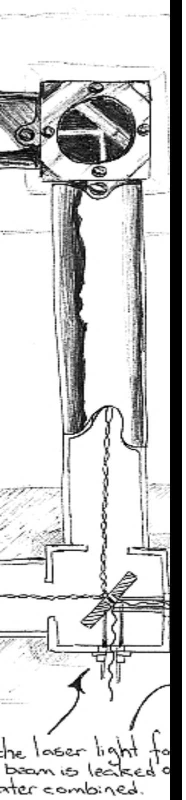

Rotational Ground Motions: A New Observable for Seismology?

H. Igel[1], A. Cochard[1,4], A. Flaws[1,2], W. Suryanto[1], B. Schuberth[1], U. Schreiber[3] , Pham Dinh Nguyen[1], A. Velikoseltsev[3]

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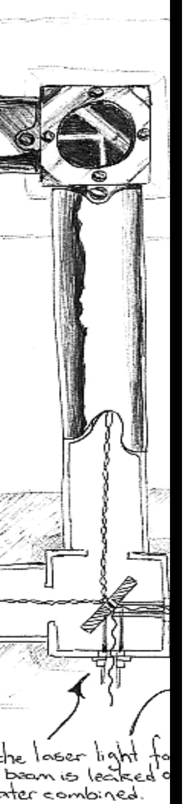
[3]Fundamentalstation Wettzell, Kötzting
[4]EOST Strasbourg

- What is *rotation* in seismology? (Why bother?)
- The ring laser instrument
- Broadband observations of rotations
  - Peak rotation rates
  - Waveform comparison with translations
  - Horizontal phase velocities
  - Love wave dispersion
  - P-coda
- Array-derived vs. directly measured rotations
- Conclusions and future

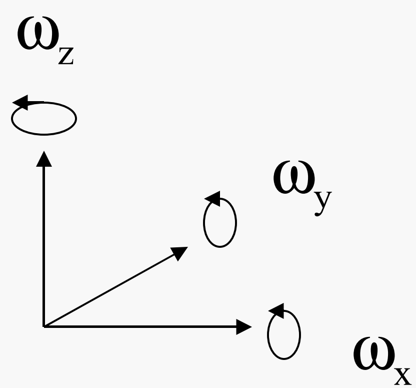
ringlaser.geophysik.uni-muenchen.de

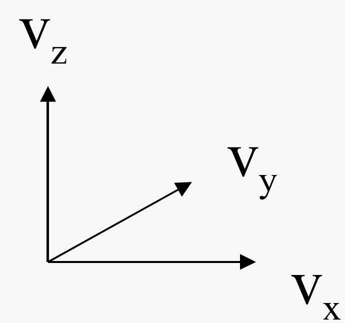


Rotation is the curl of the wavefield

... it separates P- and S-wave in isotropic media

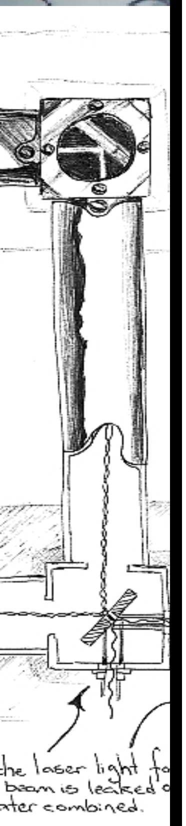
$$\begin{pmatrix}\omega_x\\ \omega_y\\ \omega_z\end{pmatrix} = \frac{1}{2}\nabla\times\underline{\mathbf{v}} = \frac{1}{2}\begin{pmatrix}\partial_y v_z - \partial_z v_y\\ \partial_z v_x - \partial_x v_z\\ \partial_x v_y - \partial_y v_x\end{pmatrix}$$





Rotation rate
Rotation sensor

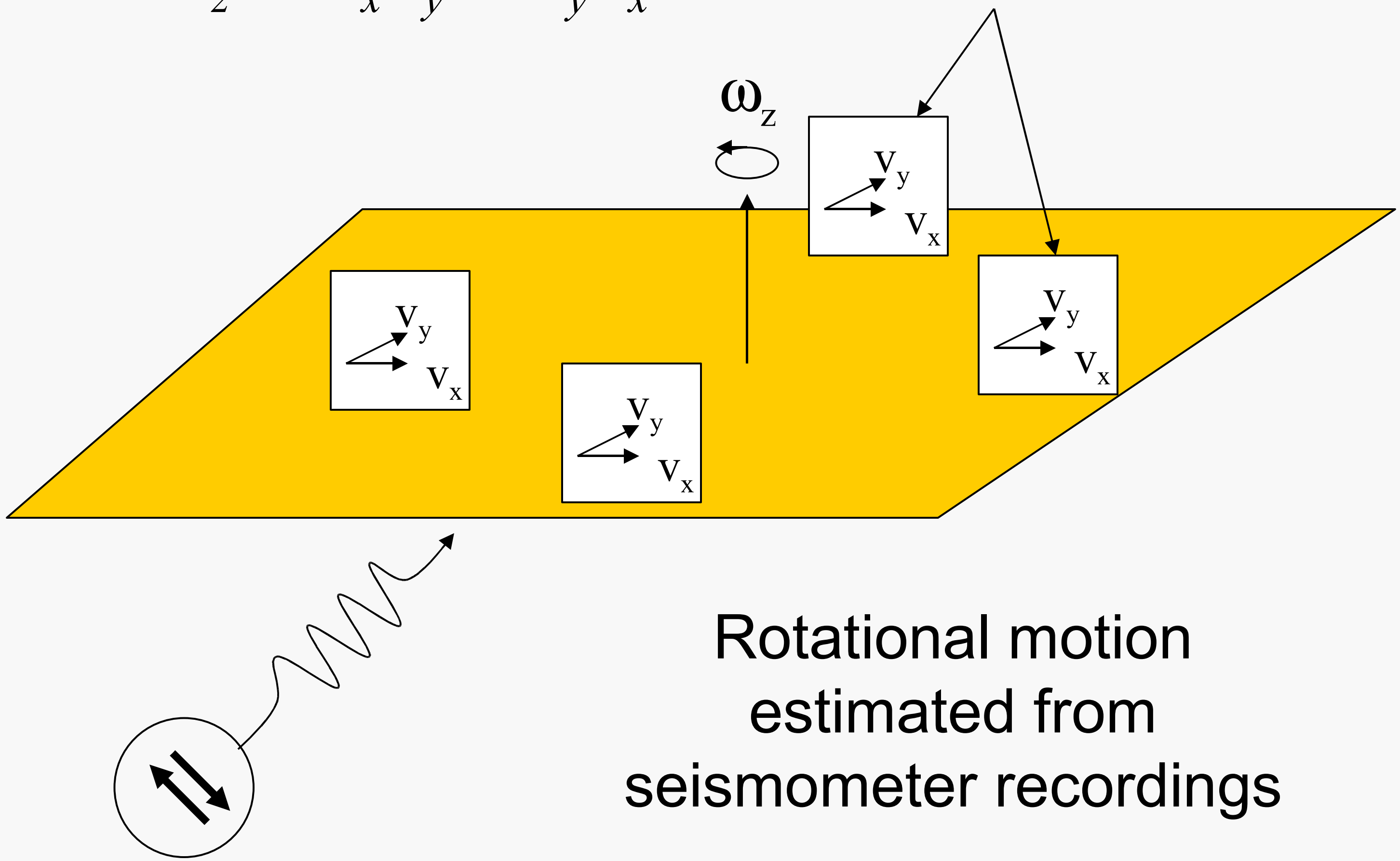
Ground velocity
Seismometer



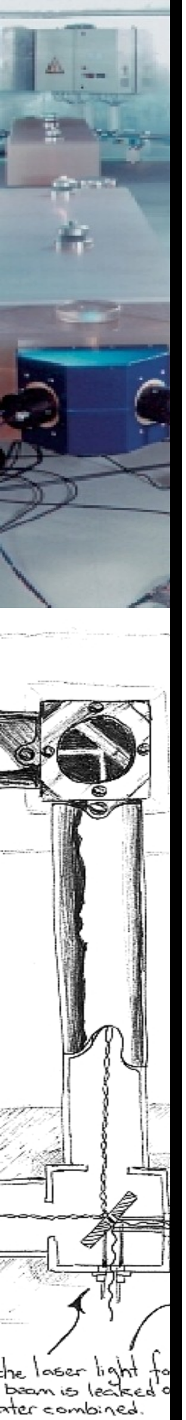
Rotation from seismic arrays?

... by finite differencing ...

$$\omega_z \approx \partial_x v_y - \partial_y v_x$$



Rotational motion estimated from seismometer recordings



Radiation from a double-couple point source

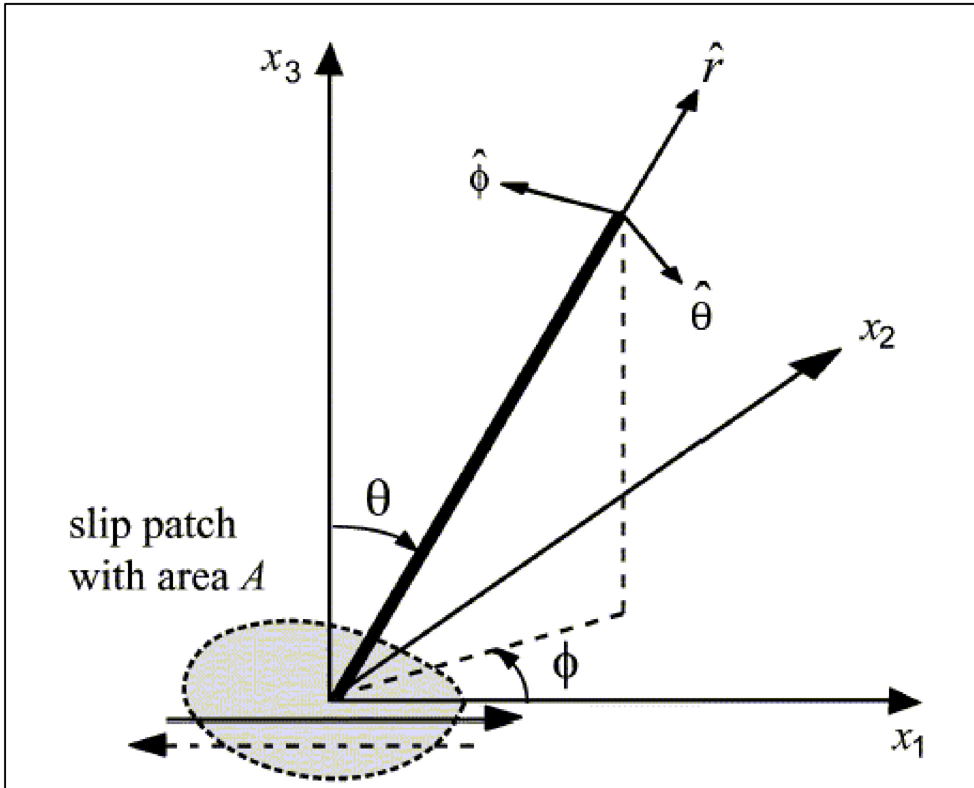
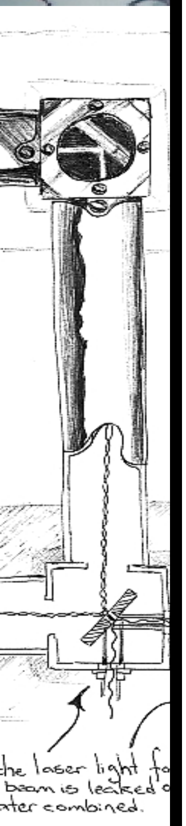


FIGURE 5 Cartesian and polar coordinate systems for analysis of radiation by a slip patch with area A and average slip $\langle \Delta u(t) \rangle$.

Geometry we use to express the seismic wavefield radiated by point double-couple source with area A and slip Δu

Here the fault plane is the x_1x_2-plane and the slip is in x_1-direction.

Aki and Richards (2002)



Radiation from a point source

Ground displacement

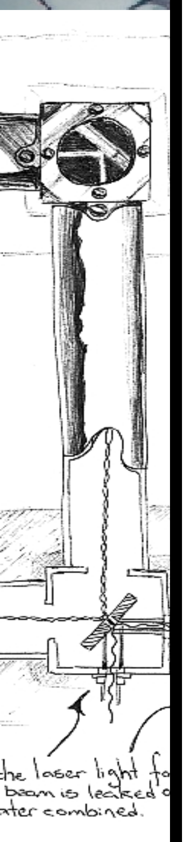
Near field term contains the static deformation

$$
\begin{aligned}
\boldsymbol{u}(\boldsymbol{x}, t) = {} & \frac{1}{4\pi\rho} \boldsymbol{A}^{N} \frac{1}{r^4} \int_{r/v_P}^{r/v_S} \tau M_0(t-\tau)\, d\tau \\
& + \frac{1}{4\pi\rho v_P^2} \boldsymbol{A}^{IP} \frac{1}{r^2} M_0(t - r/v_P) \\
& + \frac{1}{4\pi\rho v_S^2} \boldsymbol{A}^{IS} \frac{1}{r^2} M_0(t - r/v_S) \\
& + \frac{1}{4\pi\rho v_P^3} \boldsymbol{A}^{FP} \frac{1}{r} \dot{M}_0(t - r/v_P) \\
& + \frac{1}{4\pi\rho v_S^3} \boldsymbol{A}^{FS} \frac{1}{r} \dot{M}_0(t - r/v_S).
\end{aligned}
$$

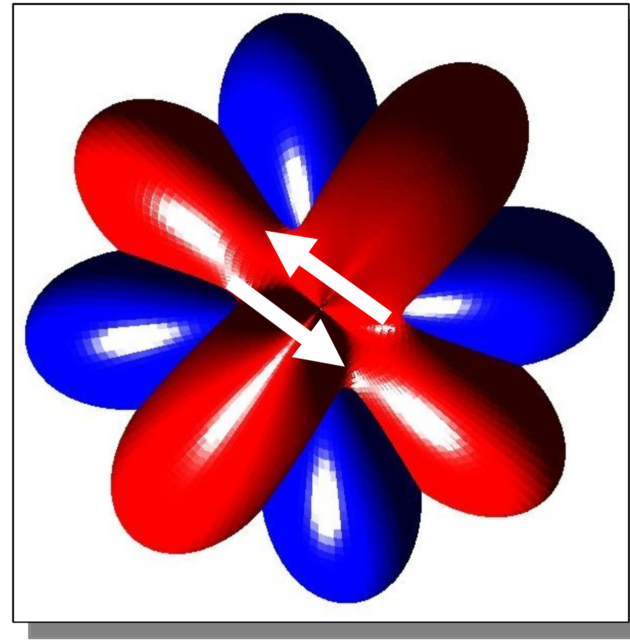
Intermediate terms

Far field terms: the main ingredient for source inversion, ray theory, etc.

Aki and Richards (2002)



Radiation pattern



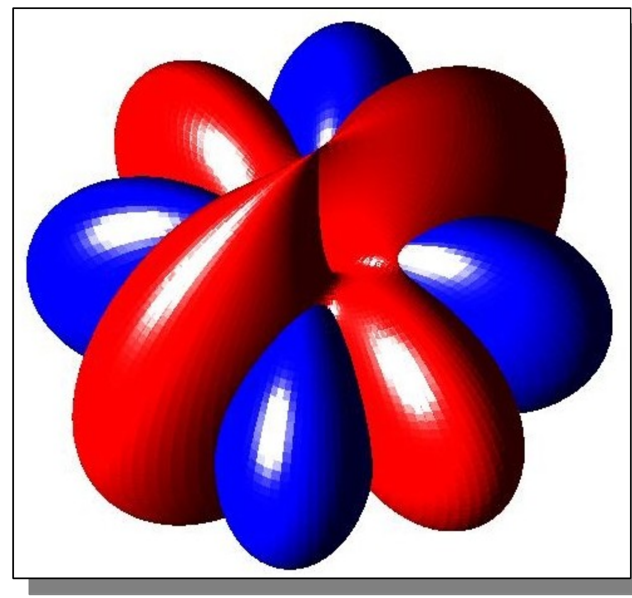
$$\mathbf{A}^{N} = 9 \sin 2\theta \cos\phi \hat{\mathbf{r}} - 6(\cos 2\theta \cos\phi \hat{\boldsymbol{\theta}} - \cos\theta \sin\phi \hat{\boldsymbol{\phi}}),$$

$$\mathbf{A}^{IP} = 4 \sin 2\theta \cos\phi \hat{\mathbf{r}} - 2(\cos 2\theta \cos\phi \hat{\boldsymbol{\theta}} - \cos\theta \sin\phi \hat{\boldsymbol{\phi}}),$$

$$\mathbf{A}^{IS} = -3 \sin 2\theta \cos\phi \hat{\mathbf{r}} + 3(\cos 2\theta \cos\phi \hat{\boldsymbol{\theta}} - \cos\theta \sin\phi \hat{\boldsymbol{\phi}}),$$

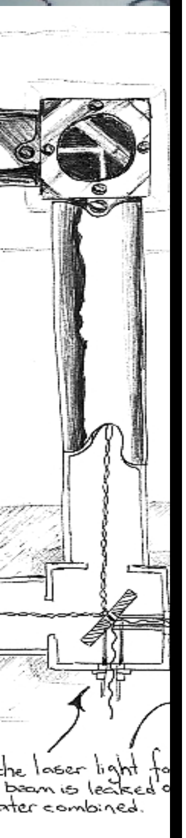
$$\mathbf{A}^{FP} = \sin 2\theta \cos\phi \hat{\mathbf{r}},$$

$$\mathbf{A}^{FS} = \cos 2\theta \cos\phi \hat{\boldsymbol{\theta}} - \cos\theta \sin\phi \hat{\boldsymbol{\phi}},$$



Far field P – blue
Far field S - red

Aki and Richards (2002)



The rotational part

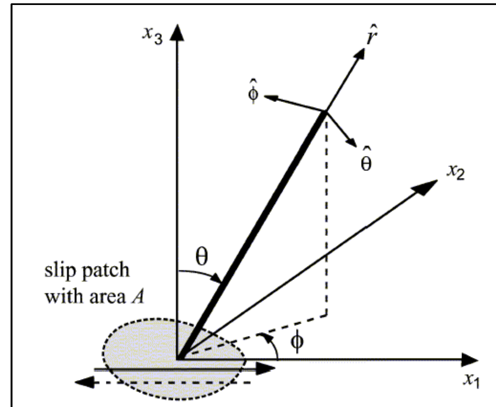


FIGURE 5 Cartesian and polar coordinate systems for analysis of radiation by a slip patch with area A and average slip $\langle \Delta u(t) \rangle$.

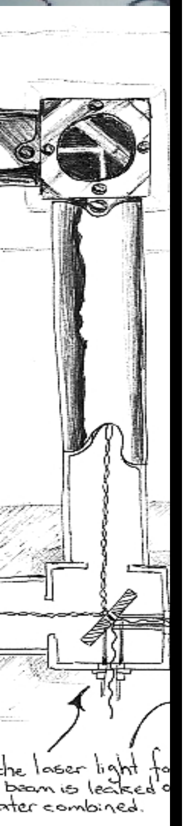
$$\boldsymbol{\omega}(\mathbf{x},t) = \frac{1}{2}\nabla \times \mathbf{u}(\mathbf{x},t)$$
$$= \frac{-\mathbf{A}^R}{8\pi\rho}\left[\frac{3}{\beta^2 r^3}M_0\left(t-\frac{r}{\beta}\right) + \frac{3}{\beta^3 r^2}\dot{M}_0\left(t-\frac{r}{\beta}\right) + \frac{1}{\beta^4 r}\ddot{M}_0\left(t-\frac{r}{\beta}\right)\right]$$

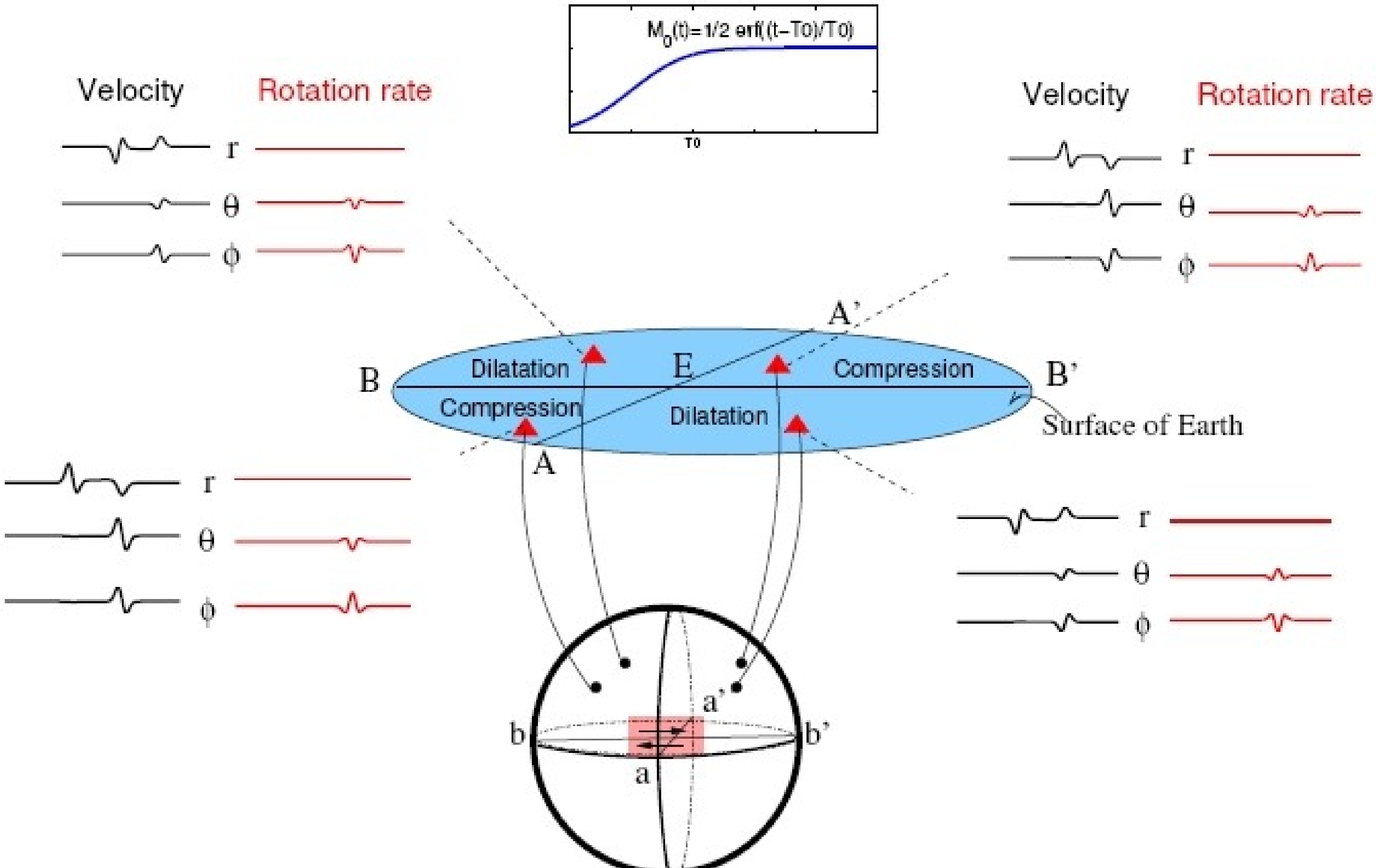
$$\mathbf{A}^R = \cos\theta\ \sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\ \cos 2\theta\,\hat{\boldsymbol{\phi}}$$

- Rotations are zero before S arrival
- This includes the near field!
- Far-field P-rotation is not zero! Only the sum of all contributions cancel!

