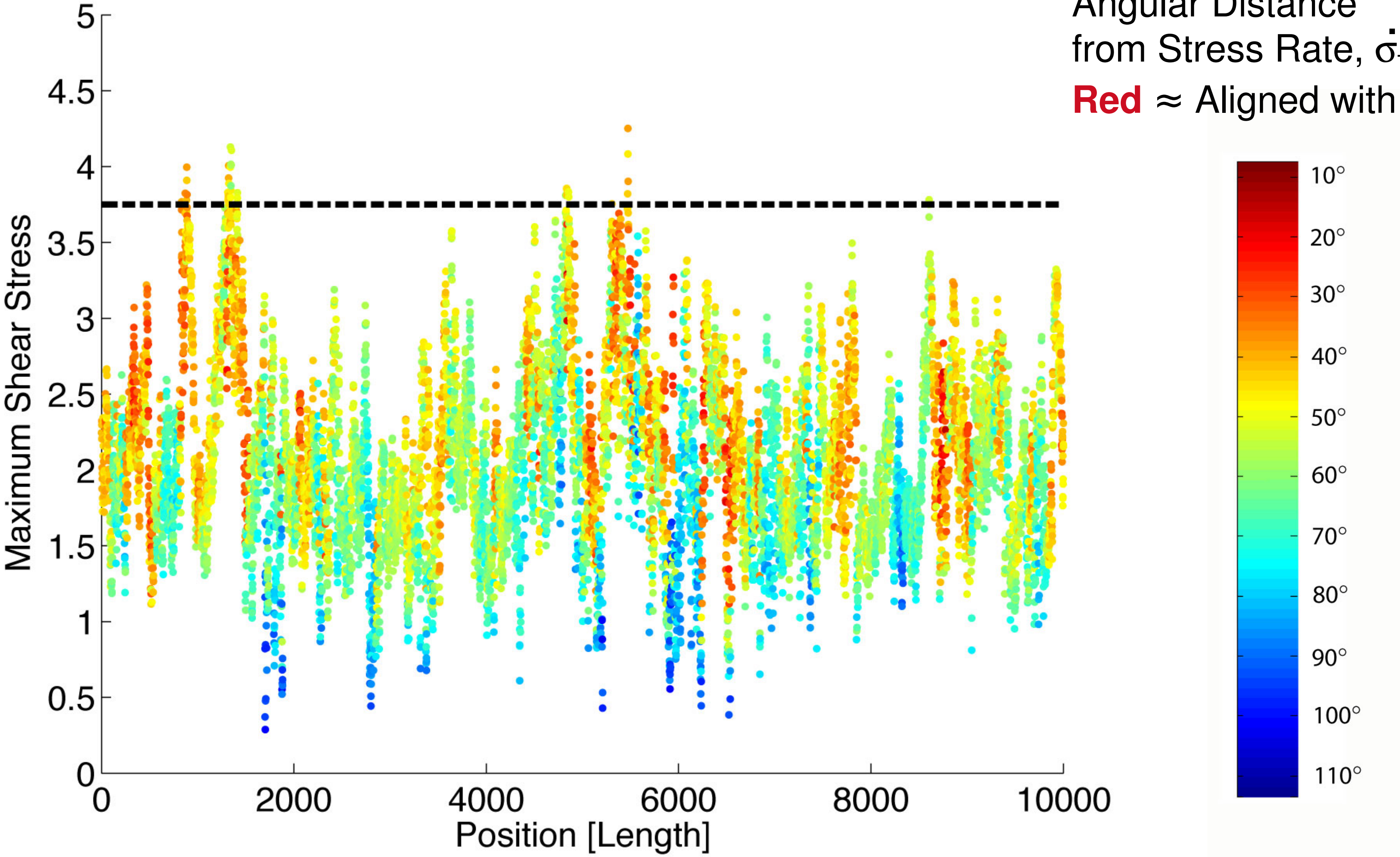
Red ≈ Aligned with $\dot{\sigma}_T$.



Time = 13.0 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

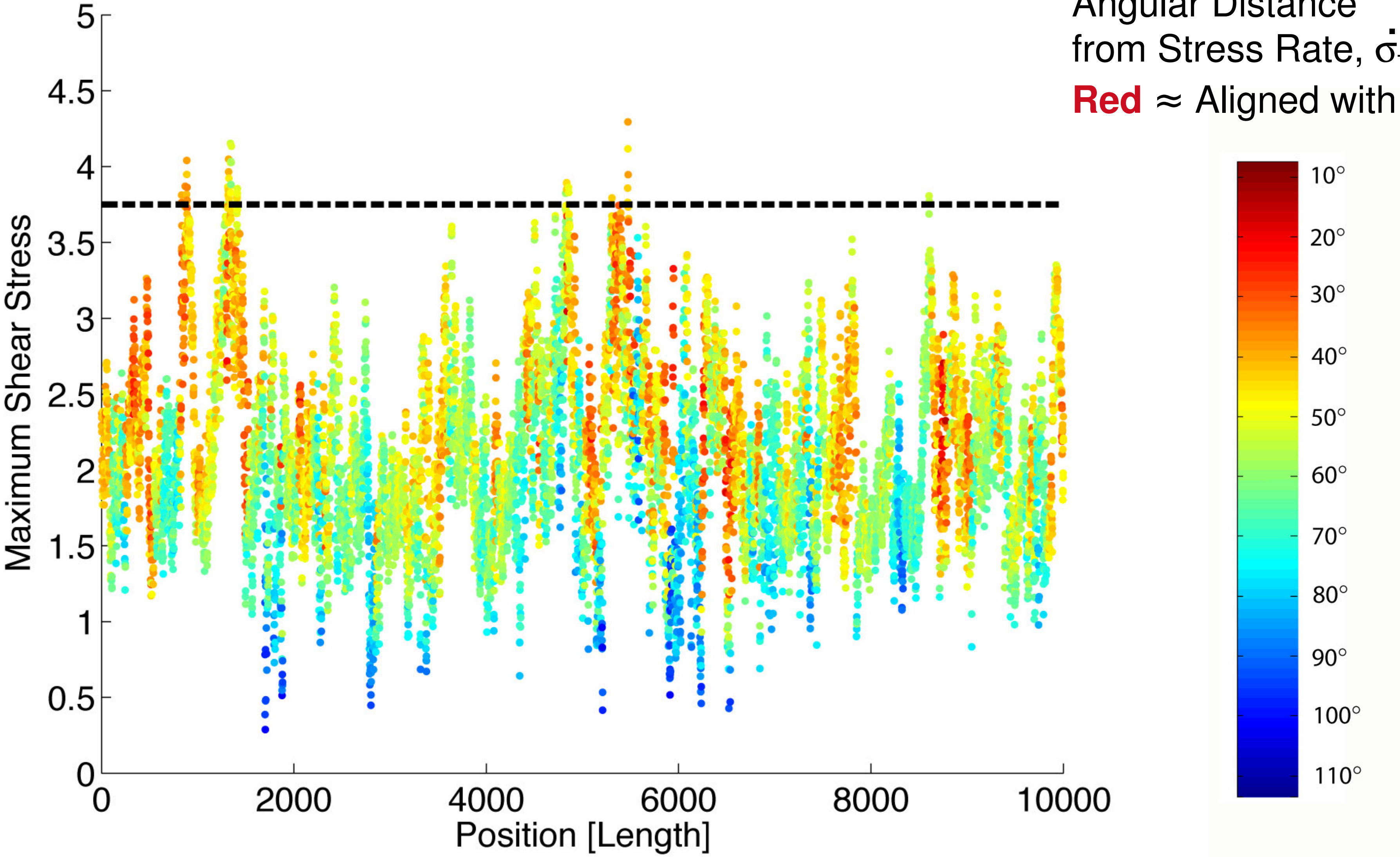
Red ≈ Aligned with $\dot{\sigma}_T$.



Time = 14.0 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

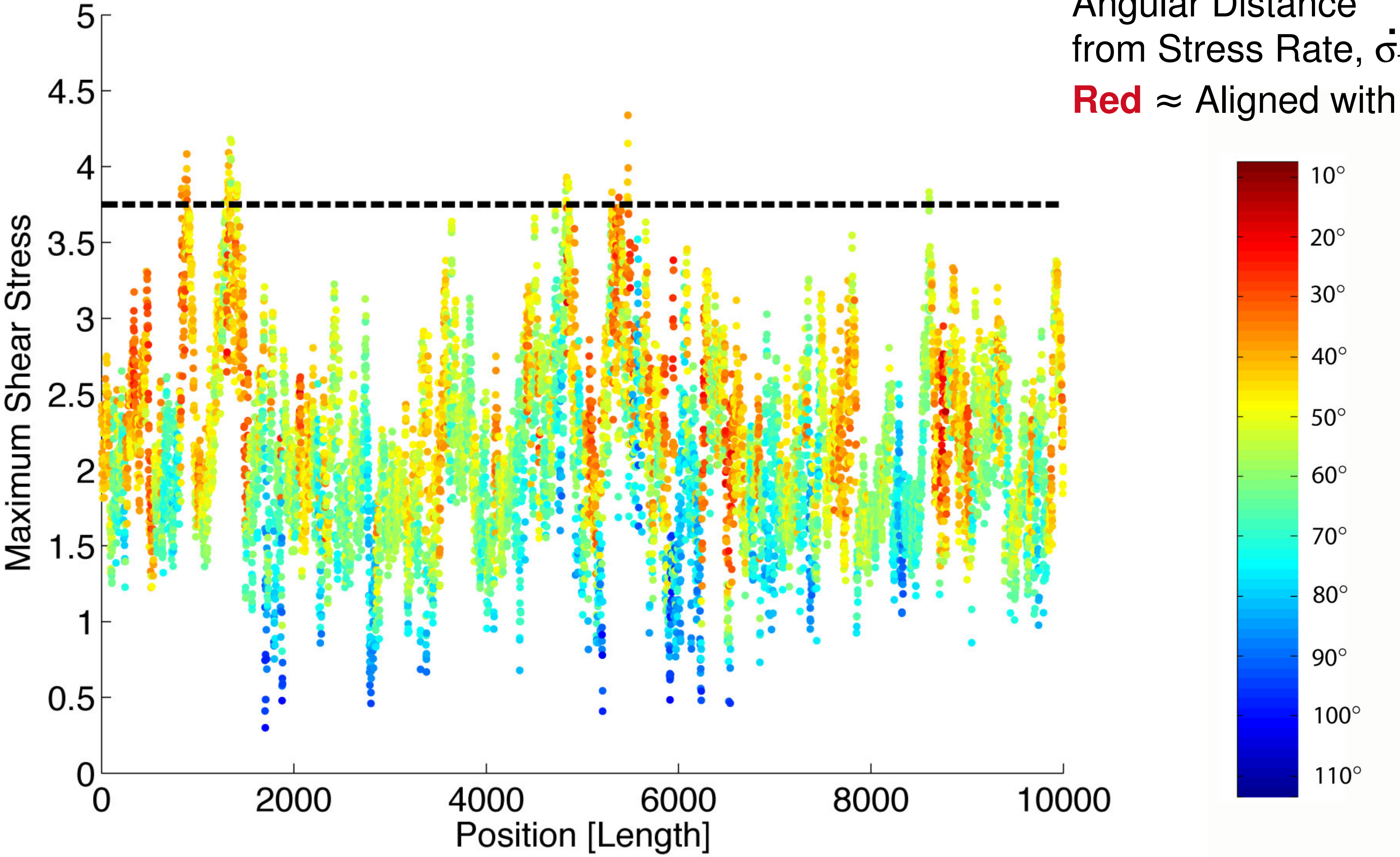
Red $\approx$ Aligned with $\dot{\sigma}_T$.



Time = 15.0 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

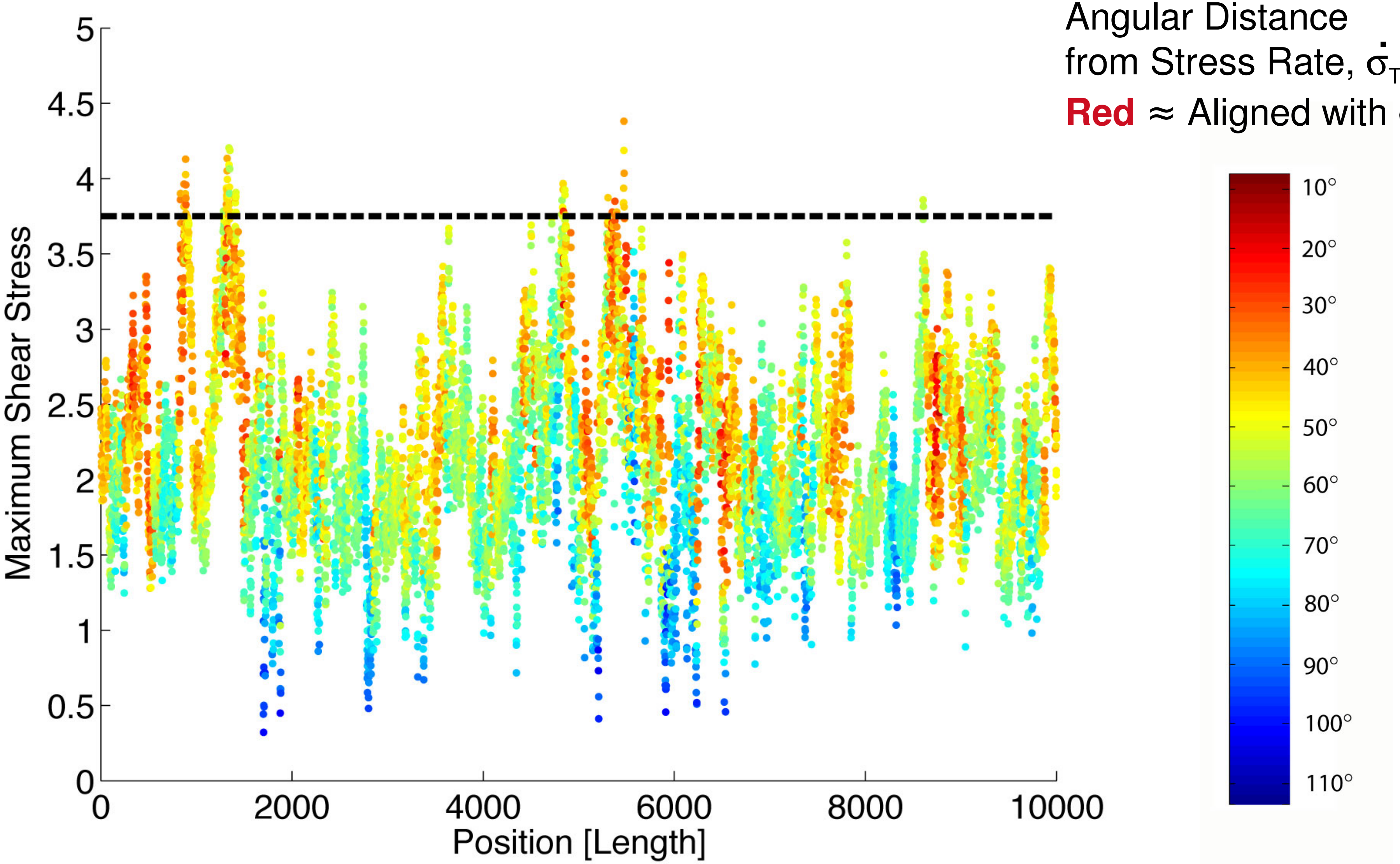
Red $\approx$ Aligned with $\dot{\sigma}_T$.



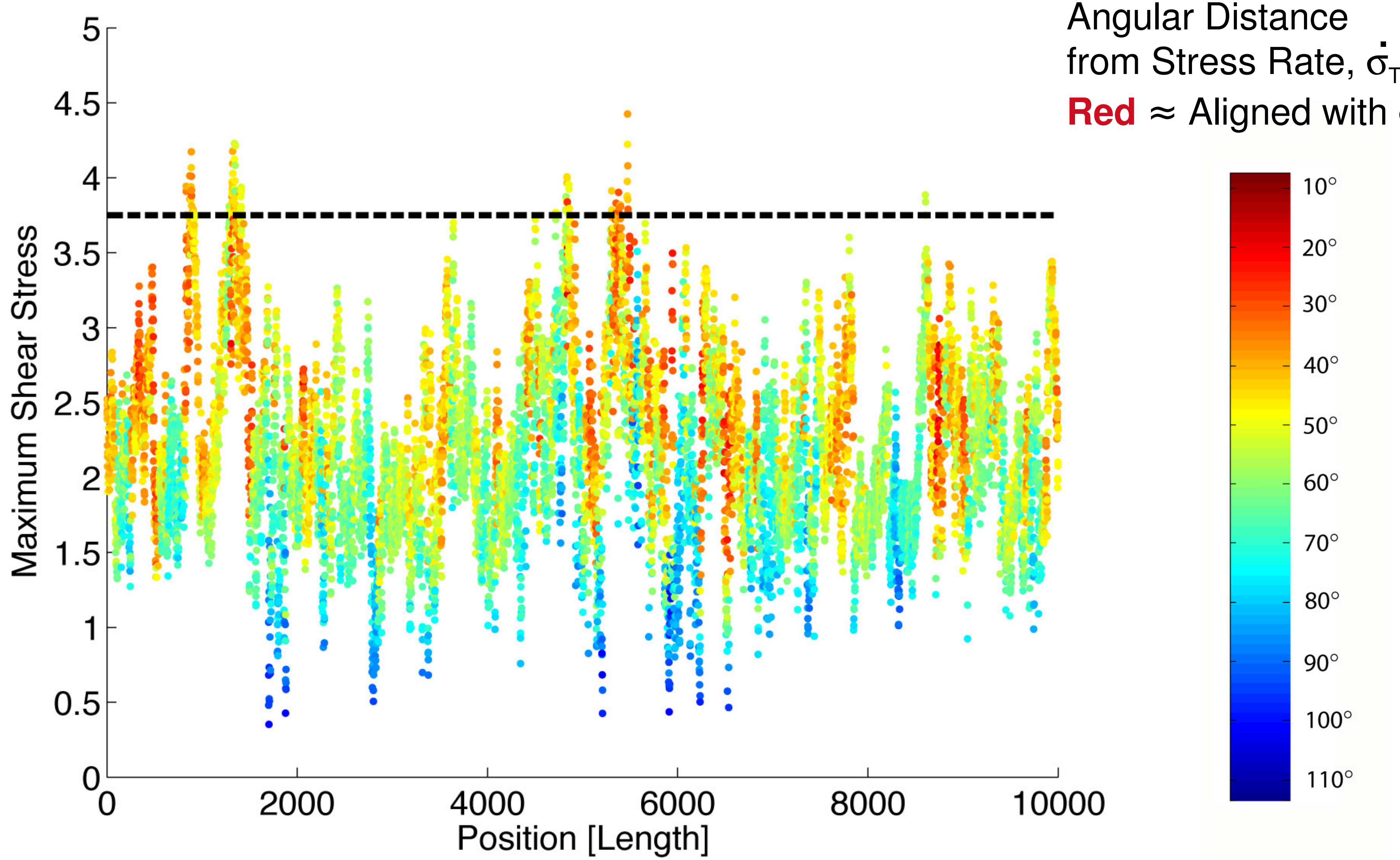
Time = 16.0 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

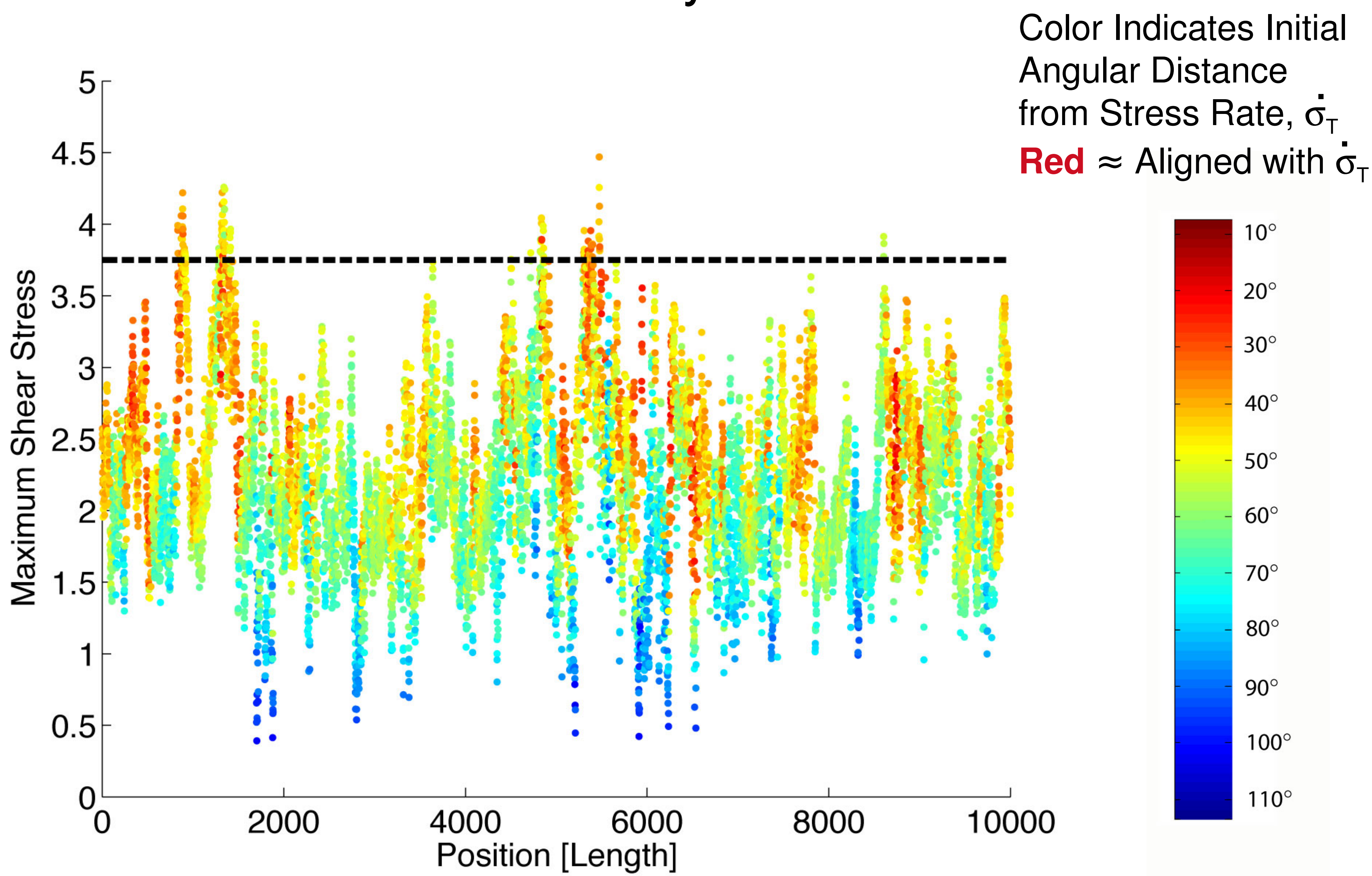
Red $\approx$ Aligned with $\dot{\sigma}_T$.



Time = 17.0 years



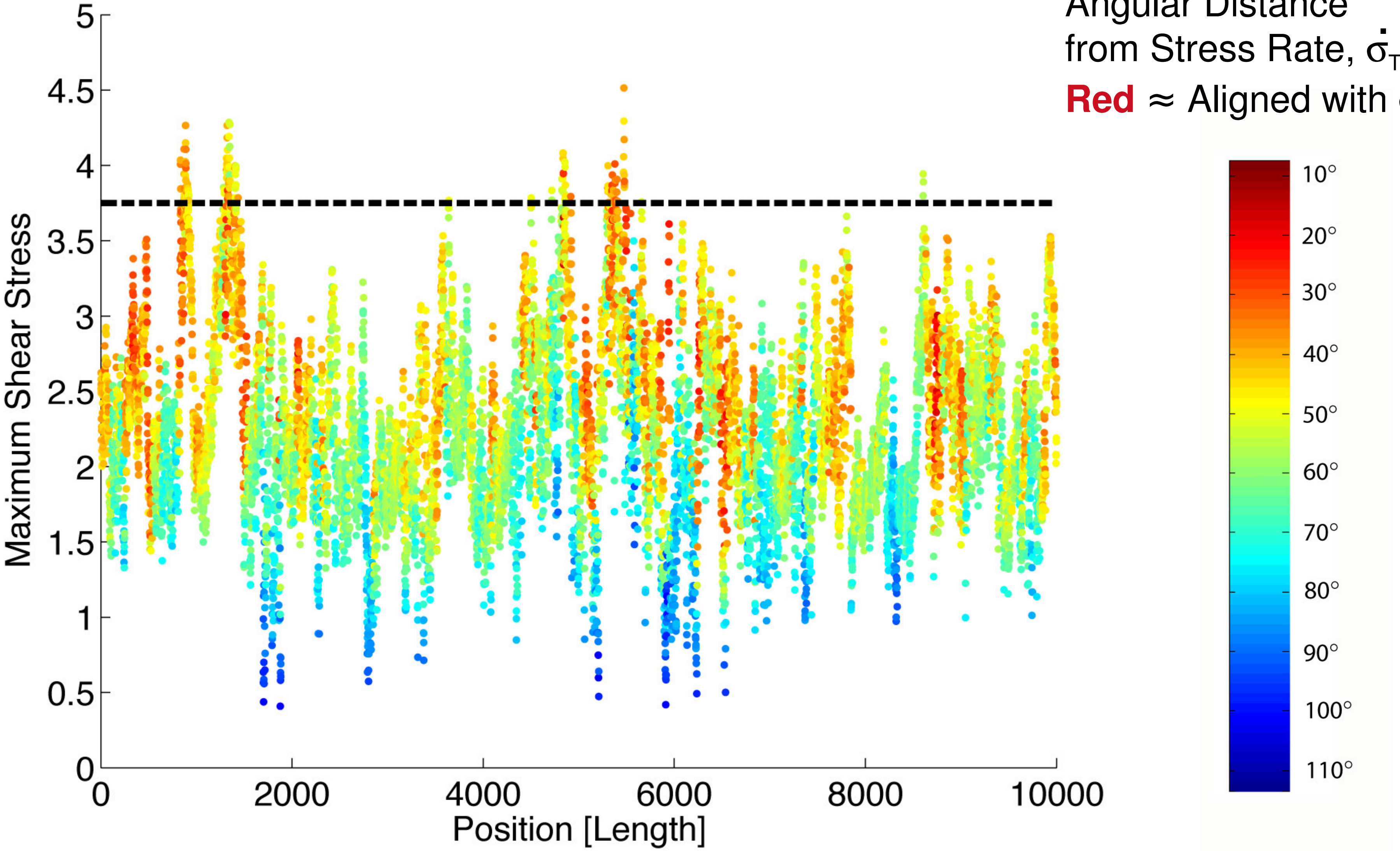
Time = 18.0 years



Time = 19.0 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

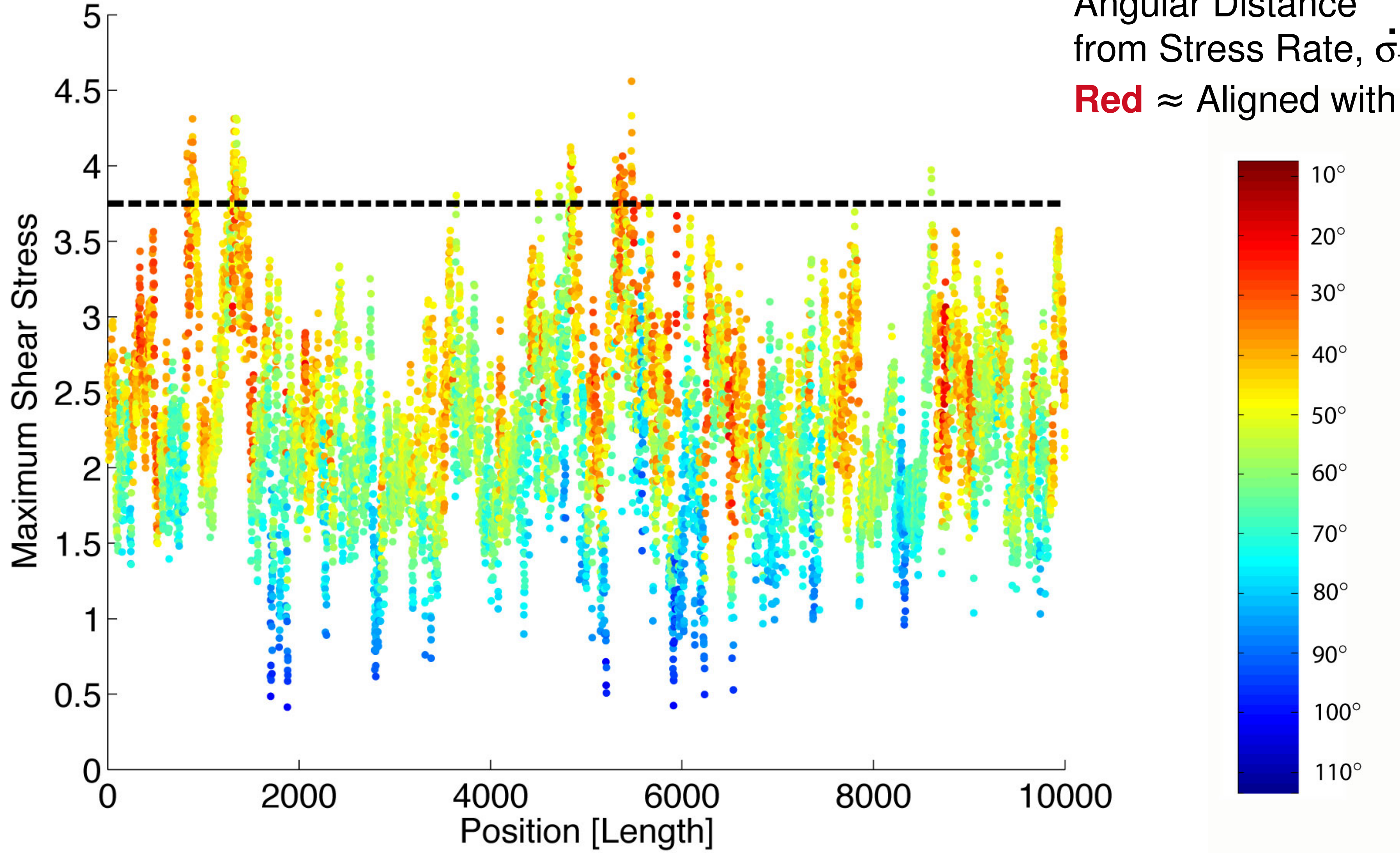
Red ≈ Aligned with $\dot{\sigma}_T$.



