Problems with the energy budget of the Earth - constraints from geodesy and geomagnetism

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Acknowledgement: Phil Livermore
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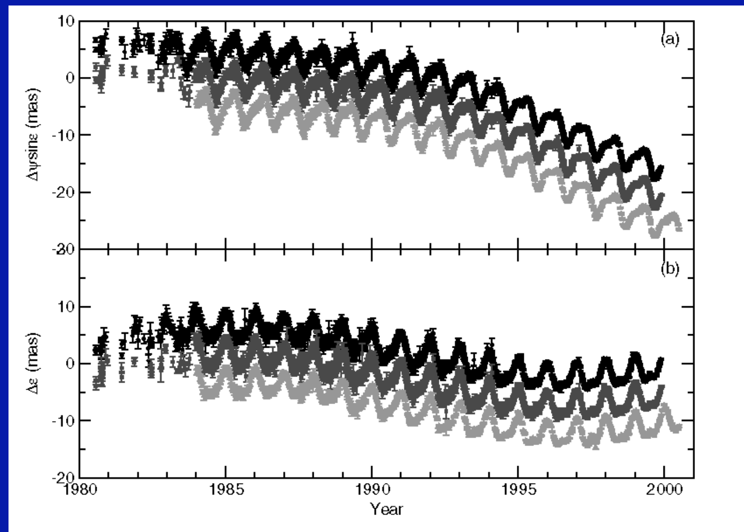
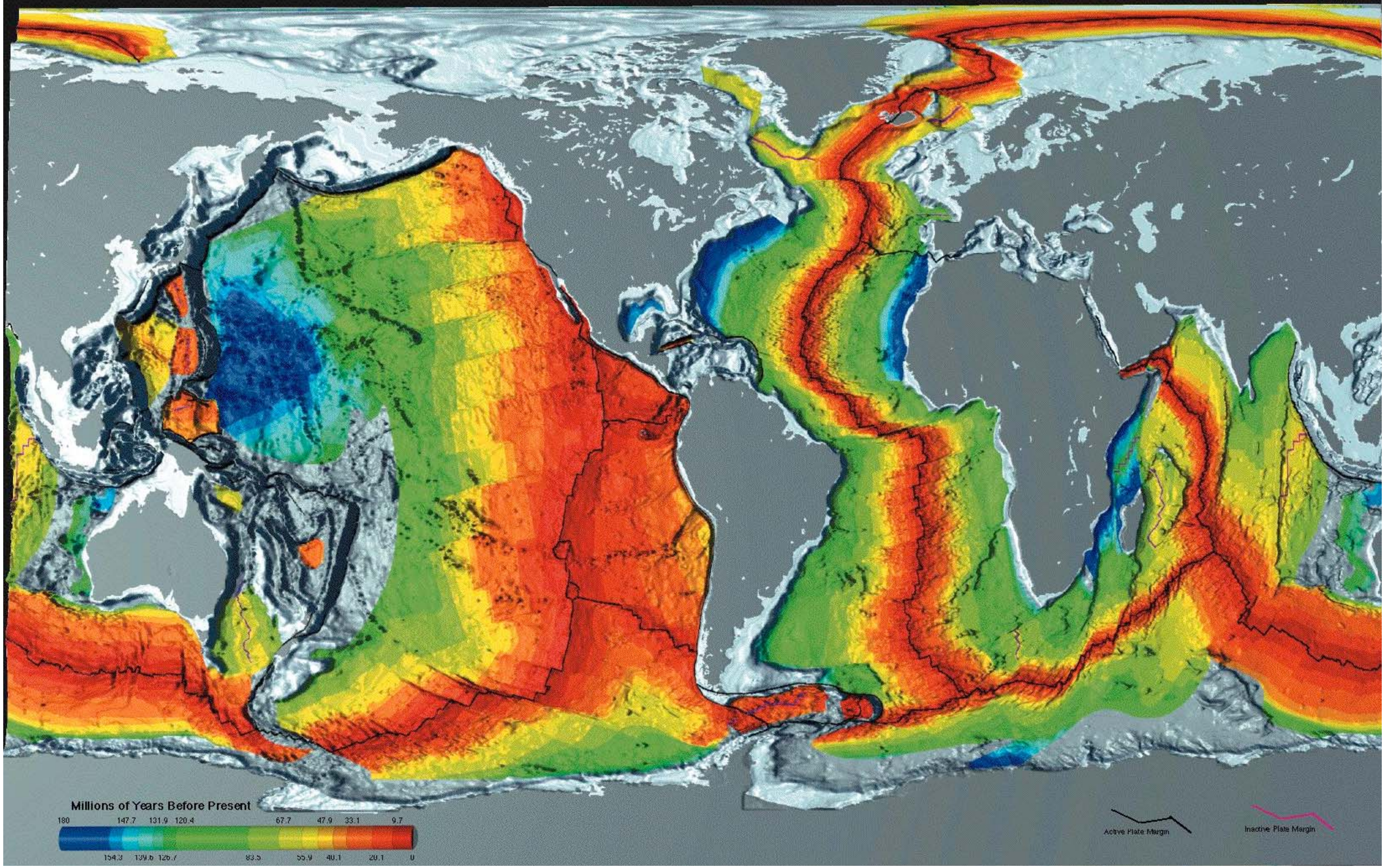
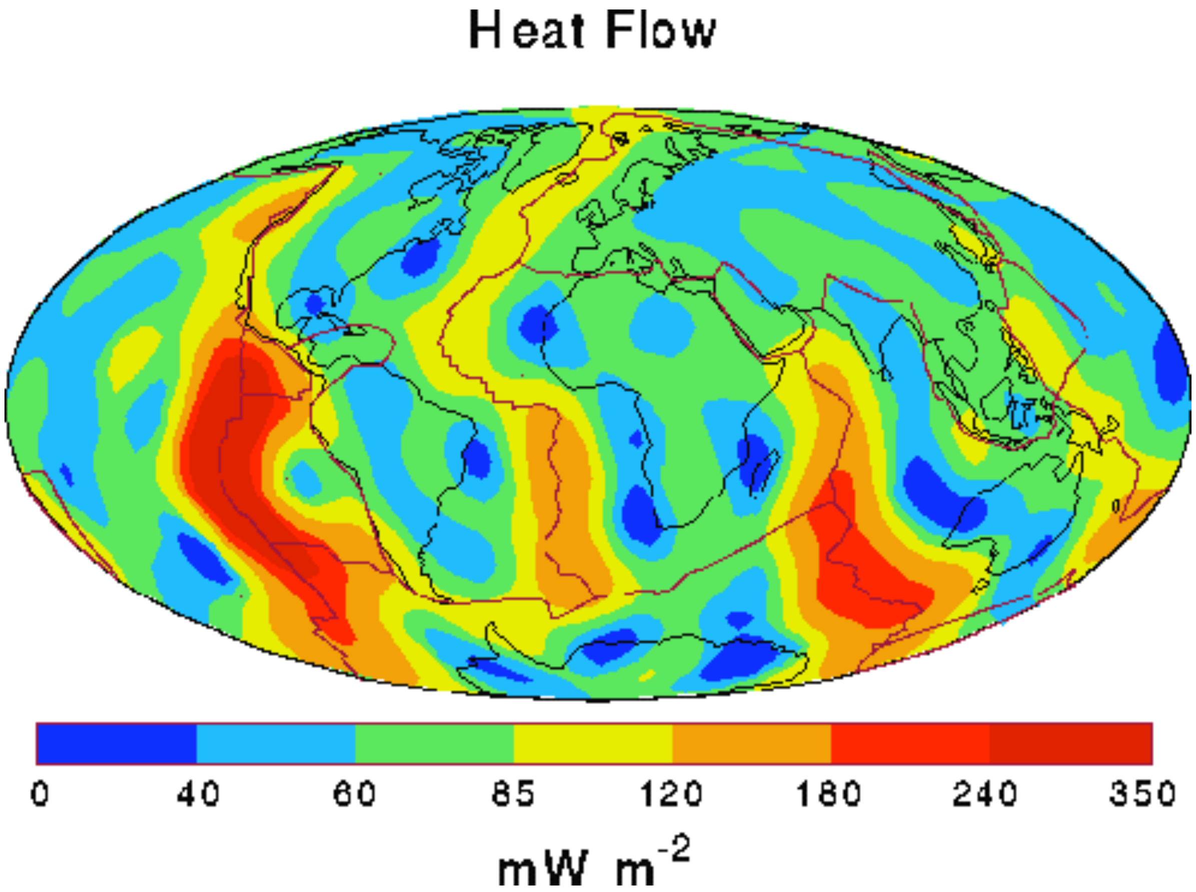
Plan