Cochard et al. (2006)

Basic seismograms, full space





Acceleration vs. Rotation rate

$$u(x,t) = \frac{1}{4\pi\rho} A^N \frac{1}{r^4} \int_{r/v_P}^{r/v_S} \tau M_0(t-\tau)\, d\tau$$

$$+ \frac{1}{4\pi\rho v_P^2} A^{IP} \frac{1}{r^2} M_0(t - r/v_P)$$

$$+ \frac{1}{4\pi\rho v_S^2} A^{IS} \frac{1}{r^2} M_0(t - r/v_S)$$

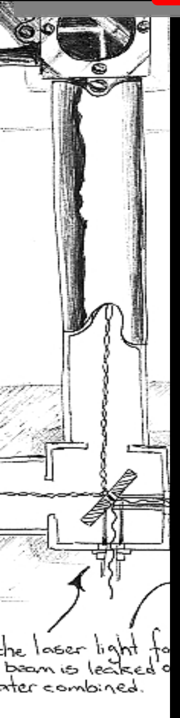
$$+ \frac{1}{4\pi\rho v_P^3} A^{FP} \frac{1}{r} \dot{M}_0(t - r/v_P)$$

$$+ \frac{1}{4\pi\rho v_S^3} A^{FS} \frac{1}{r} \dot{M}_0(t - r/v_S).$$

$$\omega(\mathbf{x},t) = \frac{1}{2}\nabla \times \mathbf{u}(\mathbf{x},t)$$

$$= \frac{-\mathbf{A}^R}{8\pi\rho}\left[\frac{3}{\beta^2 r^3} M_0\left(t - \frac{r}{\beta}\right) + \frac{3}{\beta^3 r^2}\dot{M}_0\left(t - \frac{r}{\beta}\right) + \frac{1}{\beta^4 r}\ddot{M}_0\left(t - \frac{r}{\beta}\right)\right]$$

$$\mathbf{A}^R = \cos\theta\ \sin\phi\, \hat{\boldsymbol{\theta}} + \cos\phi\ \cos 2\theta\, \hat{\boldsymbol{\phi}}$$

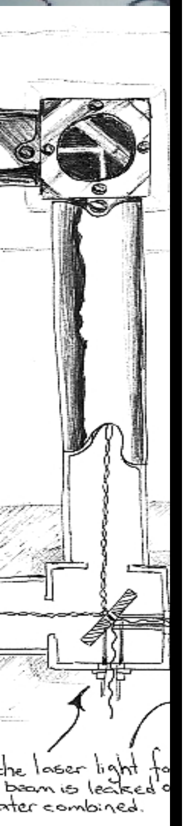


... in the far field ...

$$\ddot{u}^{FS} = \frac{1}{4\pi\rho v_s^3 r} \dddot{M}_0(t - r/v_s)$$

$$\varpi^F = -\frac{1}{8\pi\rho v_s^4 r} \dddot{M}_0(t - r/v_s)$$

$$\frac{\ddot{u}^{FS}}{\varpi^F} = -2v_s$$



Rotations – why bother?

- Standard seismological observations are polluted by rotations
- Tiltmeters (rotation around horizontal axes) are polluted by translations

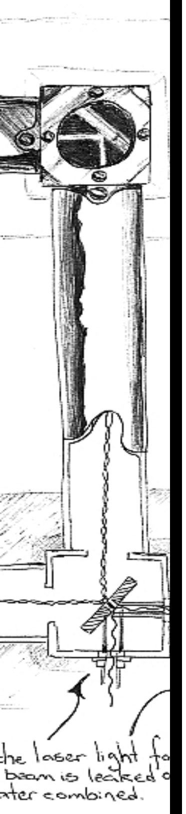
Instruments

- Rotations may contribute to co-seismic structural damage

Earthquake engineering

- Rotational measurements my provide additional wavefield information (phase velocities, etc)
- ... and may allow further constraints on rupture processes ...

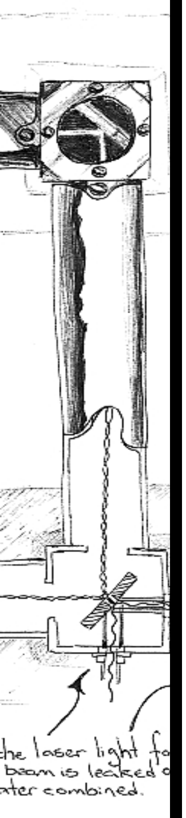
Waves and rupture



A+R's view ...

„The state-of-the-art sensitivity of the general rotation-sensor is not yet enough for a useful geophysical application“ (Aki and Richards, Quantitative Seismology, 1980)

„... note the utility of measuring rotation near a rupturing fault plane (...), but as of this writing seismology still awaits a suitable instrument for making such measurements“ (Aki and Richards, Quantitative Seismology, 2nd edition 2002)



Previous studies

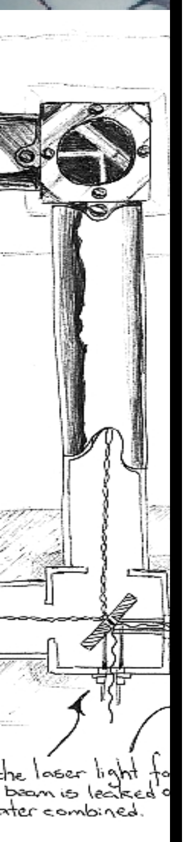
Schreiber, Stedman, and co-workers
Ring laser technology New Zealand and Germany

Takeo and co-workers
Gyroscopic rotation sensor, theoretical work

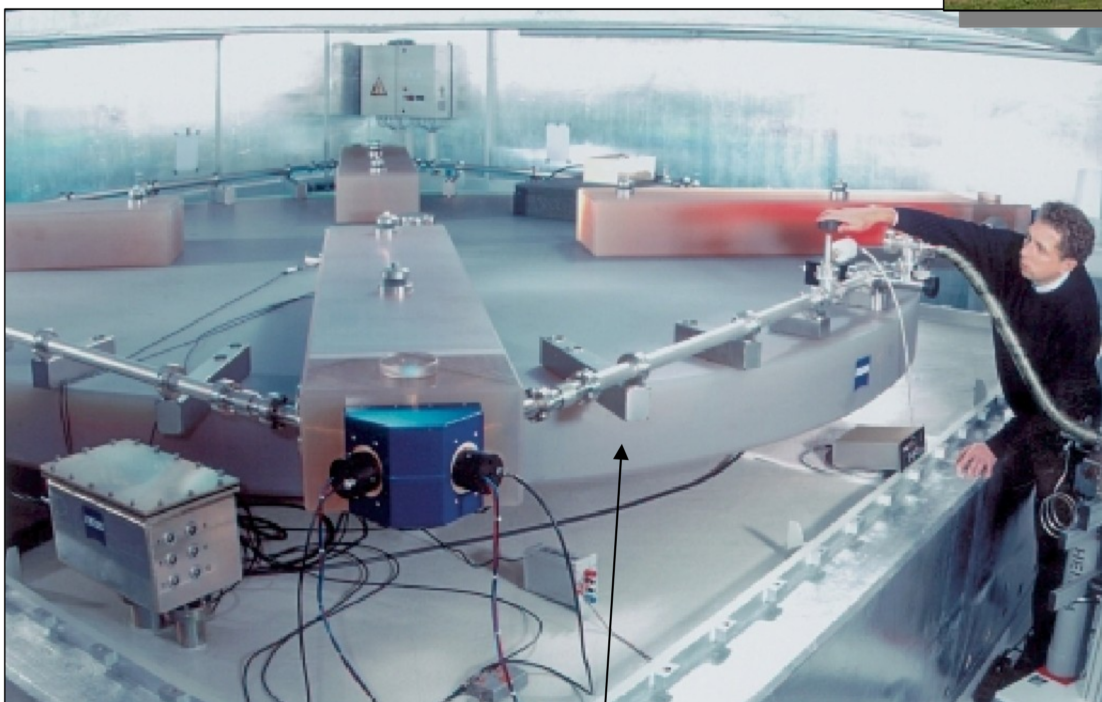
Nigbor and co-workers
rotational sensor and observation of rotational motion of nuclear blast

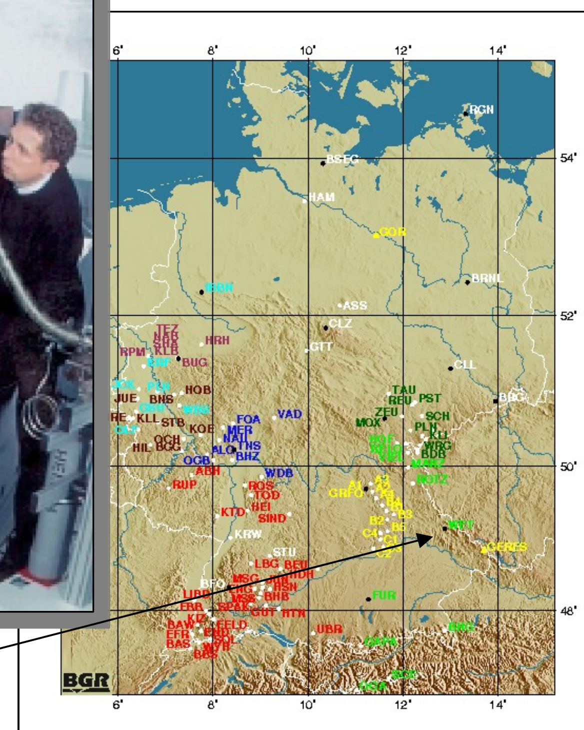
Teisseyre and co-workers
mechanical rotational sensor and observation of local events

→ **it seems that only optical technology provides the required high resolution for (tele-)seismic measurements**



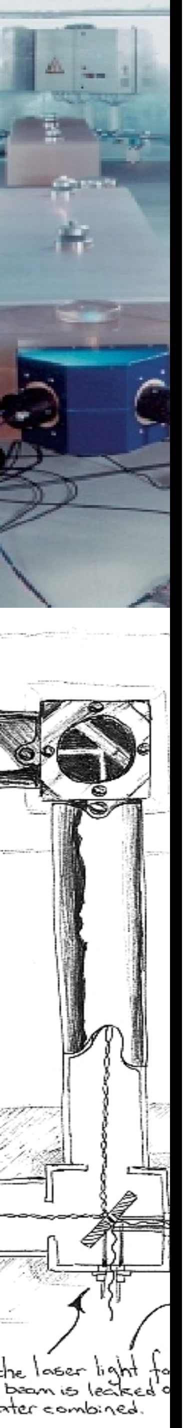
The ring laser at Wettzell

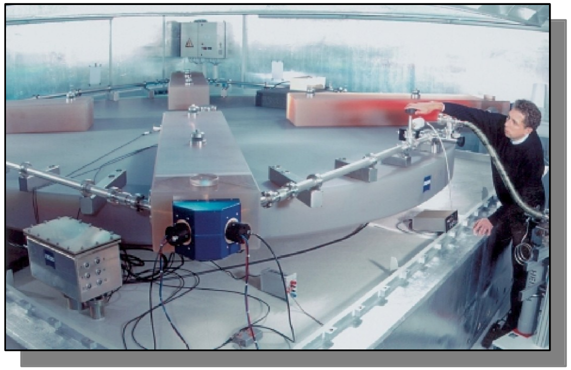


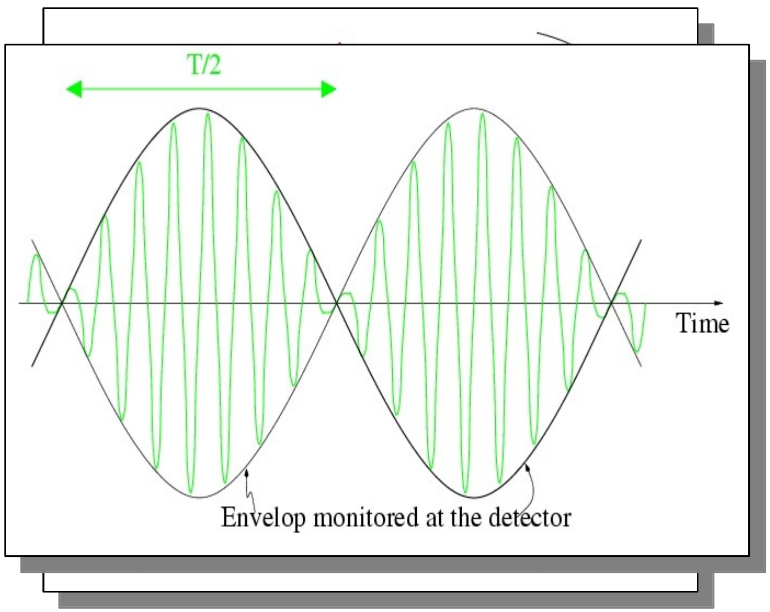


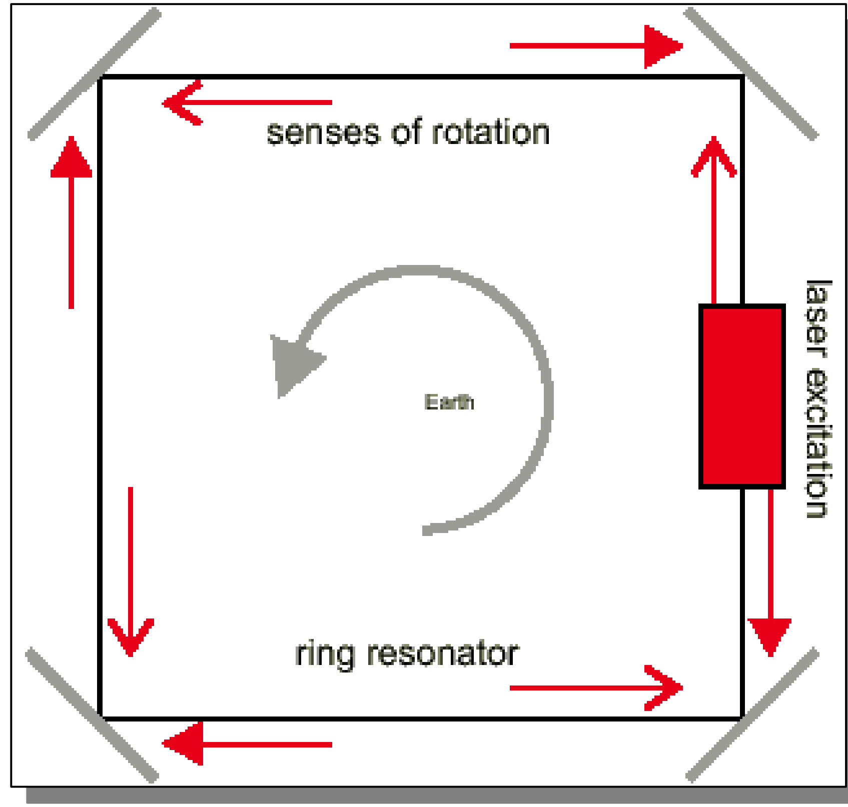
ring laser

How can we observe rotations? -> ring laser



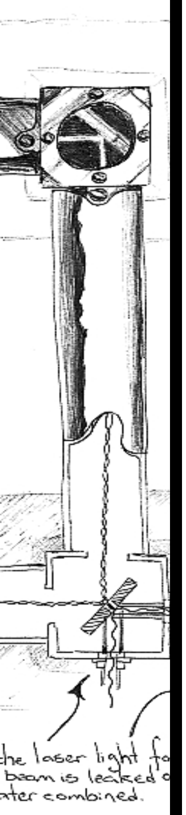


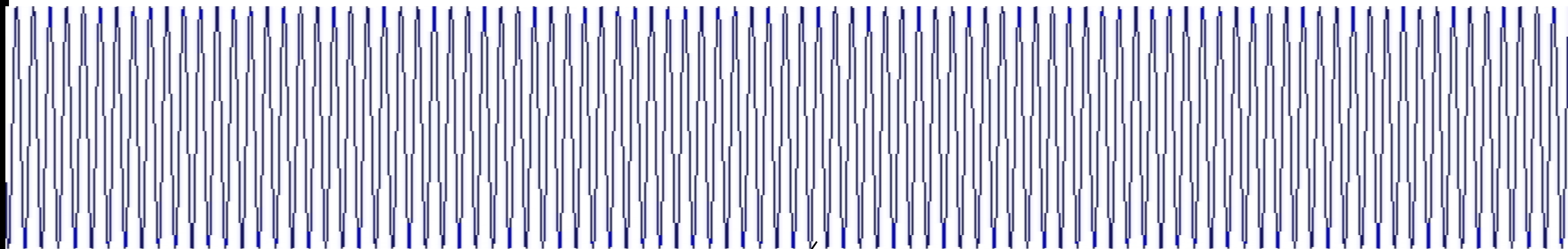




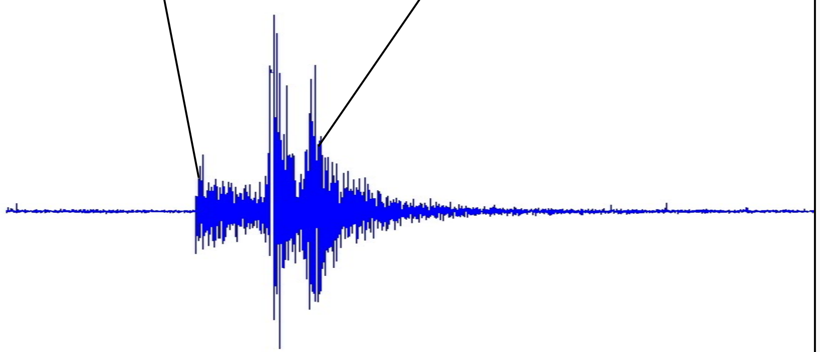
Ring laser technology developed by the groups at the Technical University Munich and the University of Christchurch, NZ

The Sagnac Frequency
(schematically)



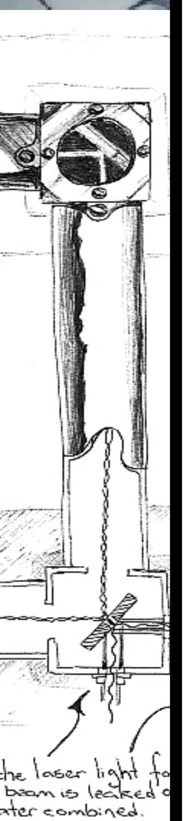


Sagnac frequency sampled with 800Hz

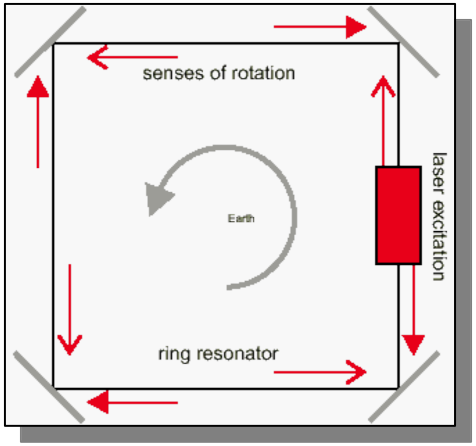


Rotation rate sampled with 4Hz

Tiny changes in the Sagnac frequencies are extracted to obtain the time series with rotation rate $\Delta f \rightarrow \Theta$