Time = 20.0 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

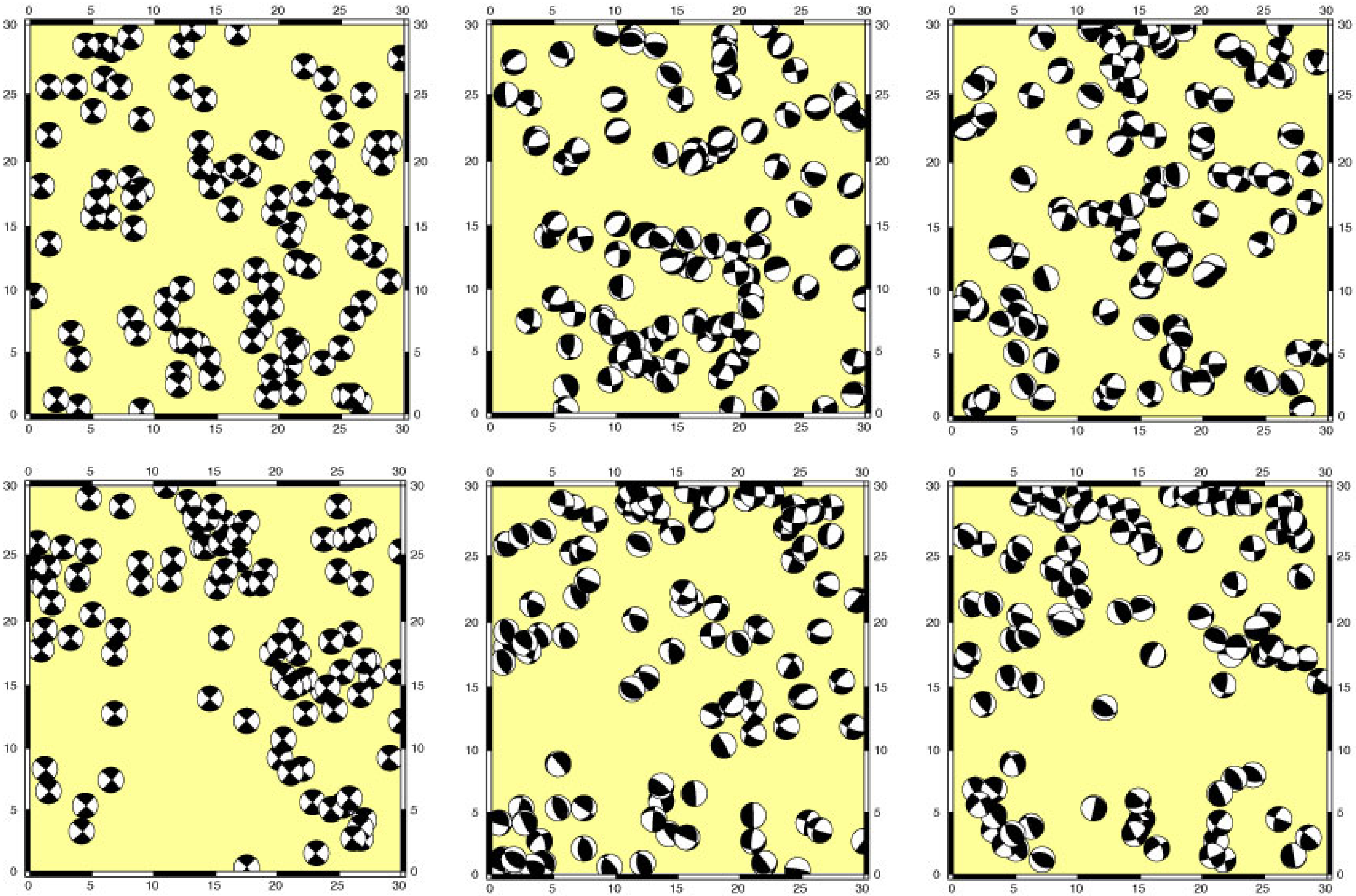
Red ≈ Aligned with $\dot{\sigma}_T$.



Effect of Stress Heterogeneity and Spatial Smoothing on Focal Mechanisms

α = 0.0

α = 1.0

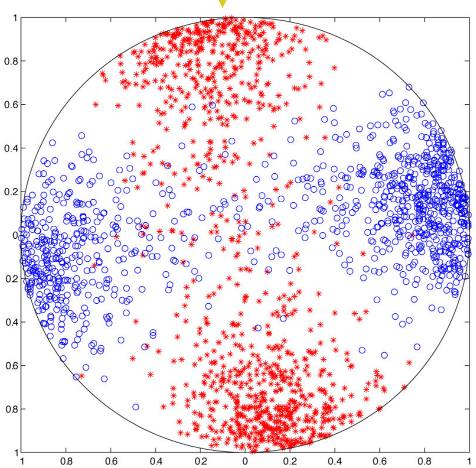


Ratio = 0.1 **Ratio = 3.5** **Ratio = 10.0**

Are Real Focal Mechansims Heterogeneous?

Synthetic Data

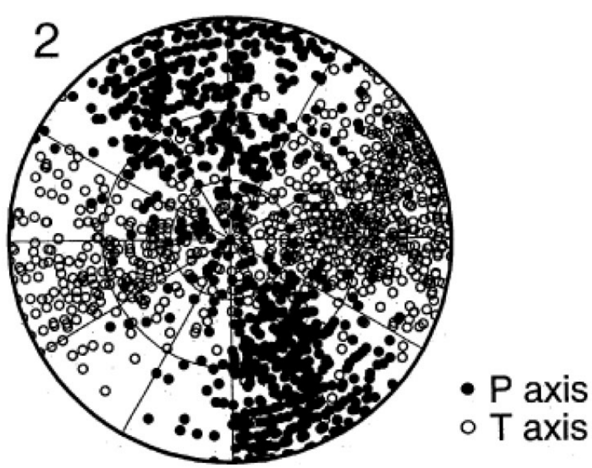
Ratio = 2.0



Real Data

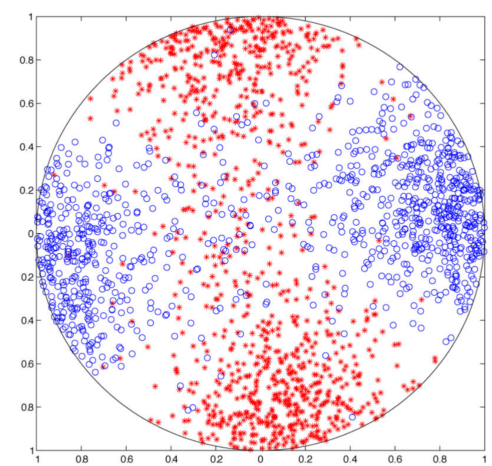
White Wolf Fault

(Hardebeck and Hauksson, 2001)



Synthetic Data

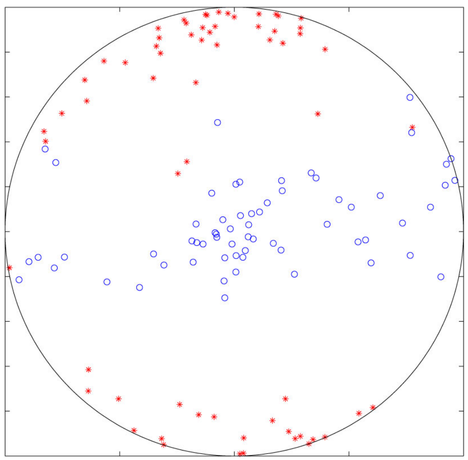
Ratio = 3.5



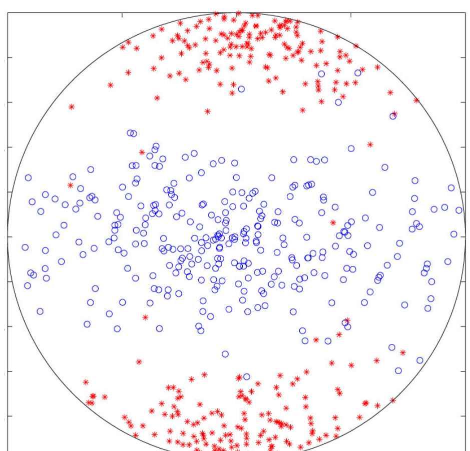
Our Simulated Data with Heterogeneity Ratio = 1.25 Looks Similar to Real Data

San Gabriel Mountains,
Reverse Fault

Real

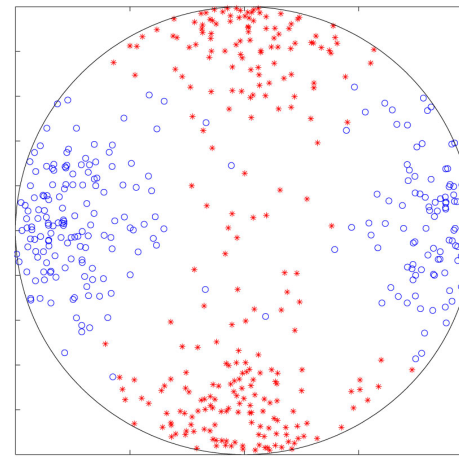


Simulated

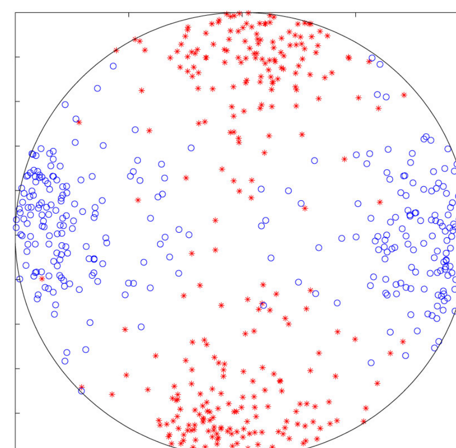


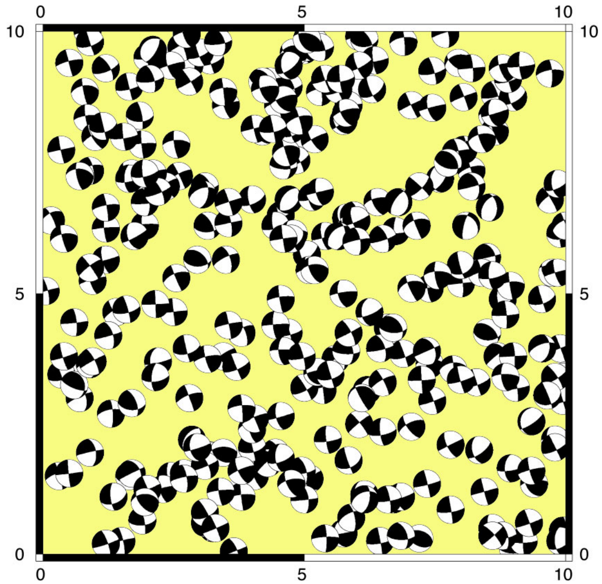
Region #4,
Strike-Slip Fault

Real

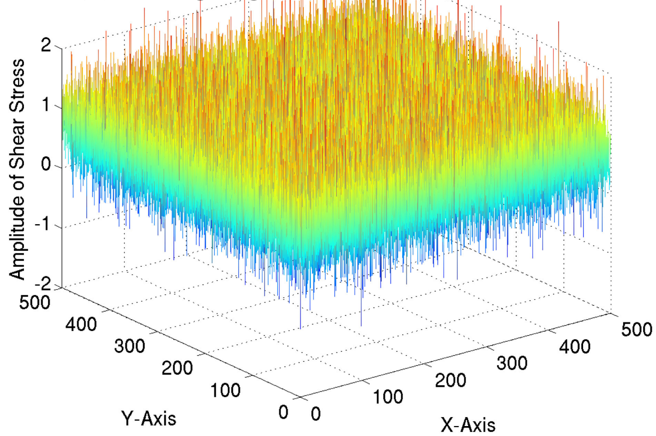


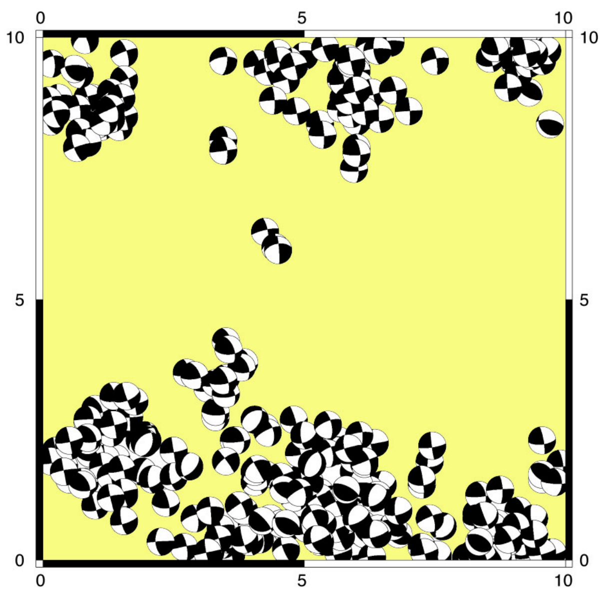
Simulated



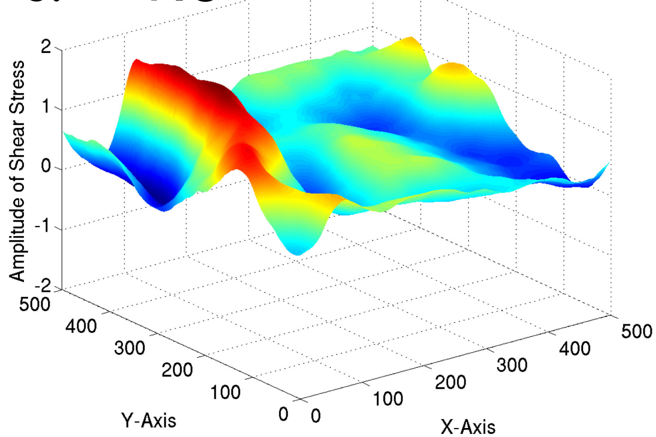


$\alpha = 0.0$





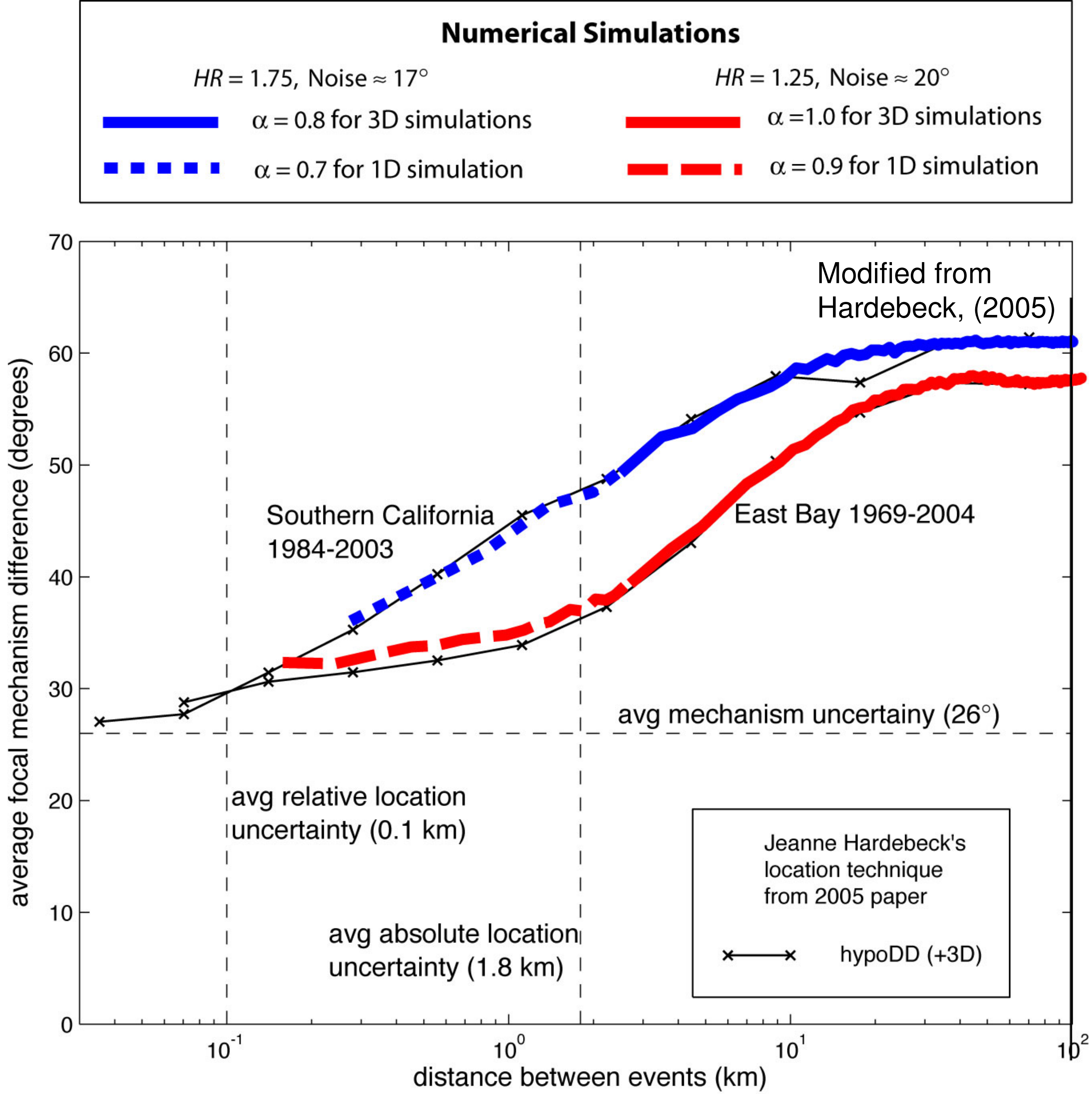
$\alpha = 1.5$



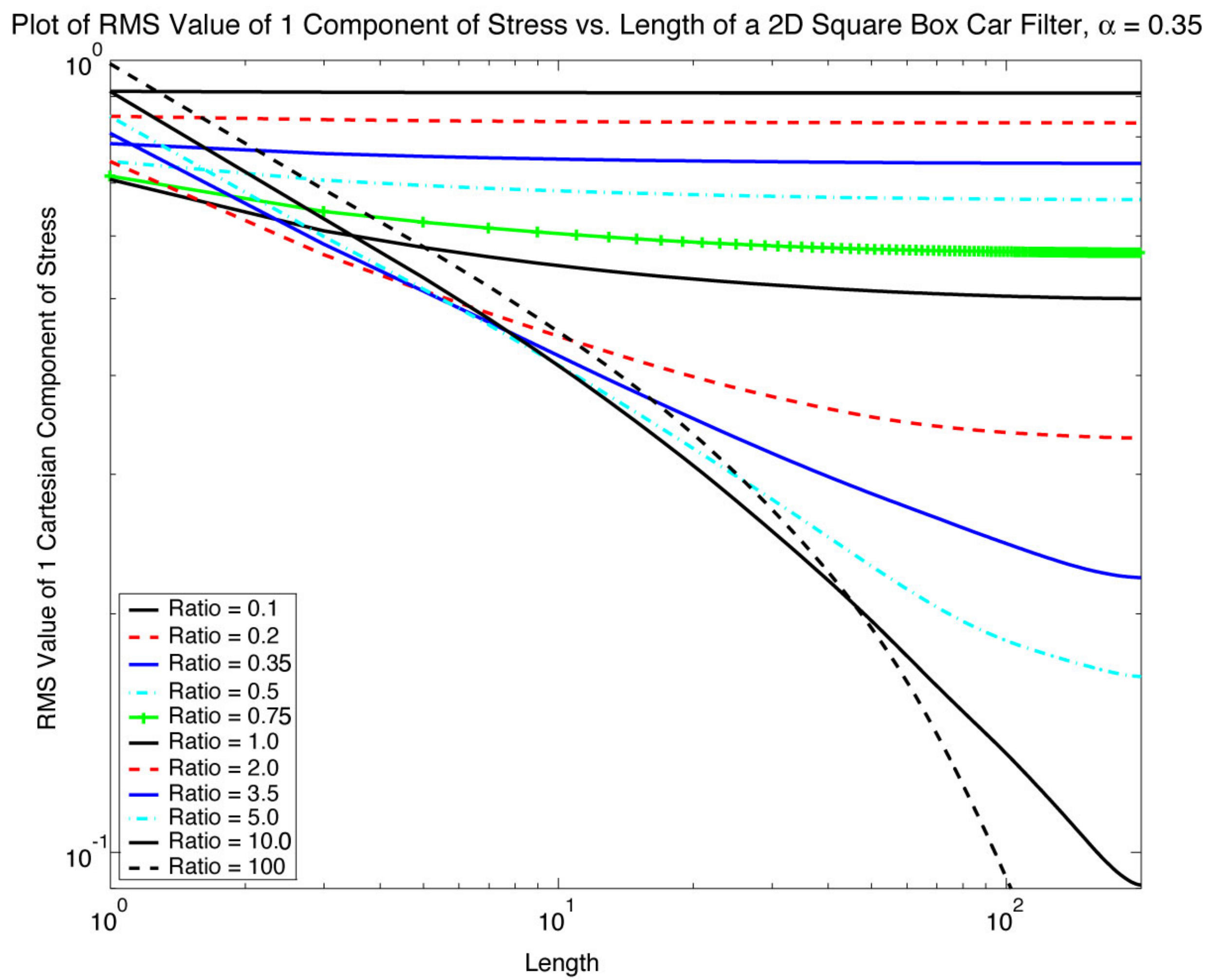
- As the spatial smoothing parameter, α, increases, two things happen:

- The failures begin clumping in space.

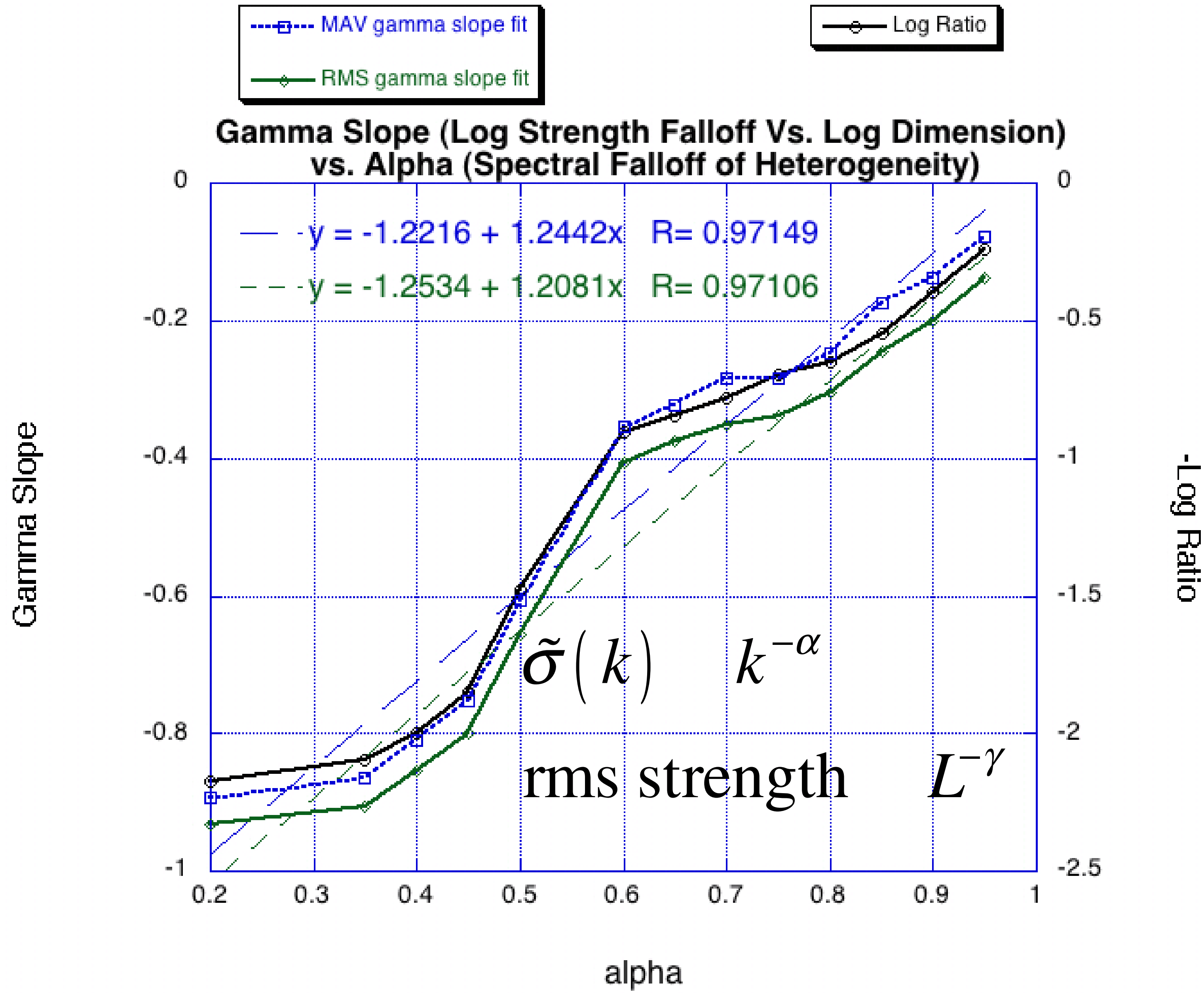
- Focal mechanisms close to one another have similar orientations.



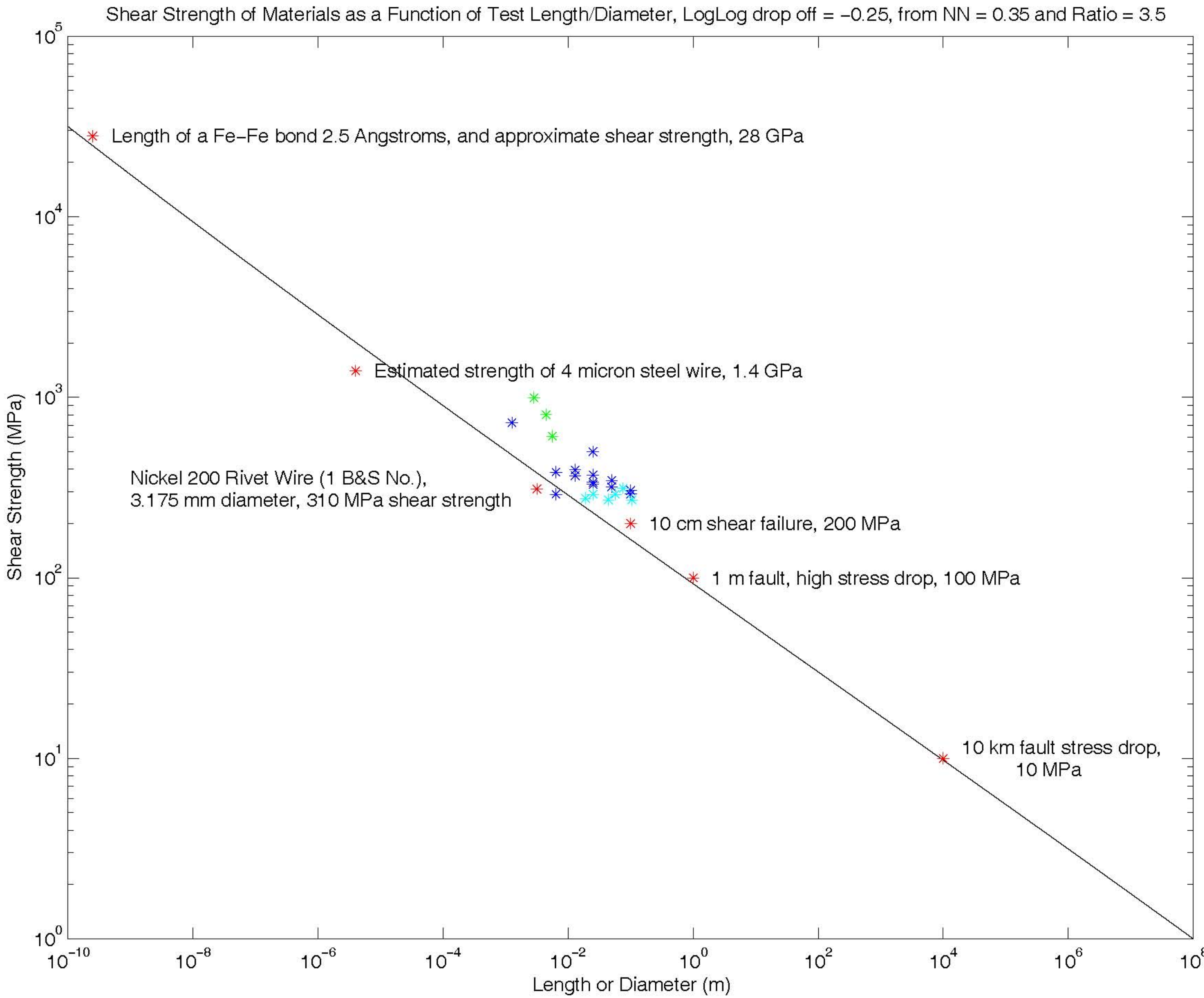
Numerical Simulations
$HR = 1.75$, Noise ≈ 17°
$HR = 1.25$, Noise ≈ 20°
α = 0.8 for 3D simulations
α =1.0 for 3D simulations
α = 0.7 for 1D simulation
α = 0.9 for 1D simulation
Modified from Hardebeck, (2005)
Southern California 1984-2003
East Bay 1969-2004
avg mechanism uncertainy (26°)
avg relative location uncertainty (0.1 km)
avg absolute location uncertainty (1.8 km)
Jeanne Hardebeck's location technique from 2005 paper
hypoDD (+3D)
average focal mechanism difference (degrees)
distance between events (km)
0
10
20
30
40
50
60
70
10^{-1}
10^{0}
10^{1}
10^{2}



- Now that we have a guess of stress distribution, what is the rms internal strength?
- strength is the expected value [rms (stress averaged over 2-d squares of length given by horizontal axis)]
- Define γ such that rms strength $\quad$ length$^{-\gamma}$