- Heat loss in the mantle
- The geodynamo
  - Background to magnetic field generation and heat transfer
- Core's energy budget estimate
  - Use observational constraints from geomagnetism & geodesy



Age of ocean floor





44 TW Total Heat Loss

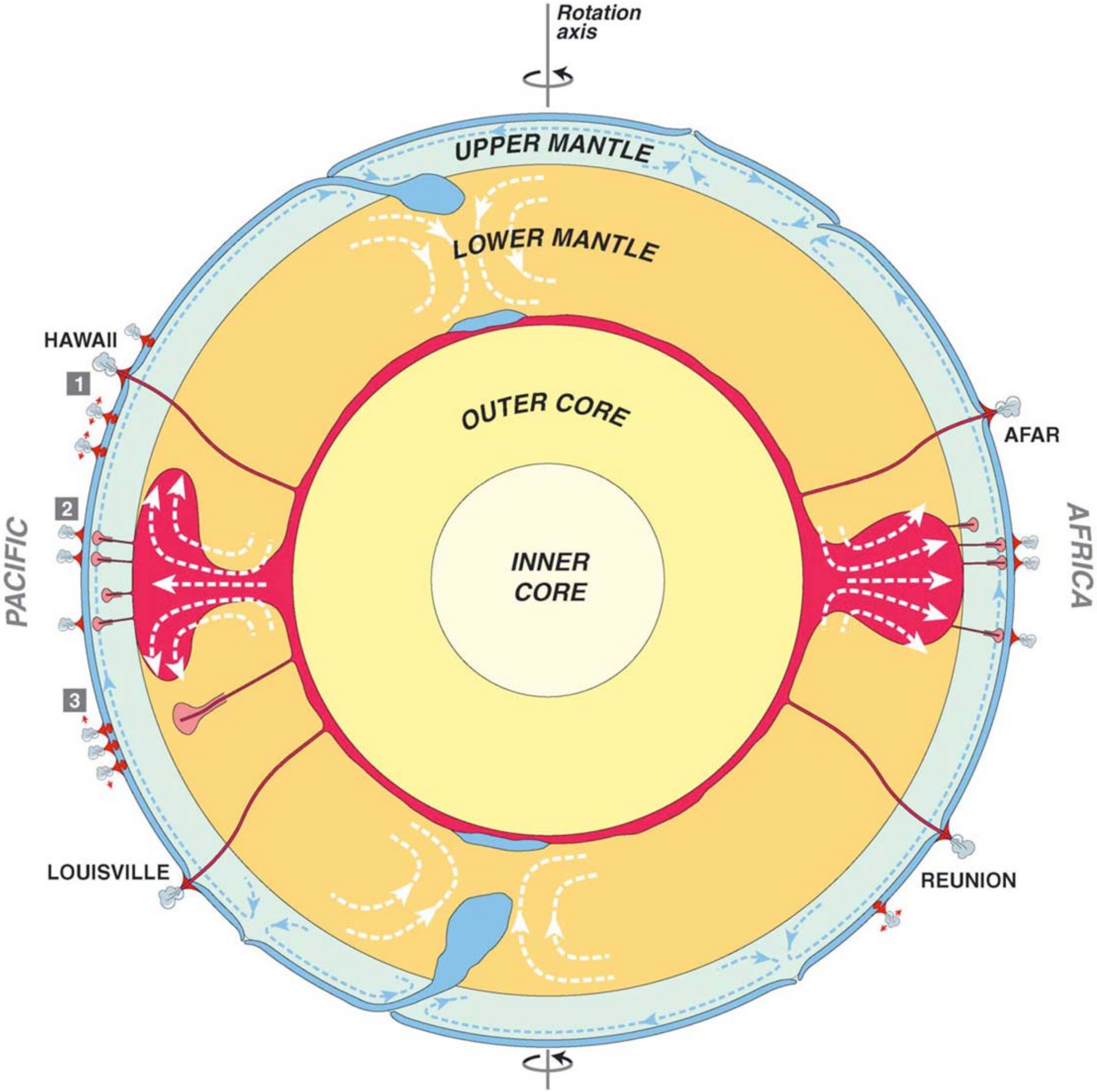
Francis Birch (1952)