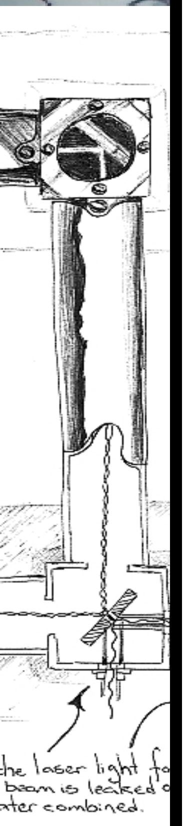


Ring laser – the principle

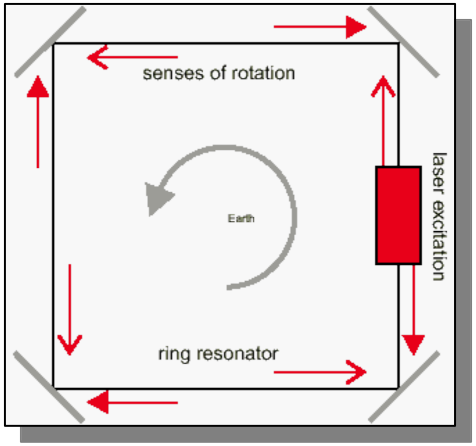


$$\Delta f_{Sagnac} = \frac{4\mathbf{\Omega} \cdot \mathbf{A}}{\lambda P}$$

A surface of the ring laser (vector)
Ω imposed rotation rate (Earth's rotation + earthquake +...)
λ laser wavelength (e.g. 633 nm)
P perimeter (e.g. 4-16m)
Δf Sagnac frequency (e.g. 287,3 Hz sampled at 800Hz)



Ring laser - resolution

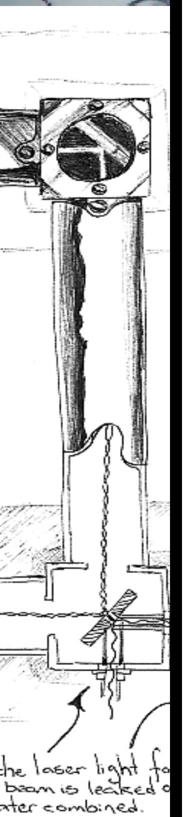


$$\Delta f_{Sagnac} = \frac{4\Omega \cdot \mathbf{A}}{\lambda P}$$

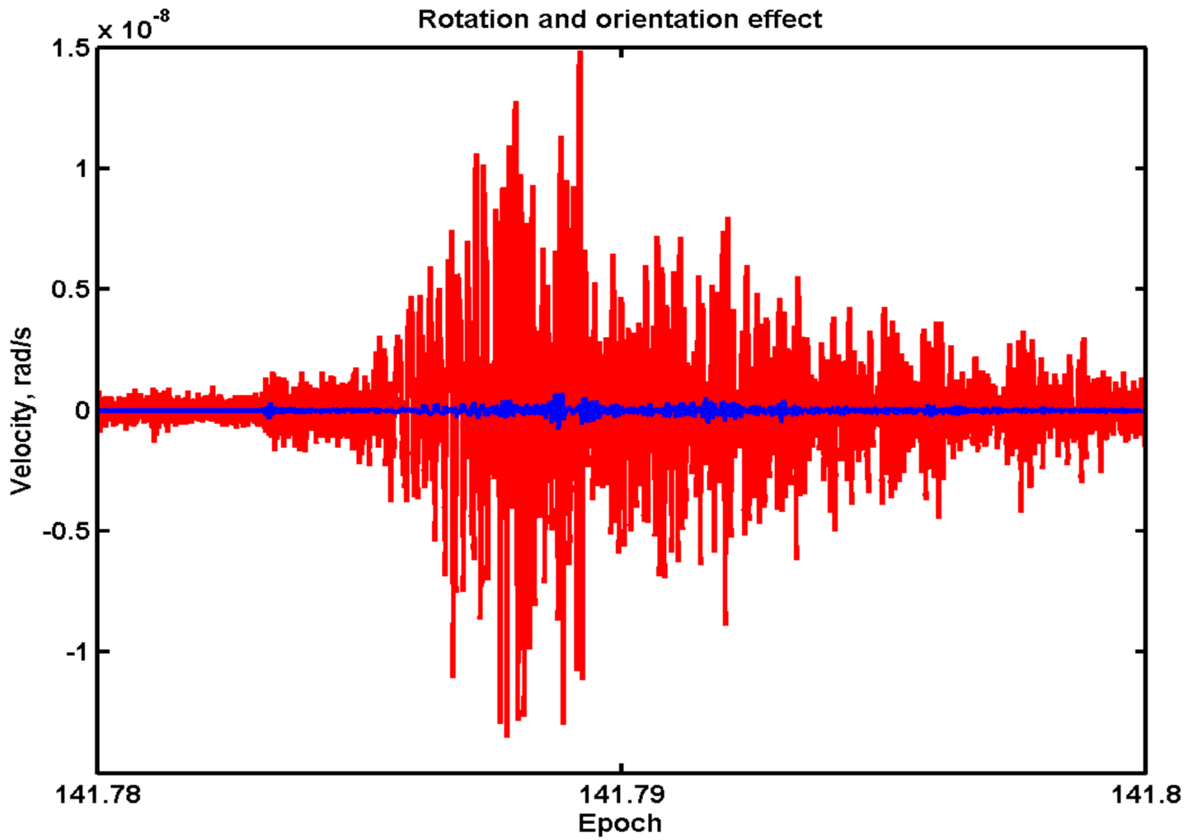
Area m^2	f_{Sagnac} (Hz)	Resolution rad/s
1	79.4	$4.8\ 10^{-10}$
16	348.6	$9.1\ 10^{-11}$
366	1512.8	$7.3\ 10^{-12}$

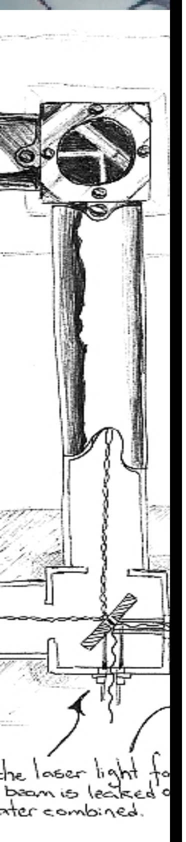
After Schreiber et al., 2002

... ring lasers are used in any commercial airplanes for stabilizing ...



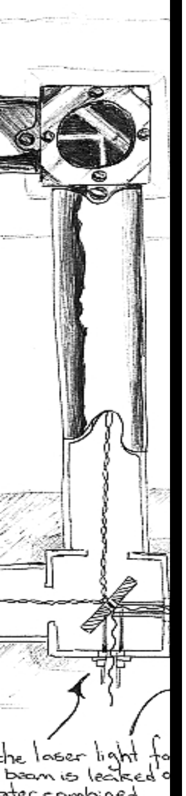
Effects of tilt on rotational measurements





... before presenting observations ...

- ... the ring laser should be sensitive to SH type motion only (S waves, Love waves) ...
- ... P-waves (or Rayleigh waves) should not lead to a signal (except via tilt coupling) ...
- ... **Rotation rate** and **transverse acceleration** should be in phase ...
- ... their amplitude ratio should be twice the local phase velocity – assuming plane non-dispersive transversely polarized wave propagation ...



Theoretical relation

rotation rate and transverse acceleration
plane-wave propagation

Plane transversely polarized wave propagating in x-direction with phase velocity

$$u_y(x,t) = f(kx - \omega t) \qquad c = \omega / k$$

Acceleration

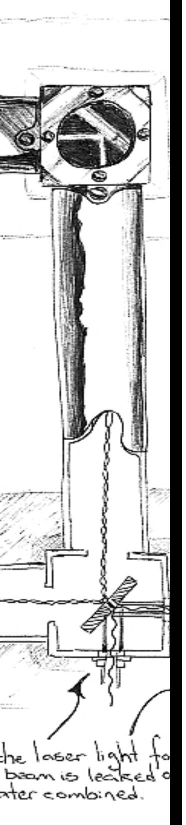
$$a_y(x,t) = \ddot{u}_y(x,t) = \omega^2 f''(kx - \omega t)$$

Rotation rate

$$\Omega(x,t) = \frac{1}{2} \nabla \times [0, \dot{u}_y, 0] = [0, -\frac{1}{2} k\omega f''(kx - \omega t), 0]$$

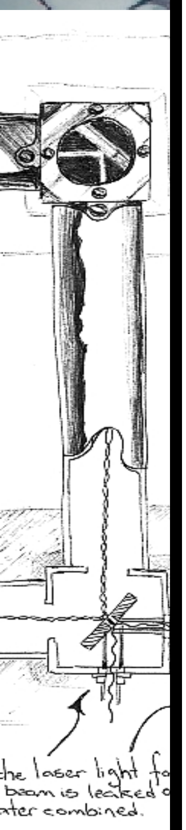
$$a(x,t) / \Omega(x,t) = -2c$$

Rotation rate and acceleration should be in phase and the amplitudes scaled by two times the horizontal phase velocity



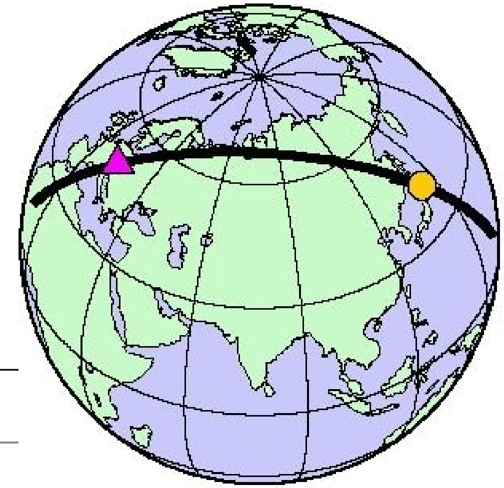
Data base 2003 + 2004

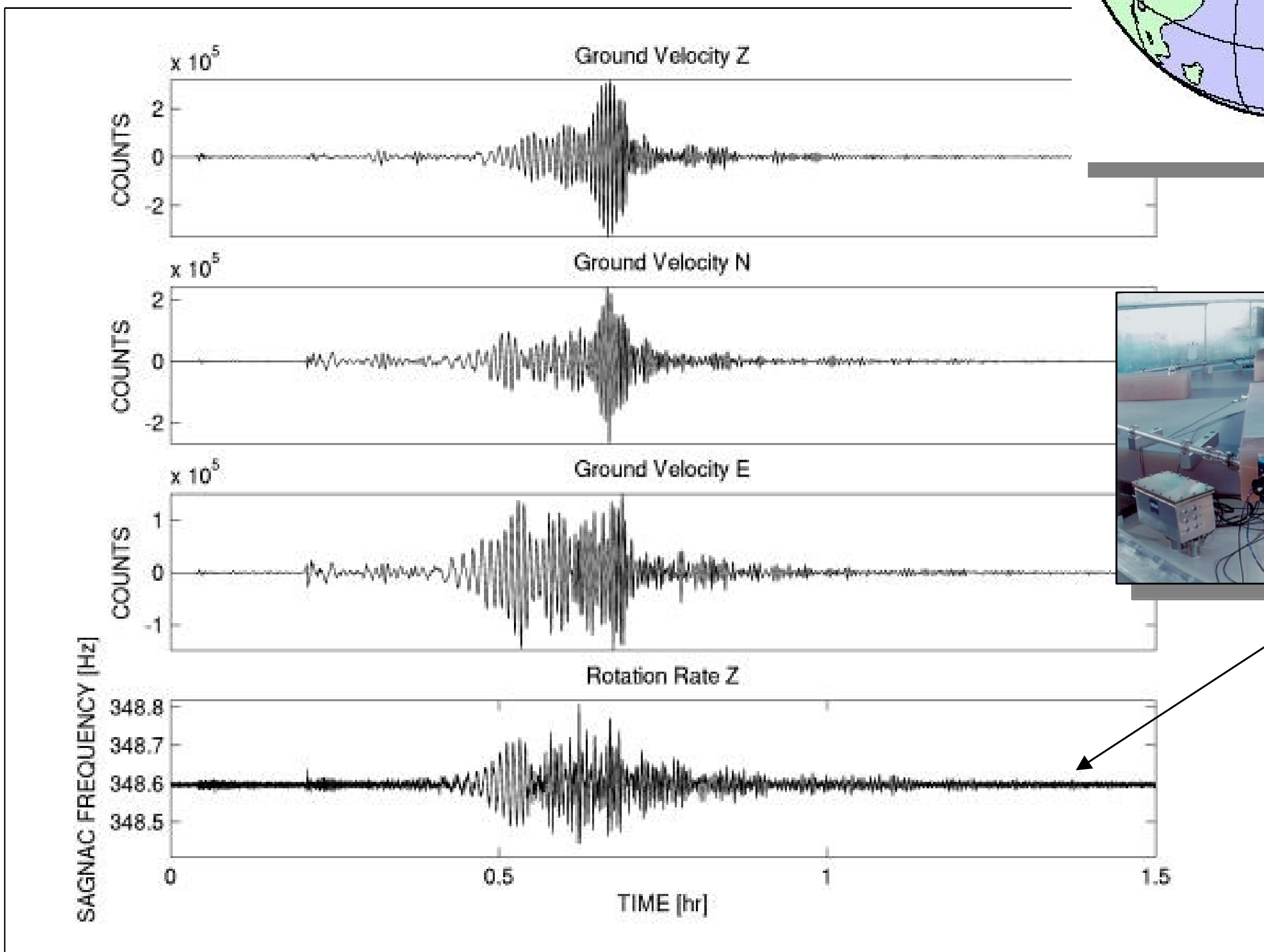
Date	Time (UTC)	Lat(°)	Lon (°)	Mag(L,b,S,w)	Region
21/05/03	18:44:20	36.964	003.634	6.9	Algeria
26/05/03	09:24:33	38.849	141.568	7.0	Honshu
06/07/03	19:10:33	40.340	026.070	5.7	Turkey
14/08/03	05:14:55	39.193	020.741	6.3	Greece
25/09/03	19:50:06	41.781	143.903	8.3	Hokkaido
27/09/03	11:33:24	50.012	087.824	7.5	Siberia
27/09/03	18:52:53	50.060	087.690	6.6	Siberia
01/10/03	01:03:25	50.218	087.685	7.1	Siberia
08/10/03	09:07:01	42.480	144.820	6.7	Hokkaido
31/10/03	01:06:40	37.890	142.680	7.0	Honshu
17/11/03	06:43:31	51.140	177.860	7.8	Rat Island
26/12/03	01:56:58	29.100	058.240	6.8	Iran
05/02/04	21:05:12	-03.620	135.530	7.1	Irian Jaya
07/02/04	02:42:43	-04.030	134.780	7.5	Irian Jaya
24/02/04	02:27:53	35.290	-003.840	6.4	Gibraltar
17/03/04	03:21:12	-21.100	-065.560	6.1	Bolivia
05/04/04	21:24:06	36.590	070.850	6.6	Afghanistan
28/05/04	12:38:50	36.520	051.810	6.4	Iran
29/05/04	20:56:14	34.220	141.790	6.6	Honshu
05/12/04	01:52:37	48.120	008.080	5.0	Germany
26/12/04	00:58:53	03.300	095.980	9.0	Sumatra

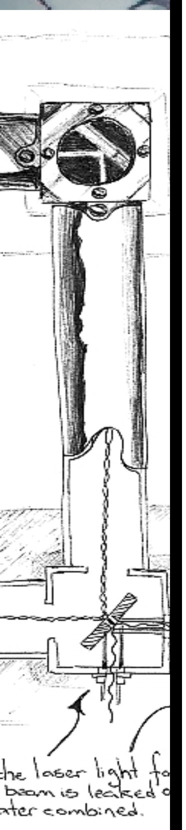


4C recordings – raw data

Mw = 8.3 Tokachi-oki earthquake
25.09.2003 19:50:38.2 GMT
Lat= 42.21 Lon= 143.84



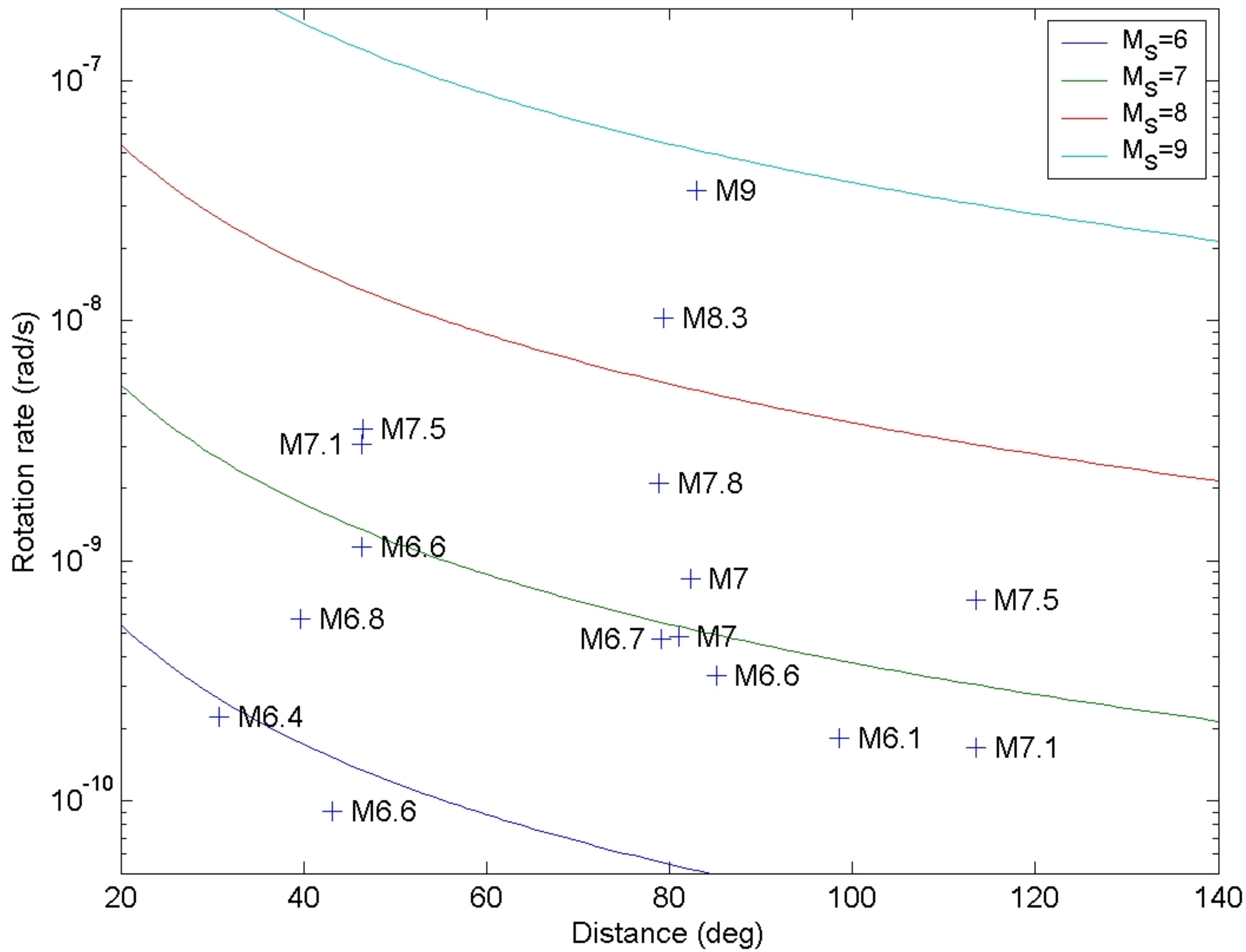




Compatibility with MS (surface wave magnitude) T=30s, c=4300m/s

$$M_S = \log_{10}\frac{A}{T} + 1.66\log_{10} D + 3.3$$

$$\Omega_z = 2\frac{\pi^2}{cT^2}A(M_S, D) = 2\frac{\pi^2}{cT}10^{M_S - 1.66\log_{10} D - 9.3}$$