MAV gamma slope fit
RMS gamma slope fit
Log Ratio
Gamma Slope (Log Strength Falloff Vs. Log Dimension)
vs. Alpha (Spectral Falloff of Heterogeneity)
y = -1.2216 + 1.2442x R= 0.97149
y = -1.2534 + 1.2081x R= 0.97106
$\tilde{\sigma}(k)$ $k^{-\alpha}$
rms strength $L^{-\gamma}$
Gamma Slope
-Log Ratio
alpha
0
-0.2
-0.4
-0.6
-0.8
-1
-0.5
-1.5
-2
-2.5
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



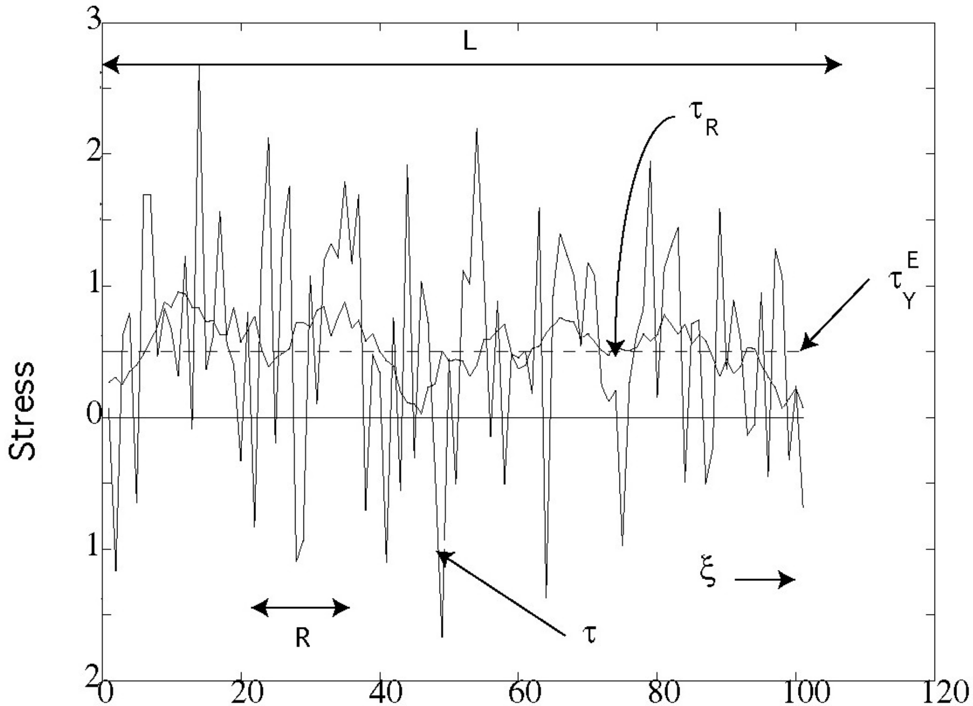
Shear Strength of Materials as a Function of Test Length/Diameter, LogLog drop off = −0.25, from NN = 0.35 and Ratio = 3.5
Length of a Fe−Fe bond 2.5 Angstroms, and approximate shear strength, 28 GPa
Estimated strength of 4 micron steel wire, 1.4 GPa
Nickel 200 Rivet Wire (1 B&S No.), 3.175 mm diameter, 310 MPa shear strength
10 cm shear failure, 200 MPa
1 m fault, high stress drop, 100 MPa
10 km fault stress drop, 10 MPa
Shear Strength (MPa)
Length or Diameter (m)

Conclusions

- If the Earth fails at multiple length scales, then the average stress (strength) must decrease with increasing length
- Focal mechanisms inversions provide stress orientations that are biased to the GPS derived stress rate tensor orientation

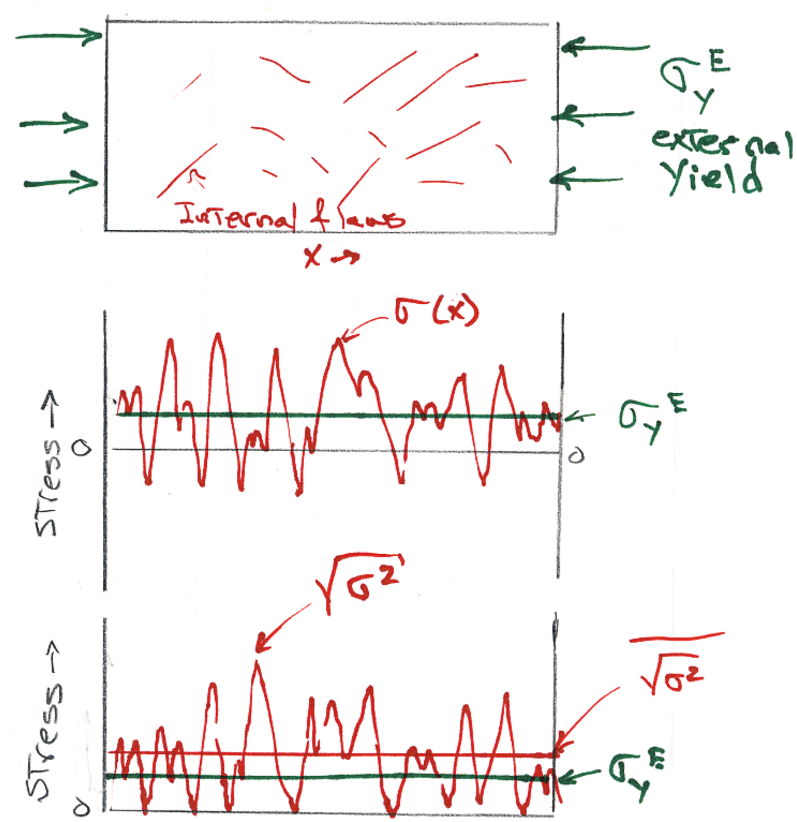
You cannot physically measure heterogeneous stress at a point

- Stress measurements are always averages over some length scale R.
- In the case of earthquakes, this is the dimension of the rupture, R. That is, strength measured from the average stress of rupture R, is similar to a low-pass filter with corner wavenumber 1/R; the low-pass filter tells us about the average stress at length scale R.
- If we could put a chunk of crust of dimension L in a laboratory press, then we would measure the same strength as derived from ruptures when R L.
- Conjecture …By characterizing the heterogeneous stress in the earth, and by then filtering at different length scales, we can determine how strength, as would be measured in a lab, changes with the dimension of the material.

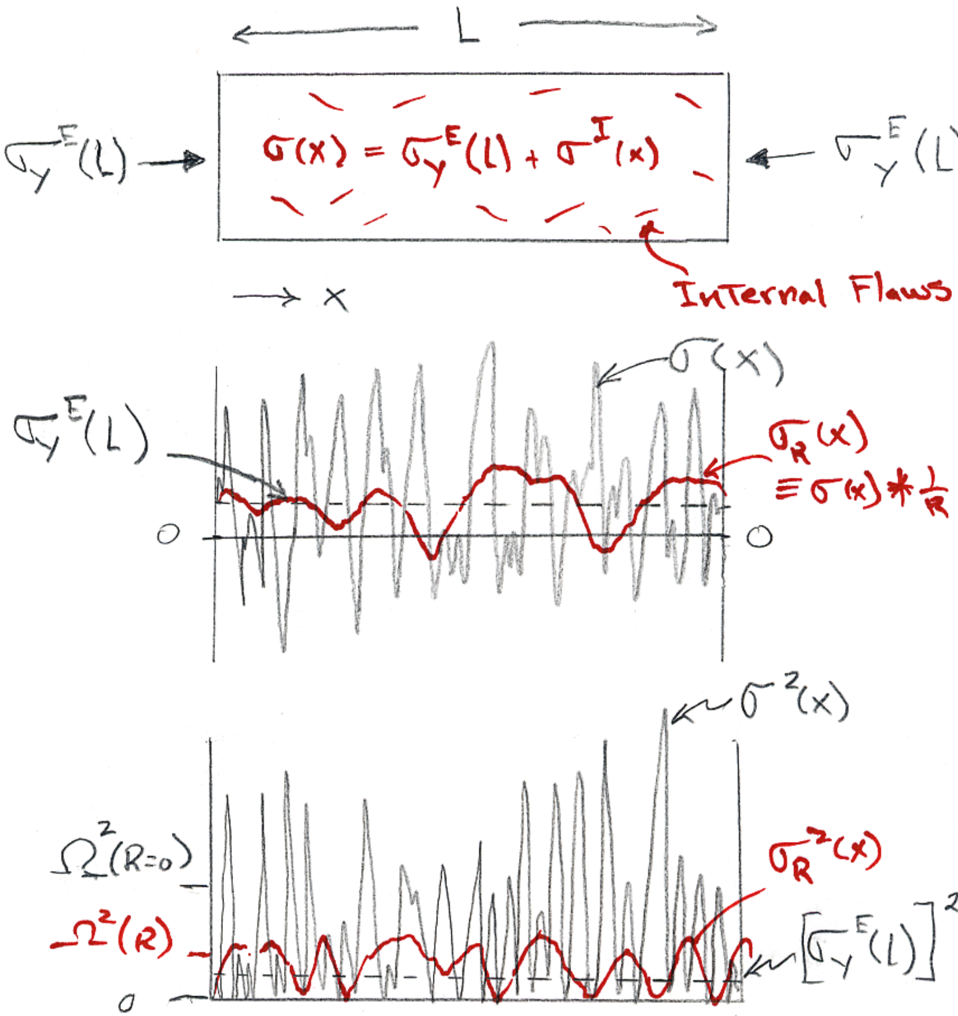


How can you define “strength” when stress is heterogeneous?

- I said take the average of the stress at the length scale of the earthquake, but strength is a **positive** quantity.
- Seismologist’s “strength” estimates are similar to determing Expected value [abs value (filtered stress)]
- Or alternatively, the Expected value [rms (filtered stress)]



What is the strength of a material with heterogeneous stress?



R dimension of rupture

EQ energy tells stress averaged over R

$\sigma_Y^E(L)$ is the "external strength" and

is measured in the Lab

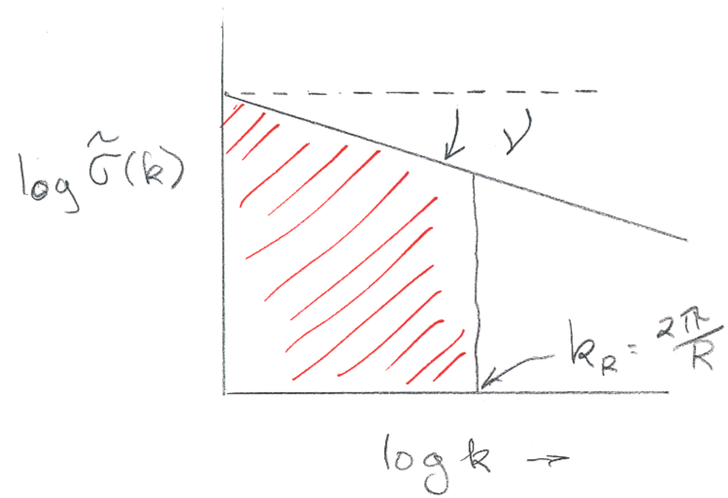
$\Omega(R)$ is the "internal rms strength"

and is estimated by seismologists

$$\Omega^2(R) \quad E \;\; \sigma_R^2 \;=\; \sigma_Y^E(L) \;\; ^2 + \mathrm{var}(\sigma_R)$$

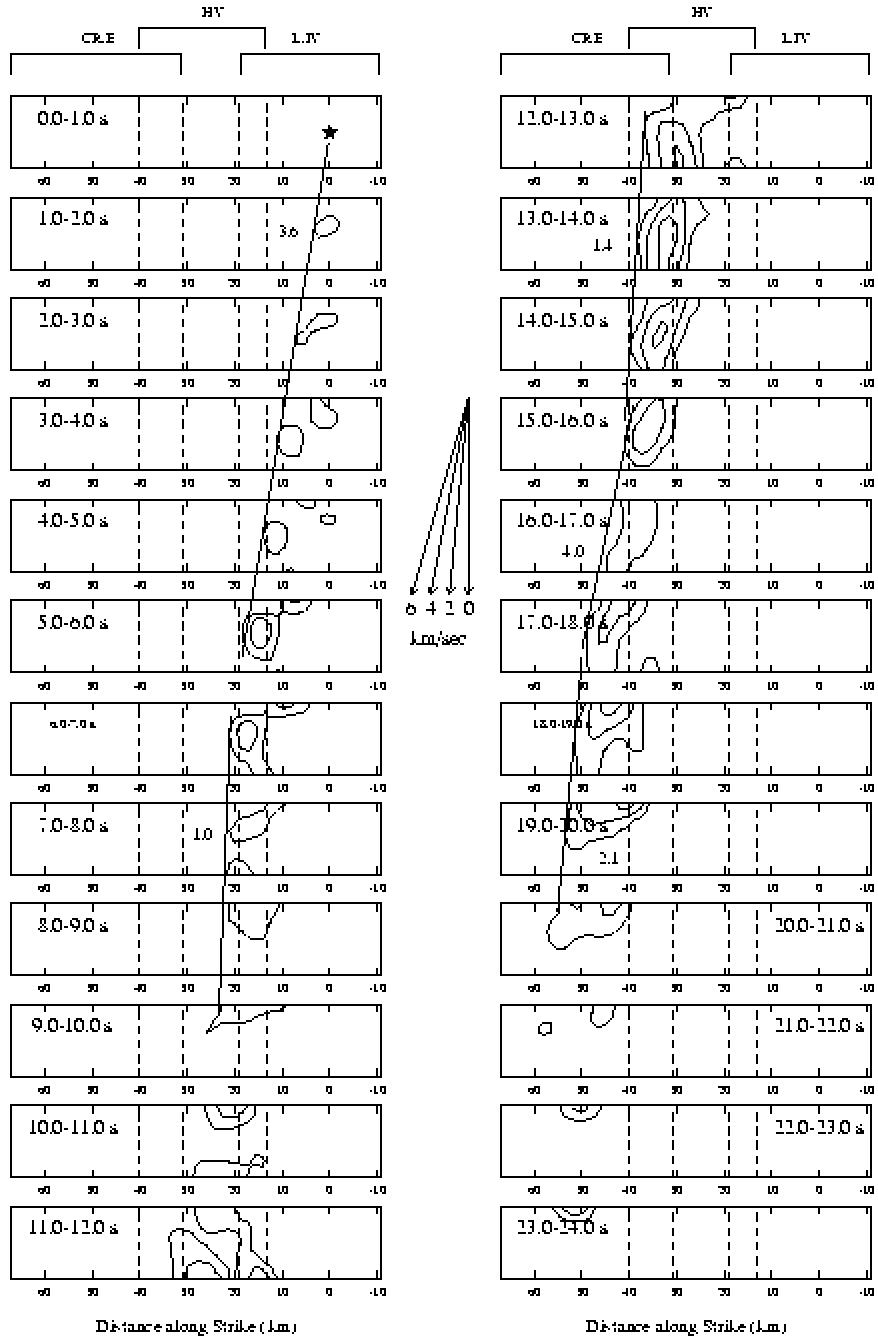
Internal rms strength can be determined from the spectrum of the stress.

- The Earth has evolved into a amplitude spectrum, which is what we really mean when we say "strength of the crust."
- If we assume that the stress is stochastic and approximately stationary with respect to position,
- And if the material fails (through time) in such a way that the internal stress at *all* values of *R* is comparable to the external yield stress at that dimension, then the material is in a critical state at all length scales.
- The amplitude of "rms strength" is determined by the integral under the power spectrum out to the wavenumber with R.
- If we can determine the spectrum of stress in the crust, we can determine its strength at all length scales!!

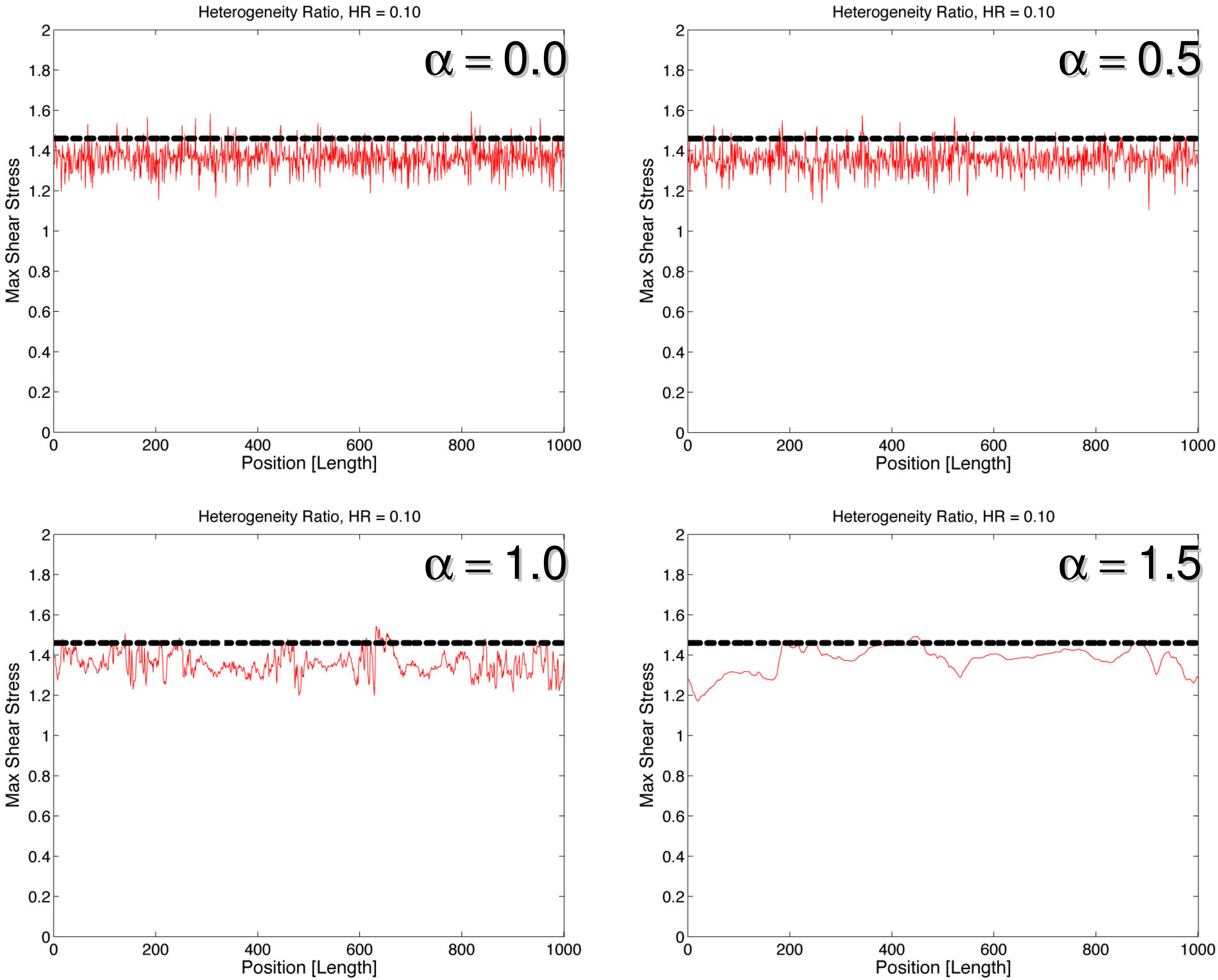


Rupture Physics Summary (Size Matters)

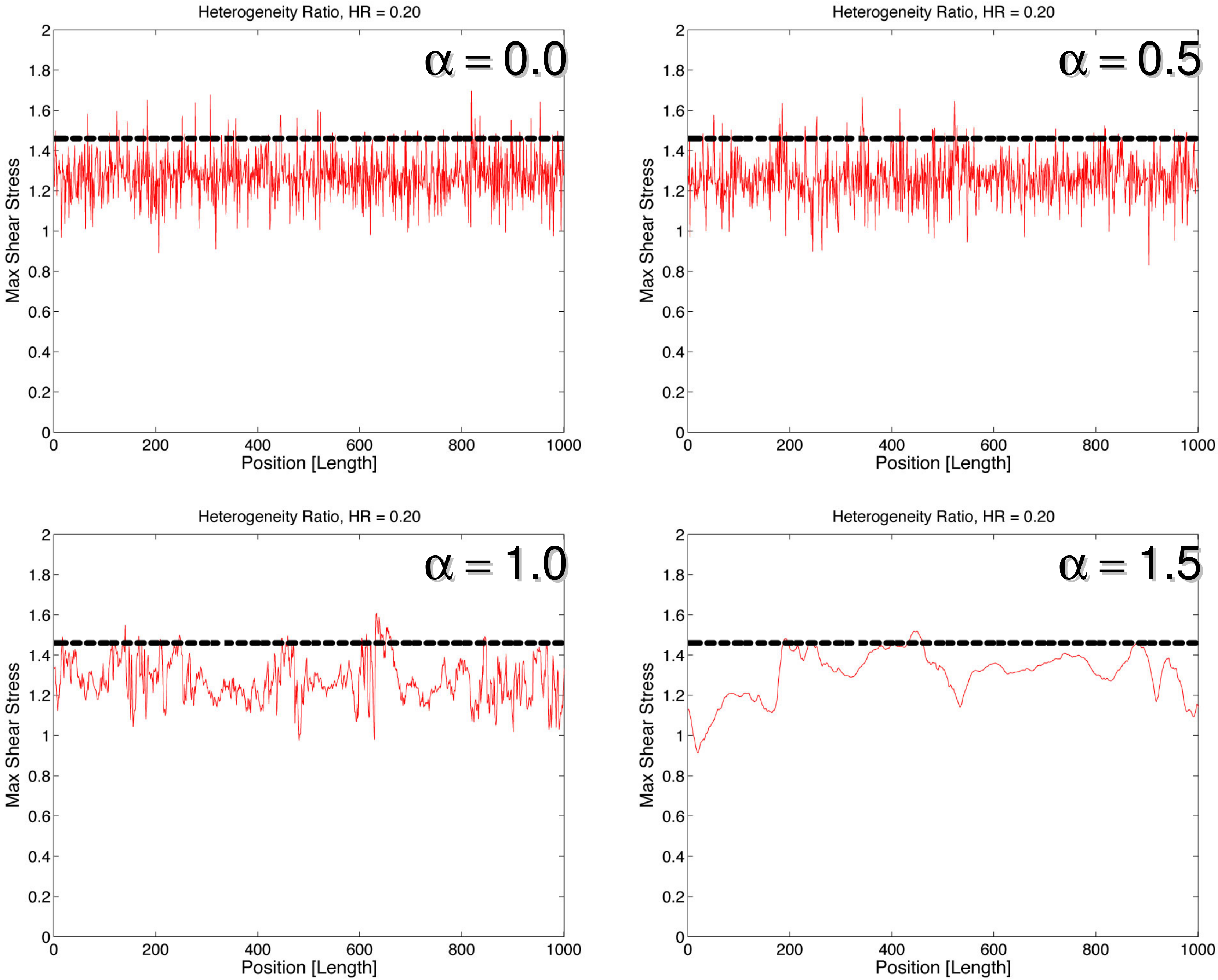
- Slip is spatially heterogeneous in earthquakes and stress in the crust is strongly heterogeneous.
- Friction changes dramatically from high static friction to very low sliding friction during rupture.
- The strength of the crust is described by the spatial power spectrum of stress in the crust.
- Strength (stress averaged over the rupture) becomes a statistical parameter that depends on the size of the rupture.
- Earthquakes are very gentle (compared to laboratory measurements) because the Earth is so large.



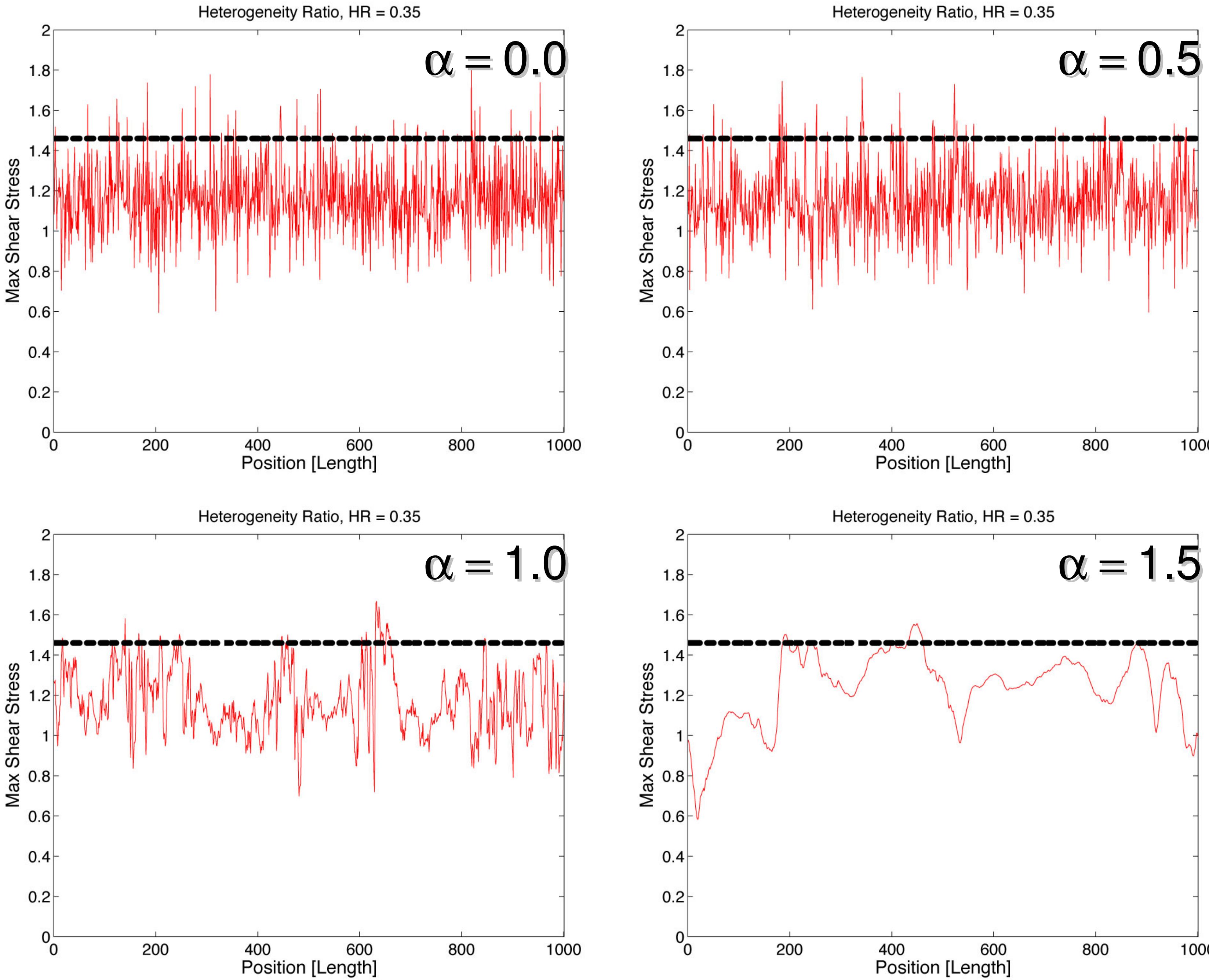
Particle Velocity-Dependent friction produces slip pulse such as the one seen for the Landers earthquake (Wald and Heaton)