Unwary readers should take warning that ordinary language undergoes modification to a high-pressure form when applied to the interior of the Earth. A few examples of equivalents follow:

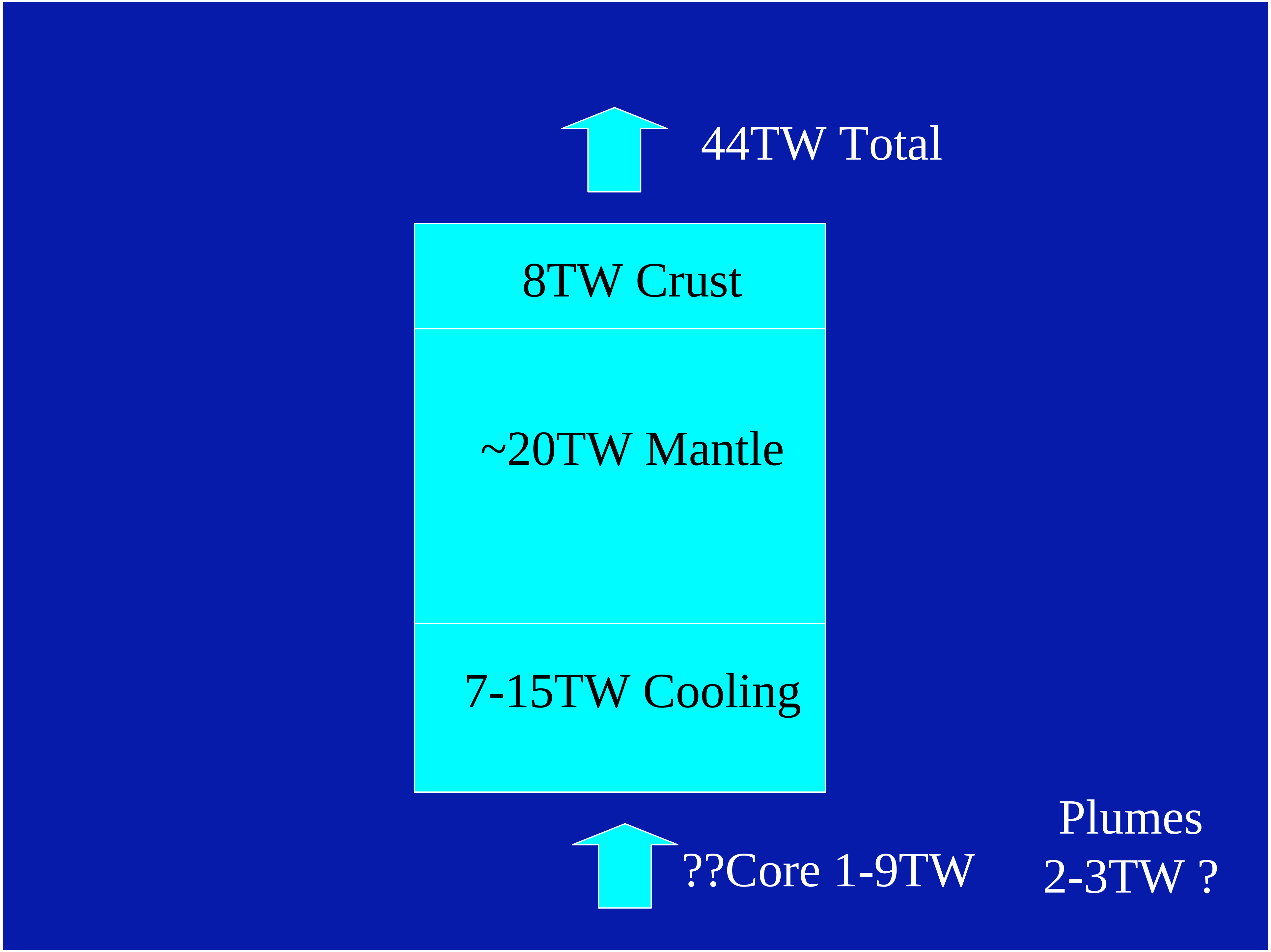
High Pressure Form	*Ordinary Meaning*
Certain	Dubious
Undoubtedly	Perhaps
Positive proof	Vague suggestion
Unanswerable argument	Trivial objection
Pure iron	Uncertain mixture of all the elements