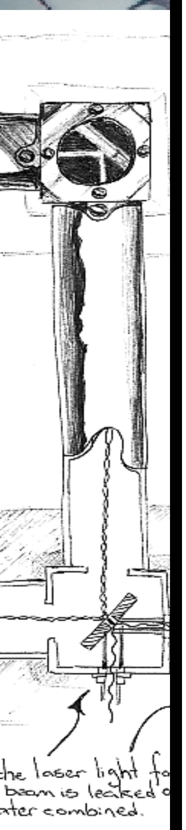


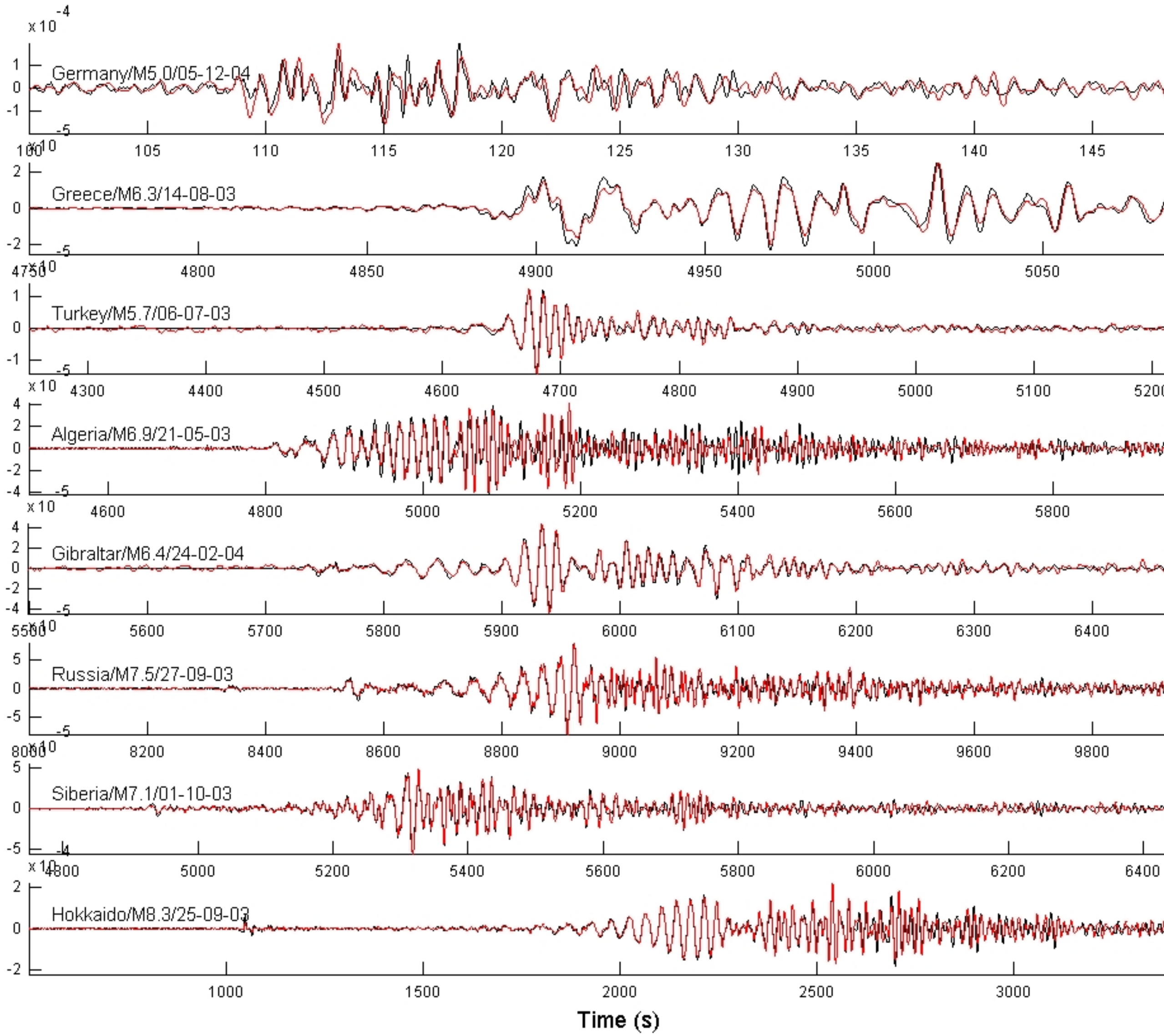
Rotational data base

events with varying distance

transverse acceleration – rotation rate

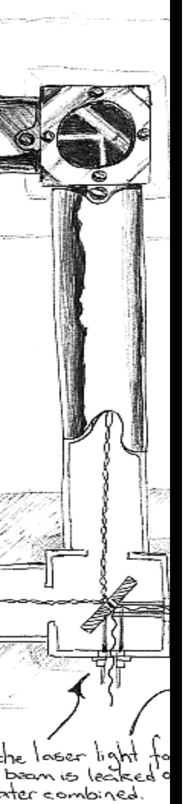
increasing epicentral distance





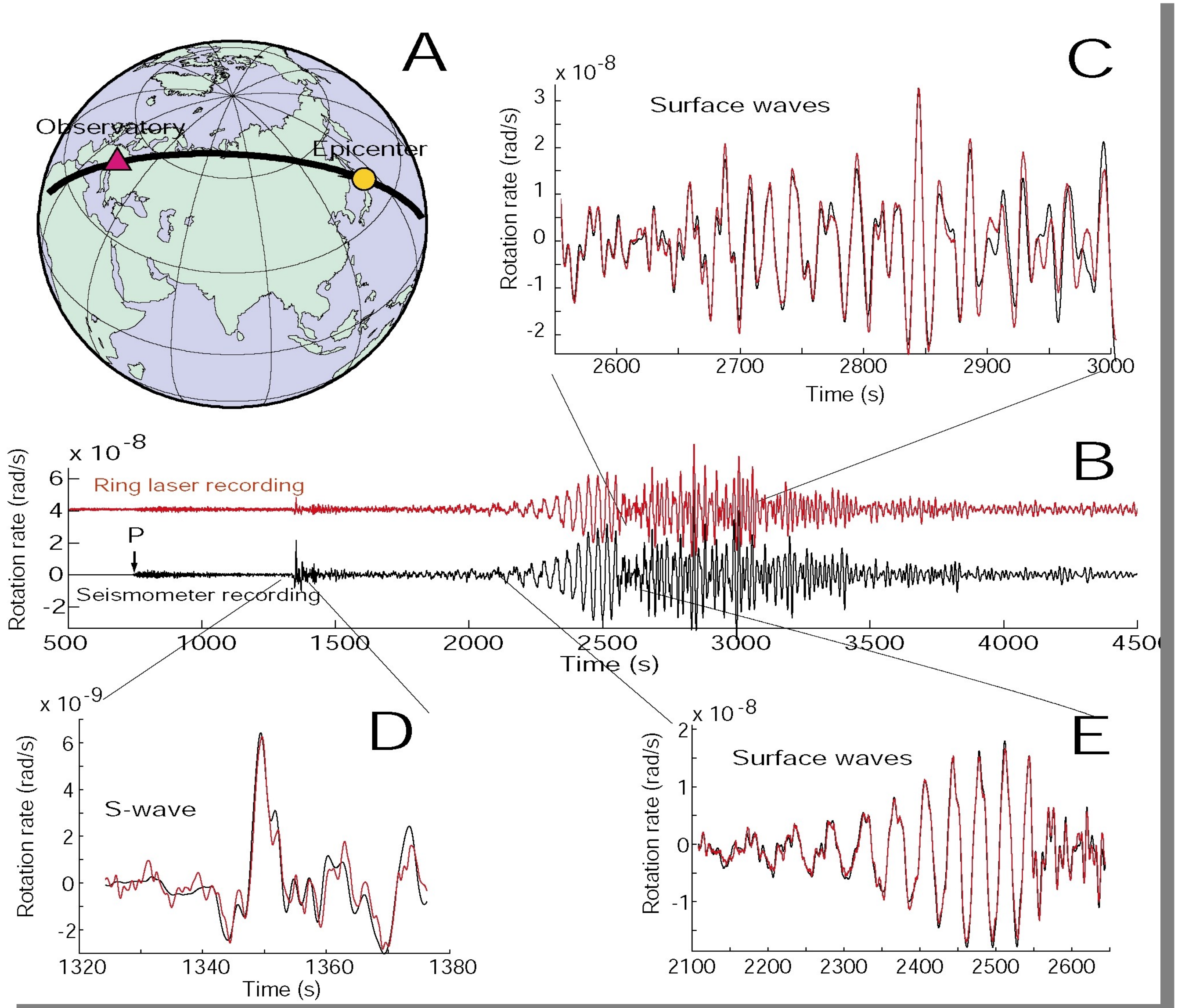
400km
M5.0

10000km
M8.3



Mw = 8.3 Tokachi-oki 25.09.2003

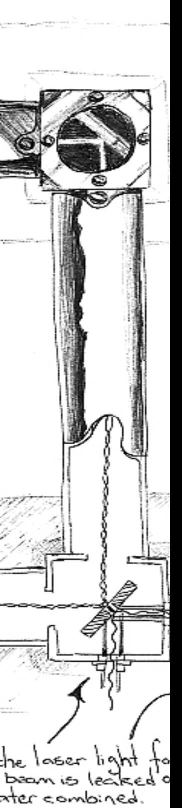
transverse acceleration – rotation rate

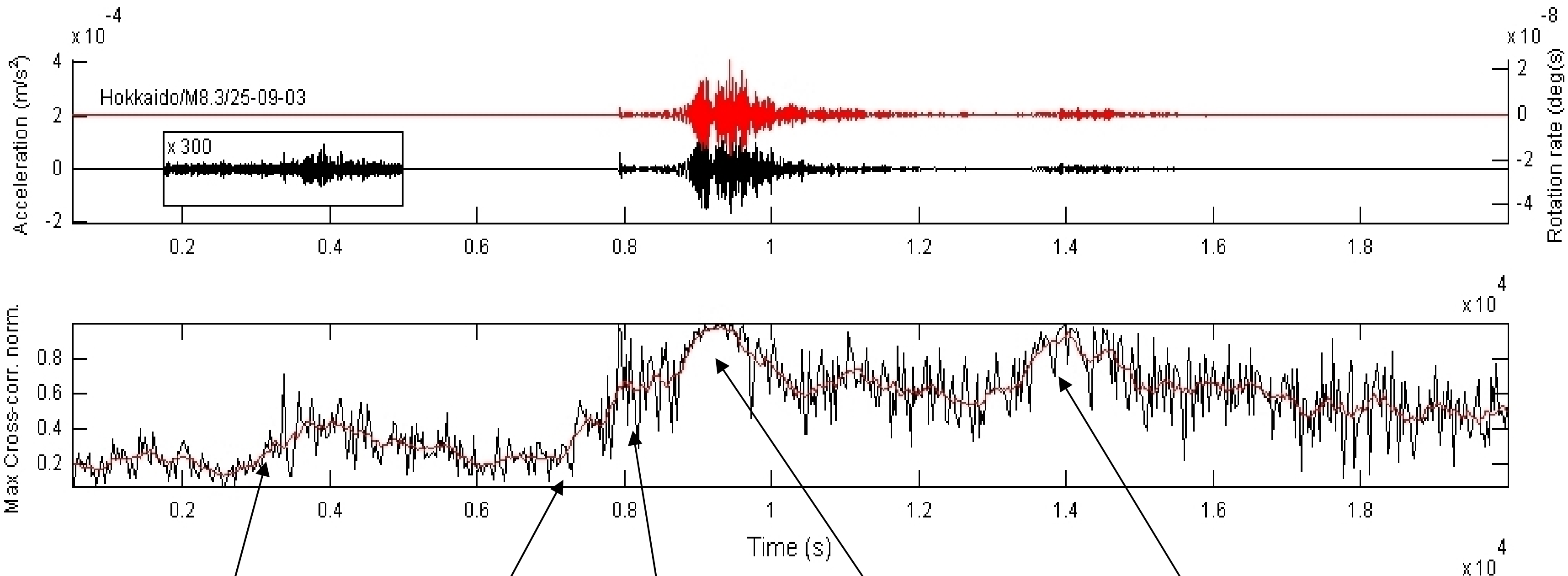


From Igel et al., GRL, 2005

Max. cross-corr. coefficient in sliding time window

transverse acceleration – rotation rate





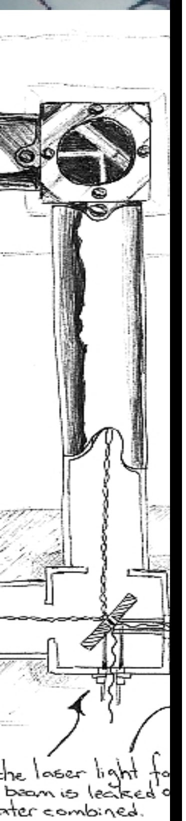
P-onset

Love waves

Aftershock

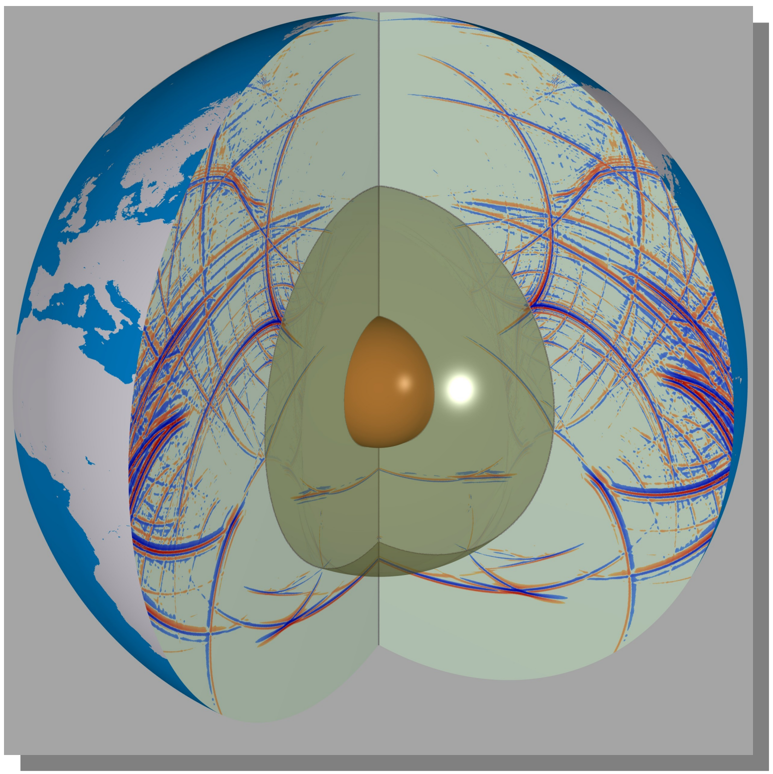
S-wave

Small tele-seismic event



Spectral element modeling of 3D global wave propagation

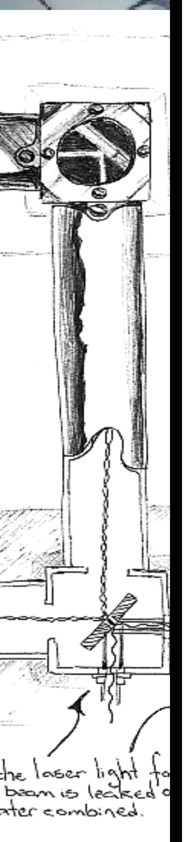
Cubed Sphere



Chunk Partitioning

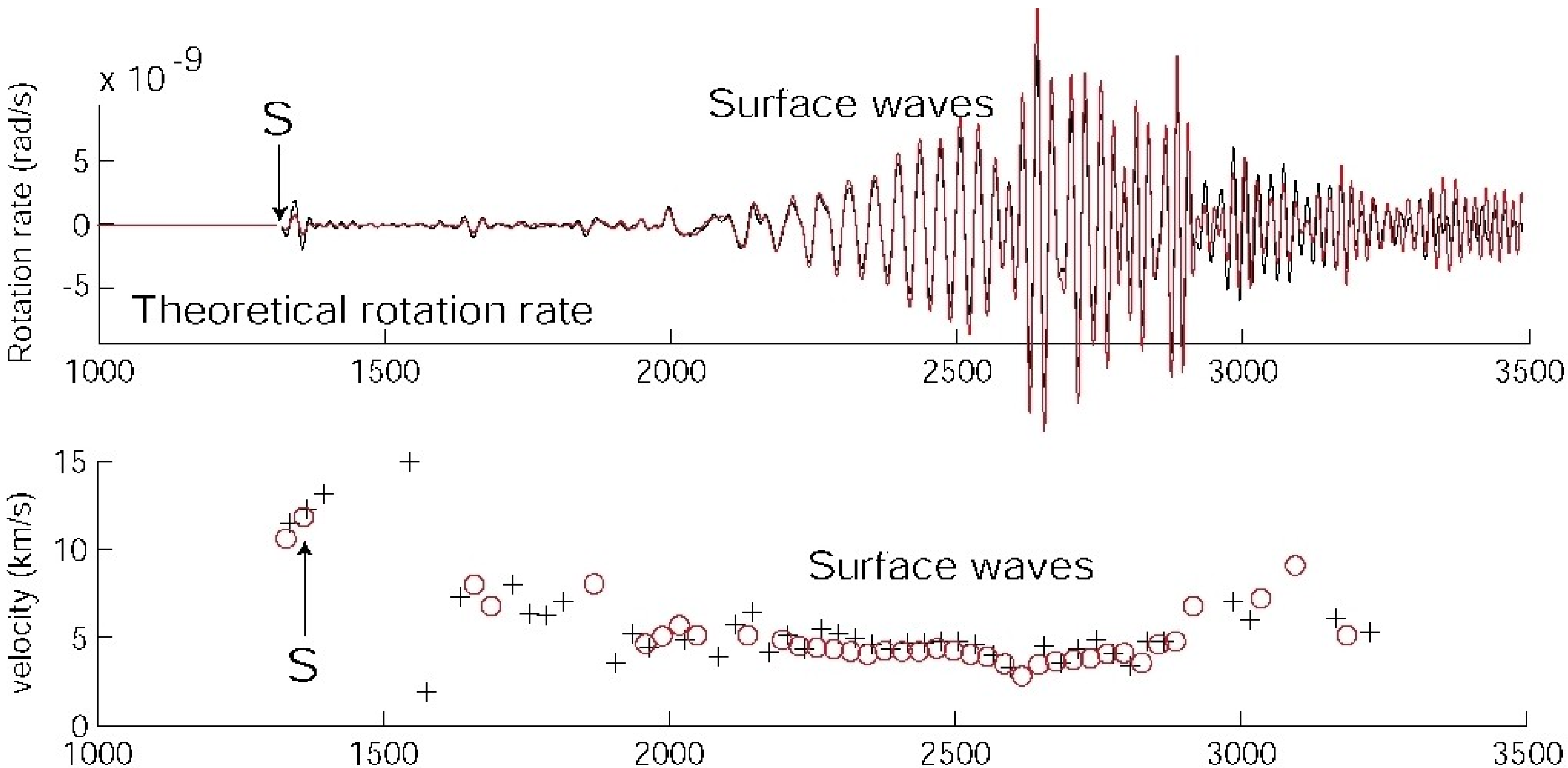
100GByte Memory
60 hours on the
Bundeshöchstleistungsrechner

Tromp and Komatitsch, 2003



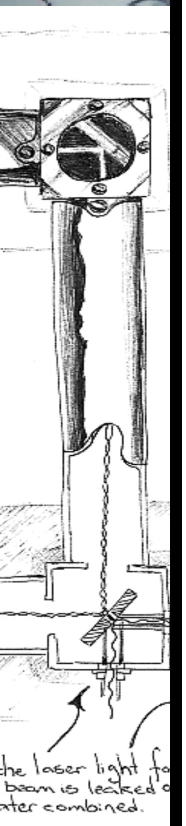
M8.3 Tokachi-oki, 25 September 2003

phase velocities (+ observations, o theory)

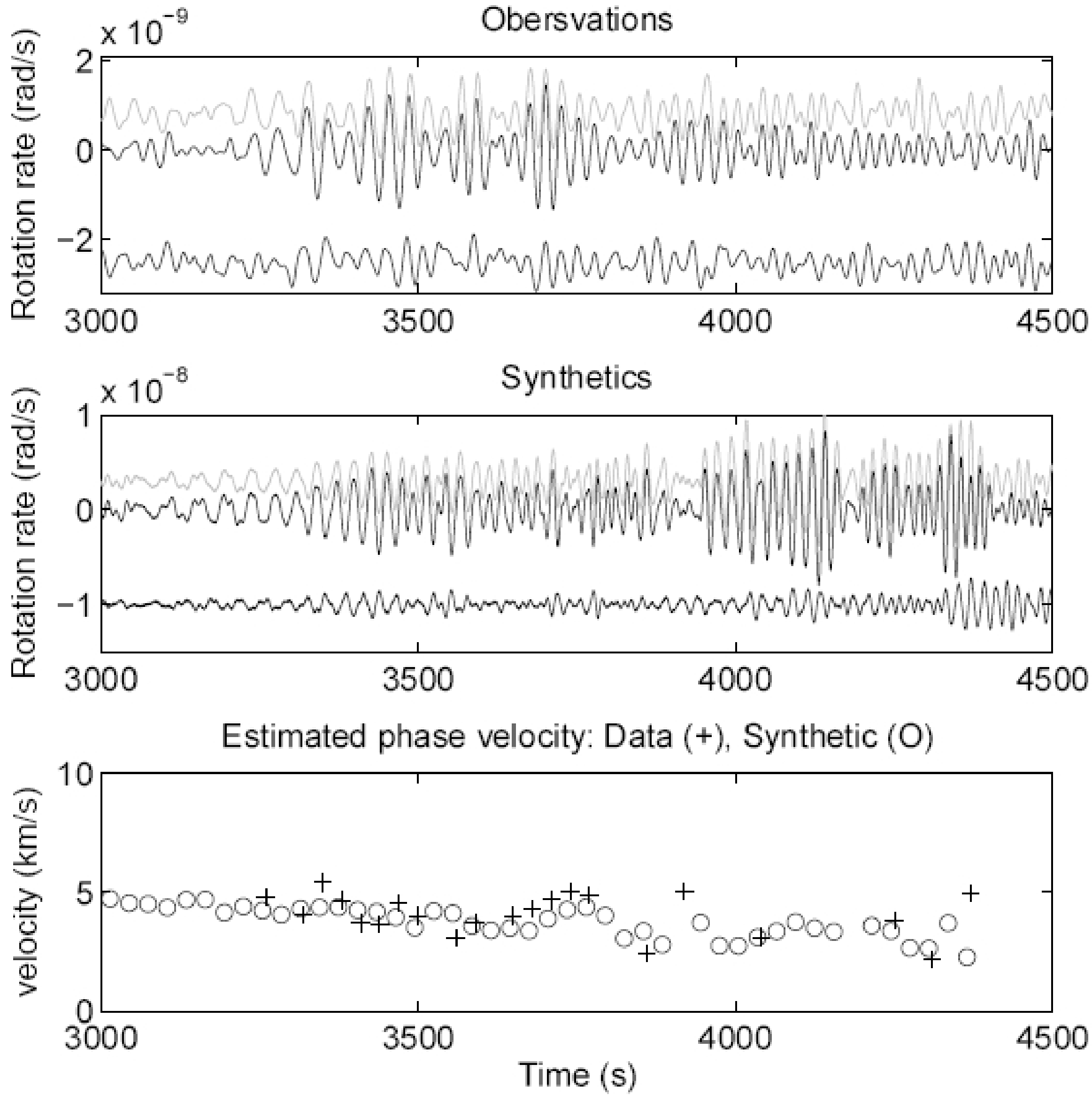


Horizontal phase velocity in sliding time window

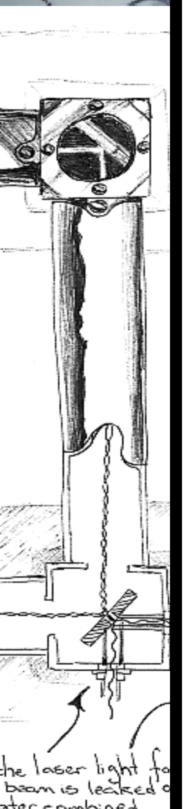
From Igel et al. (GRL, 2005)