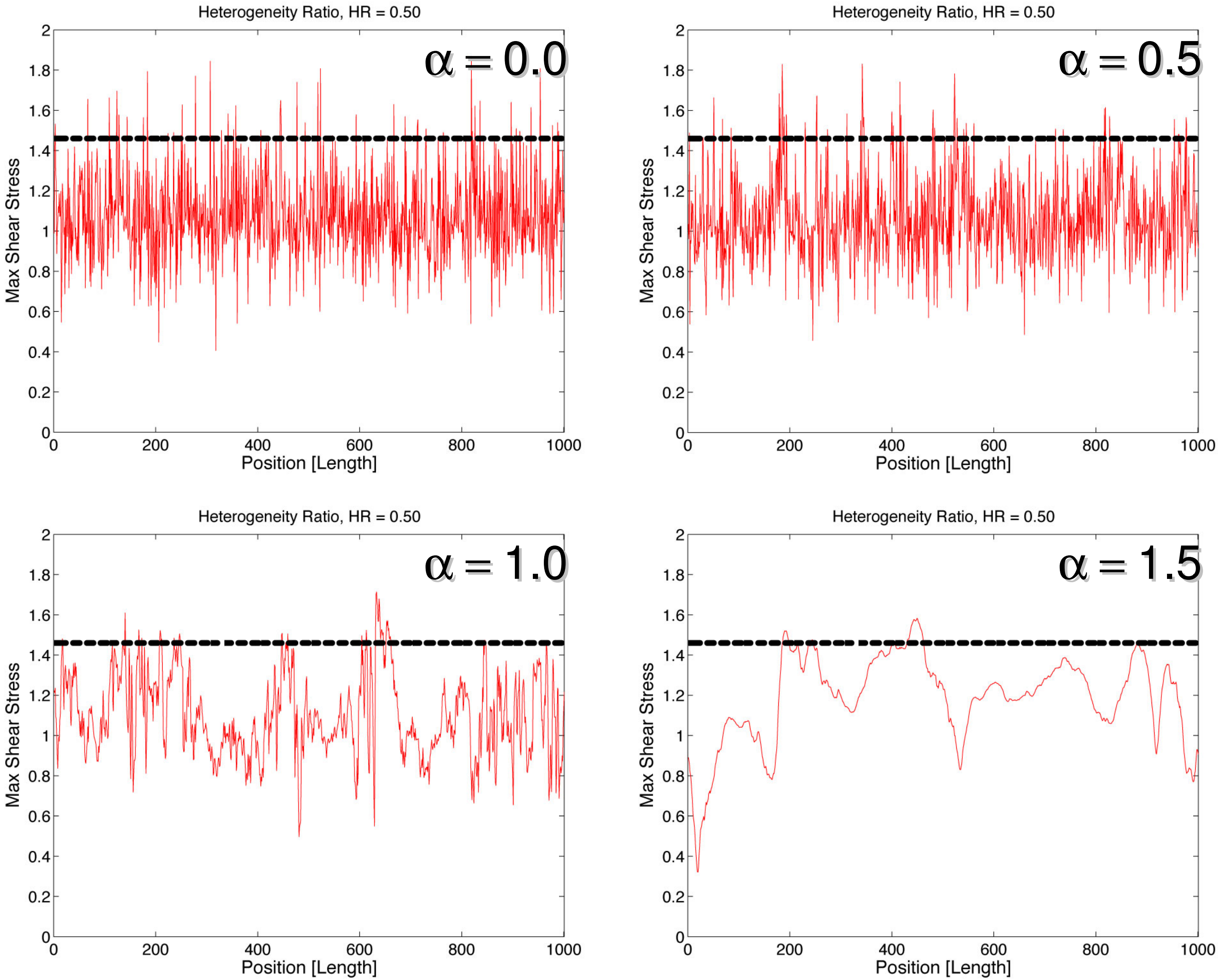


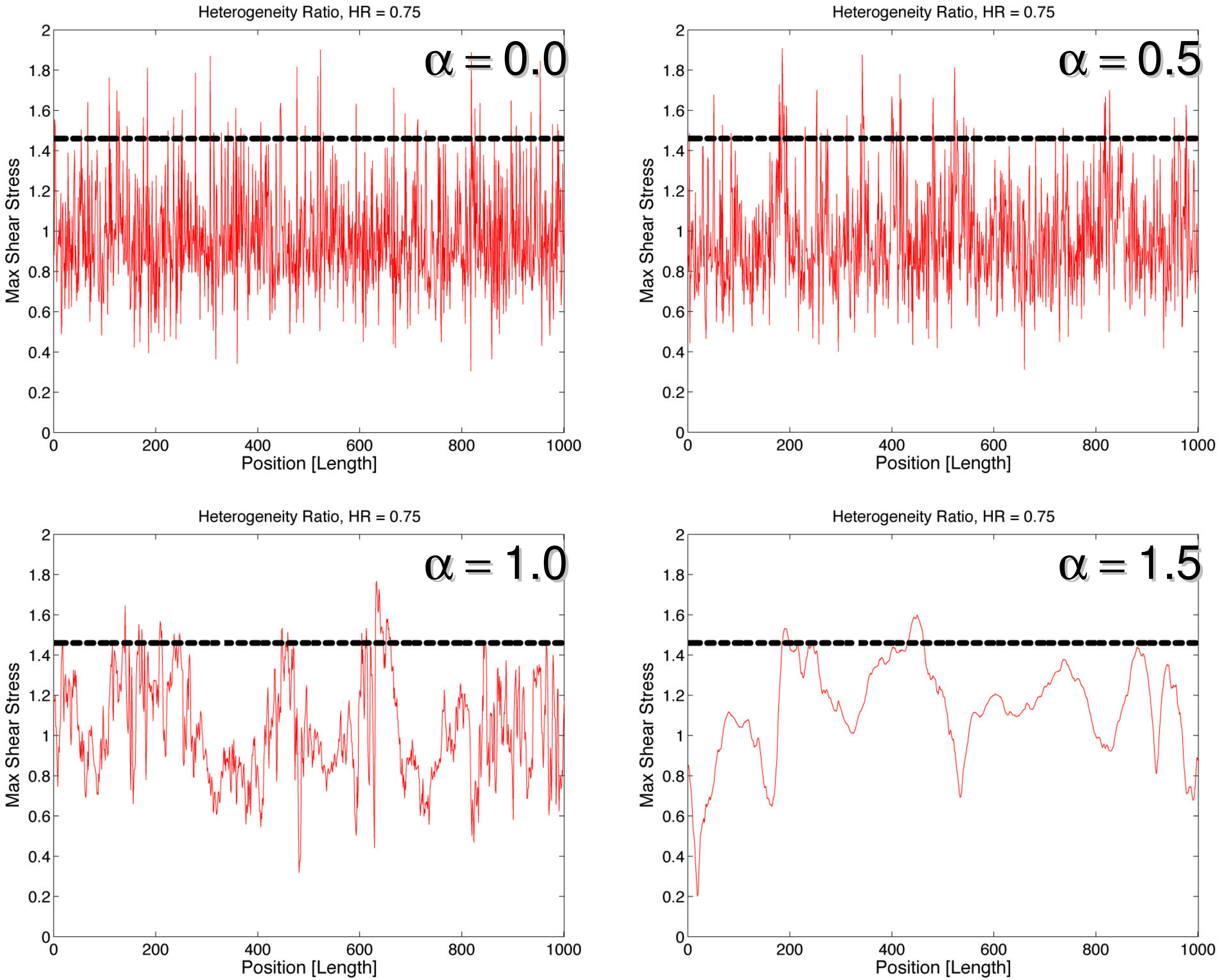
Heterogeneity Ratio, HR = 0.10
α = 0.0
Heterogeneity Ratio, HR = 0.10
α = 0.5
Heterogeneity Ratio, HR = 0.10
α = 1.0
Heterogeneity Ratio, HR = 0.10
α = 1.5
Max Shear Stress
Position [Length]

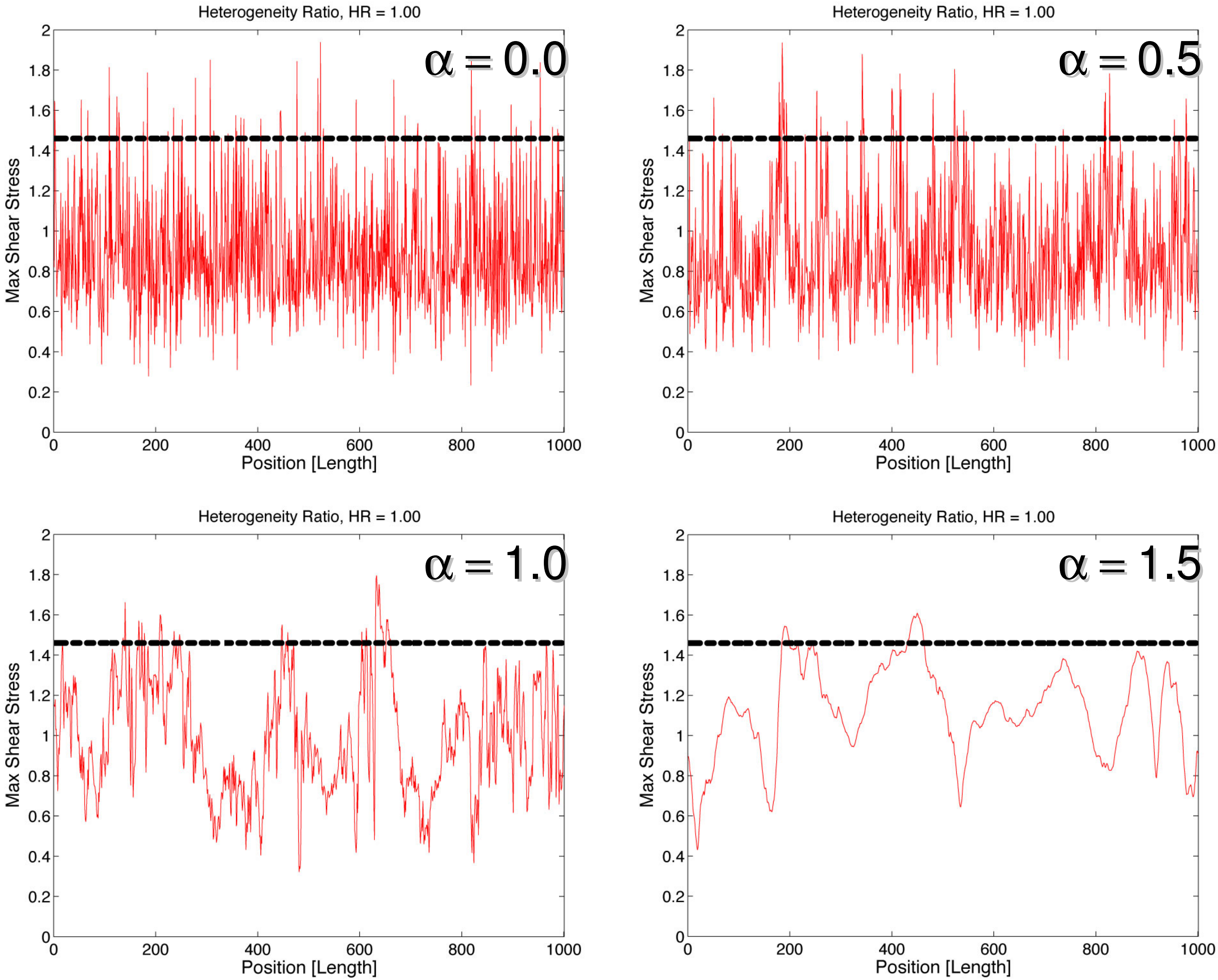


Heterogeneity Ratio, HR = 0.20
α = 0.0
Max Shear Stress
Position [Length]
Heterogeneity Ratio, HR = 0.20
α = 0.5
Max Shear Stress
Position [Length]
Heterogeneity Ratio, HR = 0.20
α = 1.0
Max Shear Stress
Position [Length]
Heterogeneity Ratio, HR = 0.20
α = 1.5
Max Shear Stress
Position [Length]

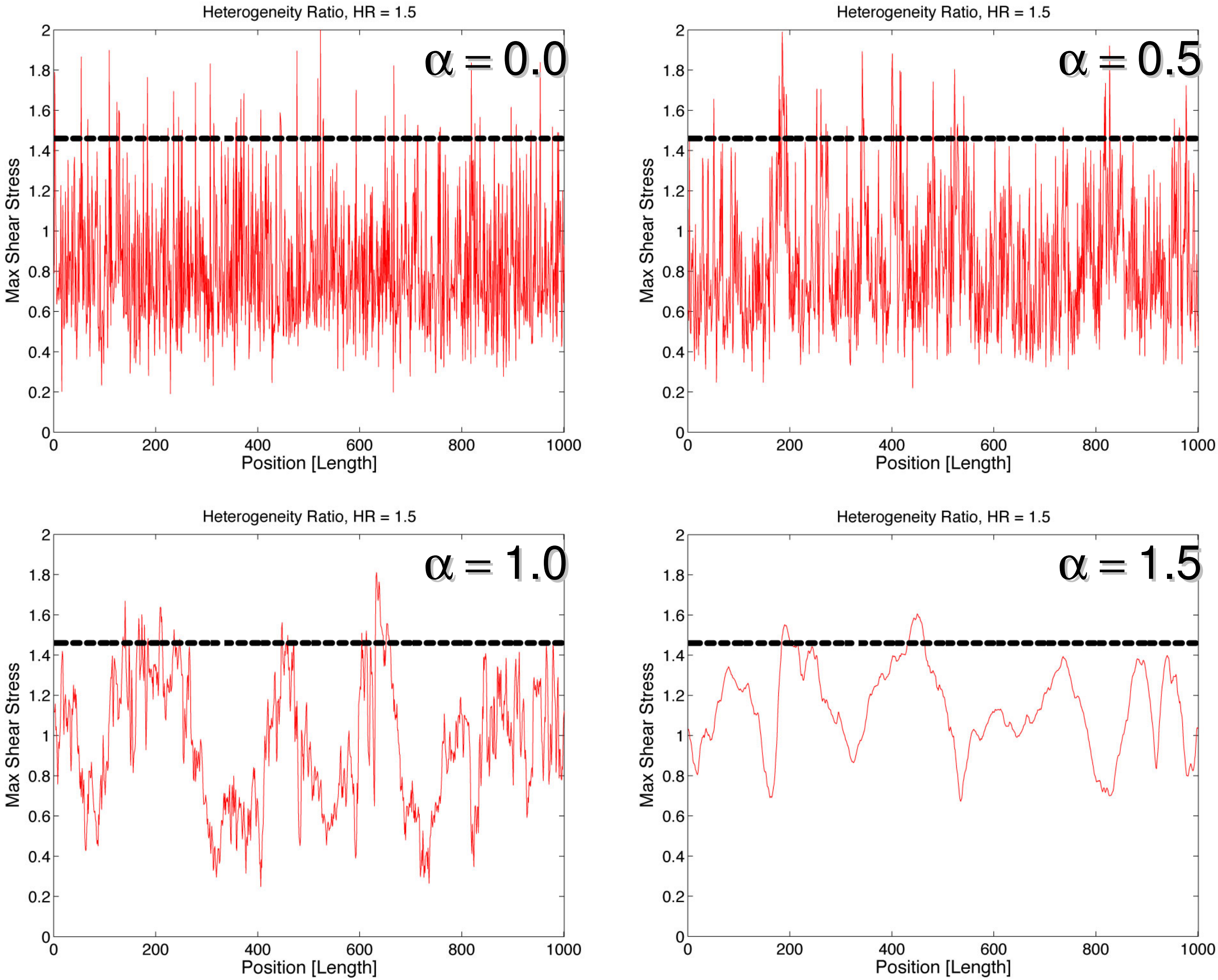

Heterogeneity Ratio, HR = 0.35
α = 0.0
Heterogeneity Ratio, HR = 0.35
α = 0.5
Heterogeneity Ratio, HR = 0.35
α = 1.0
Heterogeneity Ratio, HR = 0.35
α = 1.5
Max Shear Stress
Position [Length]

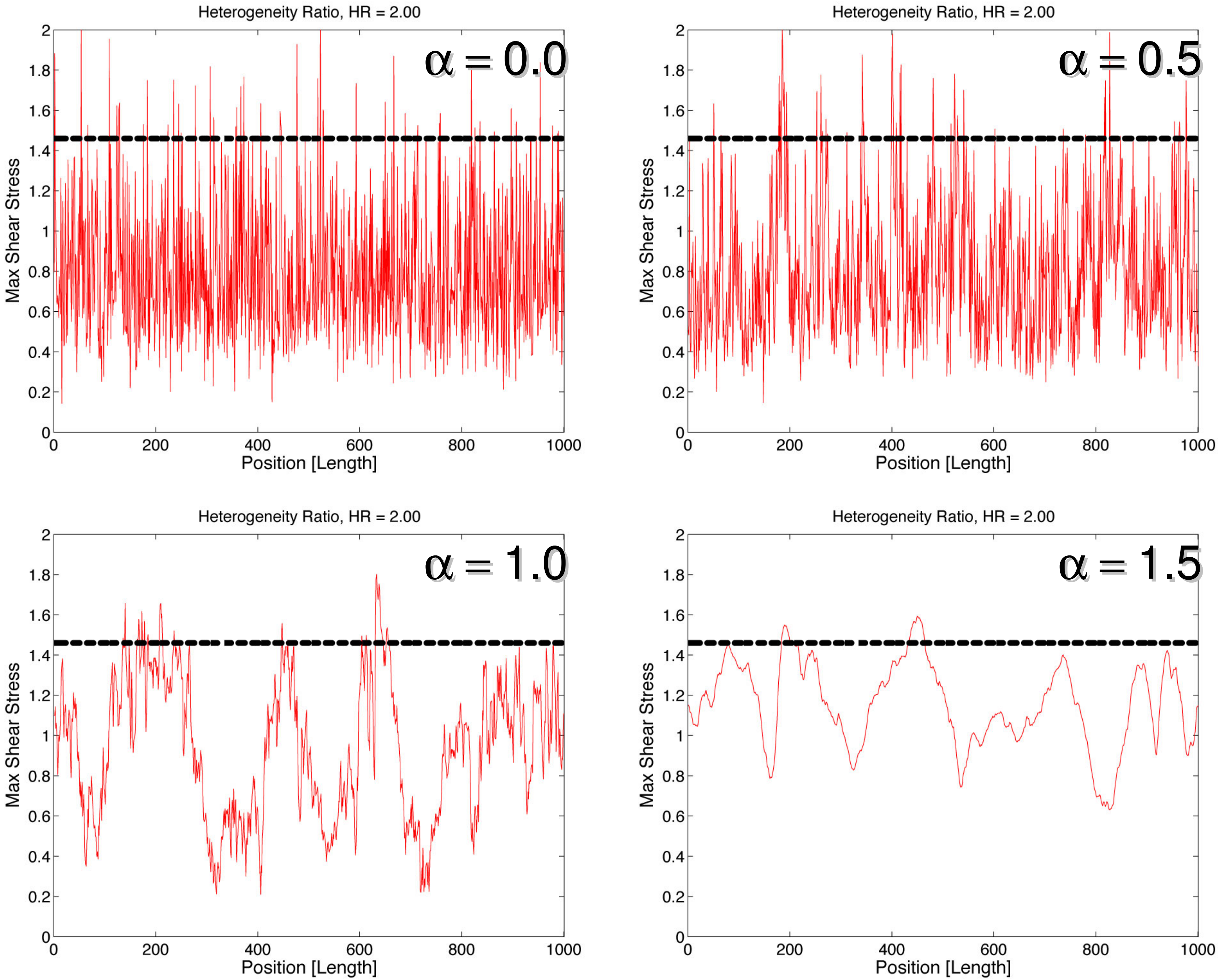

Heterogeneity Ratio, HR = 0.50
α = 0.0
α = 0.5
α = 1.0
α = 1.5
Max Shear Stress
Position [Length]


Heterogeneity Ratio, HR = 0.75
α = 0.0
Heterogeneity Ratio, HR = 0.75
α = 0.5
Heterogeneity Ratio, HR = 0.75
α = 1.0
Heterogeneity Ratio, HR = 0.75
α = 1.5
Max Shear Stress
Position [Length]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
200
400
600
800
1000

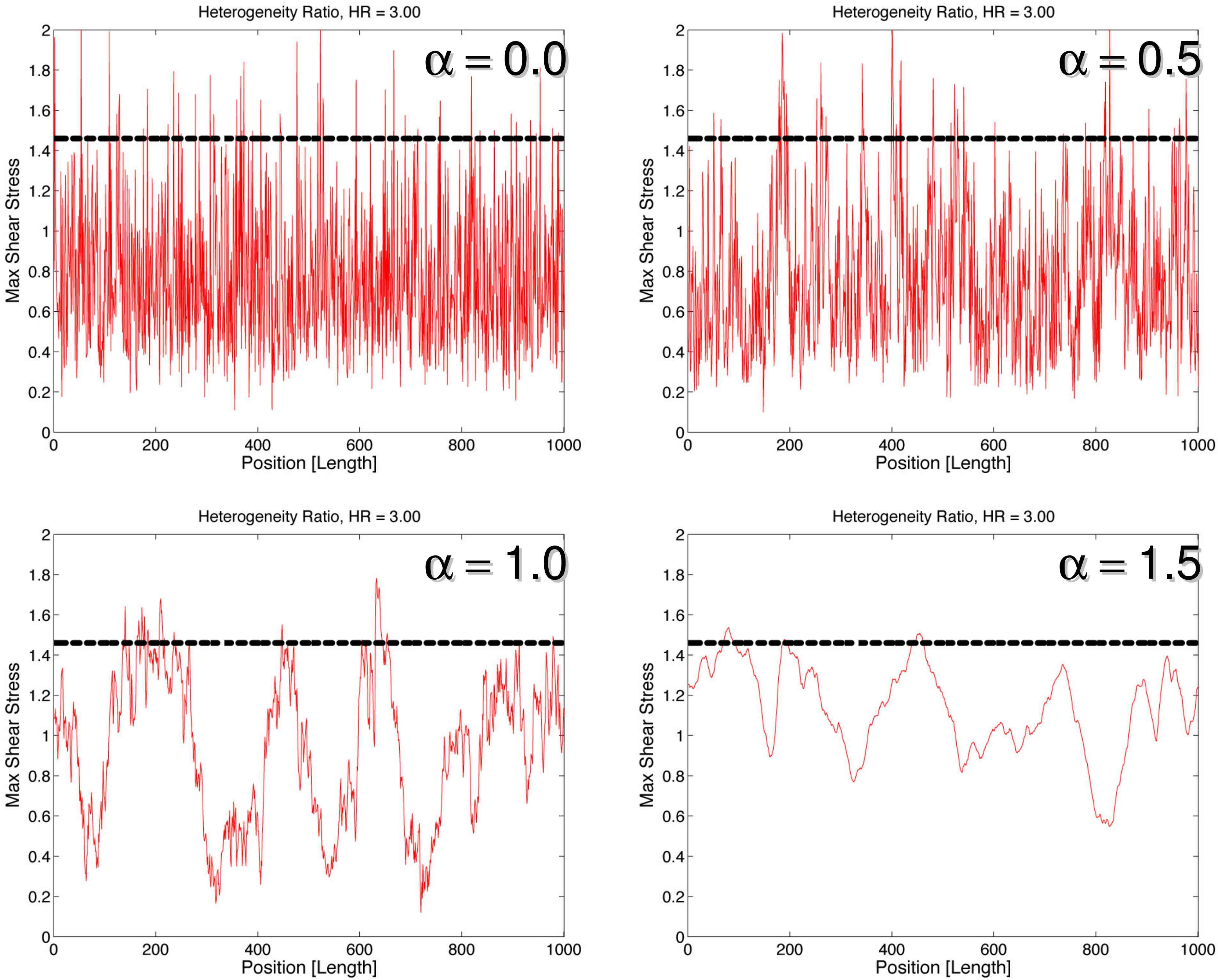


Heterogeneity Ratio, HR = 1.00
α = 0.0
α = 0.5
α = 1.0
α = 1.5
Max Shear Stress
Position [Length]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
200
400
600
800
1000

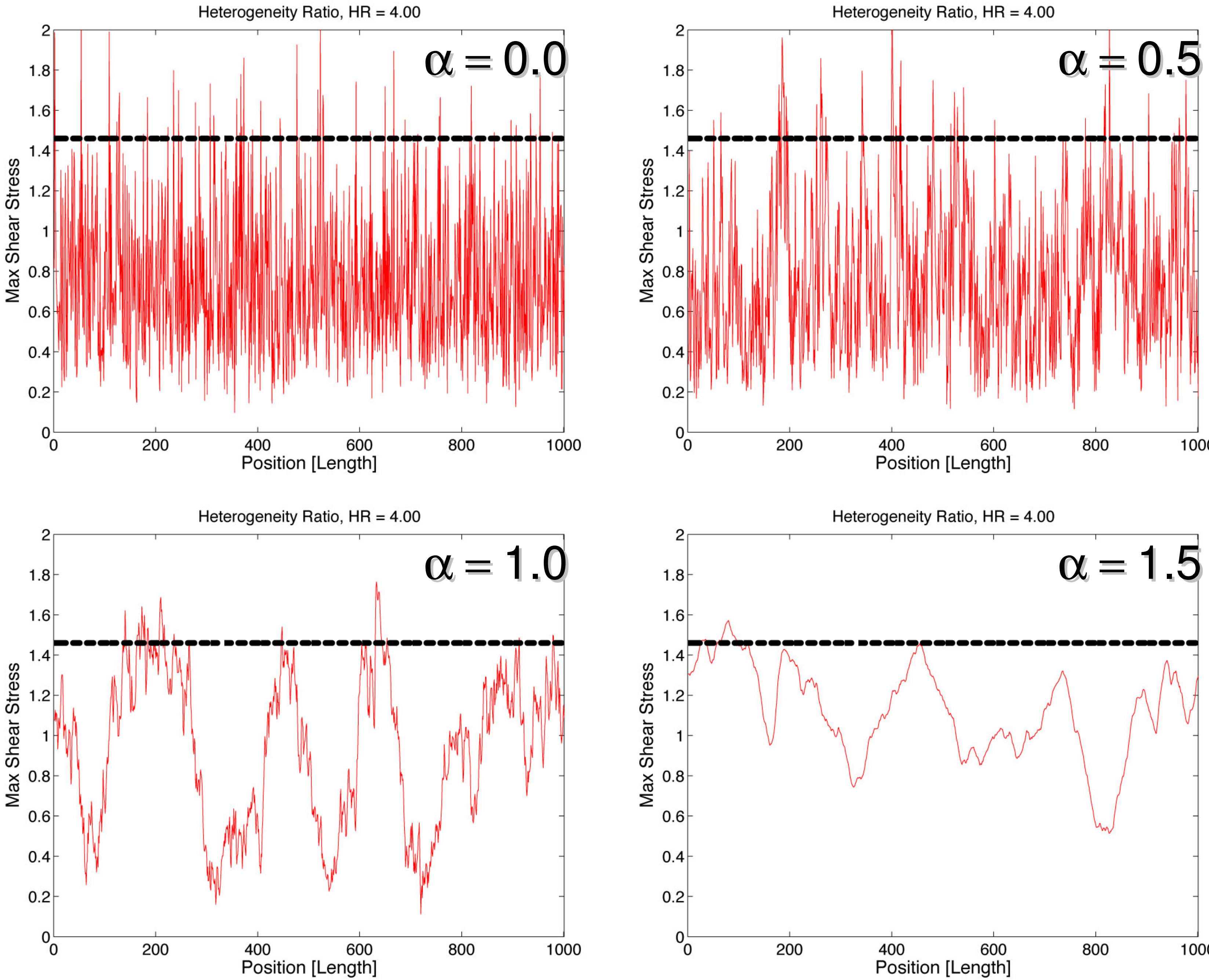

Heterogeneity Ratio, HR = 1.5
α = 0.0
Heterogeneity Ratio, HR = 1.5
α = 0.5
Heterogeneity Ratio, HR = 1.5
α = 1.0
Heterogeneity Ratio, HR = 1.5
α = 1.5
Max Shear Stress
Position [Length]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
200
400
600
800
1000



Heterogeneity Ratio, HR = 2.00
α = 0.0
Max Shear Stress
Position [Length]
Heterogeneity Ratio, HR = 2.00
α = 0.5
Max Shear Stress
Position [Length]
Heterogeneity Ratio, HR = 2.00
α = 1.0
Max Shear Stress
Position [Length]
Heterogeneity Ratio, HR = 2.00
α = 1.5
Max Shear Stress
Position [Length]



Heterogeneity Ratio, HR = 3.00
α = 0.0
Max Shear Stress
Position [Length]
Heterogeneity Ratio, HR = 3.00
α = 0.5
Max Shear Stress
Position [Length]
Heterogeneity Ratio, HR = 3.00
α = 1.0
Max Shear Stress
Position [Length]
Heterogeneity Ratio, HR = 3.00
α = 1.5
Max Shear Stress
Position [Length]



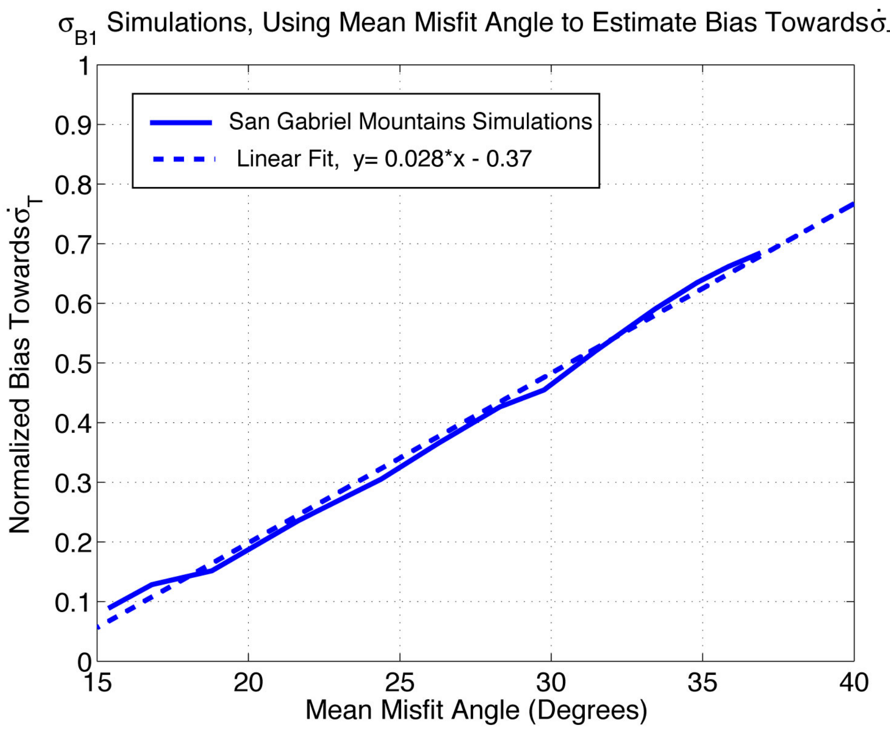
Heterogeneity Ratio, HR = 4.00
α = 0.0
Heterogeneity Ratio, HR = 4.00
α = 0.5
Heterogeneity Ratio, HR = 4.00
α = 1.0
Heterogeneity Ratio, HR = 4.00
α = 1.5
Max Shear Stress
Position [Length]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
200
400
600
800
1000

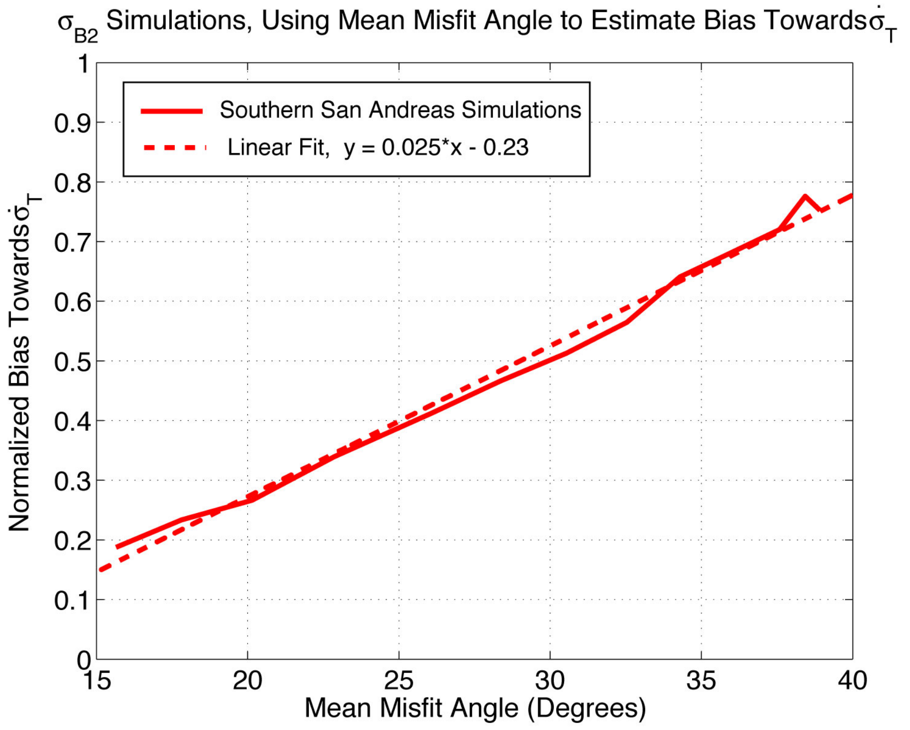
Undeterred … we guess the stress distribution

- Earthquake slip is spatially heterogeneous, which implies that stress changes are heterogeneous
- Consider a slip model in which slip is a stochastic, stationary function of location on the fault.
- Assume that an individual earthquake consist of a spatially contiguous patch.
- Statistical properties of the slip determine:
  1) (average slip)/(rupture dimension)
  2) Frequency/Magnitude statistics

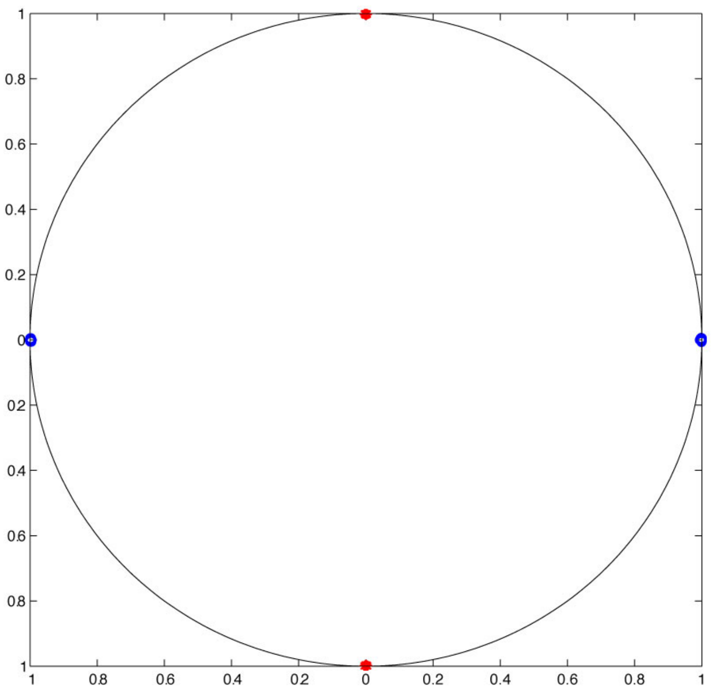
Bias Toward the Stress Rate, $\dot{\sigma}_T$ Vs. the Mean Misfit Angle

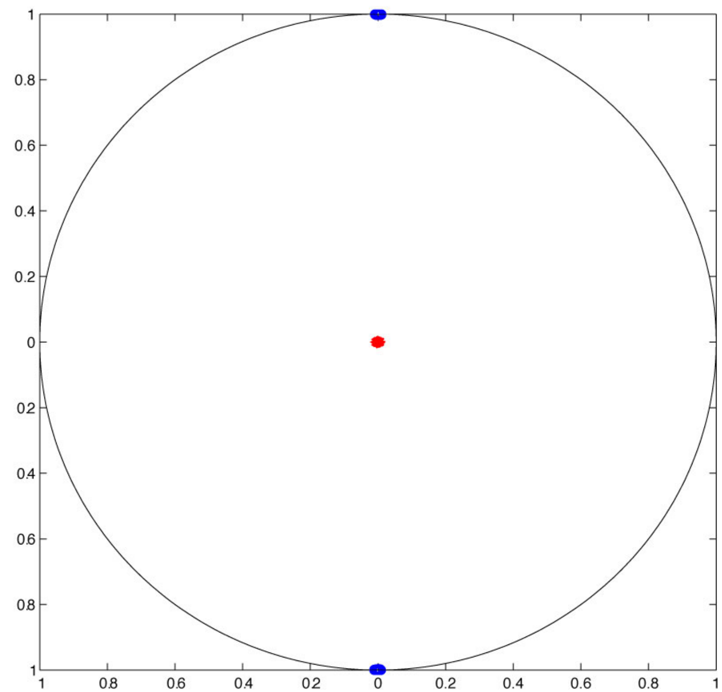
- There is almost a linear relationship between the mean misfit angle (an observable) and the % bias toward the stress rate tensor, $\dot{\sigma}_T$.





