Rotational Ground Motions: A New Observable for Seismology?

H. Igel, A. Cochard, A. Flaws, W. Suryanto, B. Schuberth, U. Schreiber, Pham Dinh Nguyen, A. Velikoseltsev

1Department of Earth and Environmental Sciences, LMU Munich
2Department of Physics and Astronomy, Christchurch, New Zealand
3Fundamentalstation Wettzell, Kötzting
4EOST Strasbourg

What is rotation in seismology? (Why bother?)

The ring laser instrument

Broadband observations of rotations
  - Peak rotation rates
  - Waveform comparison with translations
  - Horizontal phase velocities
  - Love wave dispersion
  - P-coda

Array-derived vs. directly measured rotations

Conclusions and future

ringlaser.geophysik.uni-muenchen.de
Rotation is the curl of the wavefield

... it separates P- and S-wave in isotropic media

\[
\begin{pmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{pmatrix}
= \frac{1}{2} \nabla \times \mathbf{v}
= \frac{1}{2}
\begin{pmatrix}
\partial_y v_z - \partial_z v_y \\
\partial_z v_x - \partial_x v_z \\
\partial_x v_y - \partial_y v_x
\end{pmatrix}
\]

Rotation rate
\textit{Rotation sensor}

Ground velocity
\textit{Seismometer}
Rotation from seismic arrays?
... by finite differencing ...

\[ \omega_z \approx \partial_x v_y - \partial_y v_x \]

Rotational motion estimated from seismometer recordings
Radiation from a **double-couple point source**

Geometry we use to express the seismic wavefield radiated by point double-couple source with area \( A \) and slip \( \Delta u \).

Here the fault plane is the \( x_1x_2 \)-plane and the slip is in \( x_1 \)-direction.

**FIGURE 5** Cartesian and polar coordinate systems for analysis of radiation by a slip patch with area \( A \) and average slip \( \langle \Delta u(t) \rangle \).

Aki and Richards (2002)
Radiation from a point source

**Ground displacement**

\[
u(x, t) = \frac{1}{4\pi\rho} A^N \frac{1}{r^4} \int_{r/v_p}^{r/v_S} \tau M_0(t - \tau) \, d\tau + \frac{1}{4\pi\rho v_p^2} A^{IP} \frac{1}{r^2} M_0(t - r/v_P) + \frac{1}{4\pi\rho v_S^2} A^{IS} \frac{1}{r^2} M_0(t - r/v_S) + \frac{1}{4\pi\rho v_p^3} A^{FP} \frac{1}{r} \dot{M}_0(t - r/v_P) + \frac{1}{4\pi\rho v_S^3} A^{FS} \frac{1}{r} \dot{M}_0(t - r/v_S).
\]

- **Near field term** contains the static deformation
- **Intermediate terms**
- **Far field terms**: the main ingredient for source inversion, ray theory, etc.

Aki and Richards (2002)
Radiation pattern

\[ A^N = 9 \sin 2\theta \cos \phi \hat{r} - 6(\cos 2\theta \cos \phi \hat{\theta} - \cos \theta \sin \phi \hat{\phi}), \]
\[ A^{IP} = 4 \sin 2\theta \cos \phi \hat{r} - 2(\cos 2\theta \cos \phi \hat{\theta} - \cos \theta \sin \phi \hat{\phi}), \]
\[ A^{IS} = -3 \sin 2\theta \cos \phi \hat{r} + 3(\cos 2\theta \cos \phi \hat{\theta} - \cos \theta \sin \phi \hat{\phi}), \]
\[ A^{FP} = \sin 2\theta \cos \phi \hat{r}, \]
\[ A^{FS} = \cos 2\theta \cos \phi \hat{\theta} - \cos \theta \sin \phi \hat{\phi}, \]

Far field P - blue
Far field S - red

Aki and Richards (2002)
The rotational part

\[ \omega(x, t) = \frac{1}{2} \nabla \times u(x, t) \]

\[ = \frac{-A_R}{8\pi \rho} \left[ \frac{3}{\beta^2 r^3} M_0 \left( t - \frac{r}{\beta} \right) + \frac{3}{\beta^3 r^2} \dot{M}_0 \left( t - \frac{r}{\beta} \right) + \frac{1}{\beta^4 r} \ddot{M}_0 \left( t - \frac{r}{\beta} \right) \right] \]

\[ A^R = \cos \theta \sin \phi \hat{\theta} + \cos \phi \cos 2\theta \hat{\phi} \]

- Rotations are zero before S arrival
- This includes the near field!
- Far-field P-rotation is not zero! Only the sum of all contributions cancel!