Real vs. Synthetics : Papua event



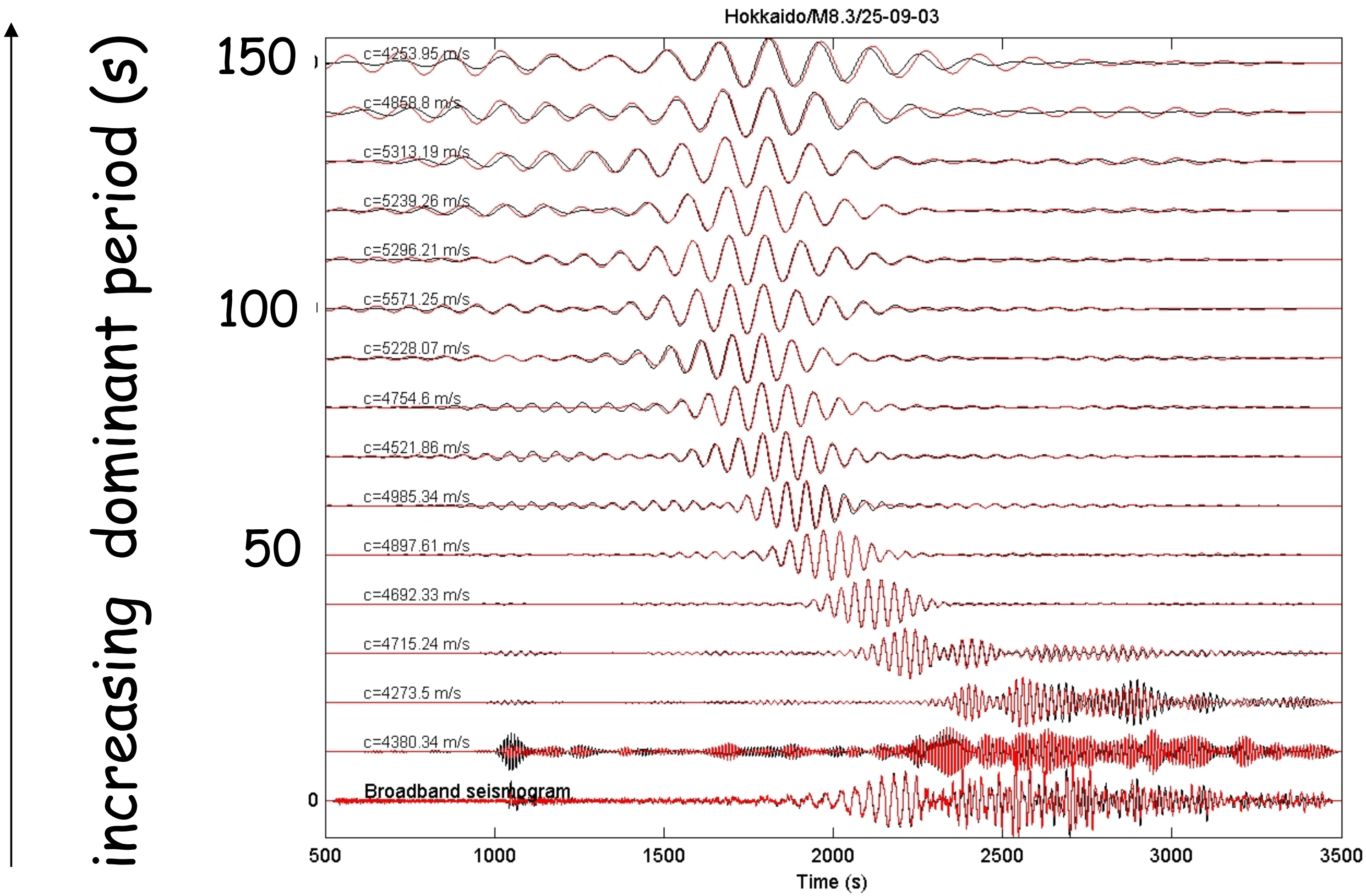
Cochard et al., 2006

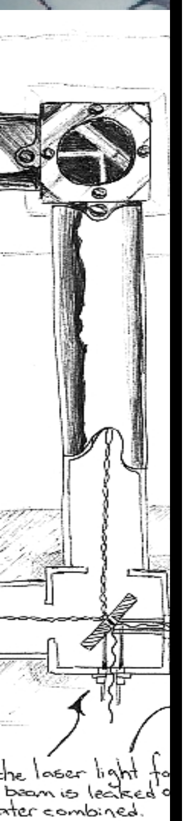


Mw = 8.3 Tokachi-oki 25.09.2003

transverse acceleration – rotation rate

narrow band-pass filtering



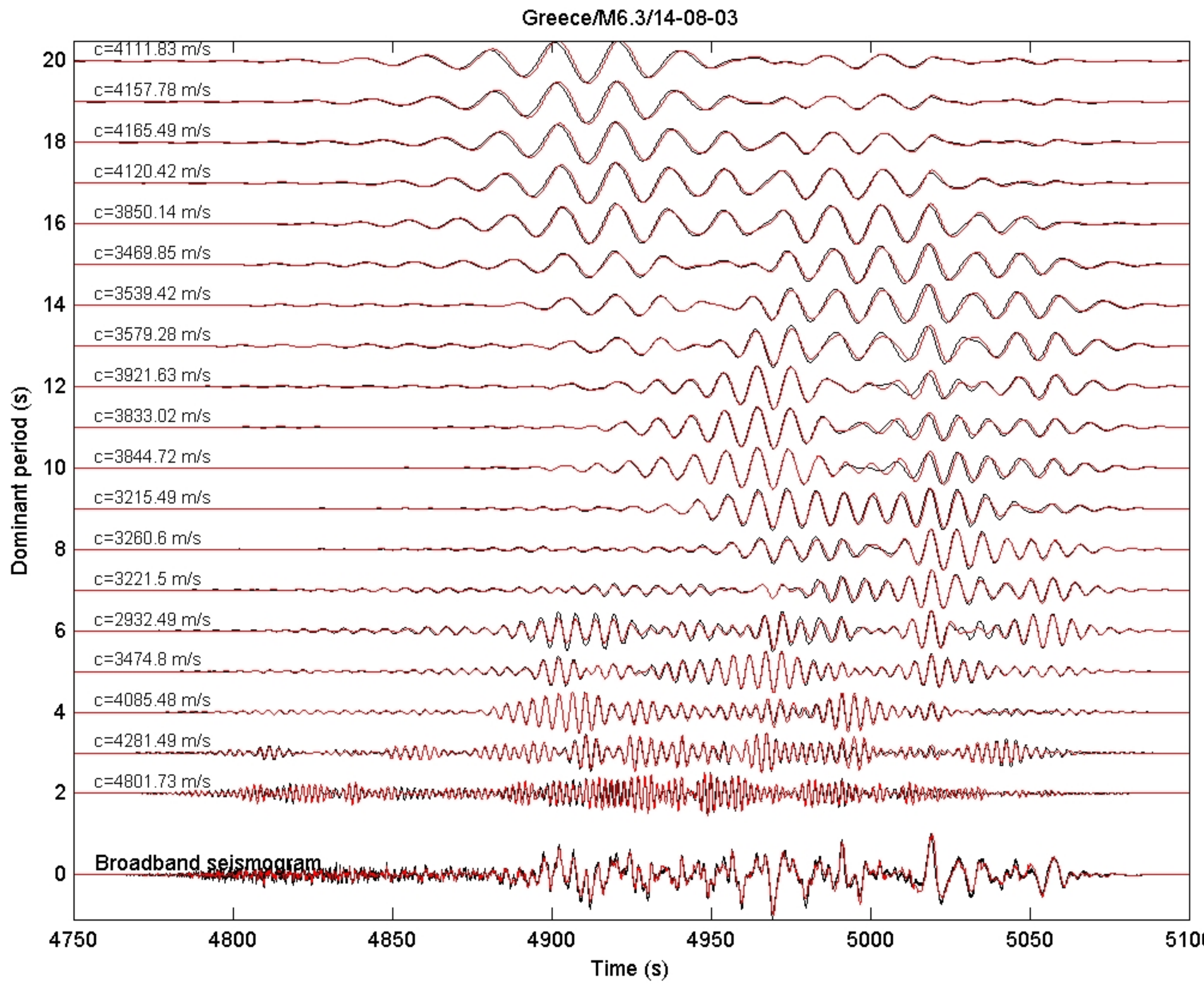


Mw = 6.3 Greece 14.08.2003

transverse acceleration – rotation rate

narrow band-pass filtering

increasing dominant period (s)

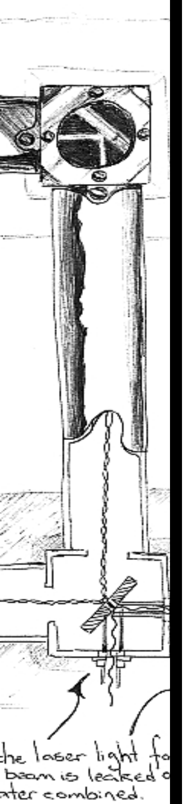


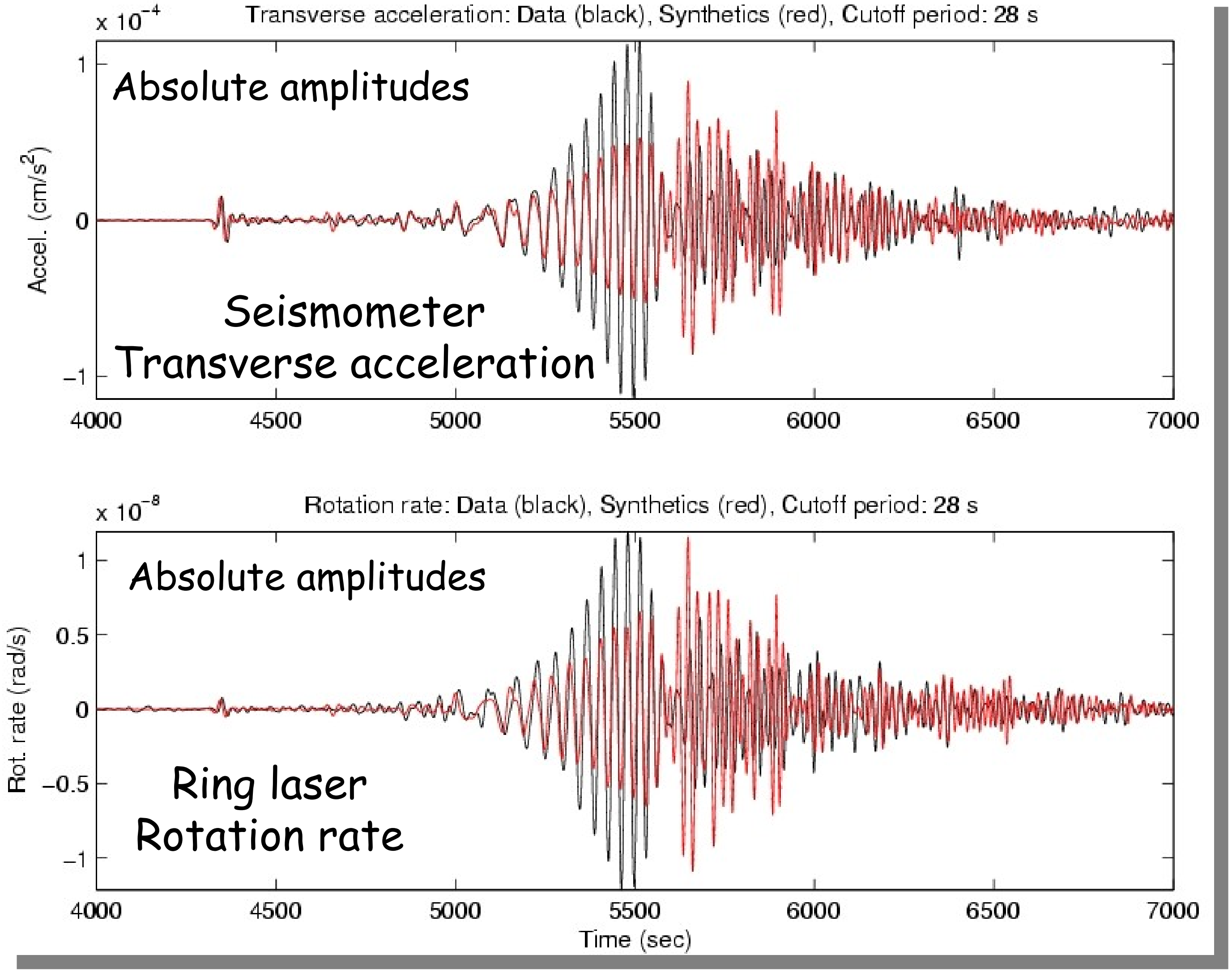
Rotational seismograms

Synthetics and Observations

M8.3 Hokkaido, 25 September 2003

(recorded in Wettzell, Germany)

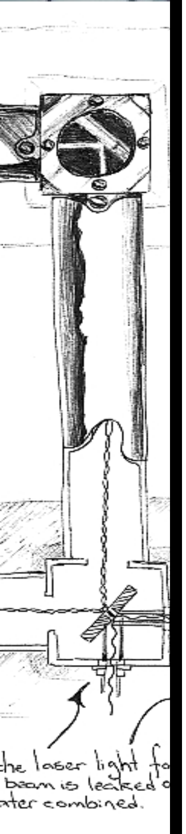


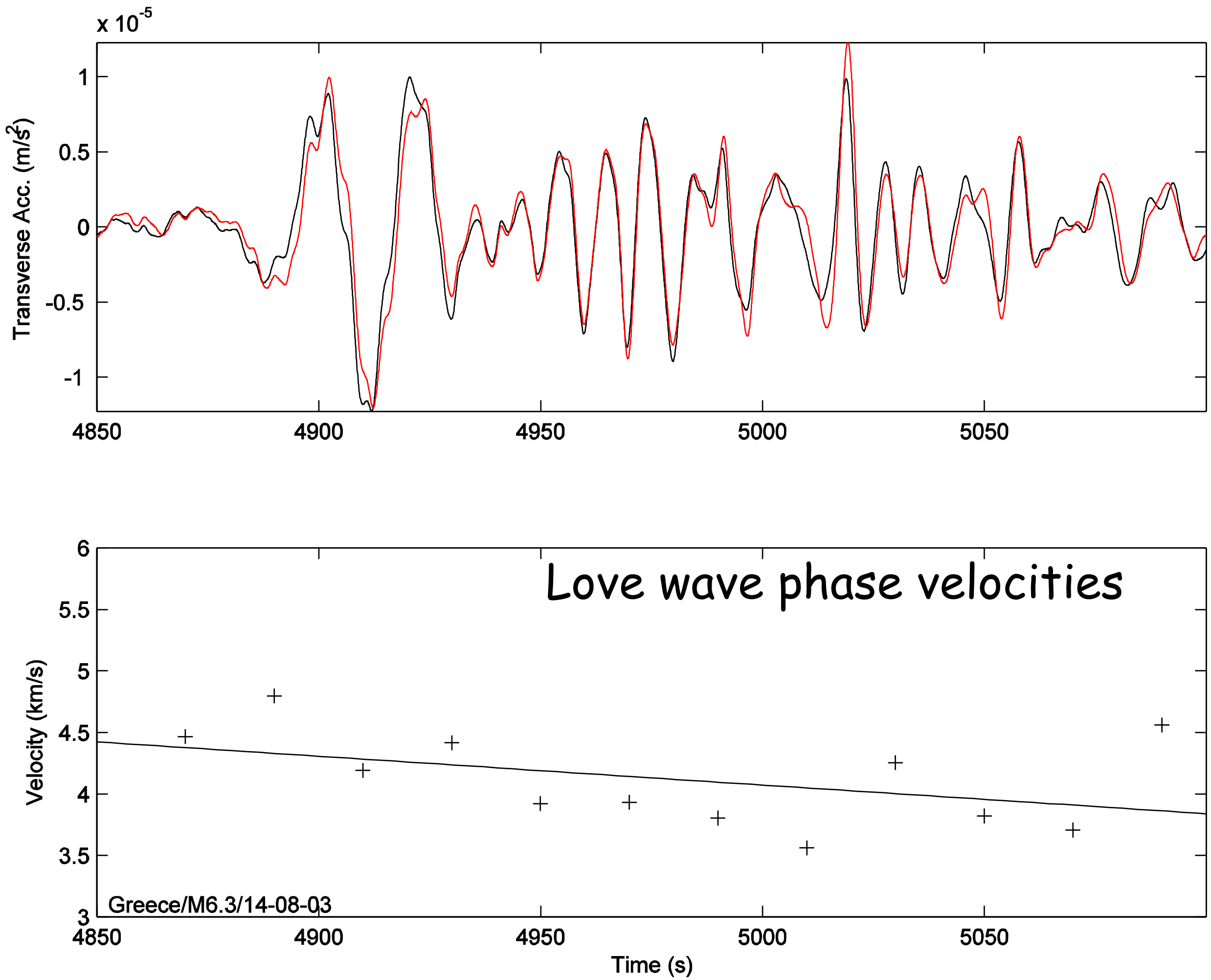


Phase velocity determination

... by dividing accelerations by rotation rates in a sliding window ...

. . . point measurement!