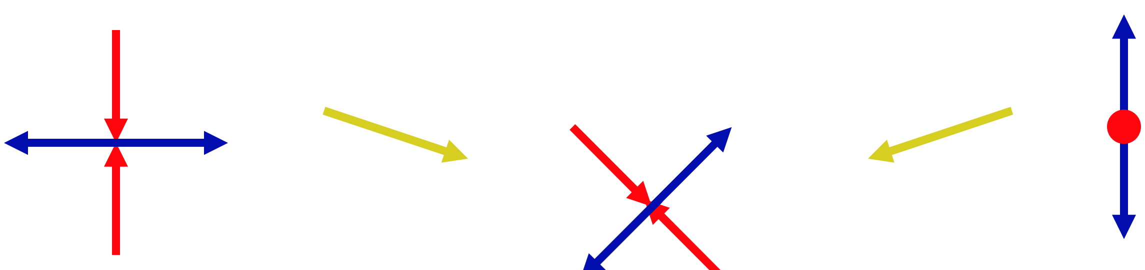
Ratio = 0.01





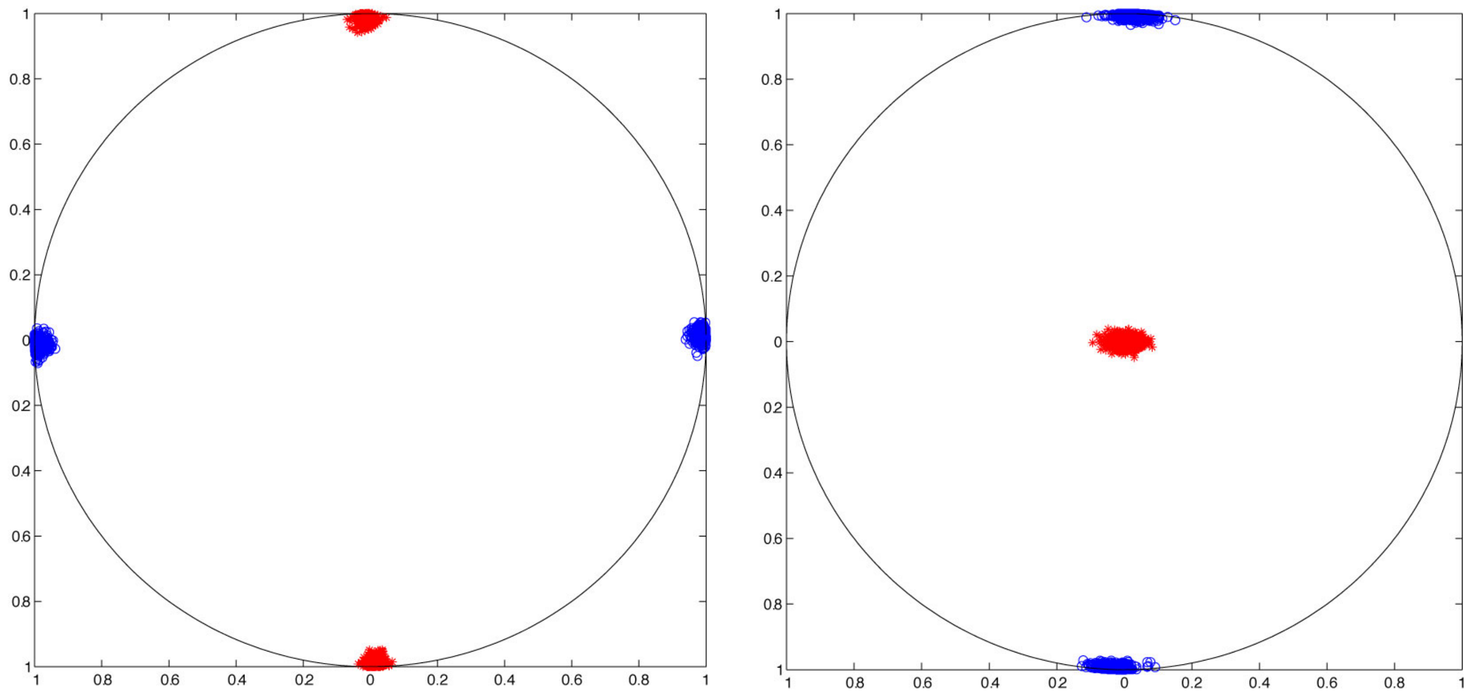
Background #1

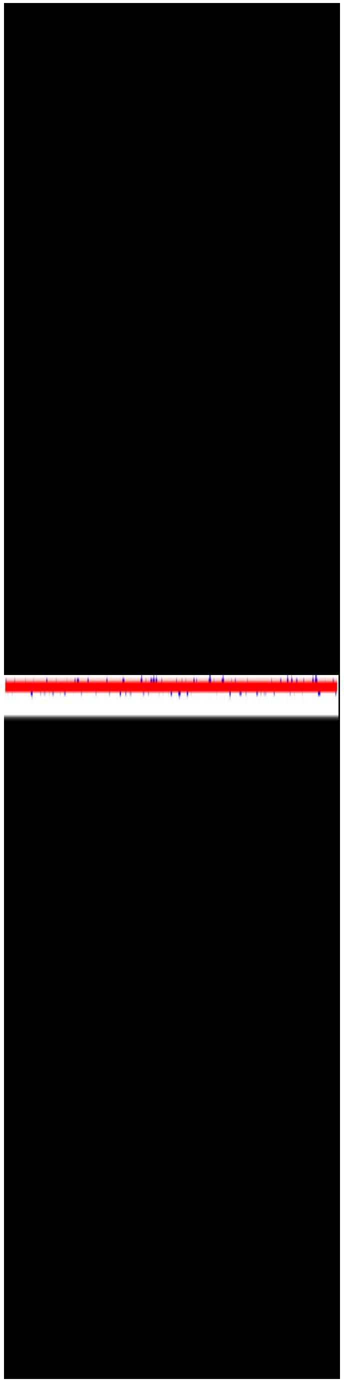
Background #2



Stress Rate Orientation

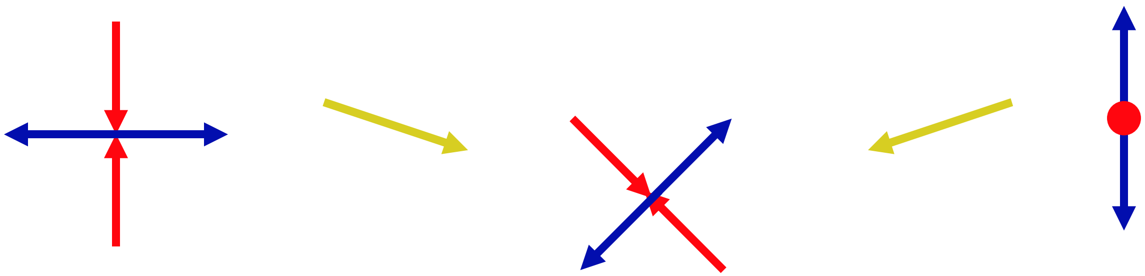
Ratio = 0.10





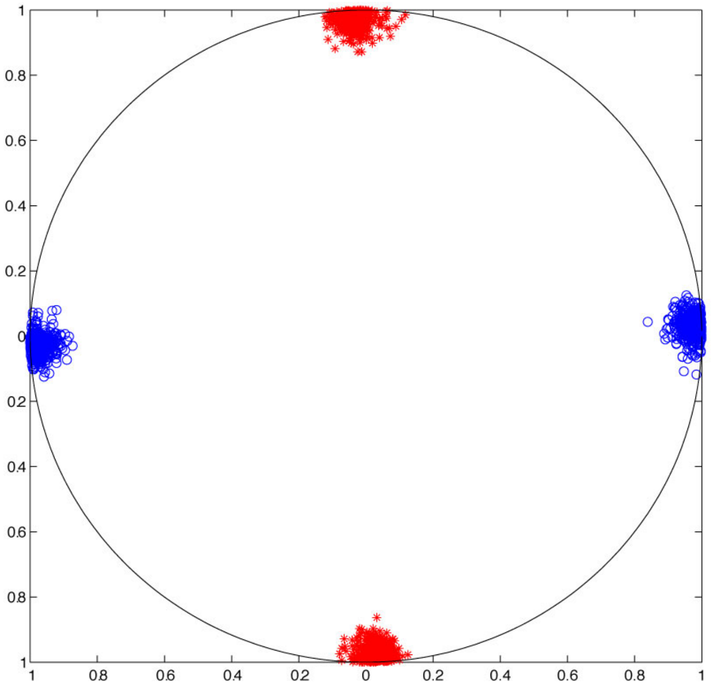
Background #1

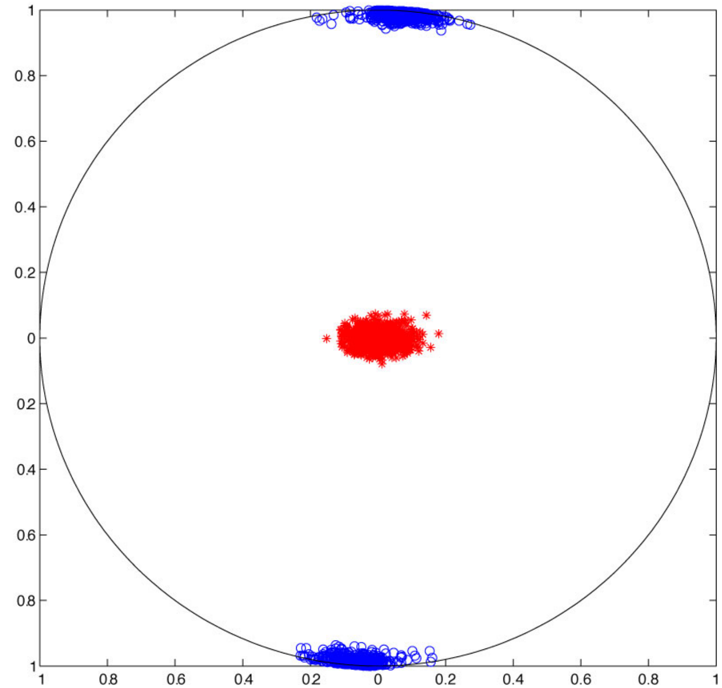
Background #2

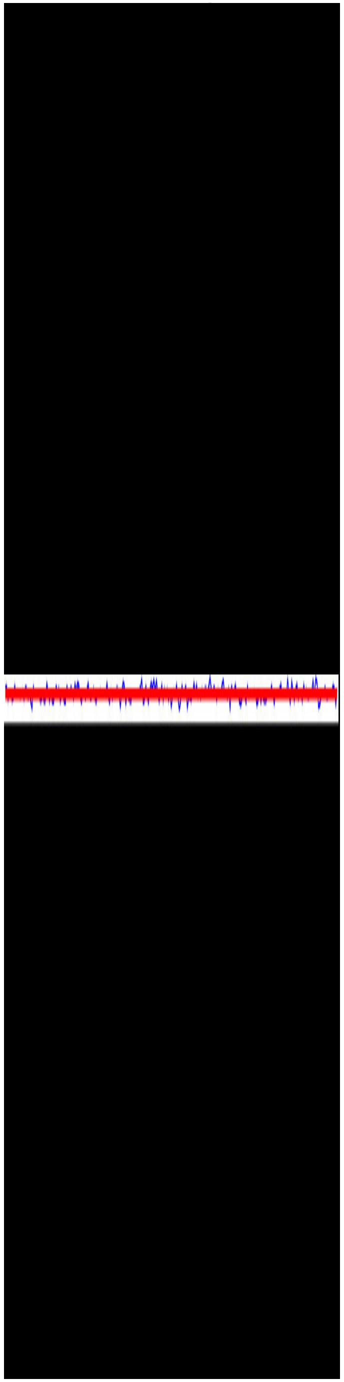


Stress Rate Orientation

Ratio = 0.20

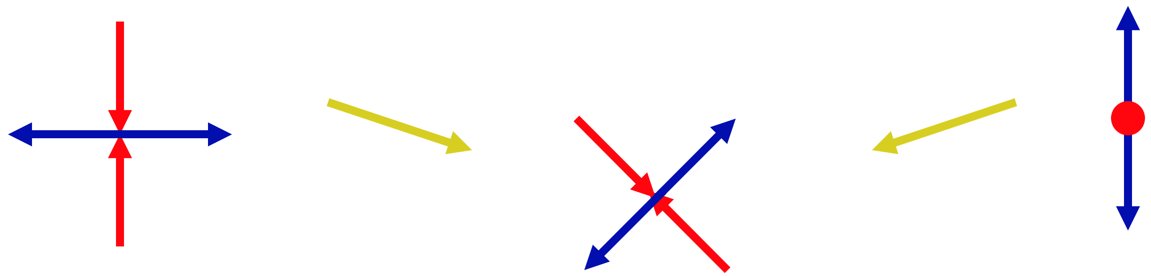






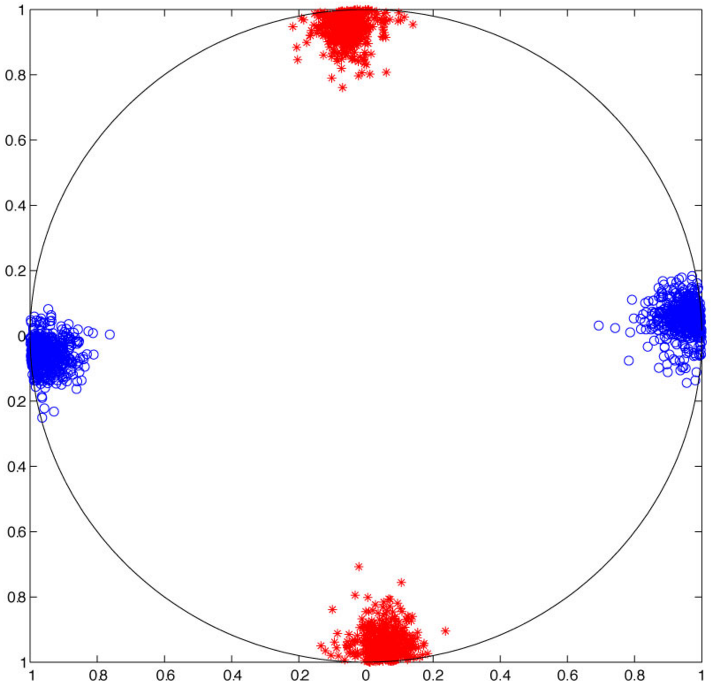
Background #1

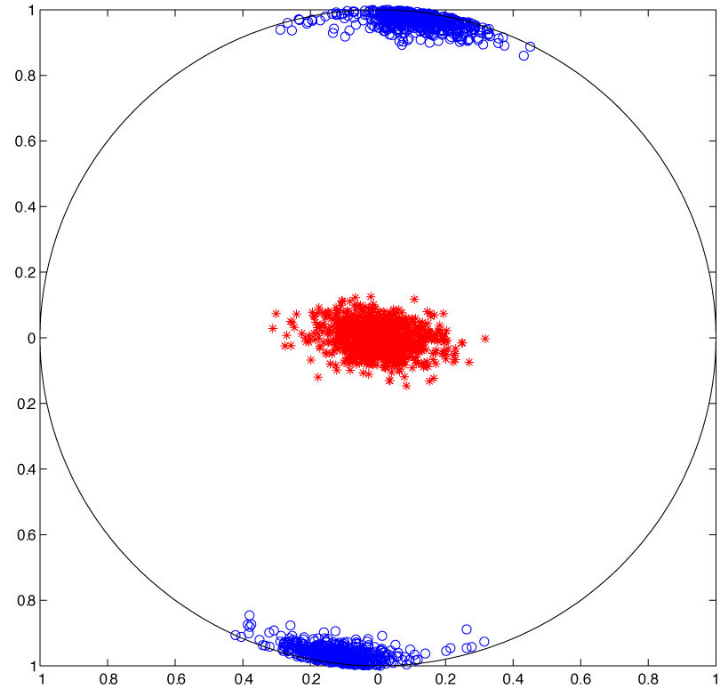
Background #2

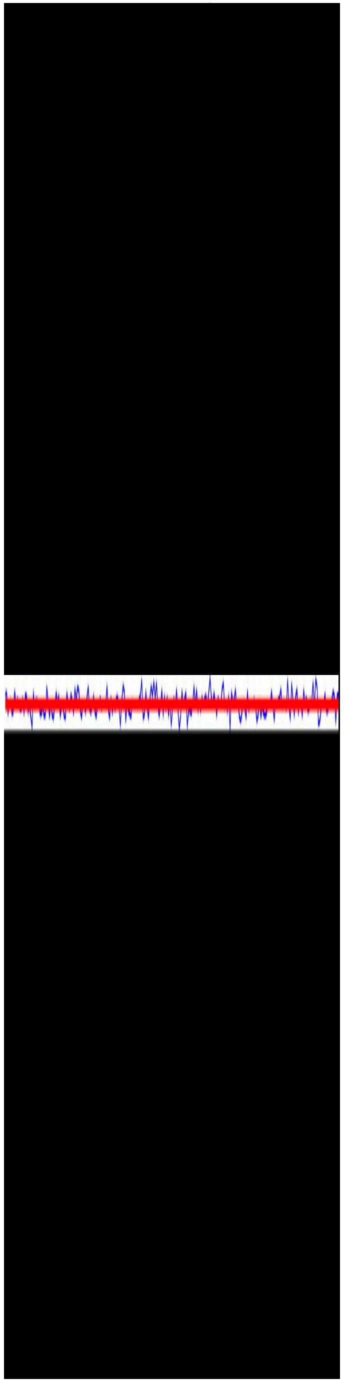


Stress Rate Orientation

Ratio = 0.35

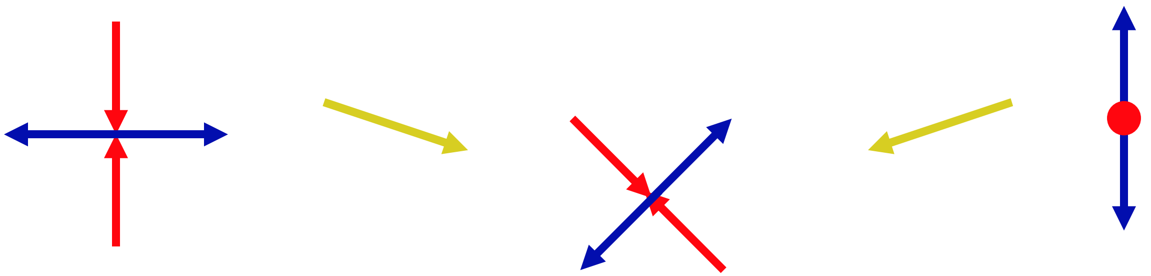






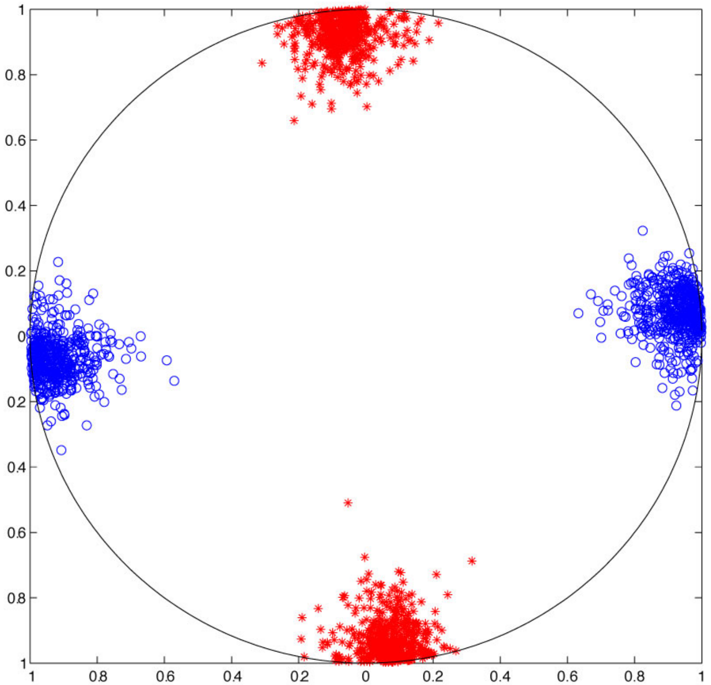
Background #1

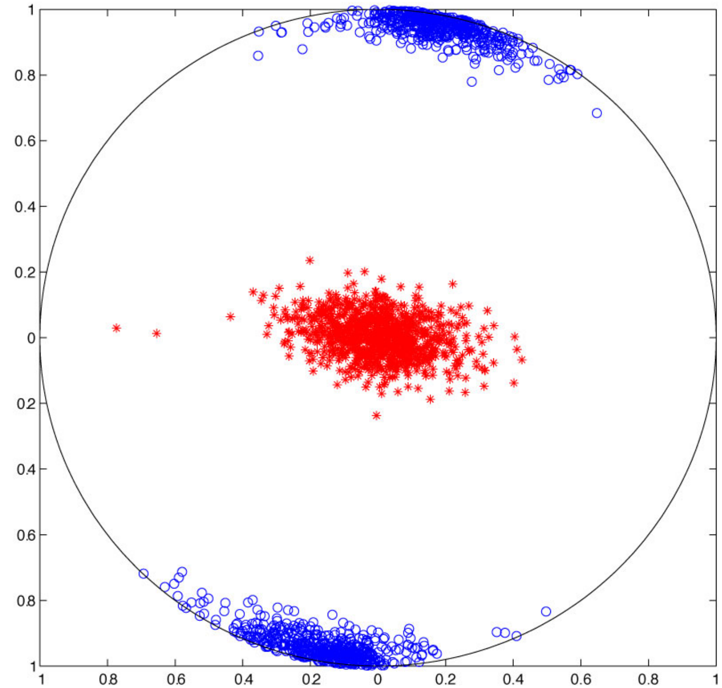
Background #2

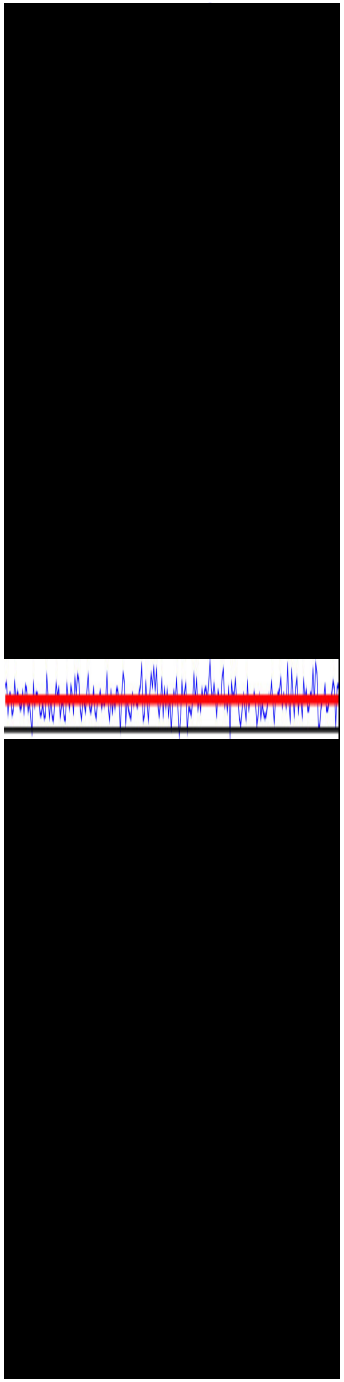


Stress Rate Orientation

Ratio = 0.50

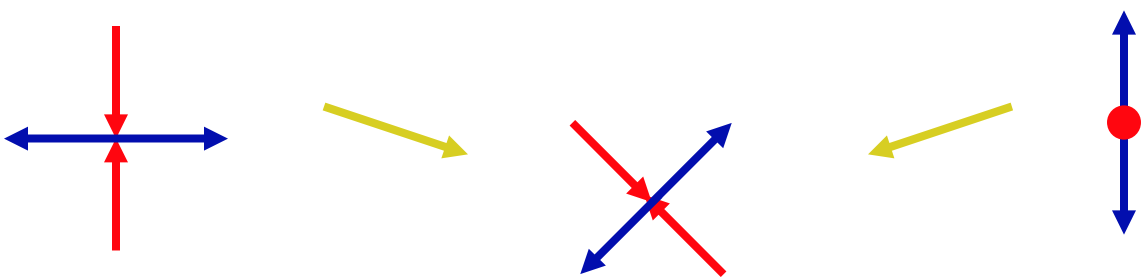






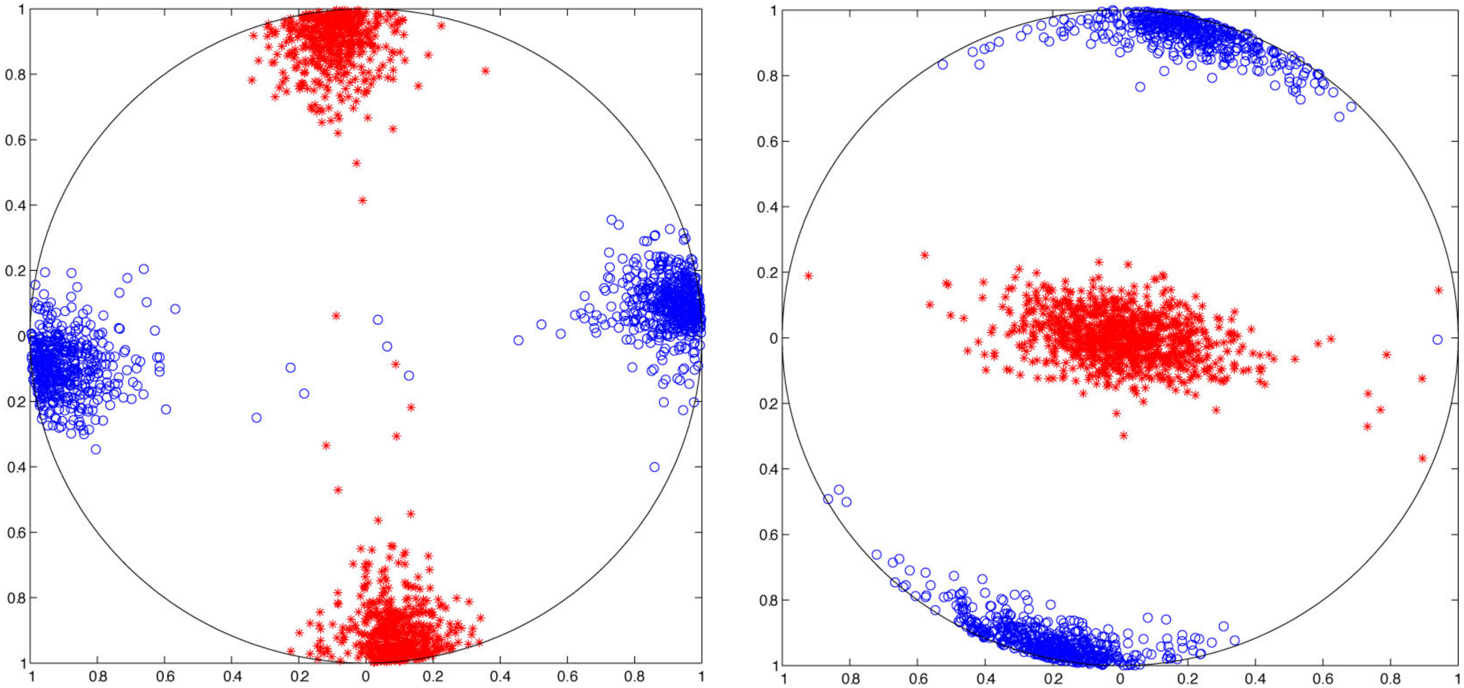
Background #1

Background #2



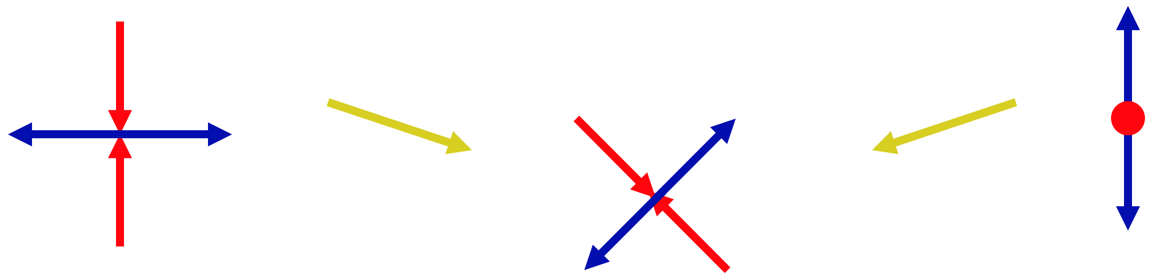
Stress Rate Orientation

Ratio = 0.62

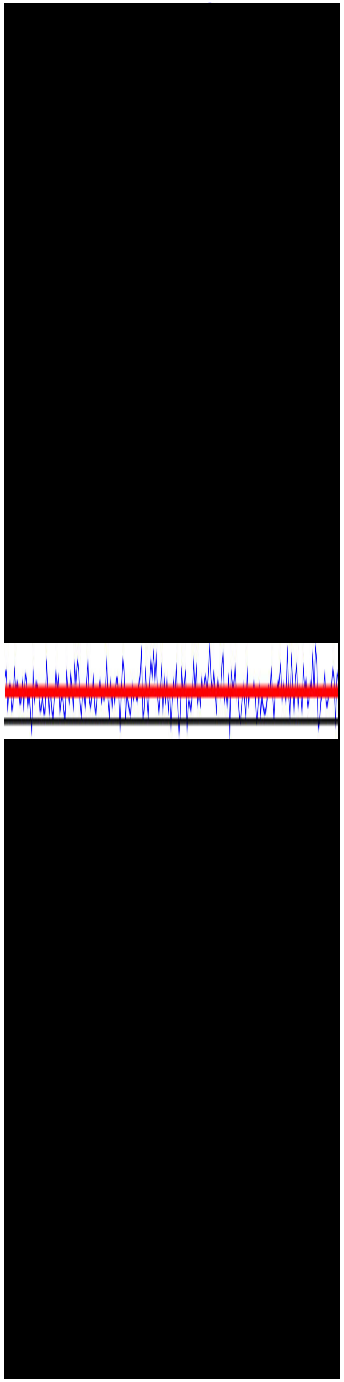


Background #1

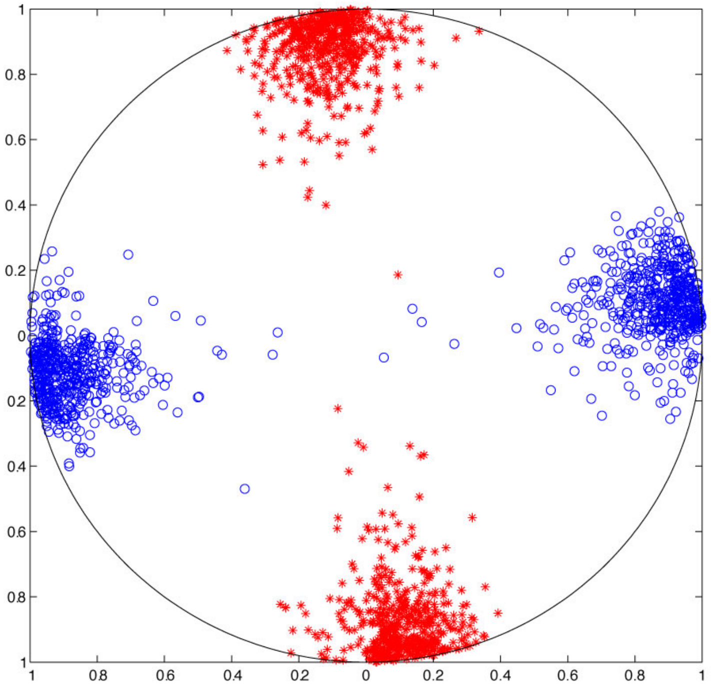