Cochard et al. (2006)
Basic seismograms, full space
Acceleration vs. Rotation rate

\[ u(x, t) = \frac{1}{4\pi\rho} A^N \frac{1}{r^4} \int_{r/v_p}^{r/v_s} \tau M_0(t - \tau) d\tau \]

\[ + \frac{1}{4\pi\rho v_{s}^2} A_{IP} \frac{1}{r^2} M_0(t - r/v_P) \]

\[ + \frac{1}{4\pi\rho v_{s}^2} A_{IS} \frac{1}{r^2} M_0(t - r/v_S) \]

\[ + \frac{1}{4\pi\rho v_{s}^3} A_{FP} \frac{1}{r} \dot{M}_0(t - r/v_P) \]

\[ + \frac{1}{4\pi\rho v_{s}^3} A_{FS} \frac{1}{r} \ddot{M}_0(t - r/v_S). \]

\[ \omega(x, t) = \frac{1}{2} \nabla \times u(x, t) \]

\[ = -\frac{A^R}{8\pi\rho} \left[ \frac{3}{\beta^2 r^3} M_0 \left( t - \frac{r}{\beta} \right) + \frac{3}{\beta^3 r^2} \dot{M}_0 \left( t - \frac{r}{\beta} \right) - \frac{1}{\beta^3 r^2} \ddot{M}_0 \left( t - \frac{r}{\beta} \right) \right] \]

\[ A^R = \cos \theta \sin \phi \dot{\theta} + \cos \phi \cos 2\theta \dot{\phi} \]

... in the far field ...

\[ \ddot{u}^{FS} = \frac{1}{4\pi\rho v_{s}^3 r} \dddot{M}_0(t - r/v_s) \]

\[ \ddot{\phi}^F = -\frac{1}{8\pi\rho v_{s}^4 r} \dddot{M}_0(t - r/v_s) \]

\[ \frac{\ddot{u}^{FS}}{\ddot{\phi}^F} = -2v_s \]
Rotations - why bother?

- Standard seismological observations are polluted by rotations
- Tiltmeters (rotation around horizontal axes) are polluted by translations
- Rotations may contribute to co-seismic structural damage
- Rotational measurements may provide additional wavefield information (phase velocities, etc)
- ... and may allow further constraints on rupture processes ...
"The state-of-the-art sensitivity of the general rotation-sensor is not yet enough for a useful geophysical application" (Aki and Richards, Quantitative Seismology, 1980)

"... note the utility of measuring rotation near a rupturing fault plane (...), but as of this writing seismology still awaits a suitable instrument for making such measurements" (Aki and Richards, Quantitative Seismology, 2nd edition 2002)
Previous studies

Schreiber, Stedman, and co-workers
Ring laser technology New Zealand and Germany

Takeo and co-workers
Gyroscopic rotation sensor, theoretical work

Nigbor and co-workers
rotational sensor and observation of rotational motion of nuclear blast

Teisseyre and co-workers
mechanical rotational sensor and observation of local events

It seems that only optical technology provides the required high resolution for (tele-)seismic measurements.
The ring laser at Wettzell
How can we observe rotations?

-> ring laser

Ring laser technology developed by the groups at the Technical University Munich and the University of Christchurch, NZ
The Sagnac Frequency
(schematically)

Sagnac frequency sampled with 800Hz
Rotation rate sampled with 4Hz

Tiny changes in the Sagnac frequencies are extracted to obtain the time series with rotation rate $\Delta f \rightarrow \Theta$. 
Ring laser - the principle

\[ \Delta f_{Sagnac} = \frac{4\Omega \cdot A}{\lambda P} \]

- \( A \): surface of the ring laser (vector)
- \( \Omega \): imposed rotation rate (Earth's rotation + earthquake +...)
- \( \lambda \): laser wavelength (e.g. 633 nm)
- \( P \): perimeter (e.g. 4-16m)
- \( \Delta f \): Sagnac frequency (e.g. 287.3 Hz sampled at 800Hz)
Ring laser - resolution

\[ \Delta f_{\text{Sagnac}} = \frac{4\Omega \cdot A}{\lambda P} \]

<table>
<thead>
<tr>
<th>Area m²</th>
<th>(f_{\text{Sagnac}}) (Hz)</th>
<th>Resolution rad/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>79.4</td>
<td>4.8 (10^{-10})</td>
</tr>
<tr>
<td>16</td>
<td>348.6</td>
<td>9.1 (10^{-11})</td>
</tr>
<tr>
<td>366</td>
<td>1512.8</td>
<td>7.3 (10^{-12})</td>
</tr>
</tbody>
</table>

After Schreiber et al., 2002

... ring lasers are used in any commercial airplanes for stabilizing ...
Effects of tilt on rotational measurements
... before presenting observations ...

- ... the ring laser should be sensitive to SH type motion only (S waves, Love waves) ...

- ... P-waves (or Rayleigh waves) should not lead to a signal (except via tilt coupling) ...

- ... Rotation rate and transverse acceleration should be in phase ...