Heat loss from the Earth

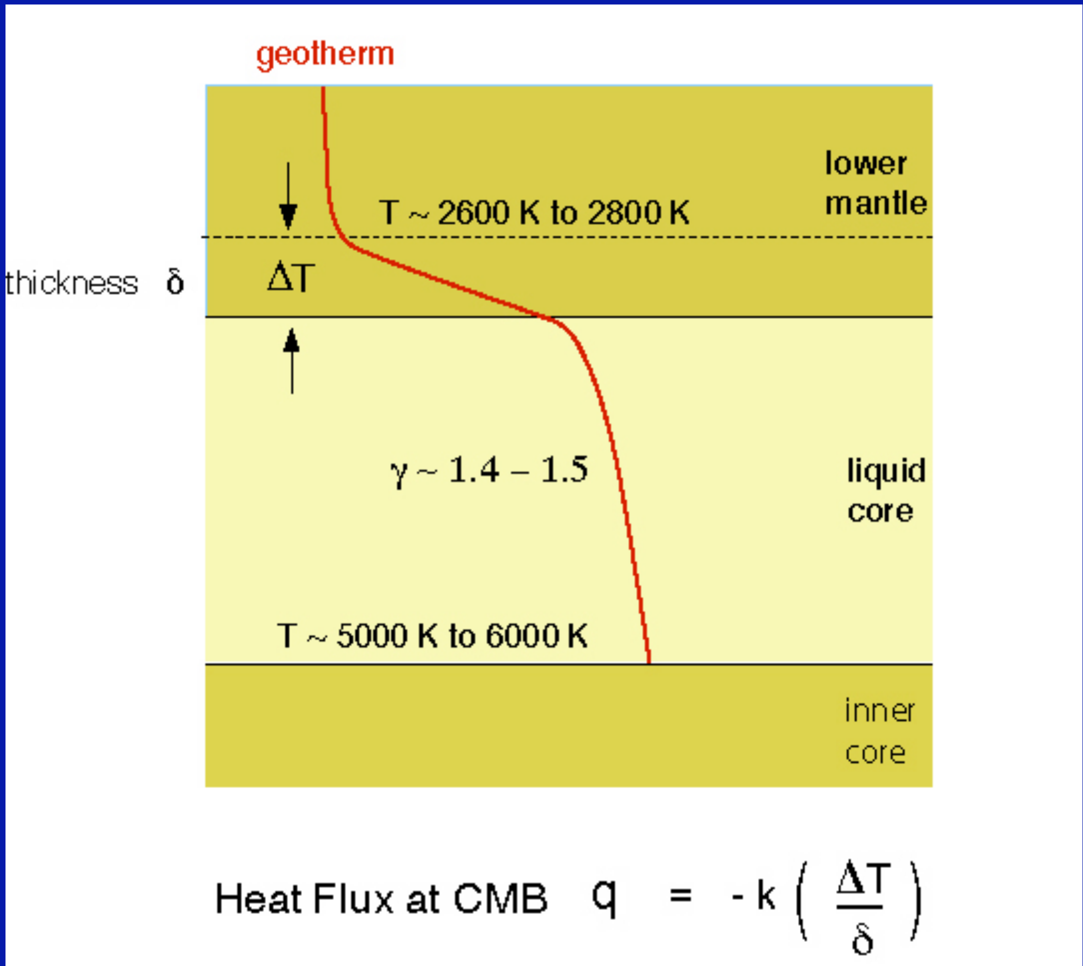
- Generally accepted that global heat loss through Earth's surface is ~ 44TW
- Wide range of estimates for heat generation from mantle radioactivity and secular cooling
- Bulk Silicate Earth of McDonough & Sun (1995) based on CI chondrites generates 20TW from radioactivity of mantle [240ppm K]
- 8TW from radioactivity of crust
- Geological arguments have Earth cooling by 50-100K/Ga => 7.5-15TW lost by cooling
- Associating hotspot flux with heat loss from core gives 2-3TW
- There is a wide range of possibilities



Rotation axis
UPPER MANTLE
LOWER MANTLE
OUTER CORE
INNER CORE
HAWAII
1
2
3
PACIFIC
LOUISVILLE
AFAR
AFRICA
REUNION


44TW Total
8TW Crust
~20TW Mantle
7-15TW Cooling
??Core 1-9TW
Plumes
2-3TW ?