Background #2

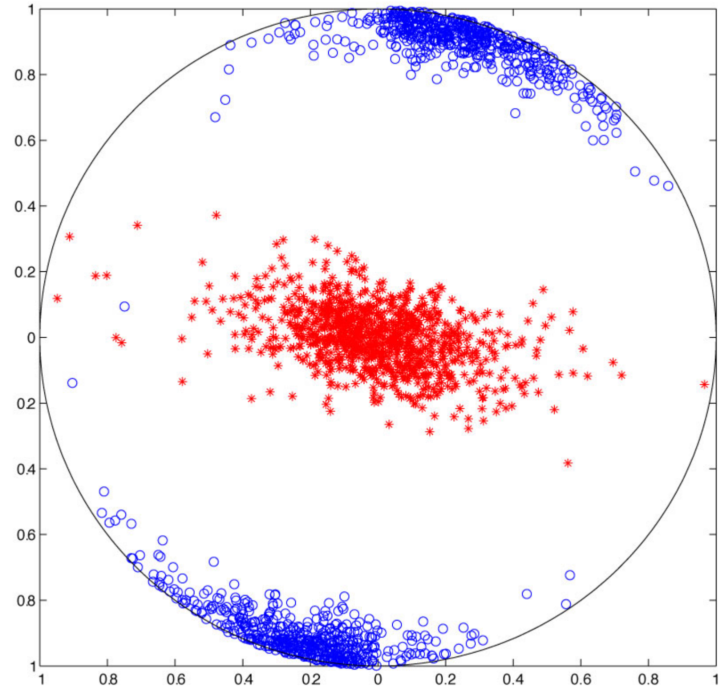


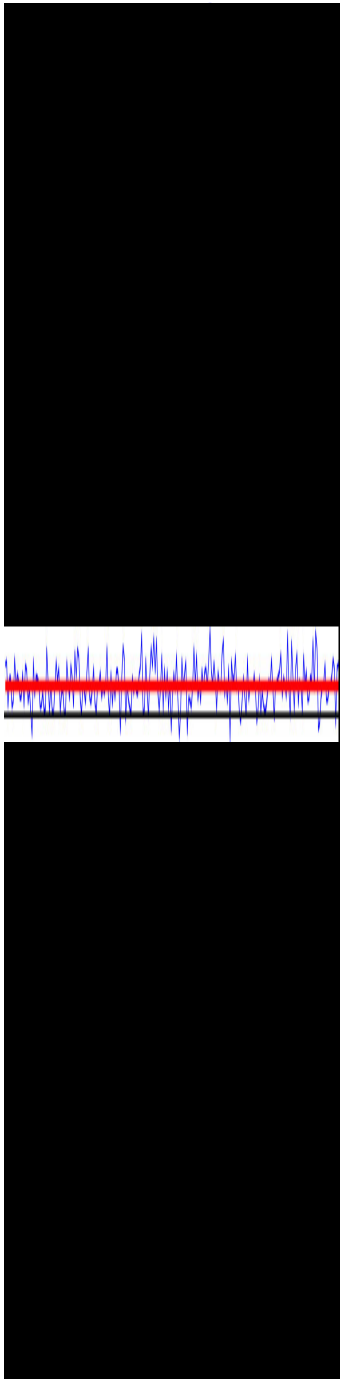
Stress Rate Orientation



Ratio = 0.75

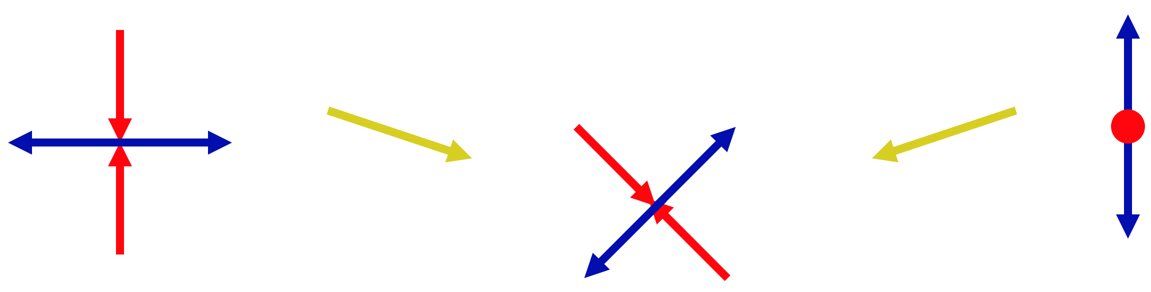






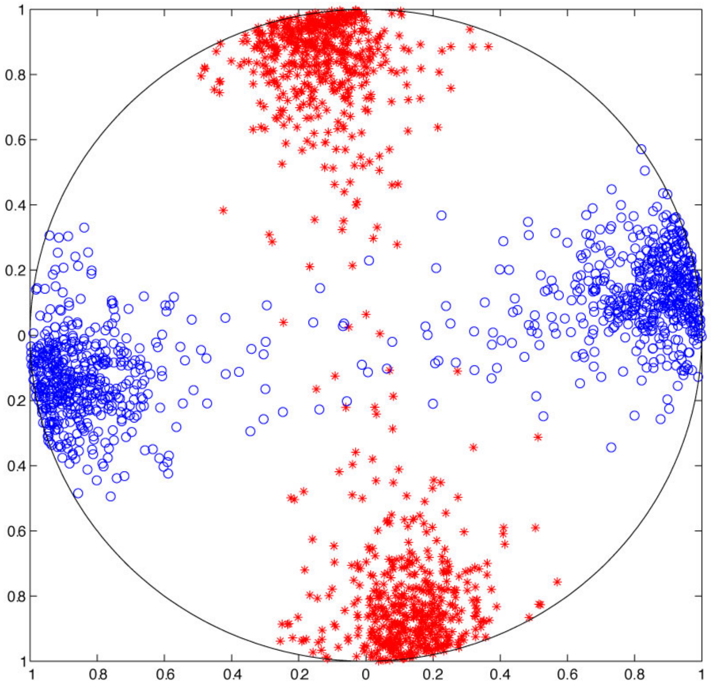
Background #1

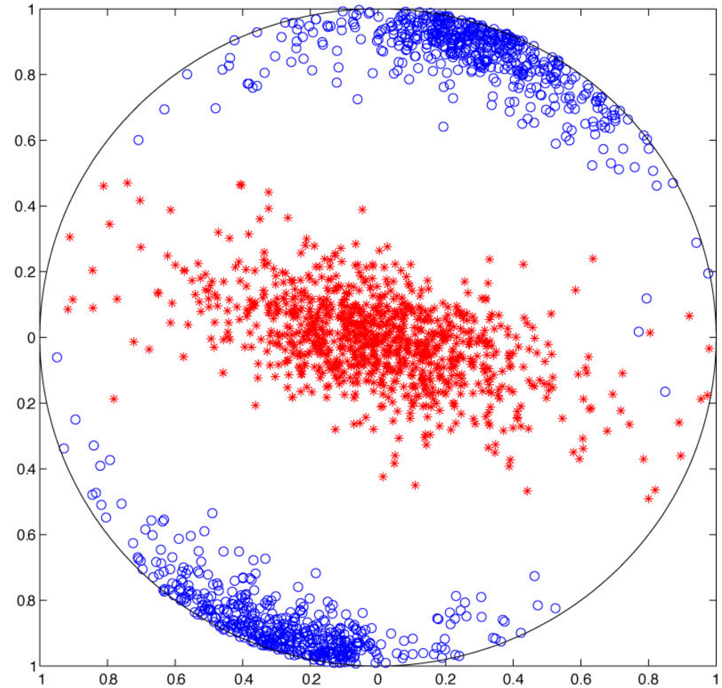
Background #2

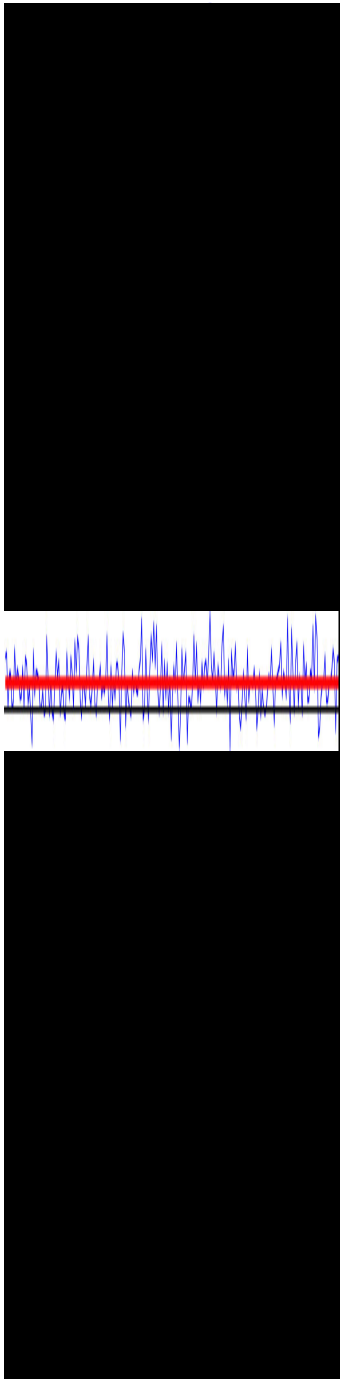


Stress Rate Orientation

Ratio = 1.00

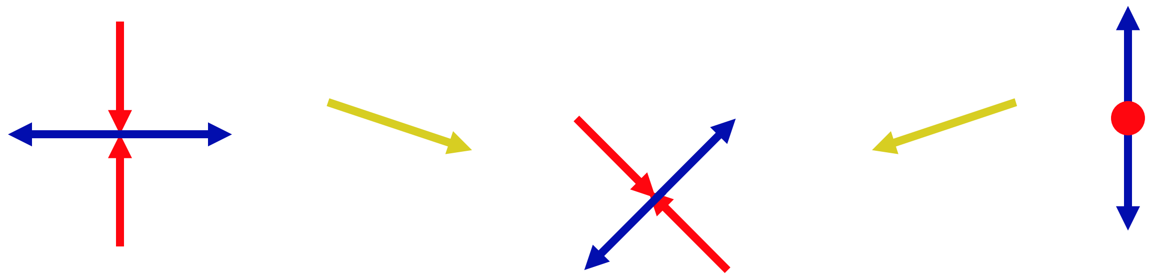






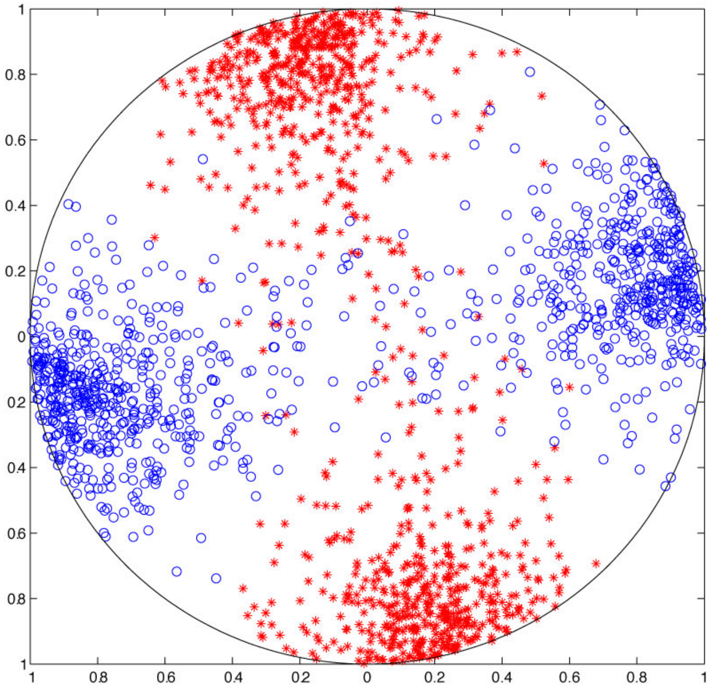
Background #1

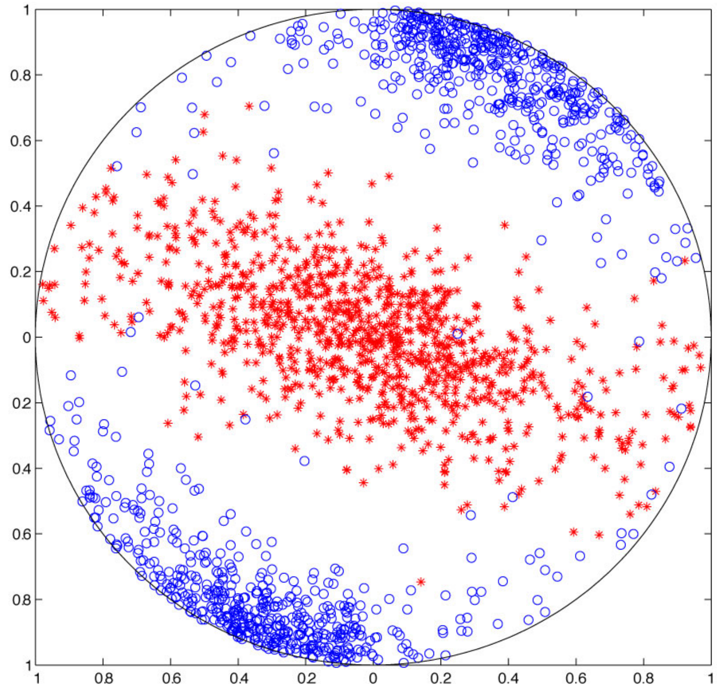
Background #2

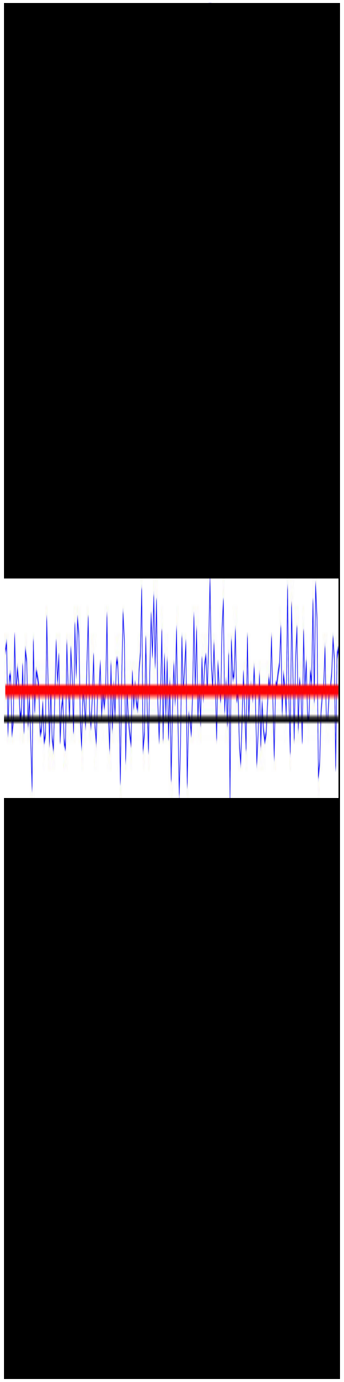


Stress Rate Orientation

Ratio = 1.50

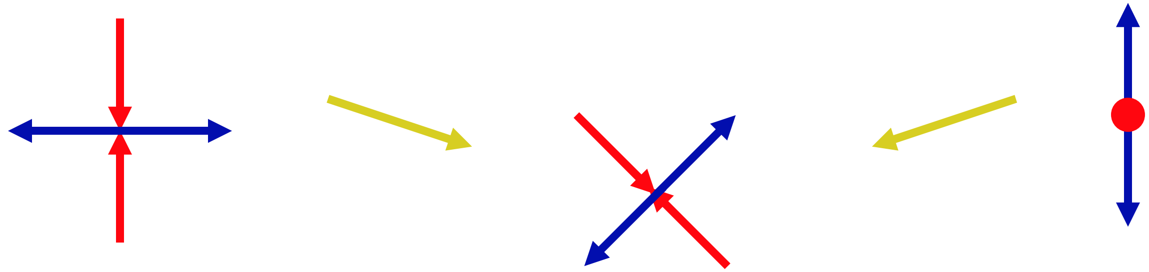






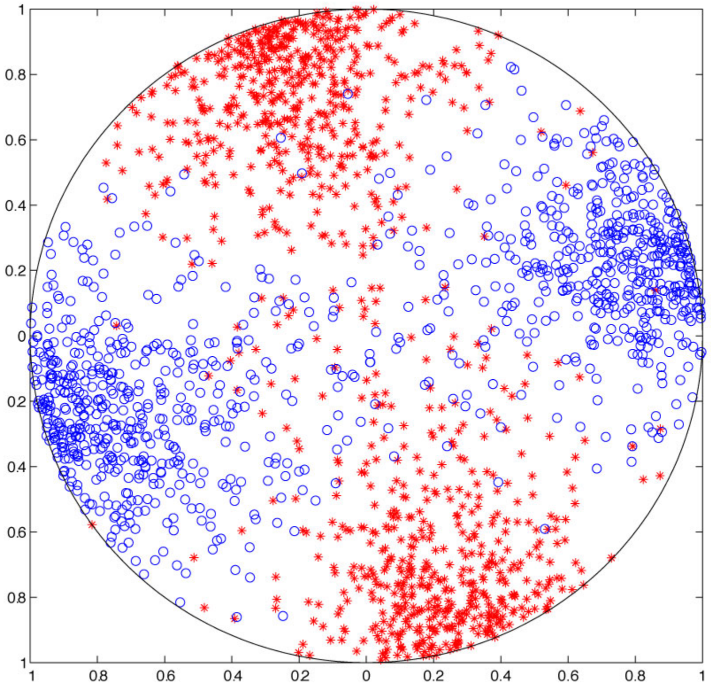
Background #1

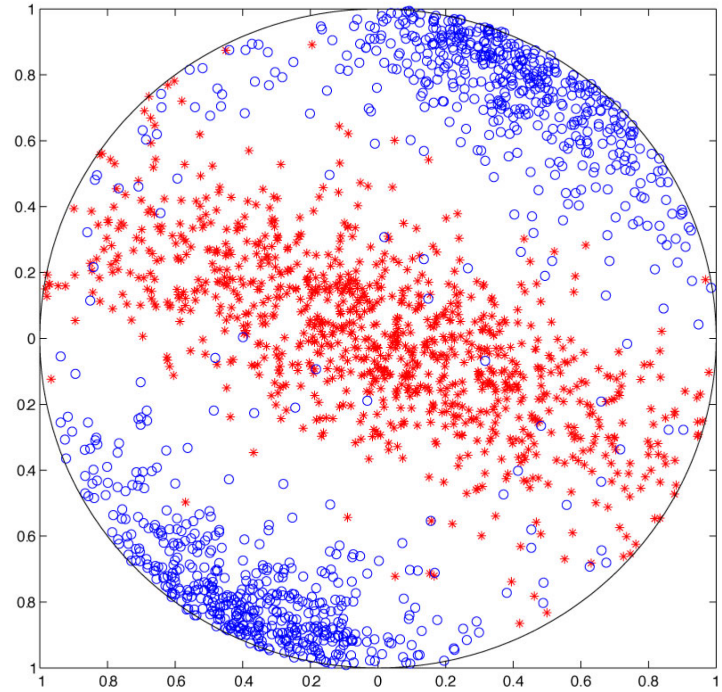
Background #2

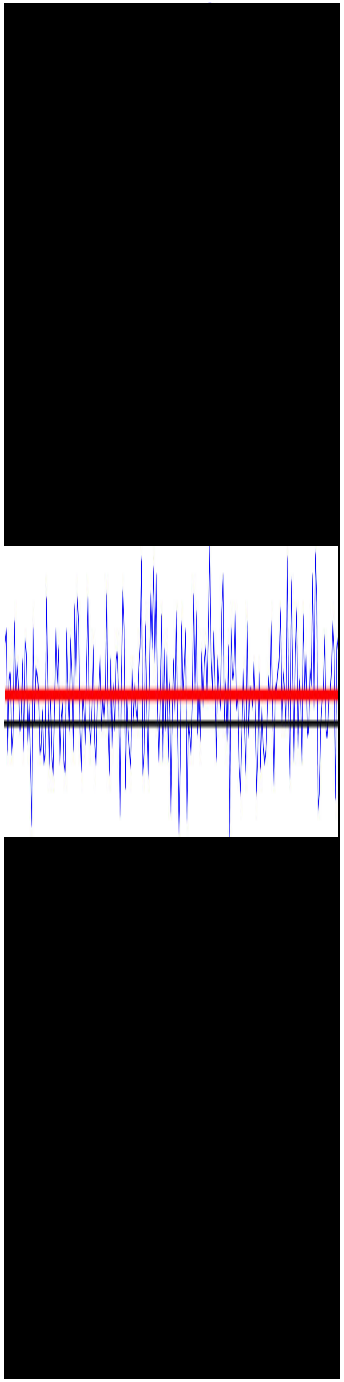


Stress Rate Orientation

Ratio = 2.00

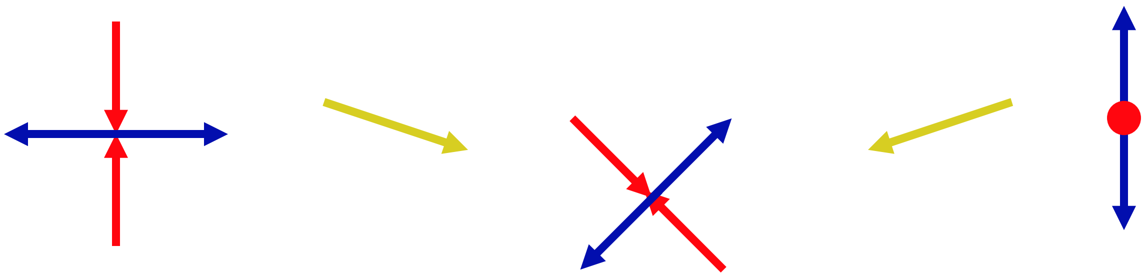






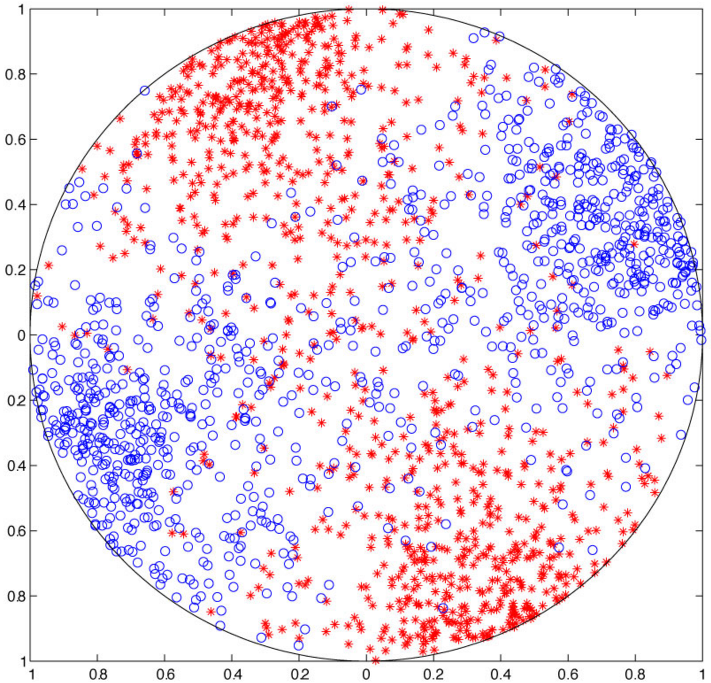
Background #1

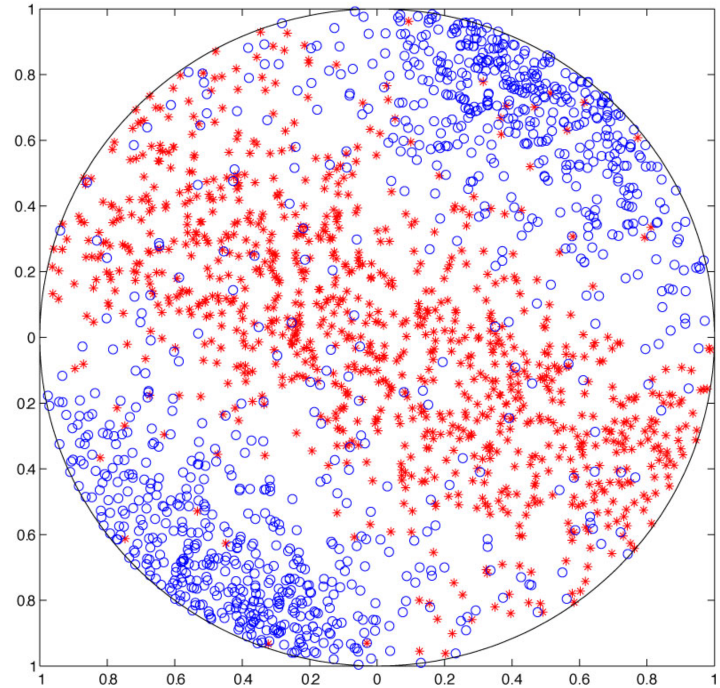
Background #2

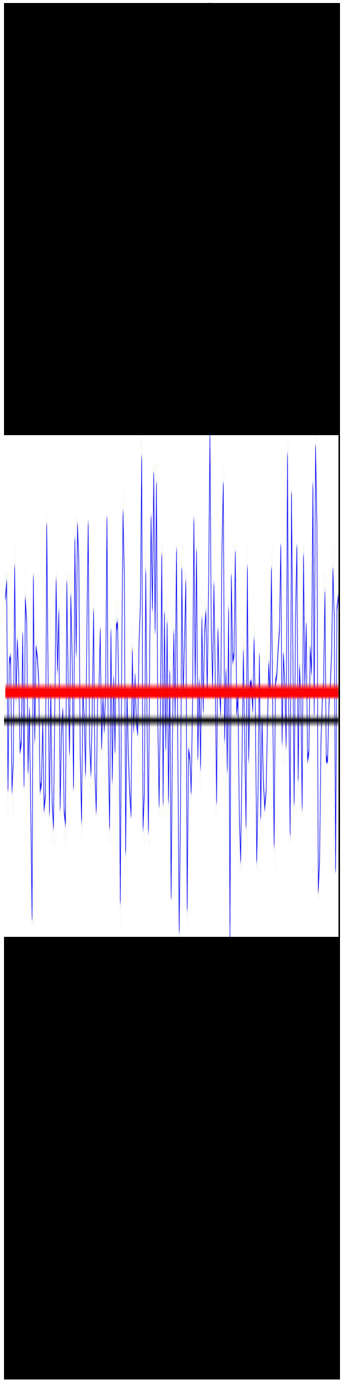


Stress Rate Orientation

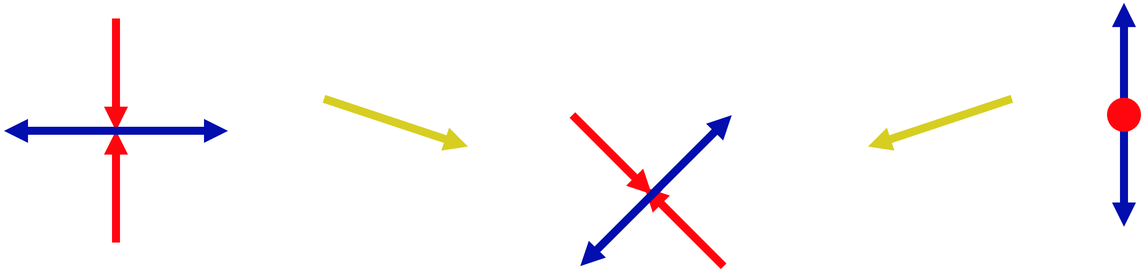
Ratio = 3.50





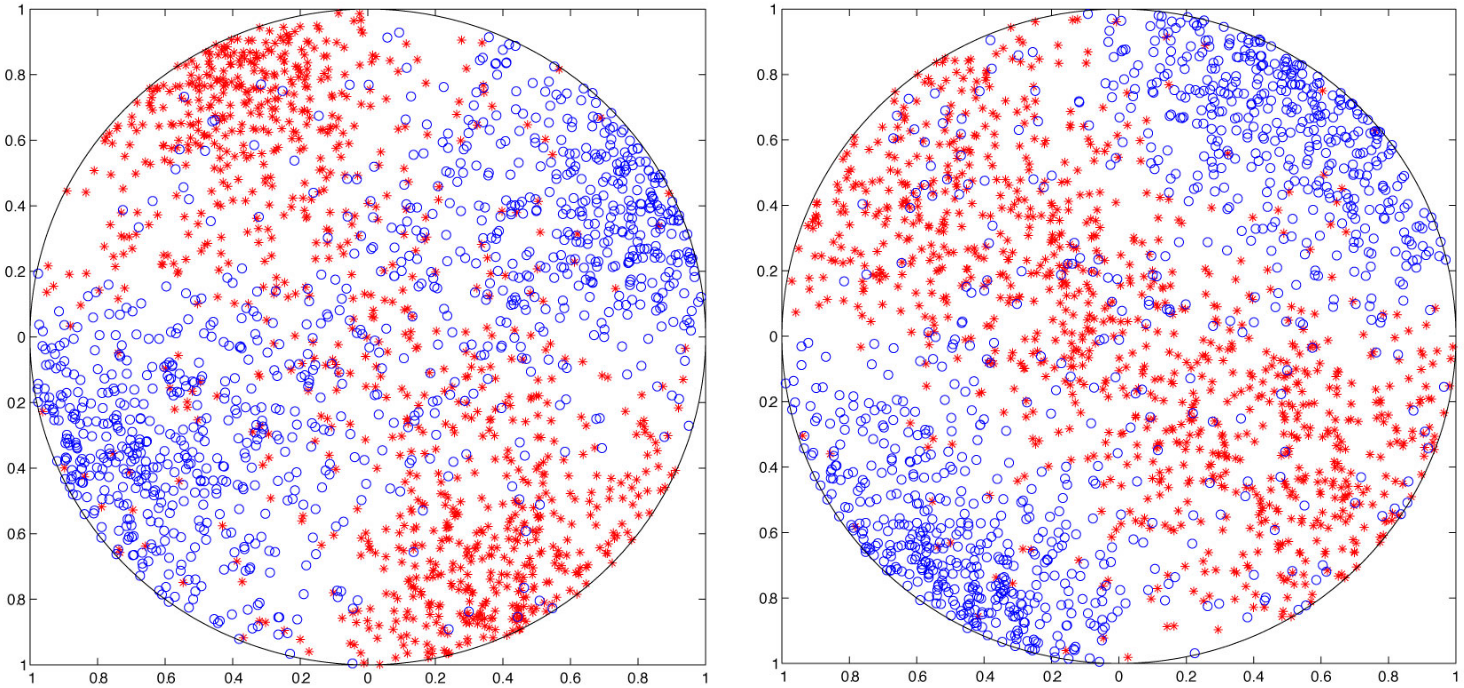