- ... their amplitude ratio should be twice the local phase velocity - assuming plane non-dispersive transversely polarized wave propagation ...
Theoretical relation
rotation rate and transverse acceleration
plane-wave propagation

Plane transversely polarized wave propagating in $x$-direction with phase velocity $c = \omega / k$

$$u_y(x,t) = f(kx - \omega t)$$

Acceleration
$$a_y(x,t) = \ddot{u}_y(x,t) = \omega^2 f''(kx - \omega t)$$

Rotation rate
$$\Omega(x,t) = \frac{1}{2} \nabla \times [0, u_y, 0] = [0, -\frac{1}{2} k \omega f''(kx - \omega t), 0]$$

Rotation rate and acceleration should be in phase and the amplitudes scaled by two times the horizontal phase velocity $a(x,t)/\Omega(x,t) = -2c$
### Data base 2003 + 2004

<table>
<thead>
<tr>
<th>Date</th>
<th>Time (UTC)</th>
<th>Lat (°)</th>
<th>Lon (°)</th>
<th>Mag(L, b, S, w)</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>21/05/03</td>
<td>18:44:20</td>
<td>36.964</td>
<td>003.634</td>
<td>6.9</td>
<td>Algeria</td>
</tr>
<tr>
<td>26/05/03</td>
<td>09:24:33</td>
<td>38.849</td>
<td>141.568</td>
<td>7.0</td>
<td>Honshu</td>
</tr>
<tr>
<td>06/07/03</td>
<td>19:10:33</td>
<td>40.340</td>
<td>026.070</td>
<td>5.7</td>
<td>Turkey</td>
</tr>
<tr>
<td>14/08/03</td>
<td>05:14:55</td>
<td>39.193</td>
<td>020.741</td>
<td>6.3</td>
<td>Greece</td>
</tr>
<tr>
<td>25/09/03</td>
<td>19:50:06</td>
<td>41.781</td>
<td>143.903</td>
<td>8.3</td>
<td>Hokkaido</td>
</tr>
<tr>
<td>27/09/03</td>
<td>11:33:24</td>
<td>50.012</td>
<td>087.824</td>
<td>7.5</td>
<td>Siberia</td>
</tr>
<tr>
<td>27/09/03</td>
<td>18:52:53</td>
<td>50.060</td>
<td>087.690</td>
<td>6.6</td>
<td>Siberia</td>
</tr>
<tr>
<td>01/10/03</td>
<td>01:03:25</td>
<td>50.218</td>
<td>087.685</td>
<td>7.1</td>
<td>Siberia</td>
</tr>
<tr>
<td>08/10/03</td>
<td>09:07:01</td>
<td>42.480</td>
<td>144.820</td>
<td>6.7</td>
<td>Hokkaido</td>
</tr>
<tr>
<td>31/10/03</td>
<td>01:06:40</td>
<td>37.890</td>
<td>142.680</td>
<td>7.0</td>
<td>Honshu</td>
</tr>
<tr>
<td>17/11/03</td>
<td>06:43:31</td>
<td>51.140</td>
<td>177.860</td>
<td>7.8</td>
<td>Rat Island</td>
</tr>
<tr>
<td>26/12/03</td>
<td>01:56:58</td>
<td>29.100</td>
<td>058.240</td>
<td>6.8</td>
<td>Iran</td>
</tr>
<tr>
<td>05/02/04</td>
<td>21:05:12</td>
<td>-03.620</td>
<td>135.530</td>
<td>7.1</td>
<td>Irian Jaya</td>
</tr>
<tr>
<td>07/02/04</td>
<td>02:42:43</td>
<td>-04.030</td>
<td>134.780</td>
<td>7.5</td>
<td>Irian Jaya</td>
</tr>
<tr>
<td>24/02/04</td>
<td>02:27:53</td>
<td>35.290</td>
<td>-003.840</td>
<td>6.4</td>
<td>Gibraltar</td>
</tr>
<tr>
<td>17/03/04</td>
<td>03:21:12</td>
<td>-21.100</td>
<td>-065.560</td>
<td>6.1</td>
<td>Bolivia</td>
</tr>
<tr>
<td>05/04/04</td>
<td>21:24:06</td>
<td>36.590</td>
<td>070.850</td>
<td>6.6</td>
<td>Afghanistan</td>
</tr>
<tr>
<td>28/05/04</td>
<td>12:38:50</td>
<td>36.520</td>
<td>051.810</td>
<td>6.4</td>
<td>Iran</td>
</tr>
<tr>
<td>29/05/04</td>
<td>20:56:14</td>
<td>34.220</td>
<td>141.790</td>
<td>6.6</td>
<td>Honshu</td>
</tr>
<tr>
<td>05/12/04</td>
<td>01:52:37</td>
<td>48.120</td>
<td>008.080</td>
<td>5.0</td>
<td>Germany</td>
</tr>
<tr>
<td>26/12/04</td>
<td>00:58:53</td>
<td>03.300</td>
<td>095.980</td>
<td>9.0</td>
<td>Sumatra</td>
</tr>
</tbody>
</table>
4C recordings - raw data