Temperature Profile

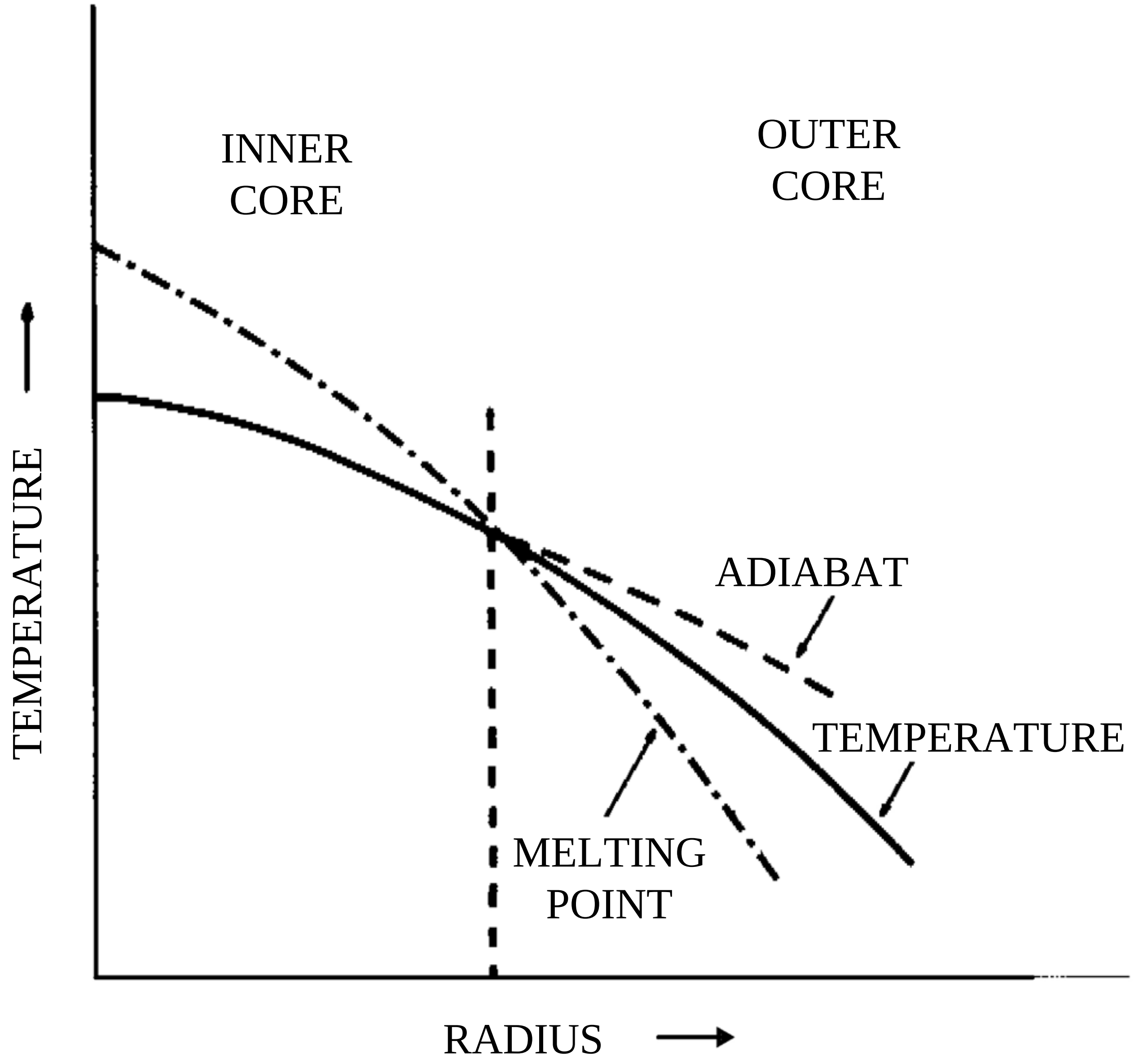


Temperature jump across D´´ really depends on knowing core melting temperature

Thermal boundary layer at base of mantle

- Layer ~200km thick
- Temperature drop 1000-2000K
- Heat flux drawn from core by conduction ~ 6-12TW