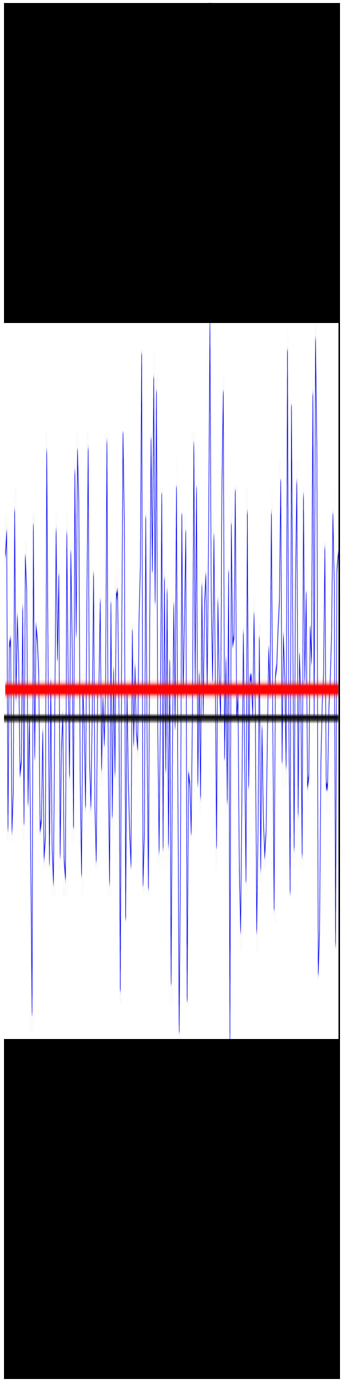
Background #1 **Background #2**



Stress Rate Orientation

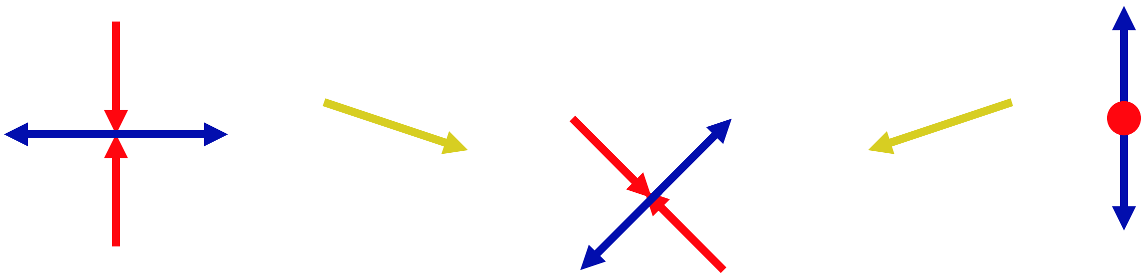
Ratio = 5.00





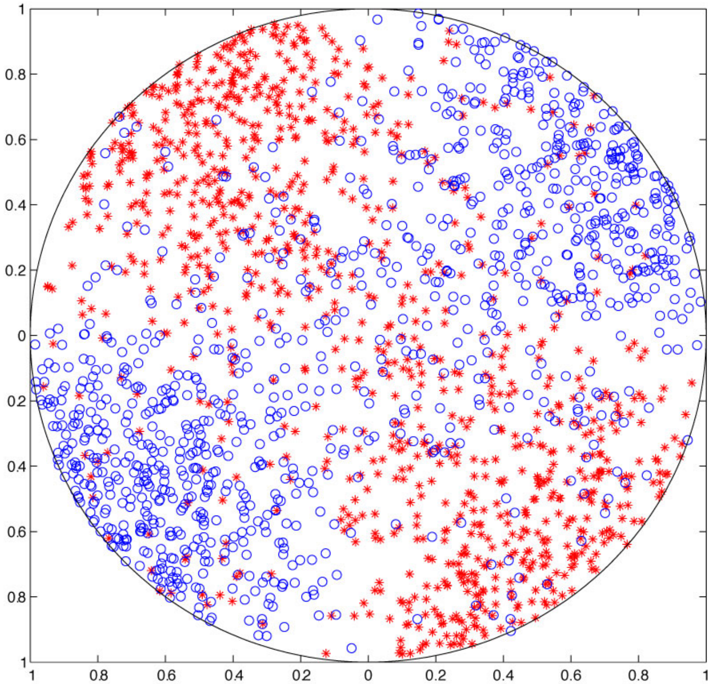
Background #1

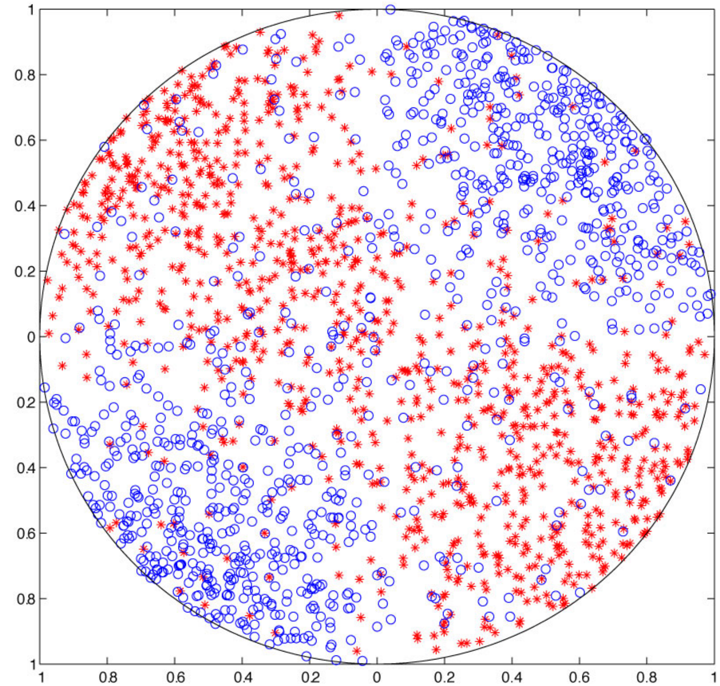
Background #2

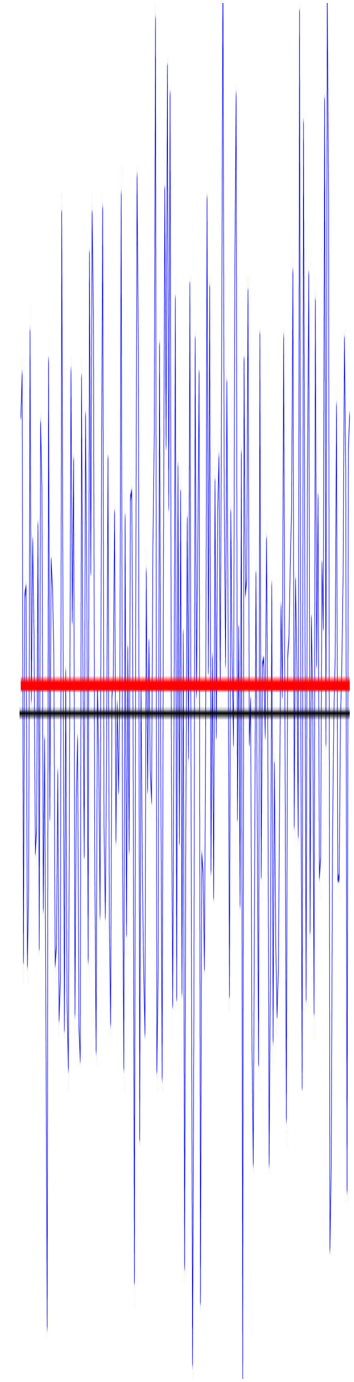


Stress Rate Orientation

Ratio = 10.0

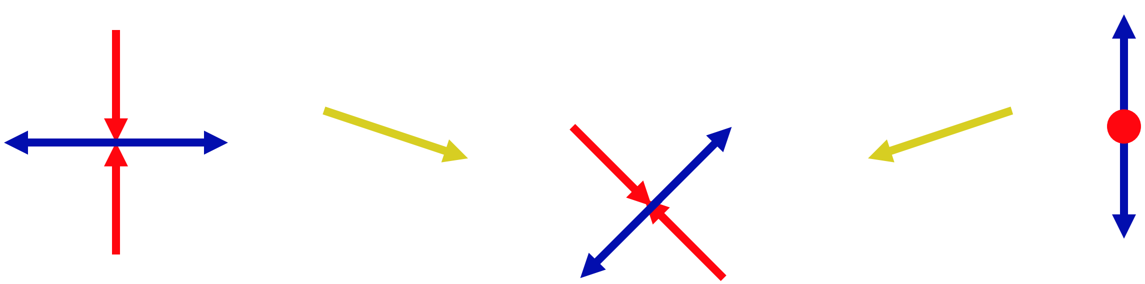






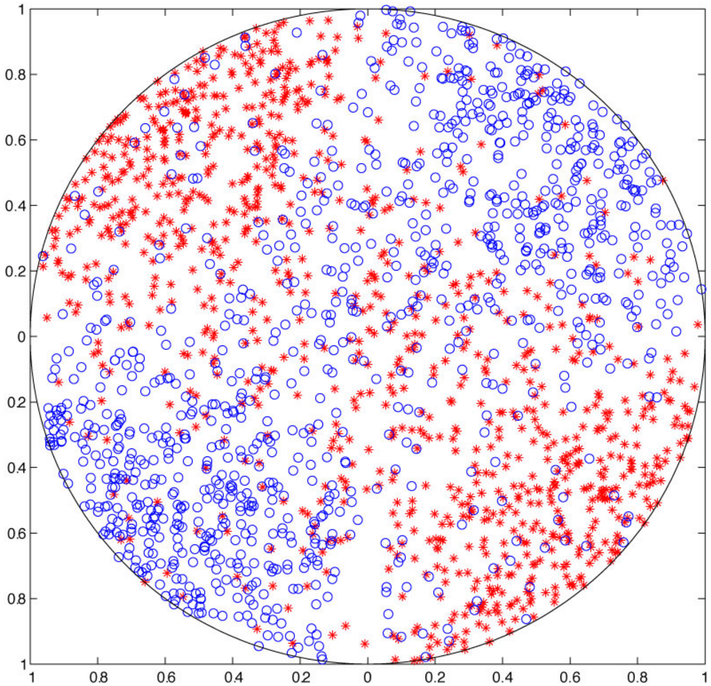
Background #1

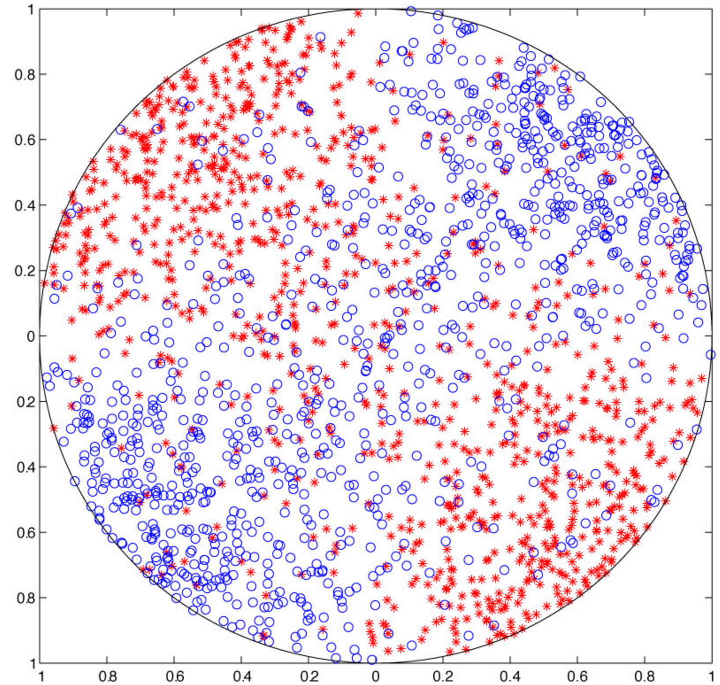
Background #2

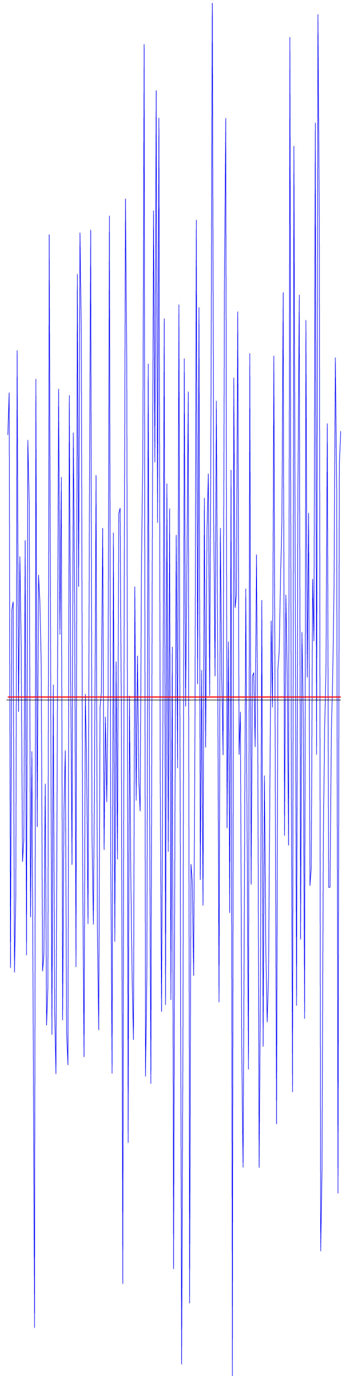


Stress Rate Orientation

Ratio = 100.

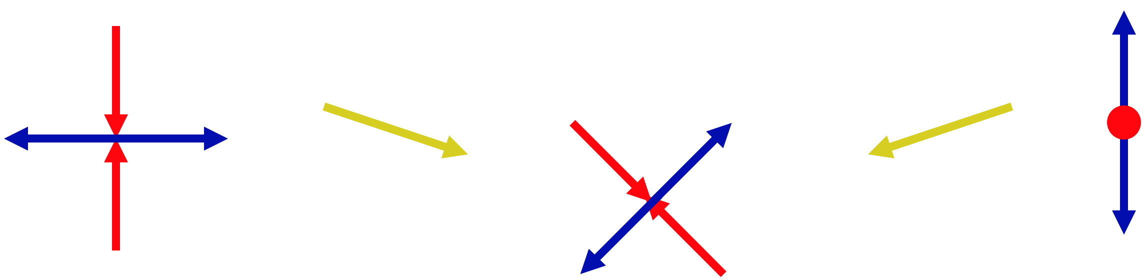




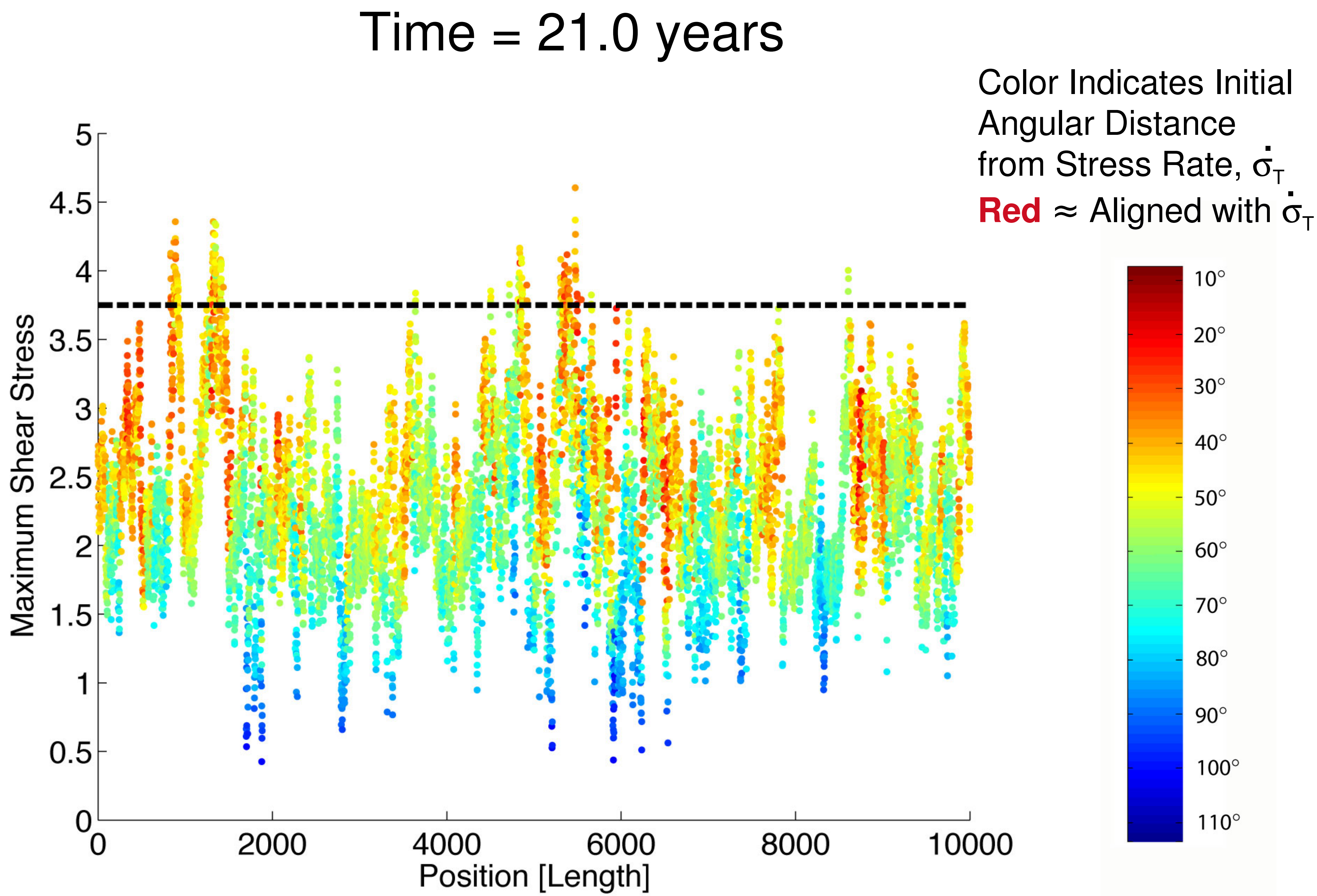


Background #1

Background #2

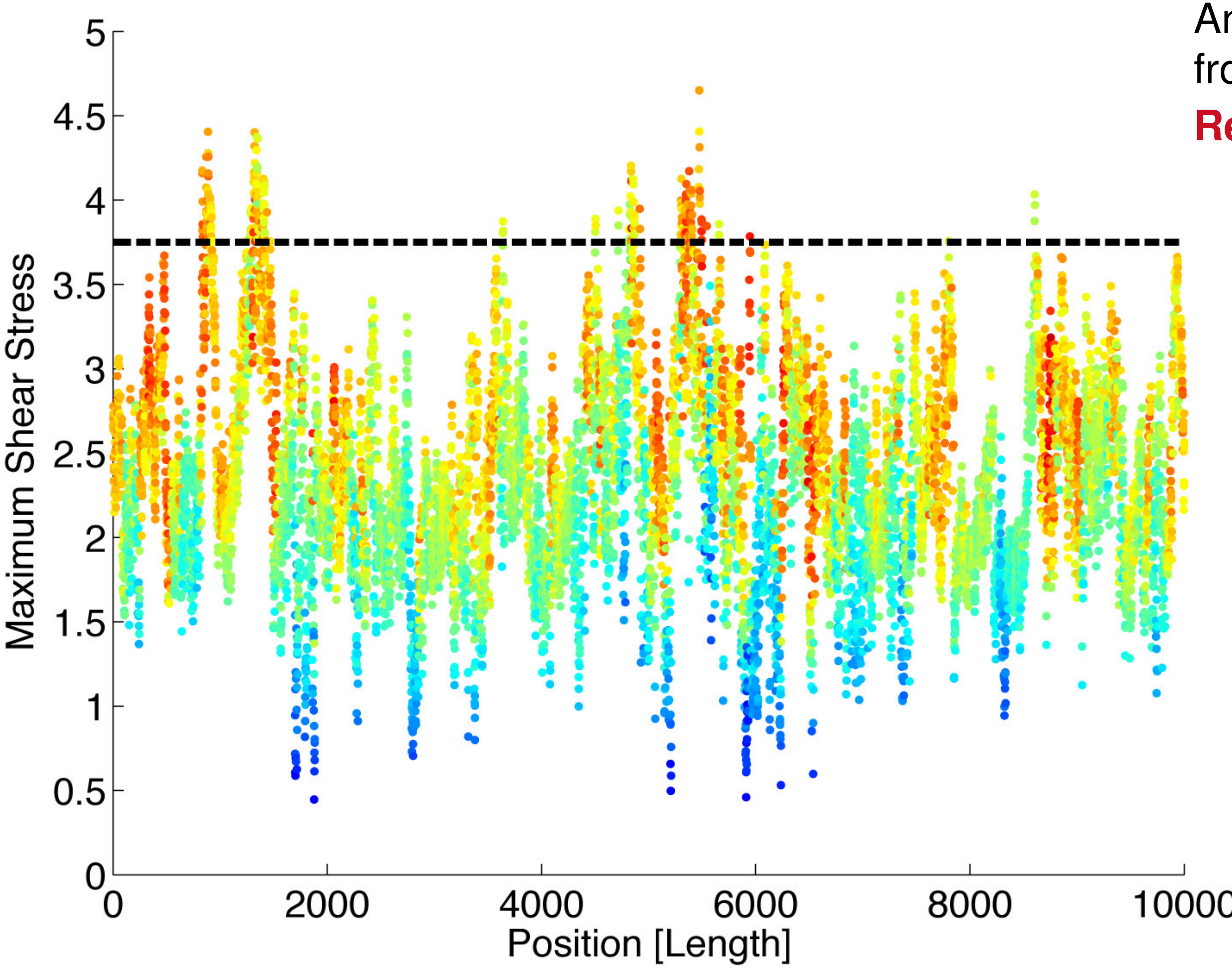


Stress Rate Orientation



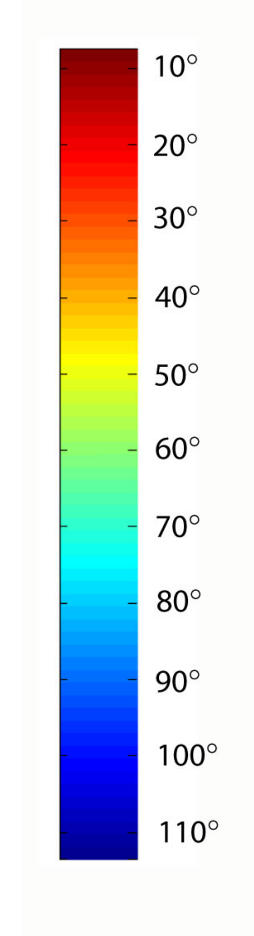
Time = 21.0 years
Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$
Red ≈ Aligned with $\dot{\sigma}_T$.
Maximum Shear Stress
Position [Length]
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
2000
4000
6000
8000
10000
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°

Time = 22.0 years



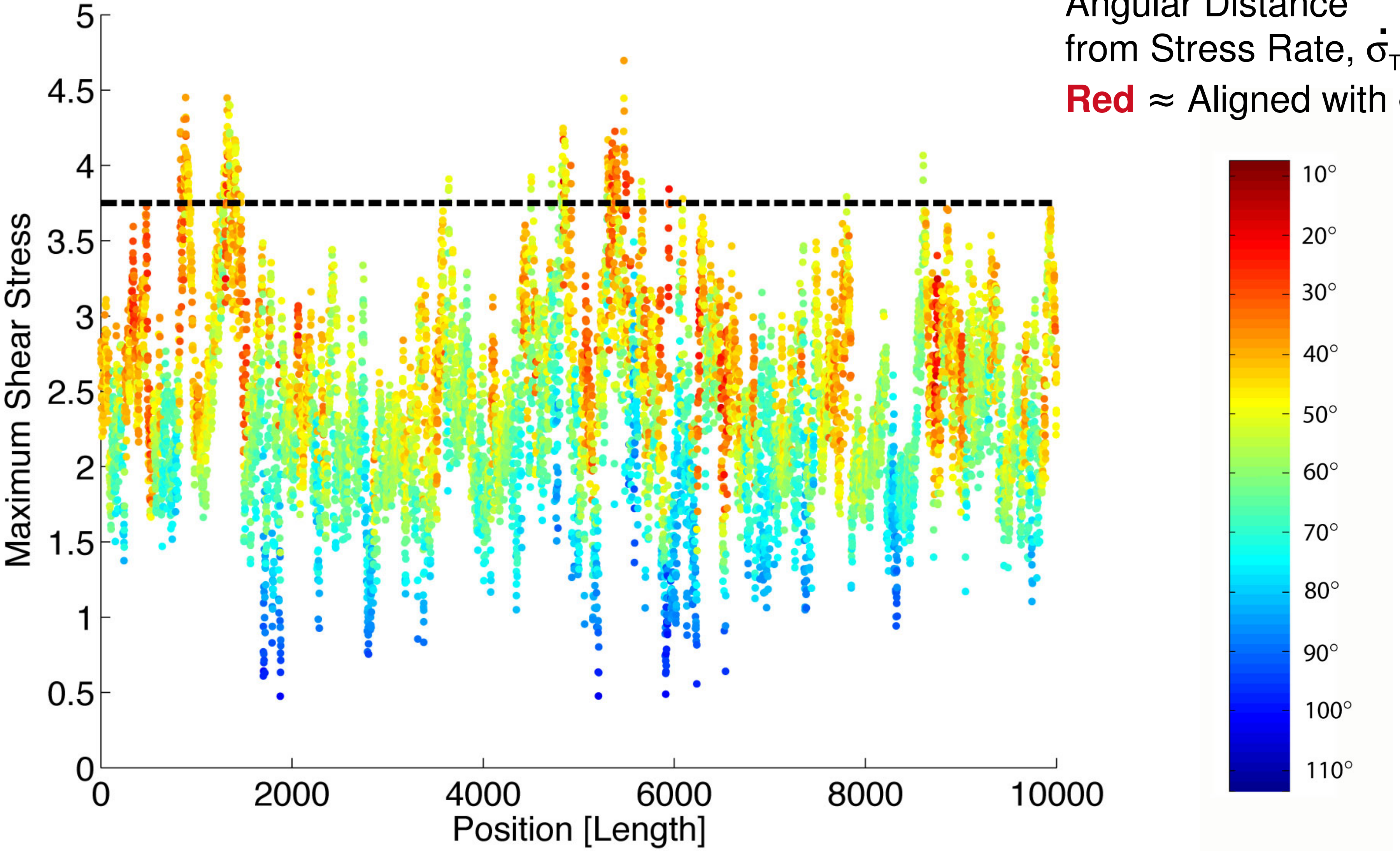
Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