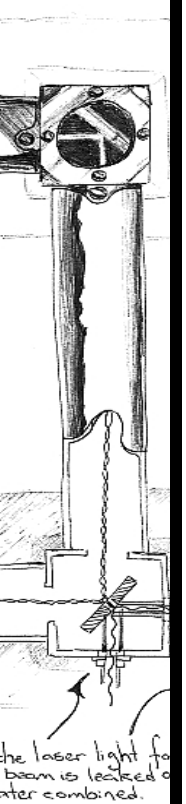


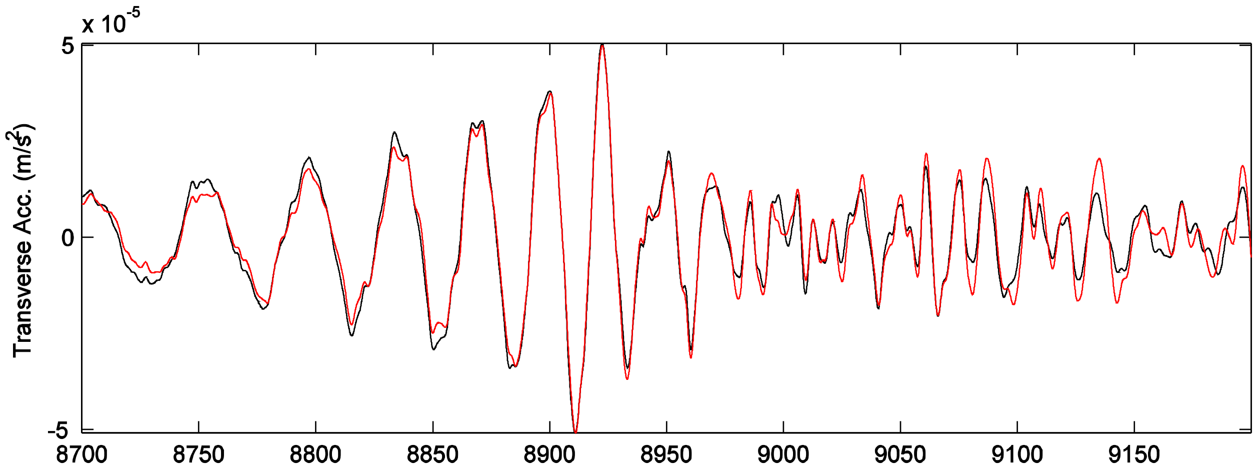
Note the decreasing velocities with time (and increasing frequency)

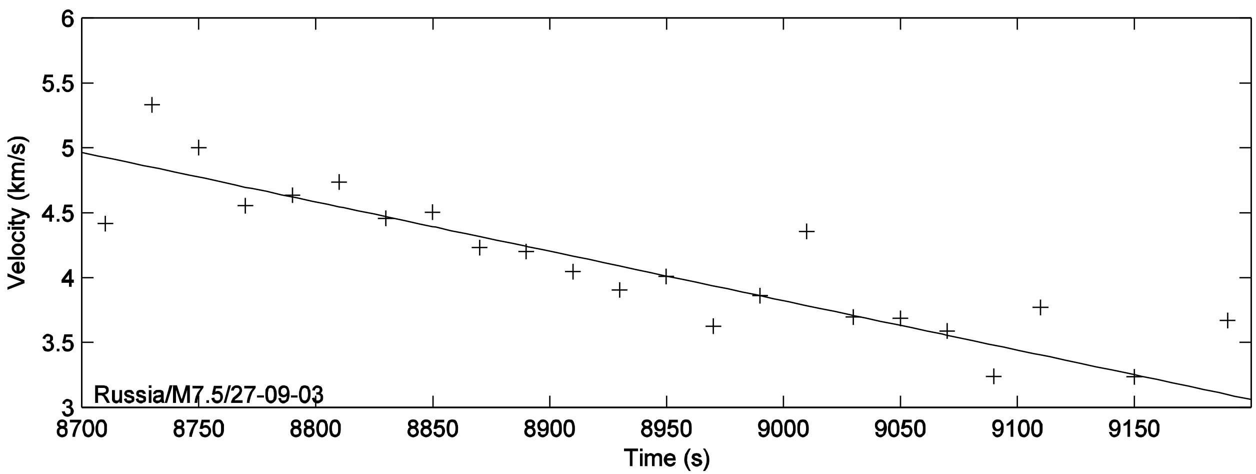
Phase velocity determination

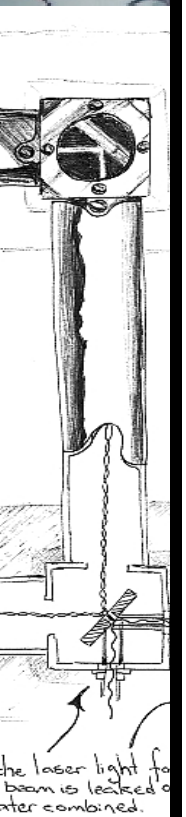
... by dividing accelerations by rotation rates in a sliding window ...

... point measurement!





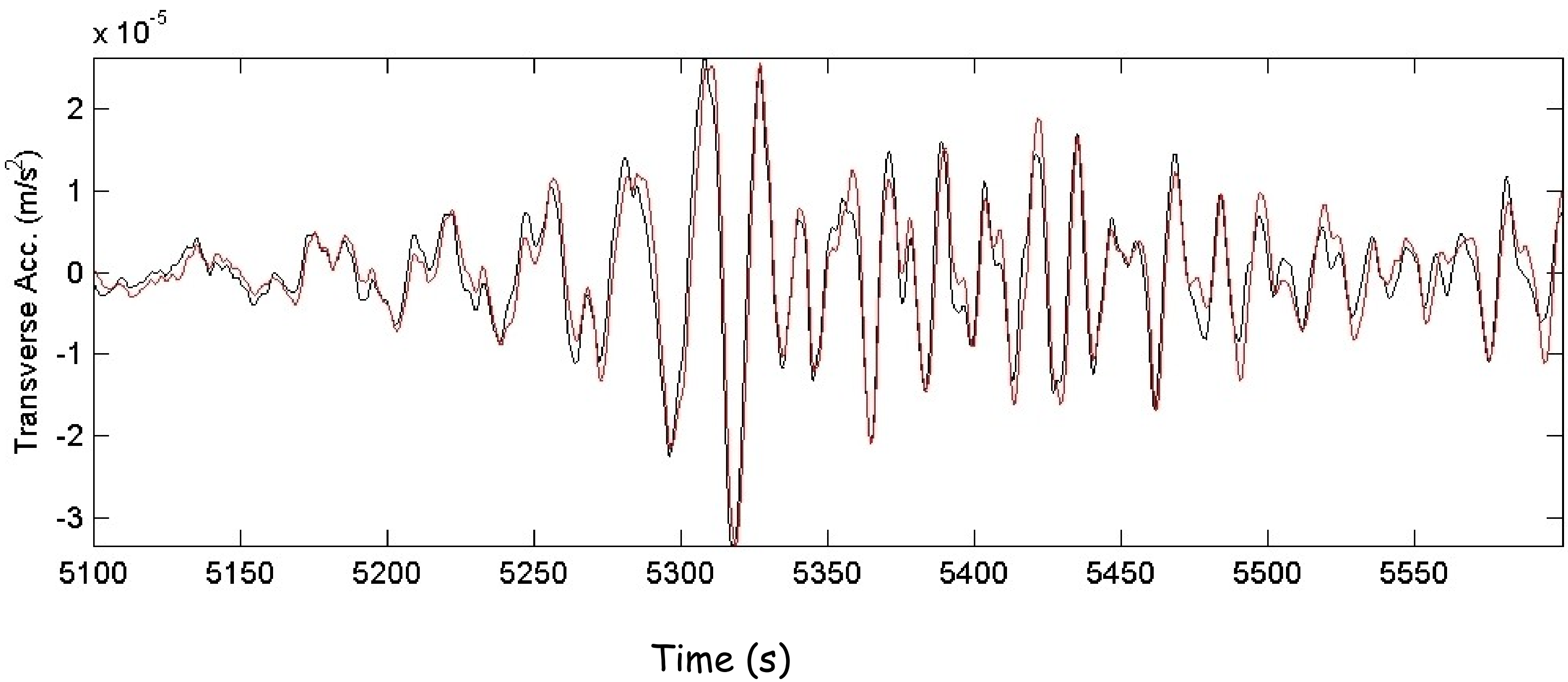




Restitute your broadband seismograms!

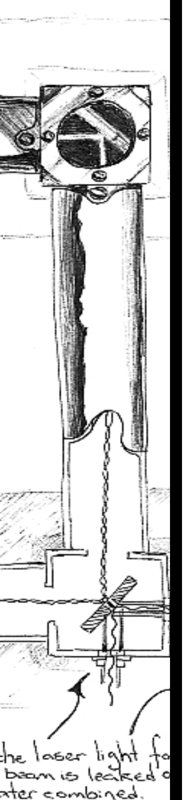
transverse acceleration – rotation rate

Before restitution

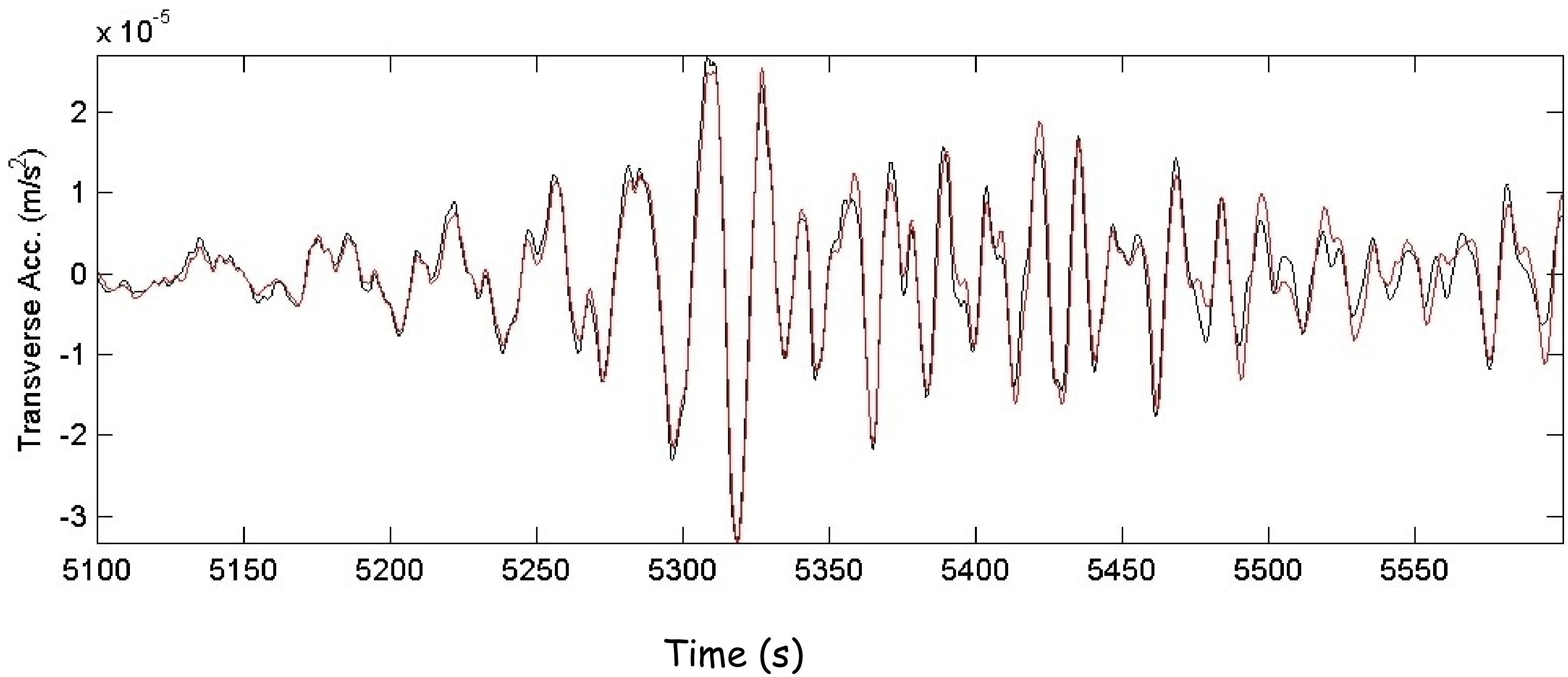


Restitute your broadband seismograms!

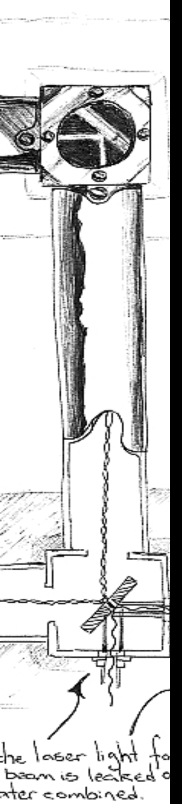
transverse acceleration – rotation rate



After restitution

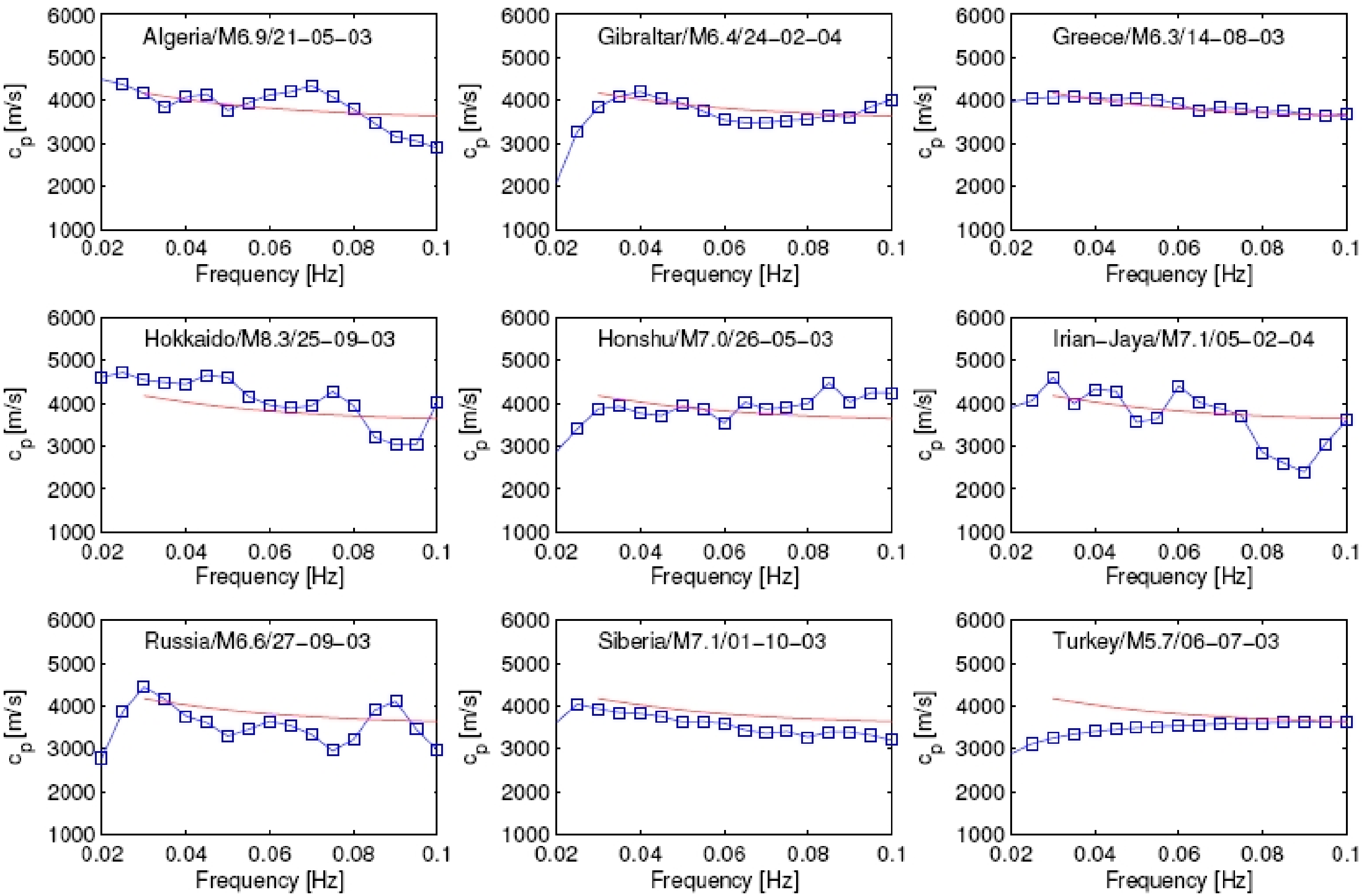


... an independent confirmation of the quality of the restitution processing ...



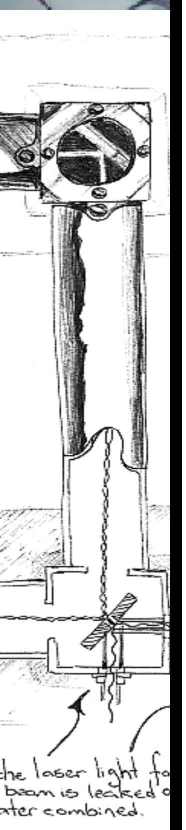
Phase velocity determination

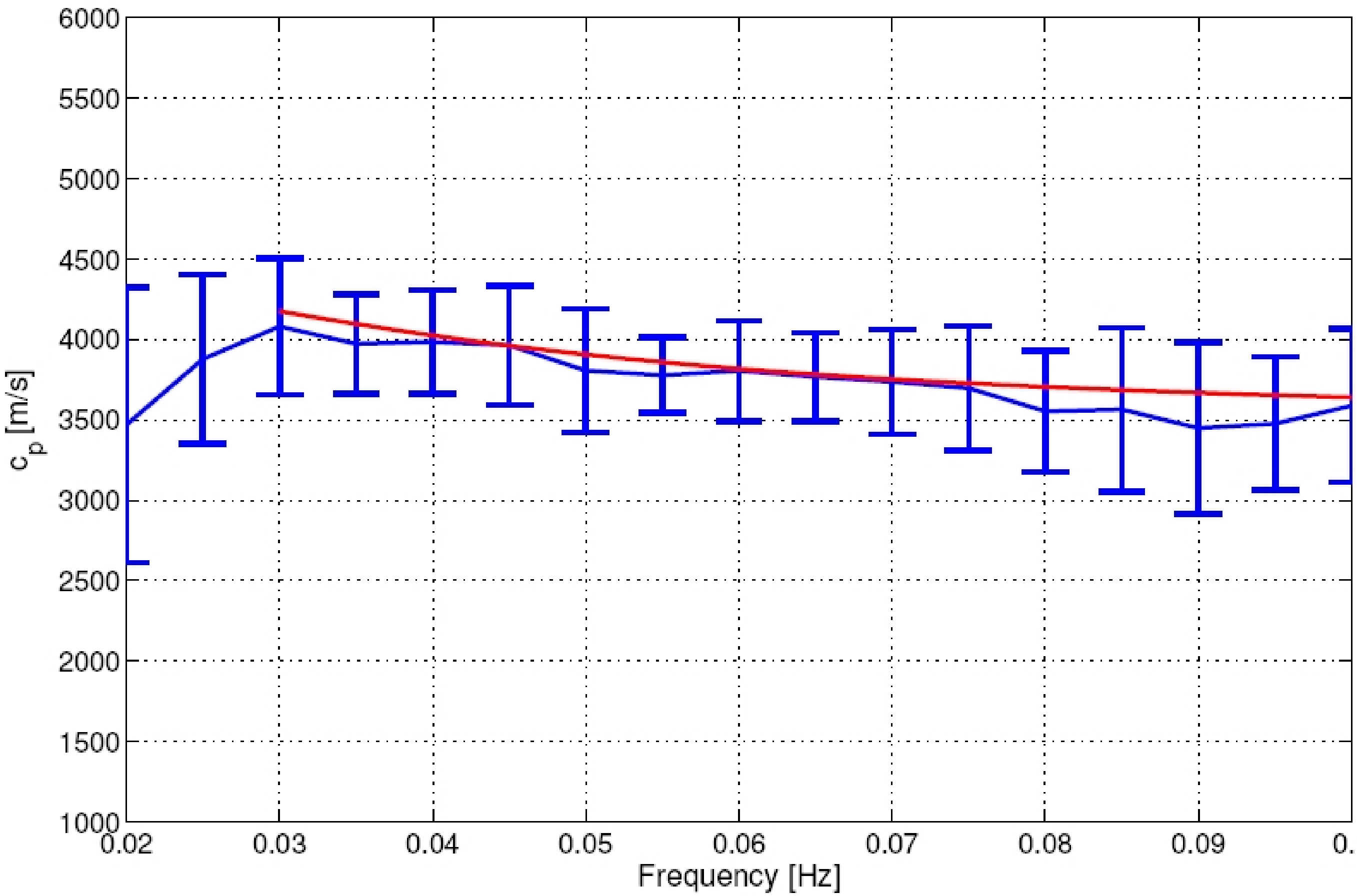
... by calculating spectral ratios ...



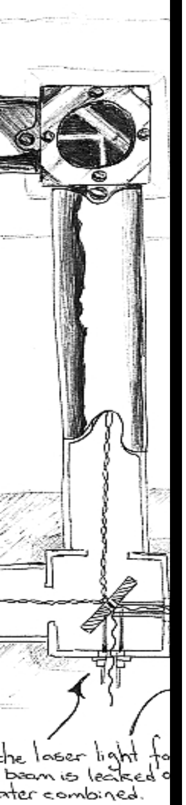
Stacked spectral ratios ...

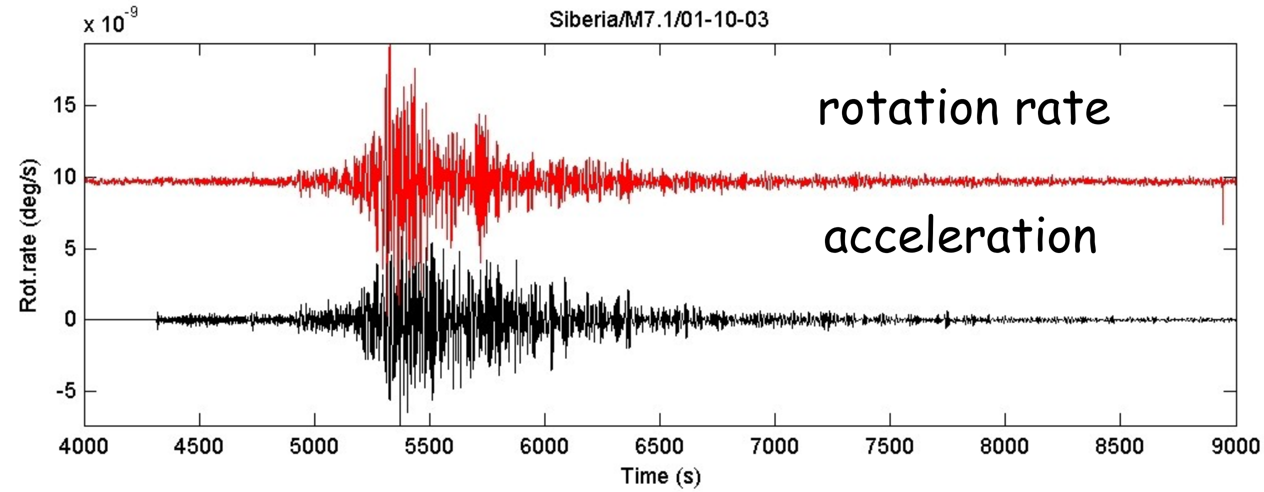
... accurate enough for structural inversion ...?