Mw = 8.3  Tokachi-oki earthquake
25.09.2003   19:50:38.2 GMT
Lat=  42.21  Lon= 143.84
Compatibility with MS (surface wave magnitude) T=30s, c=4300m/s

\[ M_s = \log_{10} \frac{A}{T} + 1.66 \log_{10} D + 3.3 \]

\[ \Omega_z = 2 \frac{\pi^2}{cT^2} A(M_s, D) = 2 \frac{\pi^2}{cT} 10^{M_s - 1.66 \log_{10} D - 9.3} \]
Rotational data base

events with varying distance

transverse acceleration - rotation rate

increasing epicentral distance

400km M5.0

10000km M8.3
$M_w = 8.3$ Tokachi-oki 25.09.2003

transverse acceleration - rotation rate

From Igel et al., GRL, 2005
Max. cross-corr. coefficient in sliding time window
transverse acceleration - rotation rate

P-onset
Love waves
Aftershock
S-wave

Small tele-seismic event
Spectral element modeling of 3D global wave propagation

Cubed Sphere

100GByte Memory
60 hours on the Bundeshöchstleistungsrechner

Chunk Partitioning

Tromp and Komatitsch, 2003
M8.3 Tokachi-oki, 25 September 2003
phase velocities (+ observations, o theory)

From Igel et al. (GRL, 2005)
Real vs. Synthetics: Papua event

Cochard et al., 2006
increasing dominant period (s)

$M_w = 8.3$  Tokachi-oki 25.09.2003

narrow band-pass filtering
$Mw = 6.3$ Greece 14.08.2003

transverse acceleration - rotation rate

narrow band-pass filtering
Rotational seismograms

Synthetics and Observations

M8.3 Hokkaido, 25 September 2003
(recorded in Wettzell, Germany)

Transverse acceleration: Data (black), Synthetics (red), Cutoff period: 28 s

Rotation rate: Data (black), Synthetics (red), Cutoff period: 28 s
Phase velocity determination

... by dividing accelerations by rotation rates in a sliding window ...

... point measurement!

Love wave phase velocities

Note the decreasing velocities with time (and increasing frequency)
Phase velocity determination

... by dividing accelerations by rotation rates in a sliding window ...

... point measurement!
Restitute your broadband seismograms!
transverse acceleration - rotation rate

Before restitution
Restitute your broadband seismograms!
transverse acceleration - rotation rate

After restitution

... an independent confirmation of the quality of the restitution processing ...
Phase velocity determination
... by calculating spectral ratios ...

Theoretical Love wave dispersion (ak135)
Stacked spectral ratios …
… accurate enough for structural inversion …?
Direction of propagation of transversely polarized energy

Max. cross-corr. coeff. as a function of time and propagation direction
Direction of propagation of transversely polarized energy

Max. cross-corr. coeff. as a function of time and propagation direction
Rotational signals in the P-coda???
Array measurements
Dec 2003-Mar 2004

\[ \omega_z \approx \frac{\partial}{\partial x} v_y - \frac{\partial}{\partial y} v_x \]
A quick-and-dirty experiment
Uniformity of rotation rate across array

Real data

-xcorr=0.76372
S5-WET-S6

-xcorr=0.86049
S6-WET-S7

-xcorr=0.63108
S7-S8-WET-S7

-xcorr=0.35993
S5-WET-S8

2x10^-8 rad/s
2 min
Effects of noise on array-derived rotation: Phase uncertainty

![Graph showing effects of noise on array-derived rotation with RMS differences of 65.4059%, 38.6247%, and 15.524% at different time intervals.](image-url)
First comparison of array-derived rotations (black) and direct ring laser measurements (red)

From Suryanto et al (2005, BSSA, submitted)
Summary
seismic ground rotations

- **Yes,** we do have a new observable for broadband seismology, that is *consistent in phase and amplitude* with collocated recordings of translations.
- The joint observations allow seismic array-type processing steps (but array-free!)
- A prototype sensor designed for seismology has been installed at **Pinon Flat, CA**
- A less sensitive (portable) sensor for near source studies and applications in *earthquake engineering* is planned.

Next steps:
- Further comparison with array observations (phase velocities)
- **Love-wave dispersion**, how accurate? -> Tomography?
- Understanding observations in data base in terms of structure, anisotropy, source, etc.
Earthquake Source Asymmetry, Structural Media and Rotation Effects

Roman Teisseyre
Minoru Takeo
Eugeniusz Majewski
Editors

Springer

Only 160€!

Info and (p)reprints: ringlaser.geophysik.uni-muenchen.de