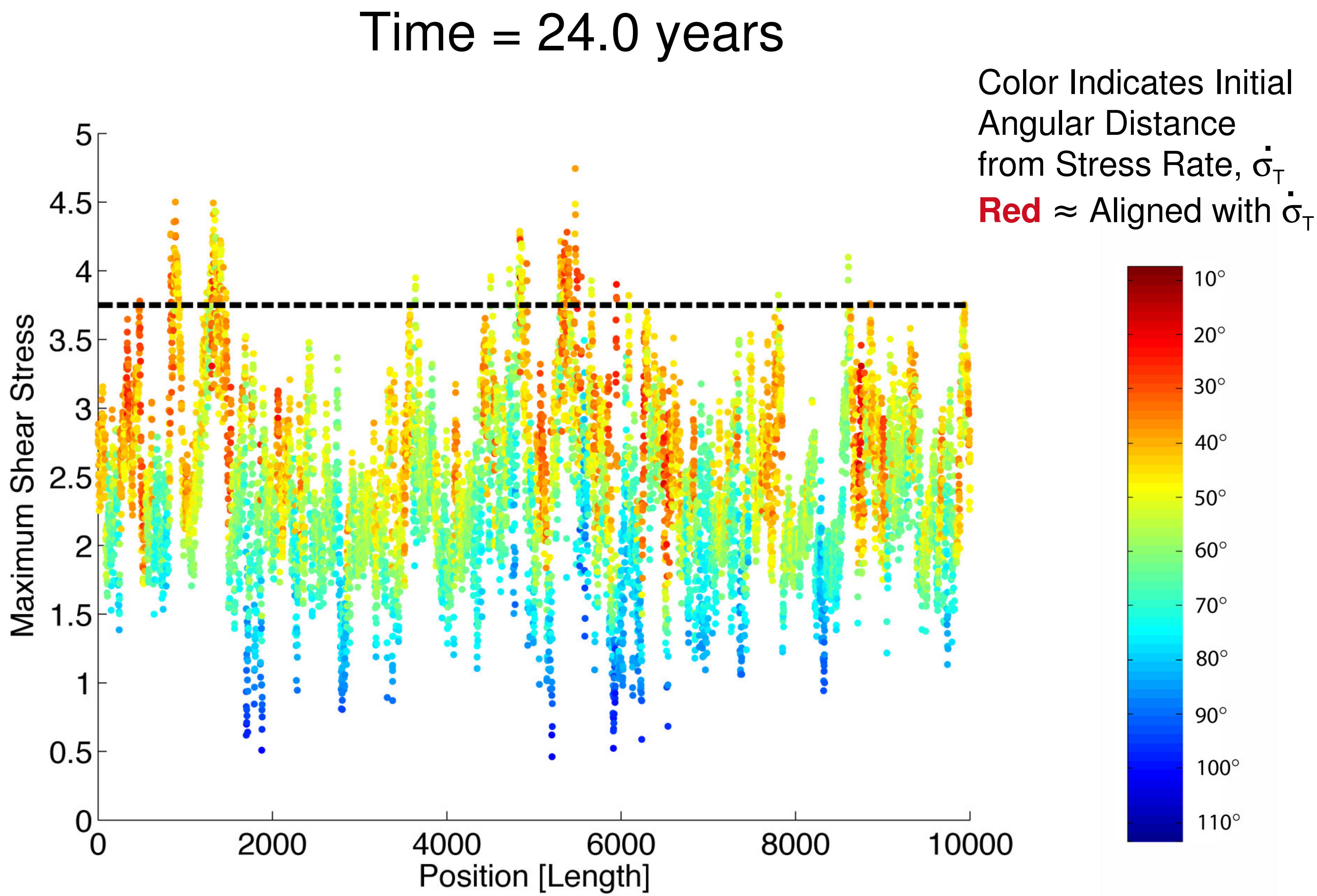
Red ≈ Aligned with $\dot{\sigma}_T$.



Time = 23.0 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$
Red $\approx$ Aligned with $\dot{\sigma}_T$.



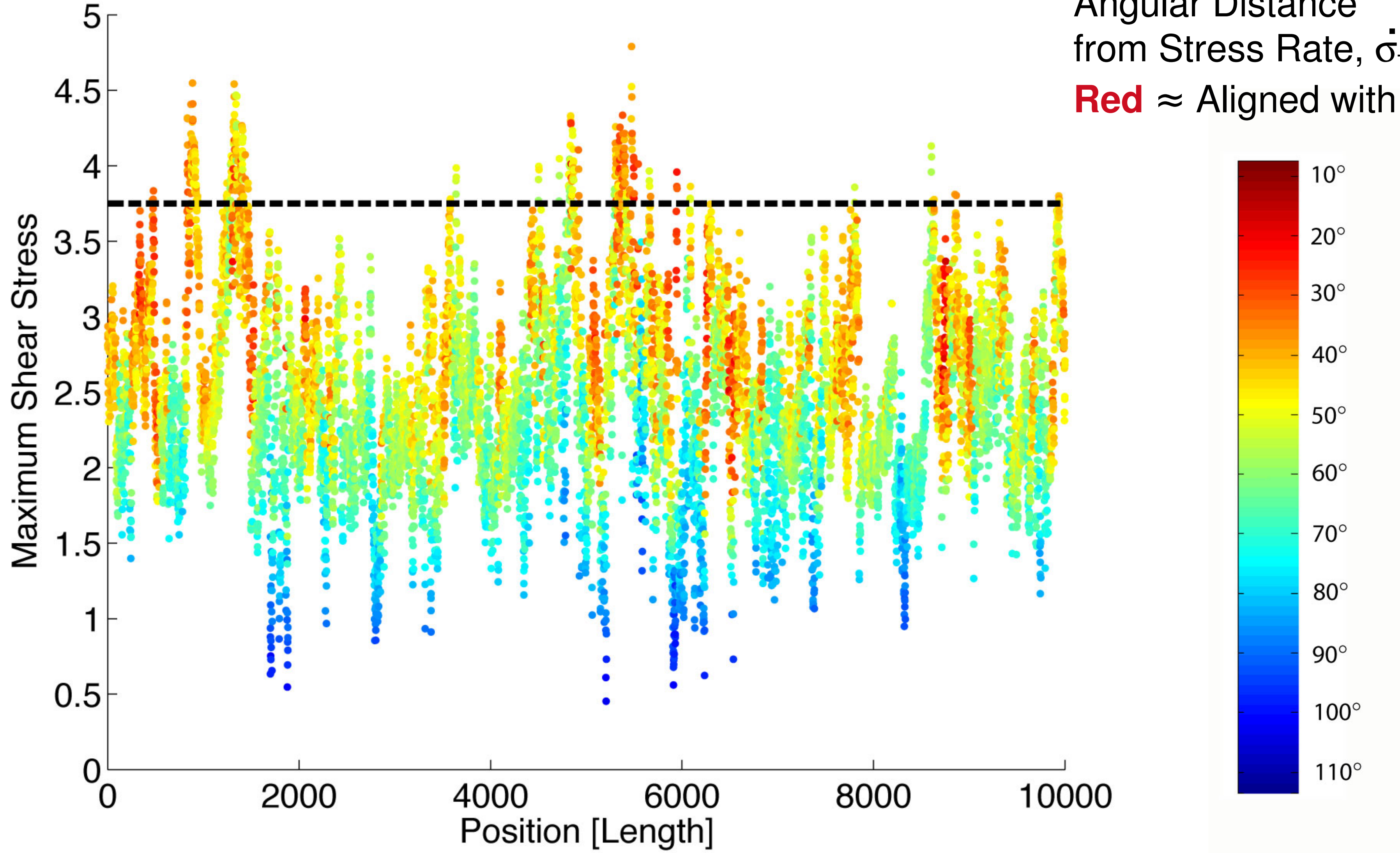


Time = 24.0 years
Color Indicates Initial
Angular Distance
from Stress Rate, $\dot{\sigma}_T$
Red ≈ Aligned with $\dot{\sigma}_T$.
Maximum Shear Stress
Position [Length]
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
2000
4000
6000
8000
10000
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°

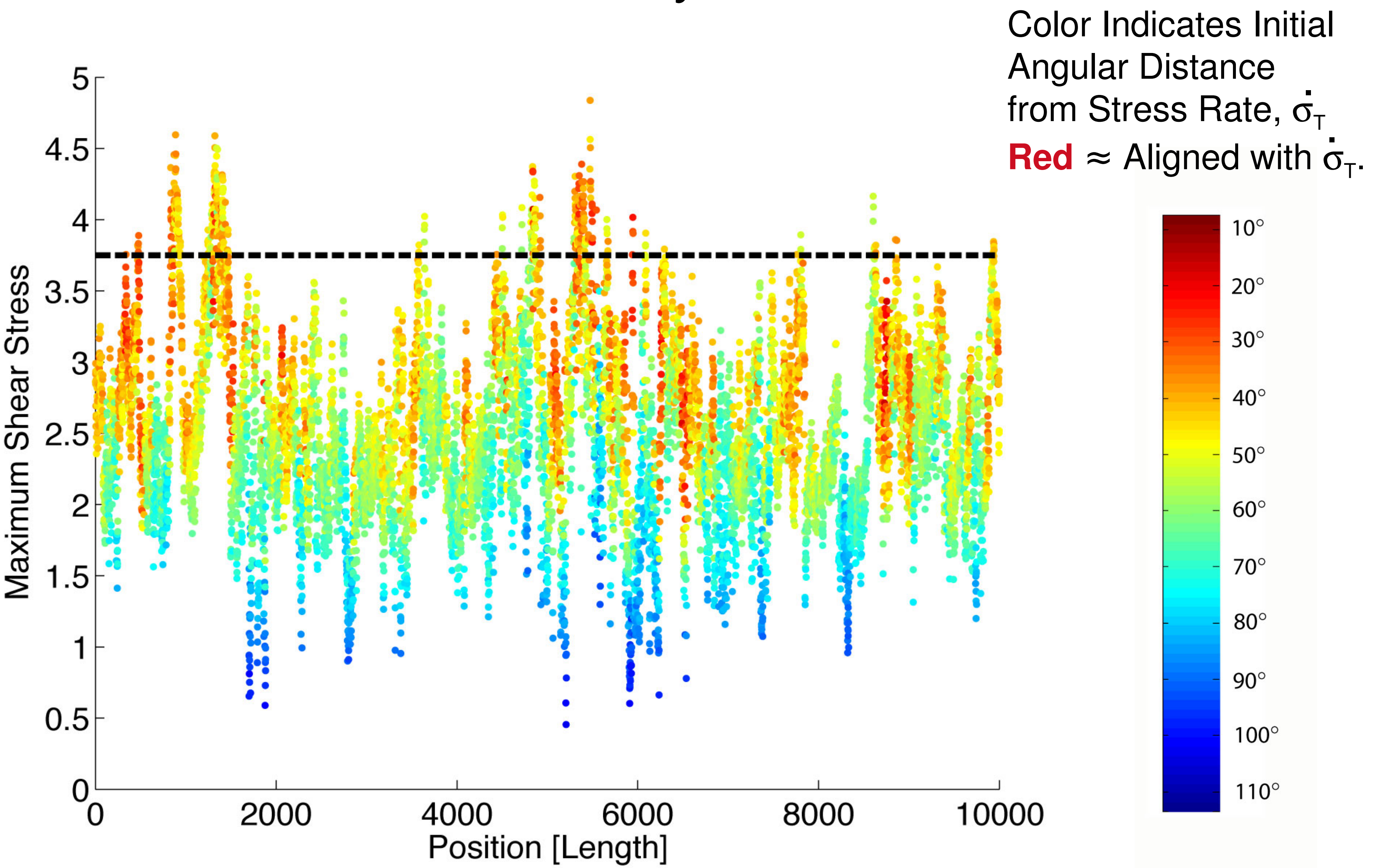
Time = 25.0 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

Red $\approx$ Aligned with $\dot{\sigma}_T$.



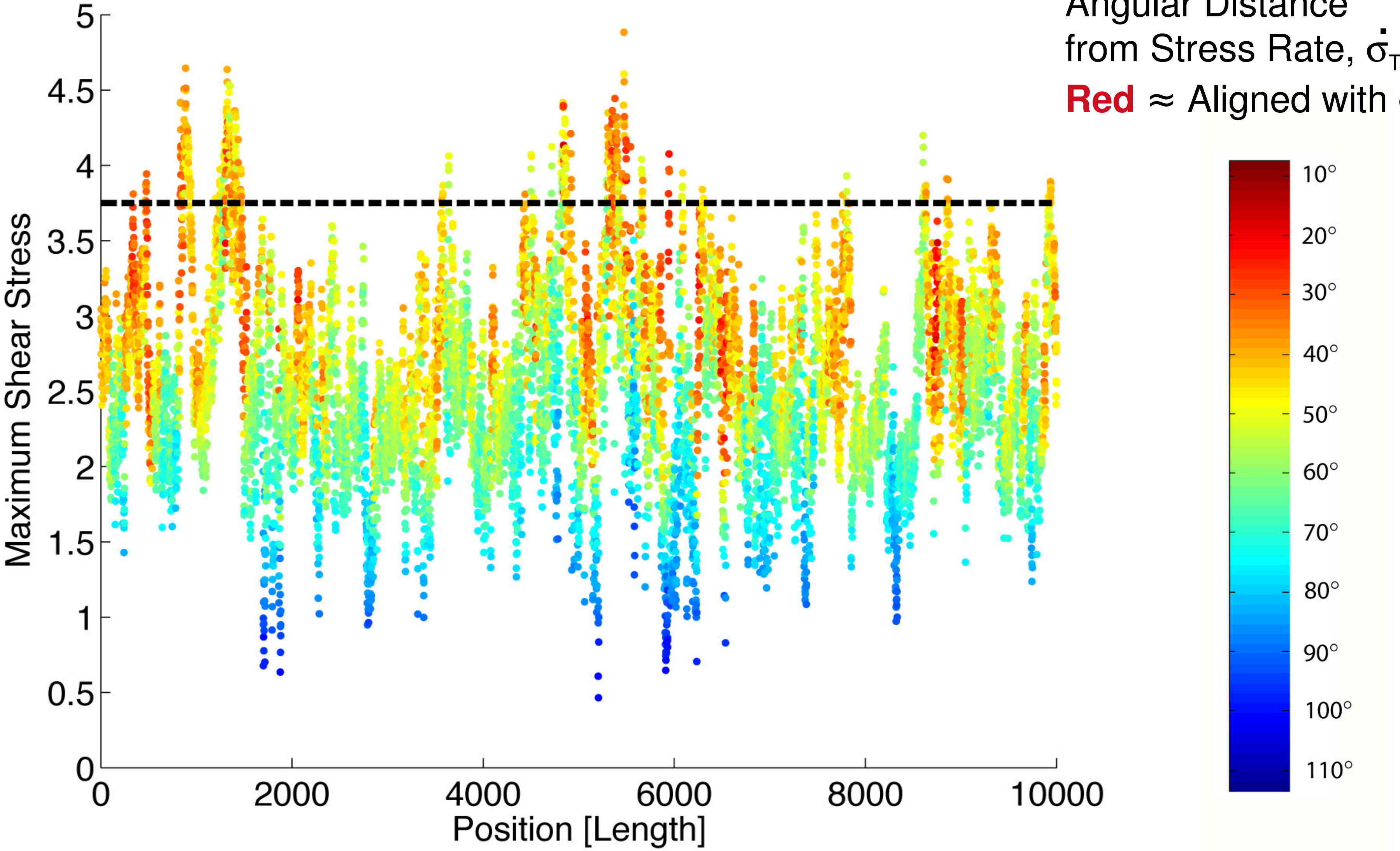
Time = 26.0 years



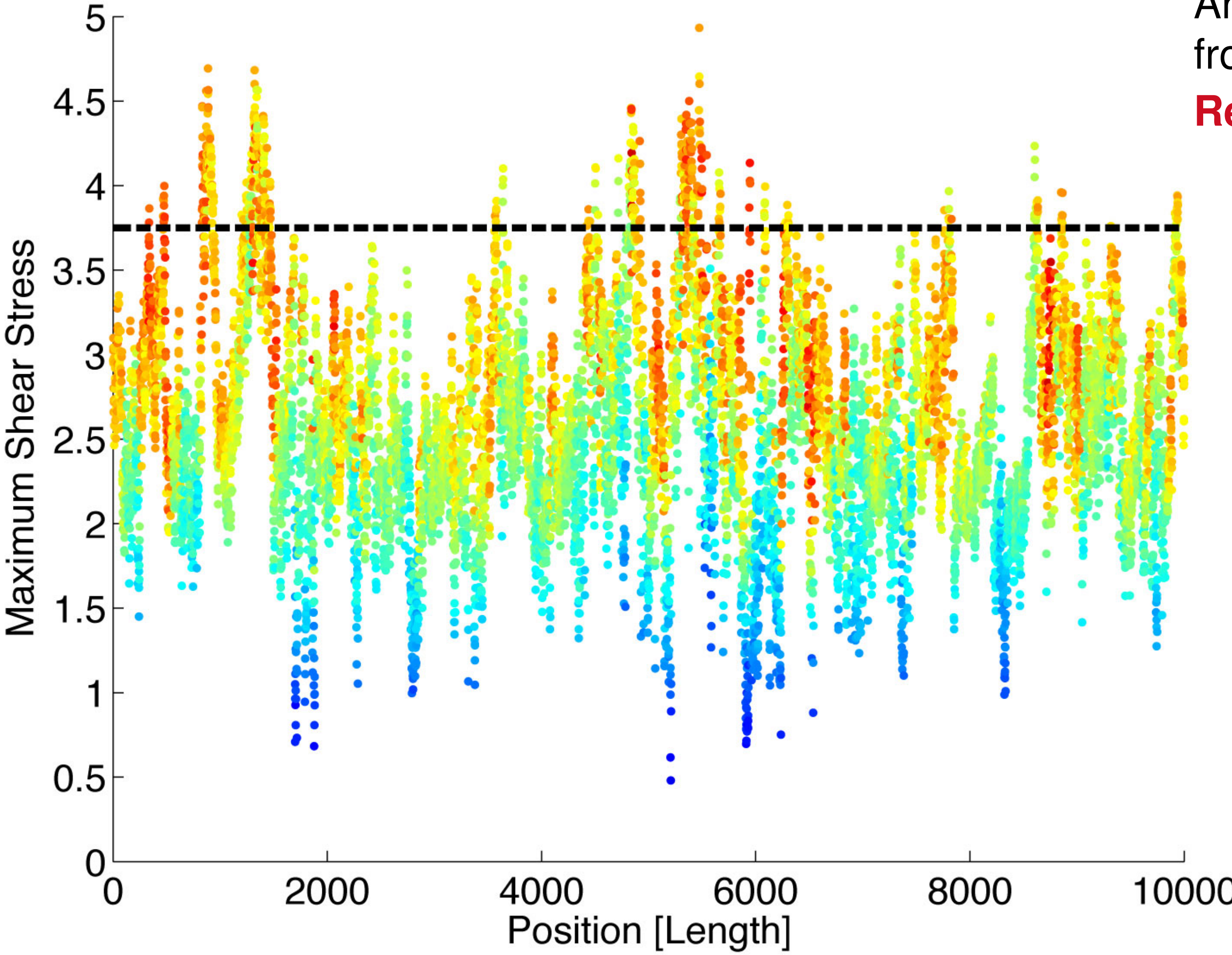
Time = 27.0 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

Red ≈ Aligned with $\dot{\sigma}_T$.

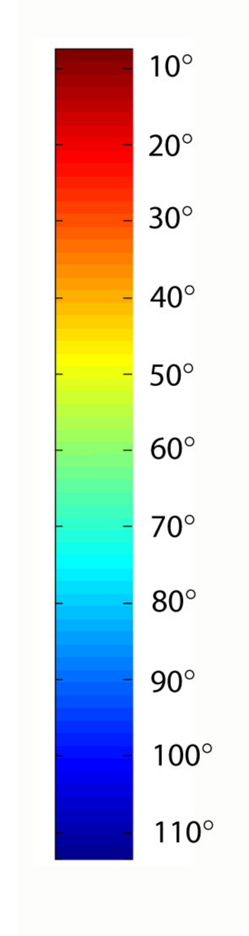


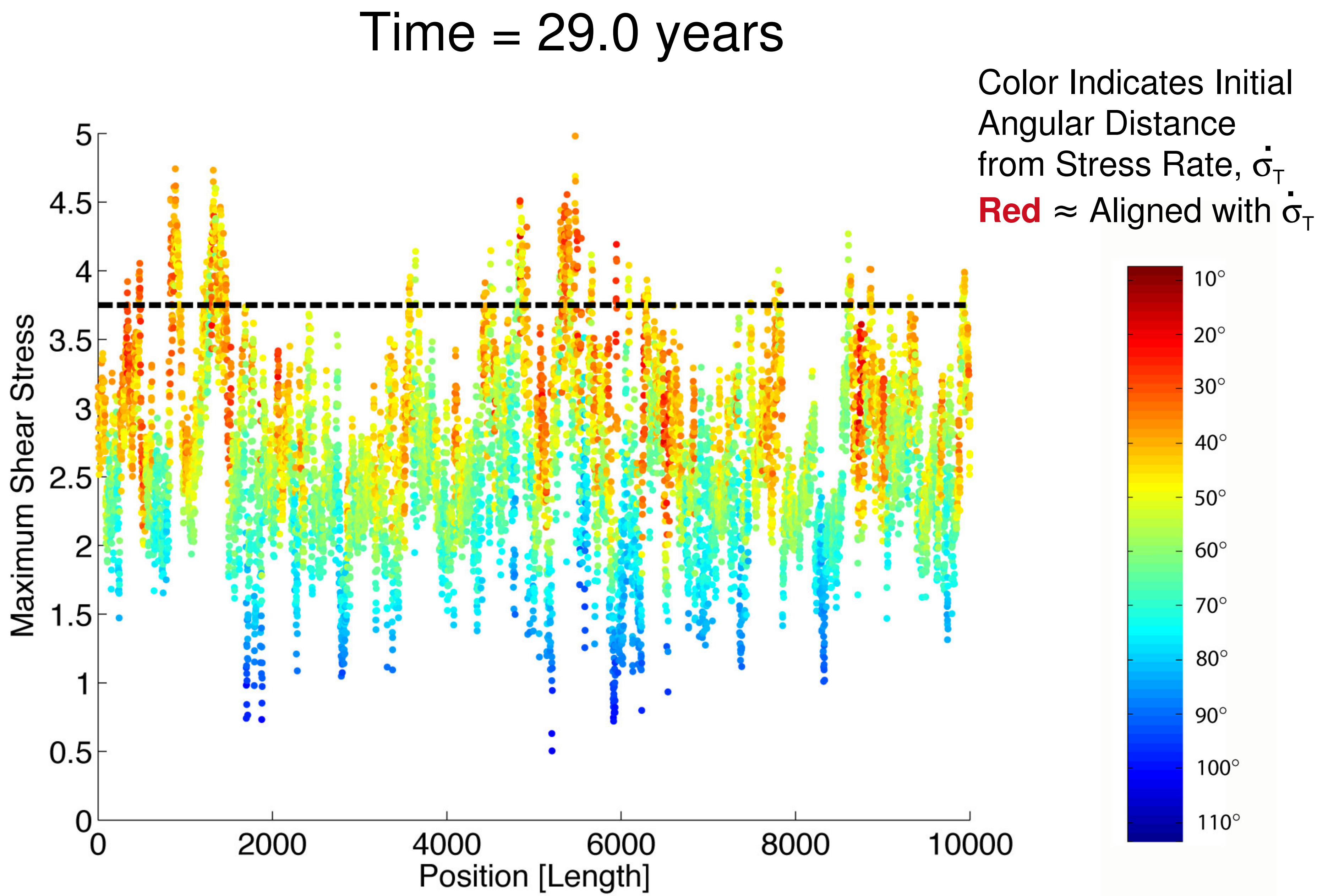
Time = 28.0 years



Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

Red ≈ Aligned with $\dot{\sigma}_T$.



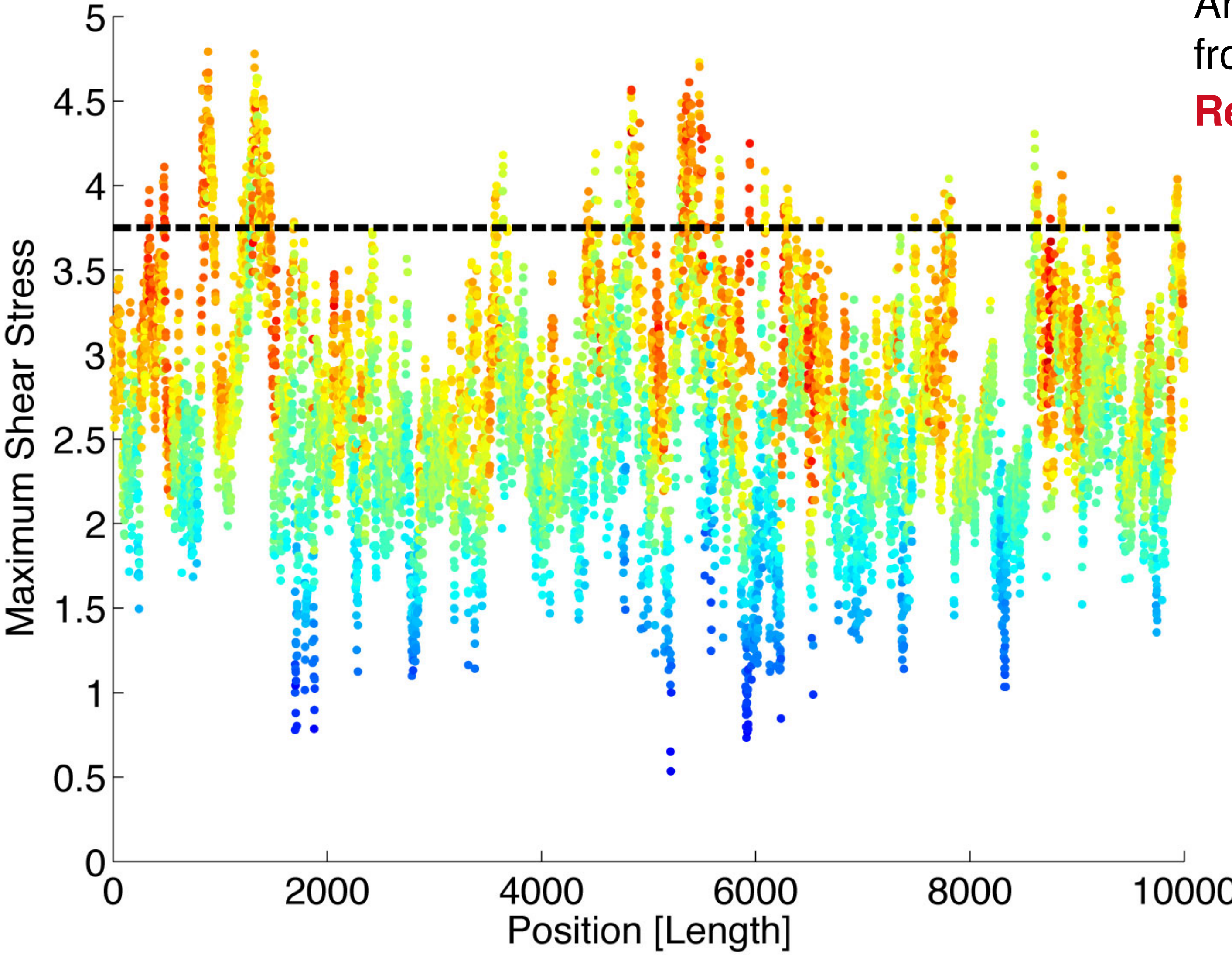


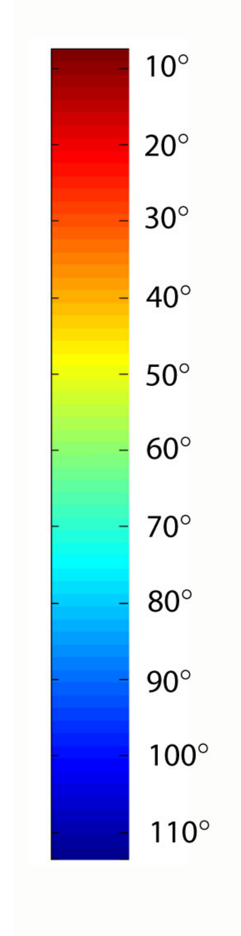
Time = 29.0 years
Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$
Red ≈ Aligned with $\dot{\sigma}_T$.
Maximum Shear Stress
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Position [Length]
0
2000
4000
6000
8000
10000
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°

Time = 30.0 years

Color Indicates Initial Angular Distance from Stress Rate, $\dot{\sigma}_T$

Red ≈ Aligned with $\dot{\sigma}_T$.





Mean Misfit Angle Increases with Increasing Heterogeneity Ratio, *HR*