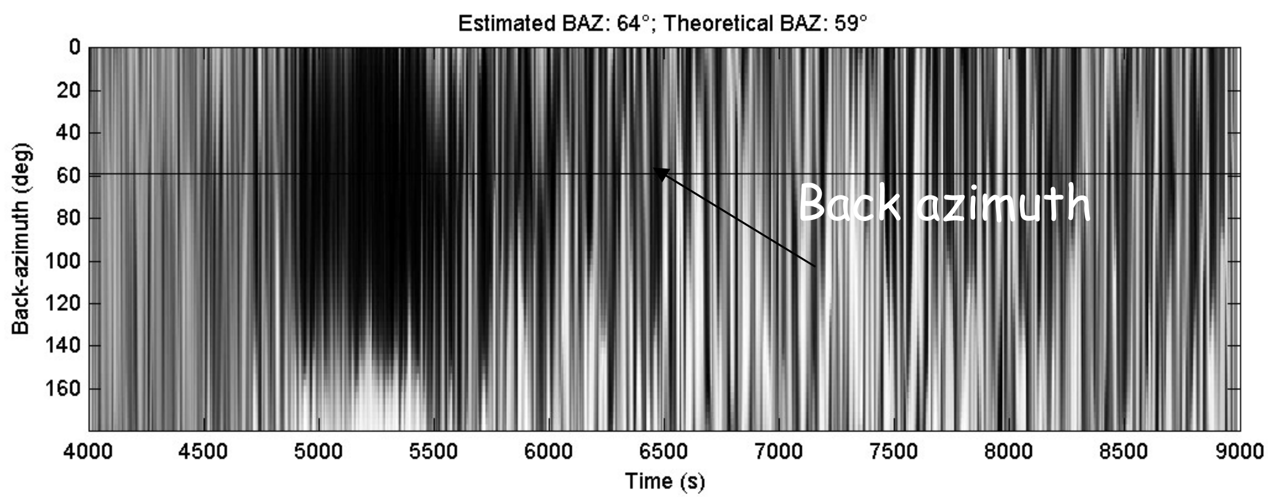




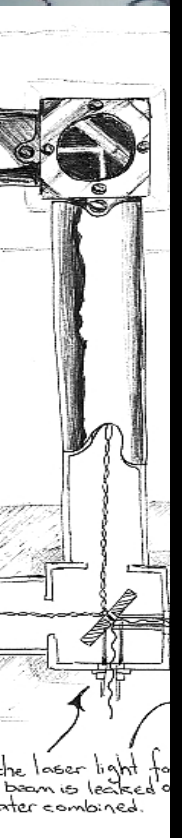
Direction of propagation of transversely polarized energy



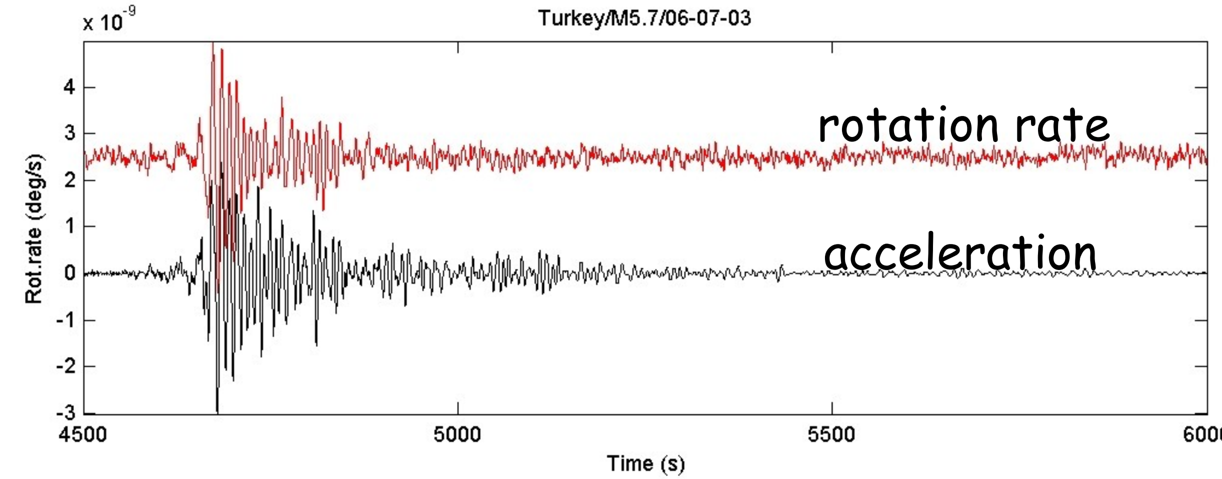


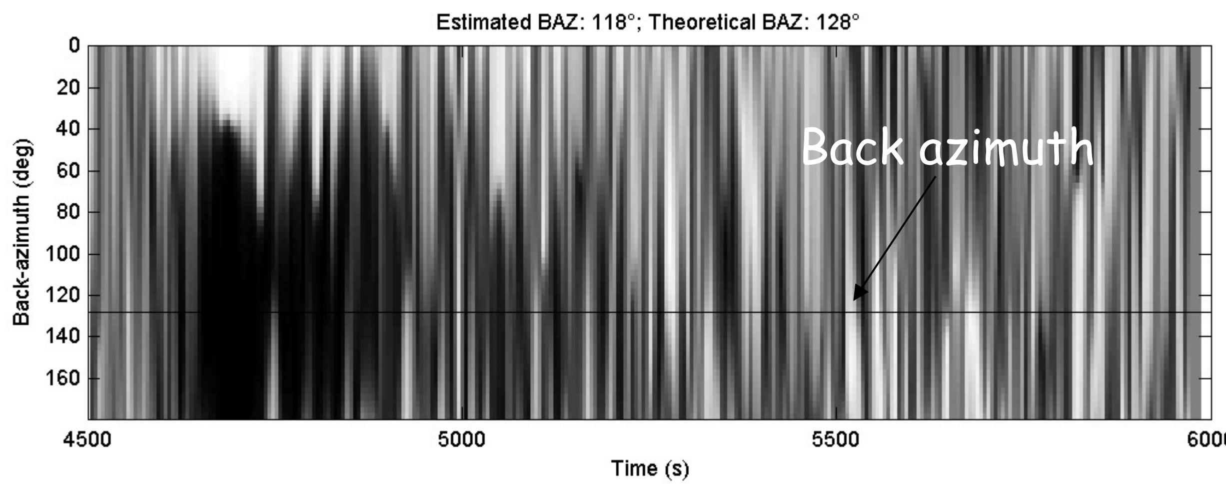


Max. cross-corr. coeff. as a function of time and propagation direction

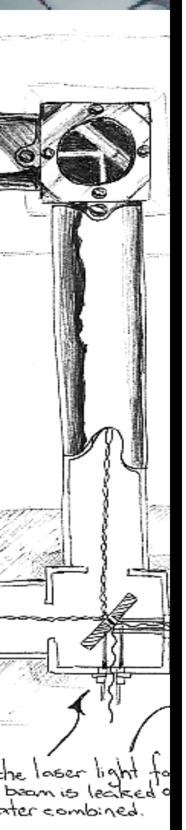


Direction of propagation of transversely polarized energy

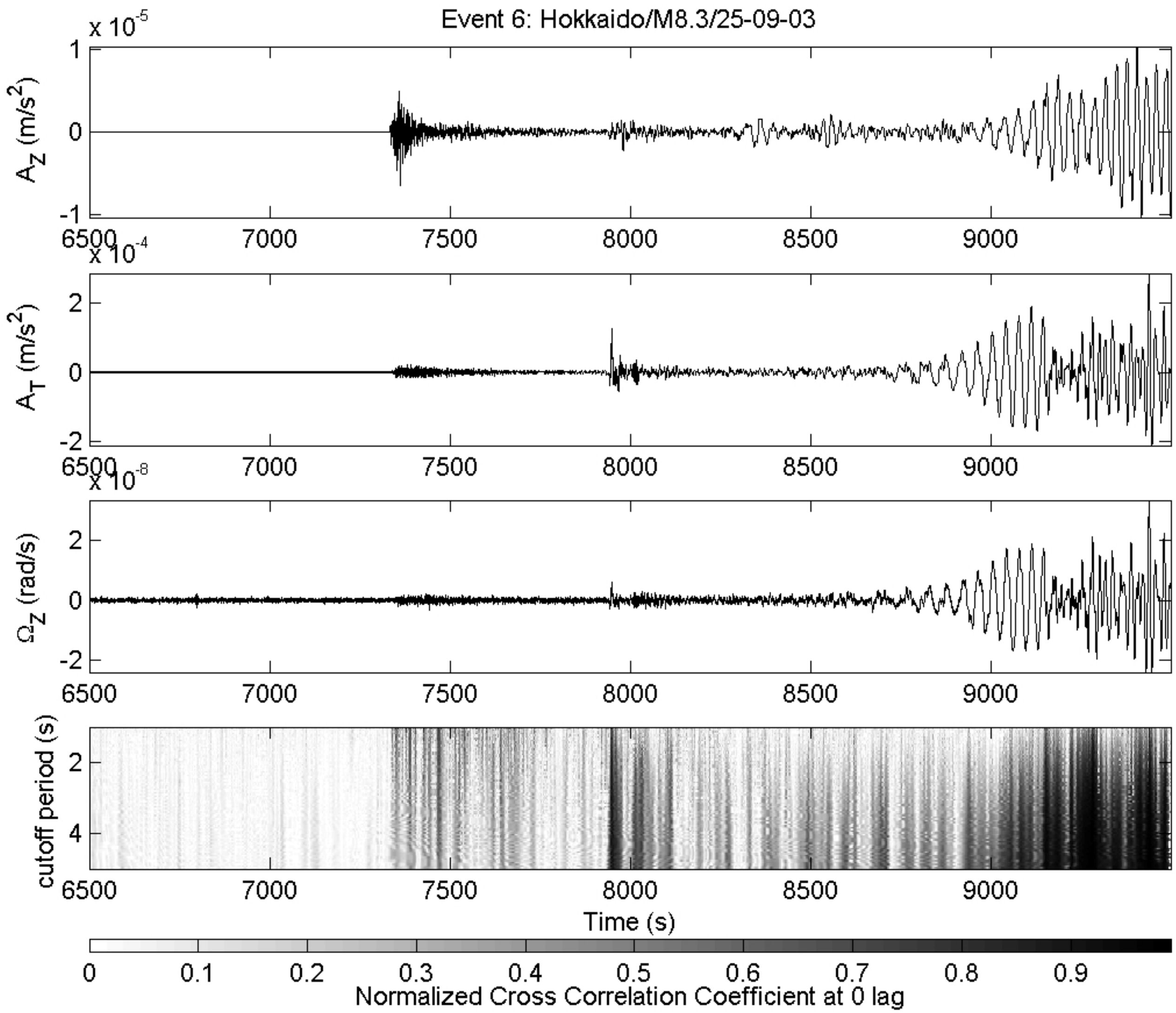




Max. cross-corr. coeff. as a function of time and propagation direction

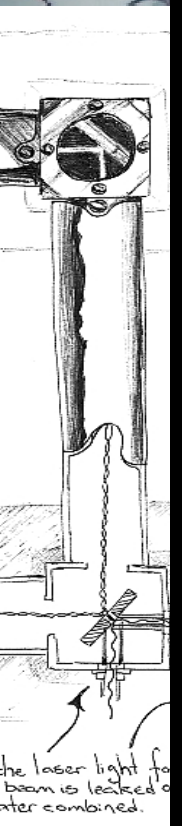


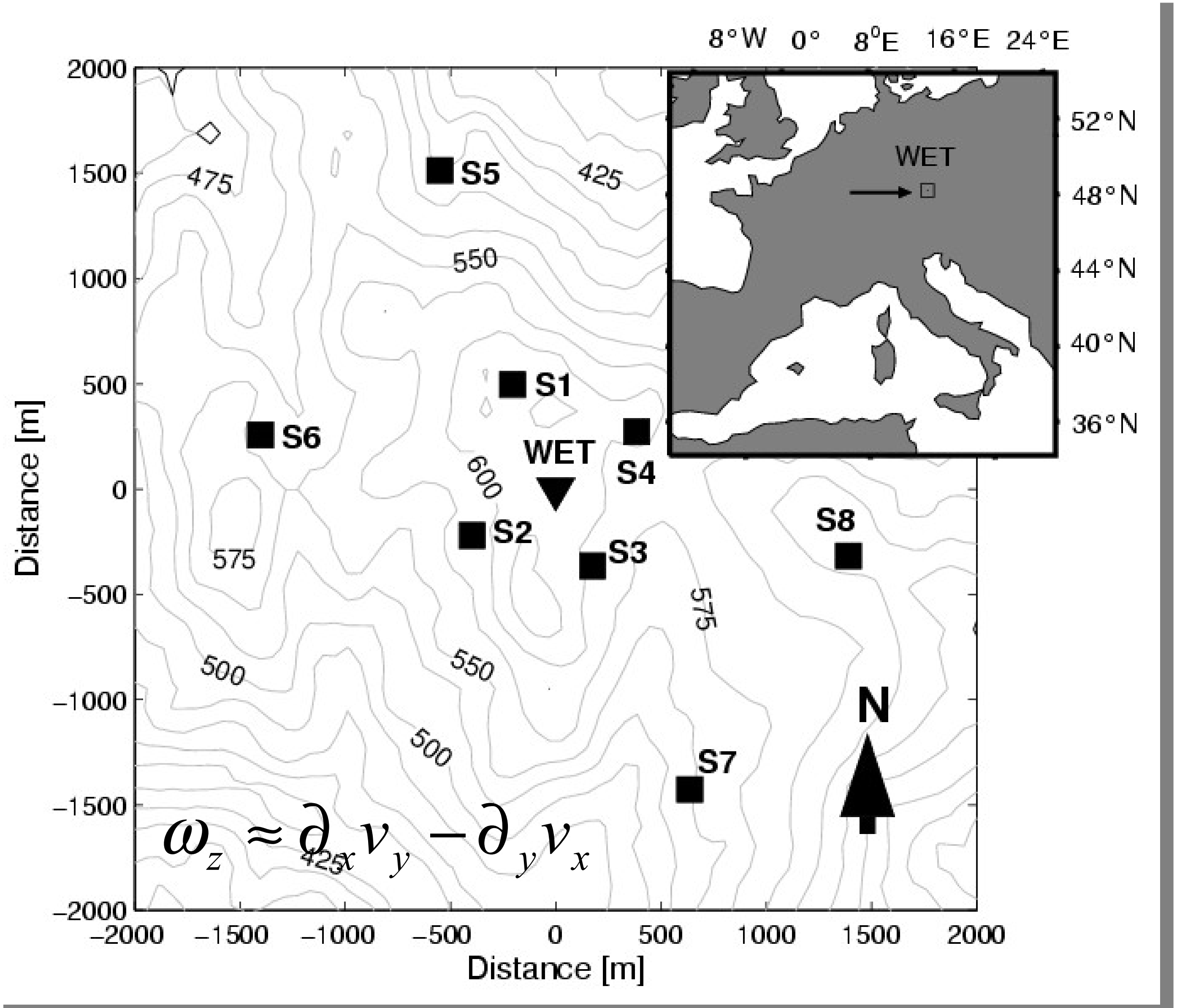
Rotational signals in the P-coda???



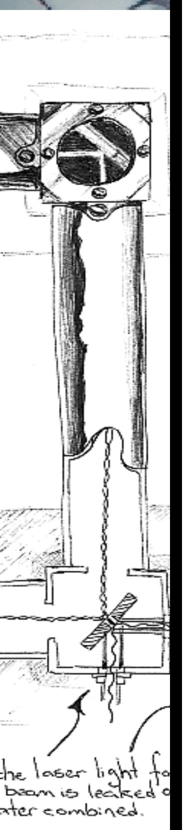
Array measurements

Dec 2003-Mar 2004

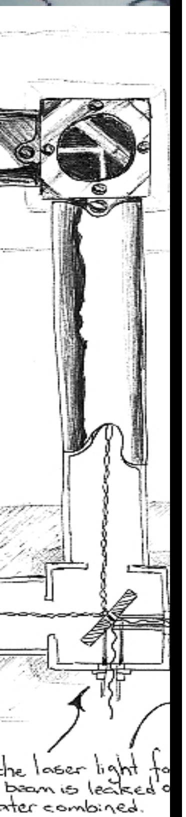


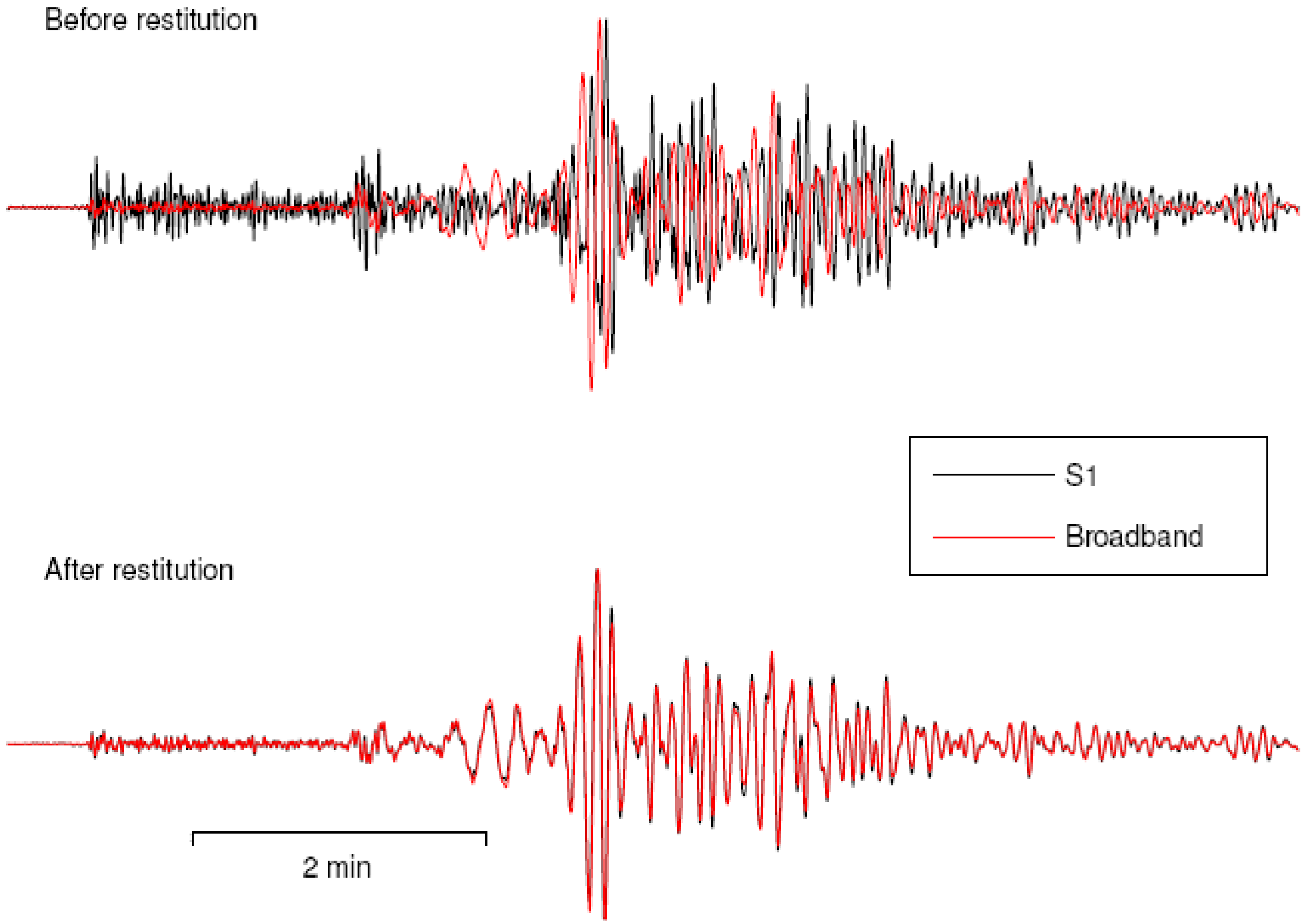


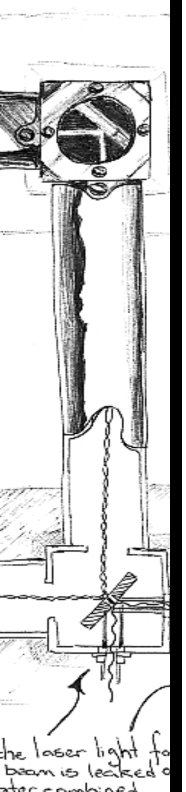
$$\omega_z \approx \partial_x v_y - \partial_y v_x$$



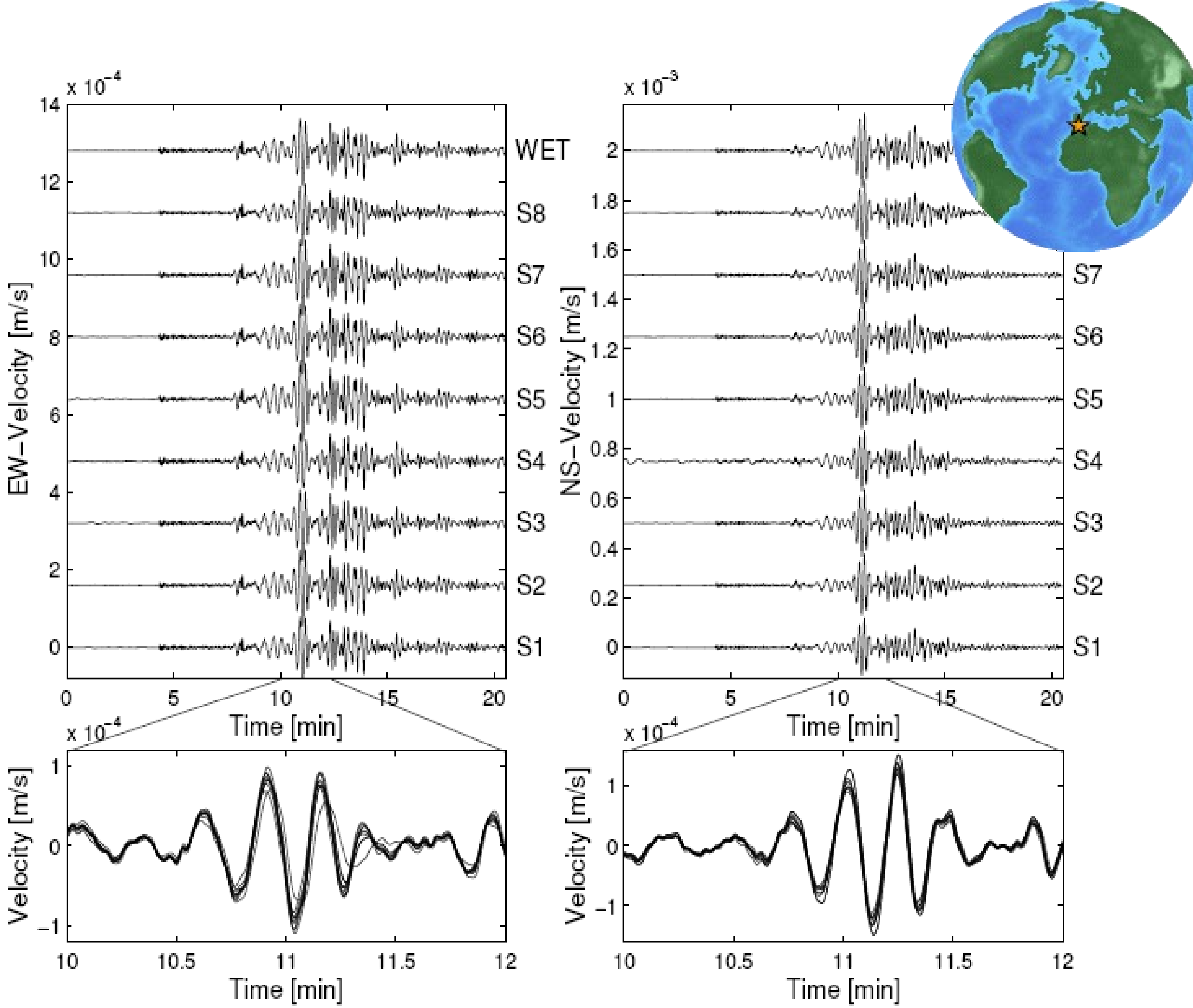
A quick-and-dirty experiment




Before restitution
After restitution
S1
Broadband
2 min

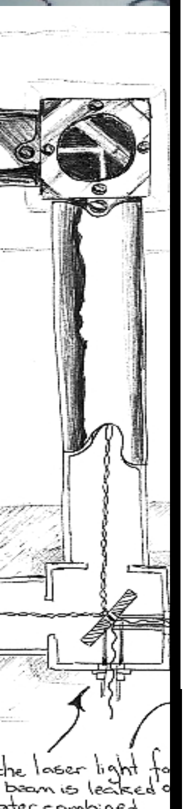


the laser light fo
beam is leaked o
ater combined.

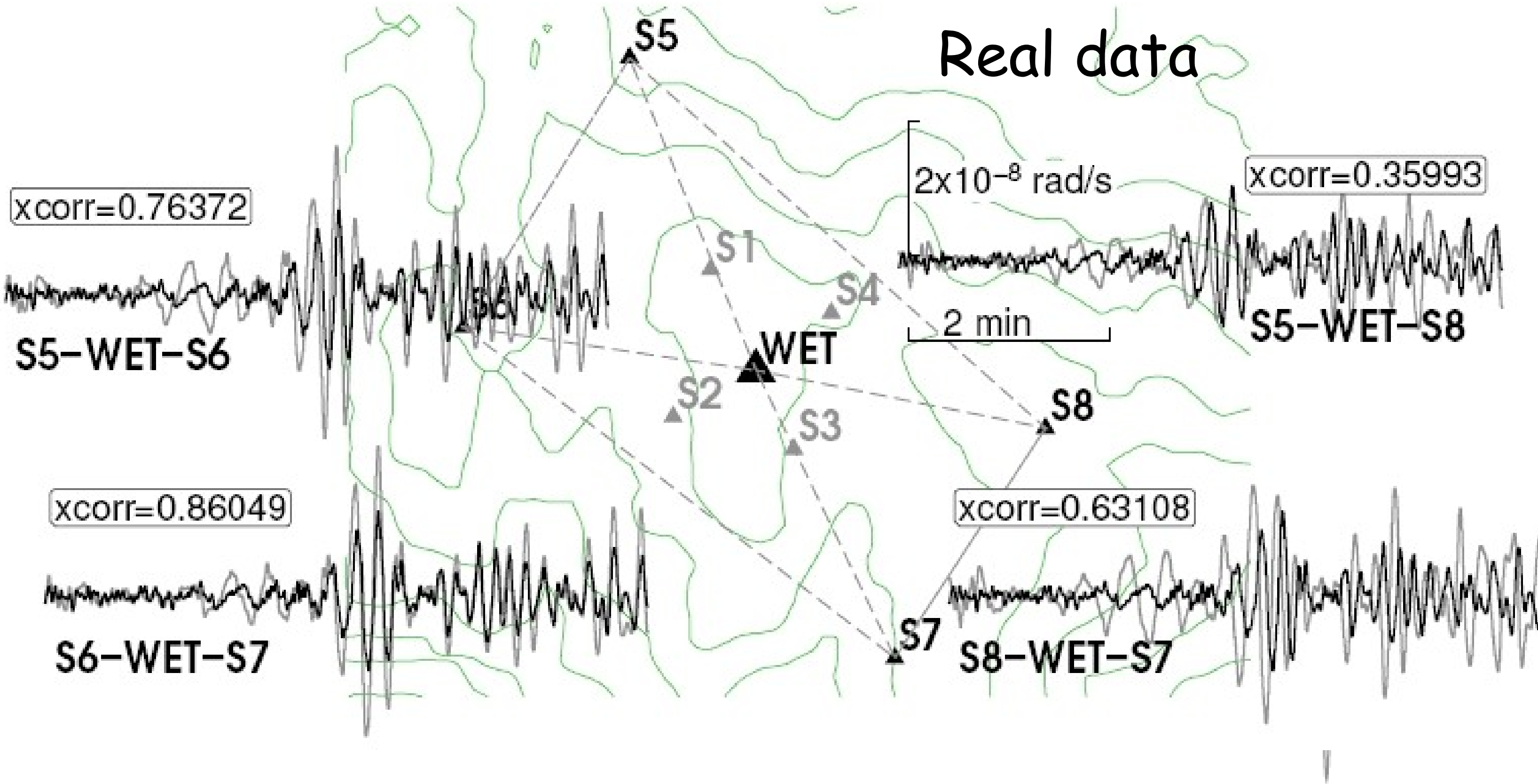


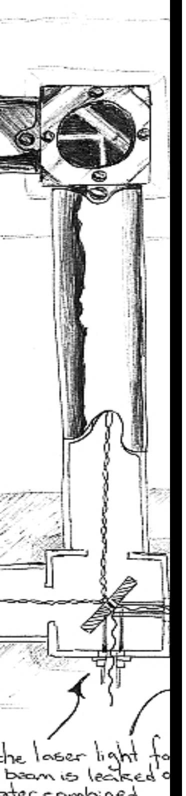
x 10^{-4}
EW–Velocity [m/s]
WET
S8
S7
S6
S5
S4
S3
S2
S1
Time [min]
x 10^{-3}
NS–Velocity [m/s]
Velocity [m/s]

Uniformity of rotation rate across array

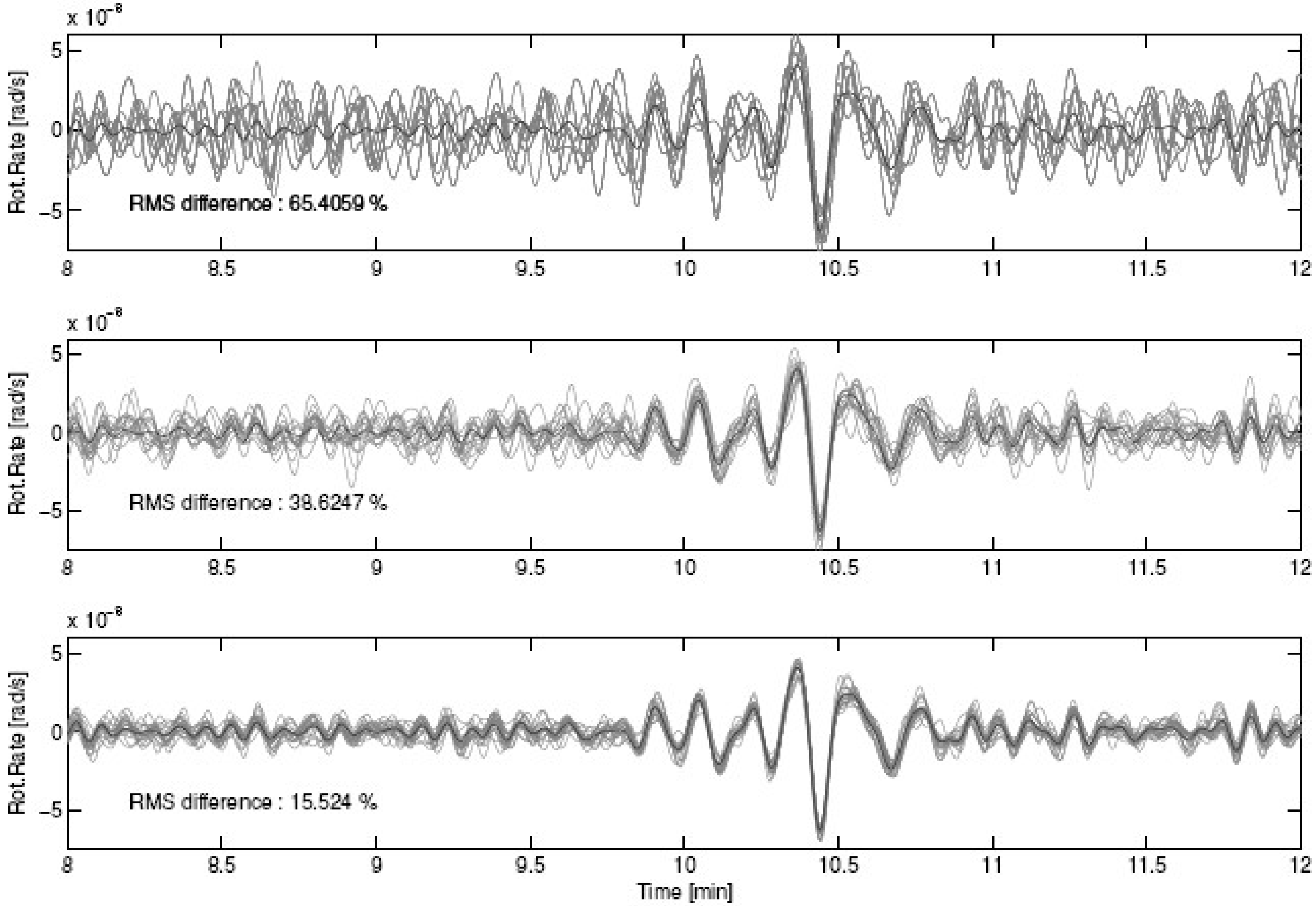


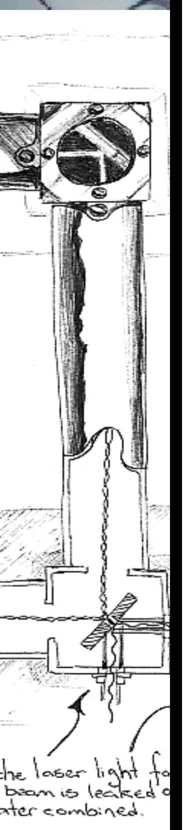
Real data



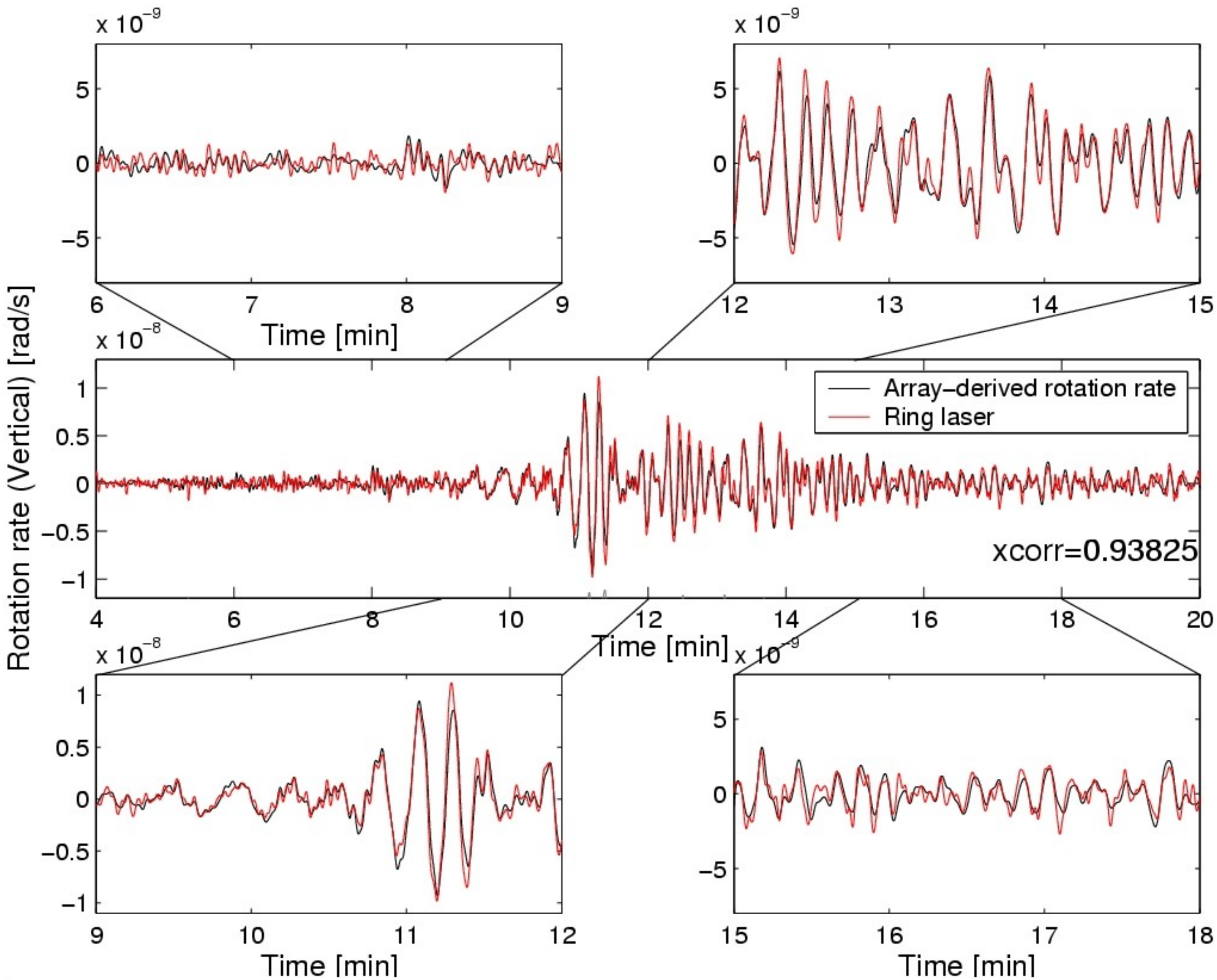


Effects of noise on array-derived rotation: Phase uncertainty

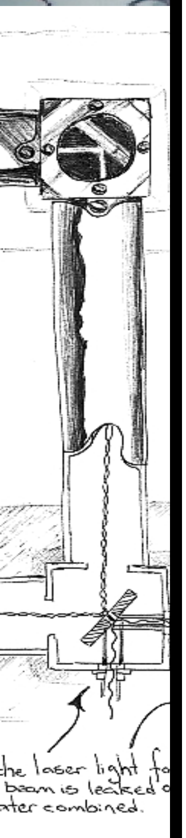




First comparison of array-derived rotations (**black**) and direct ring laser measurements (**red**)



From Suryanto et al (2005, BSSA, submitted)



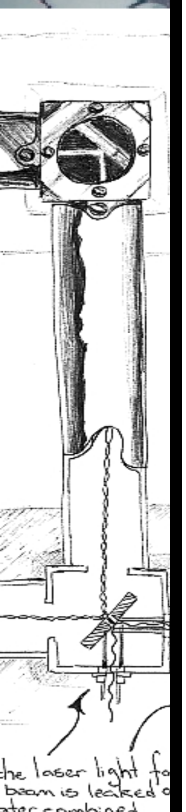
Summary

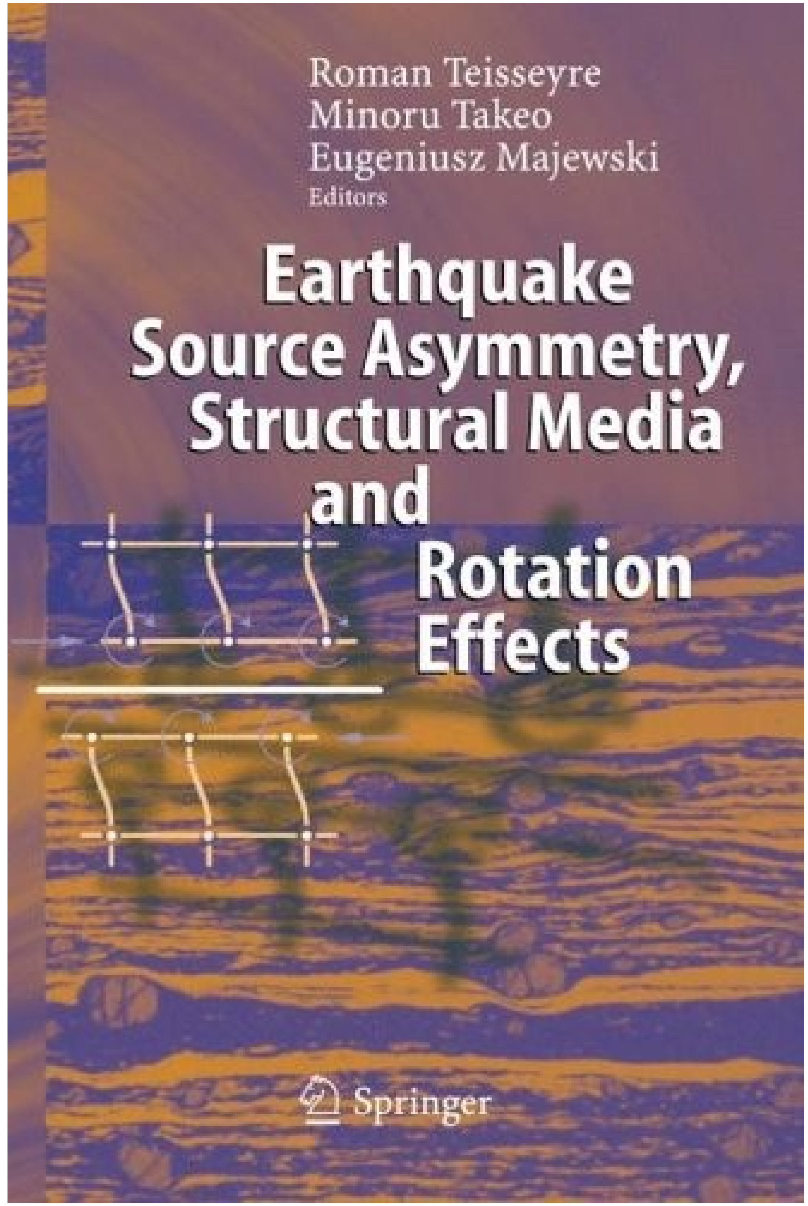
seismic ground rotations

- **Yes,** we do have a new observable for broadband seismology, that is consistent in phase and amplitude with collocated recordings of translations
- The joint observations allow seismic array-type processing steps (but array-free!)
- A prototype sensor designed for seismology has been installed at Pinon Flat, CA
- A less sensitive (portable) sensor for near source studies and applications in earthquake engineering is planned.

Next steps:

- Further comparison with array observations (phase velocities)
- Love-wave dispersion, how accurate? -> Tomography?
- Understanding observations in data base in terms of structure, anisotropy, source, etc.





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Info and (p)reprints: *ringlaser.geophysik.uni-muenchen.de*