The inner core has been growing throughout time



Adiabatic gradient -> Heat flux

Wiedemann-Franz Law (metals)

$$k = LT\sigma$$

k = thermal conductivity

σ = electrical conductivity

L= Lorentz number (very constant 2.45 E-8)

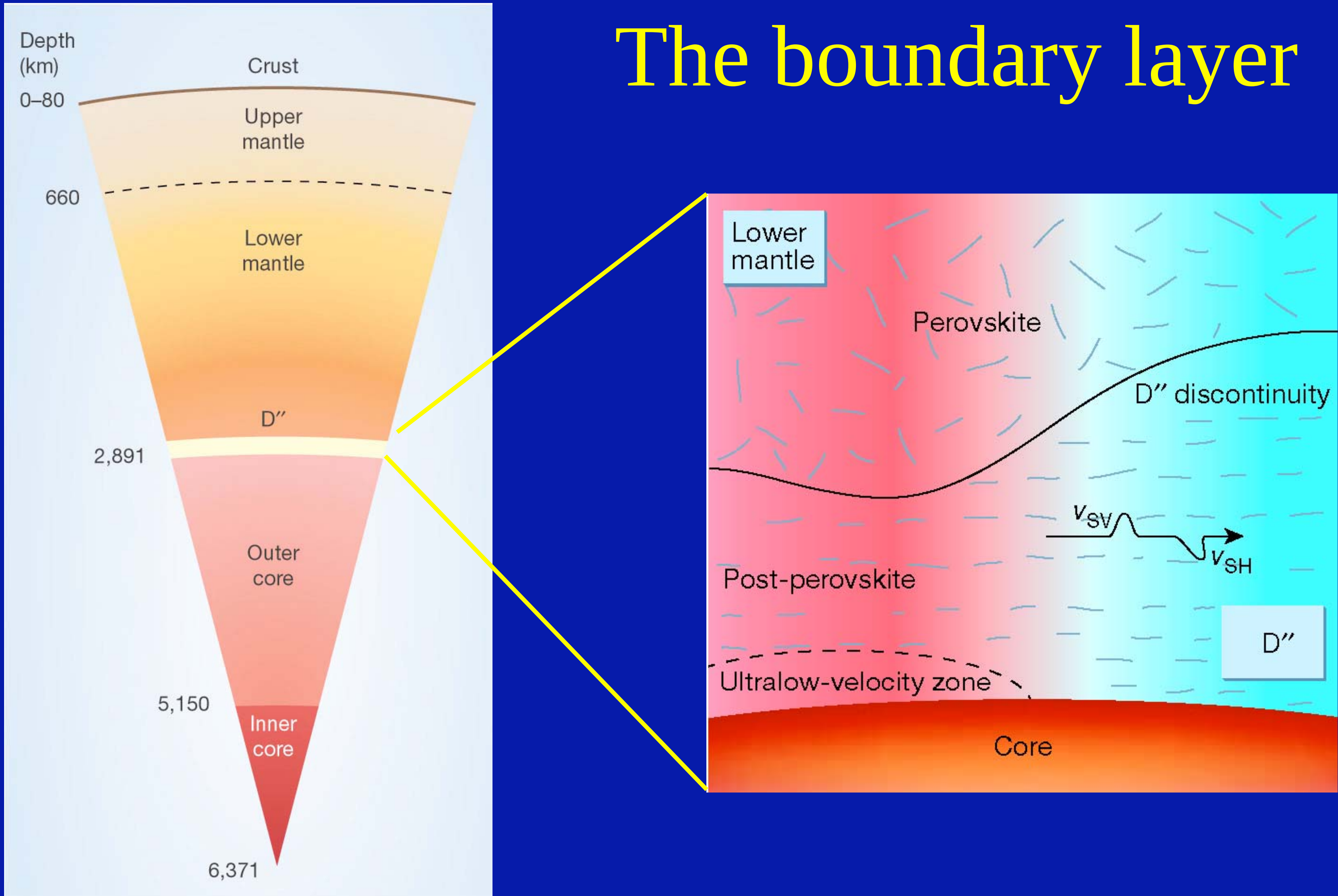
Gives k=38W/m/K from electronic part

Lattice contibution makes total ~ 45W/m/K

Adiabatic gradients

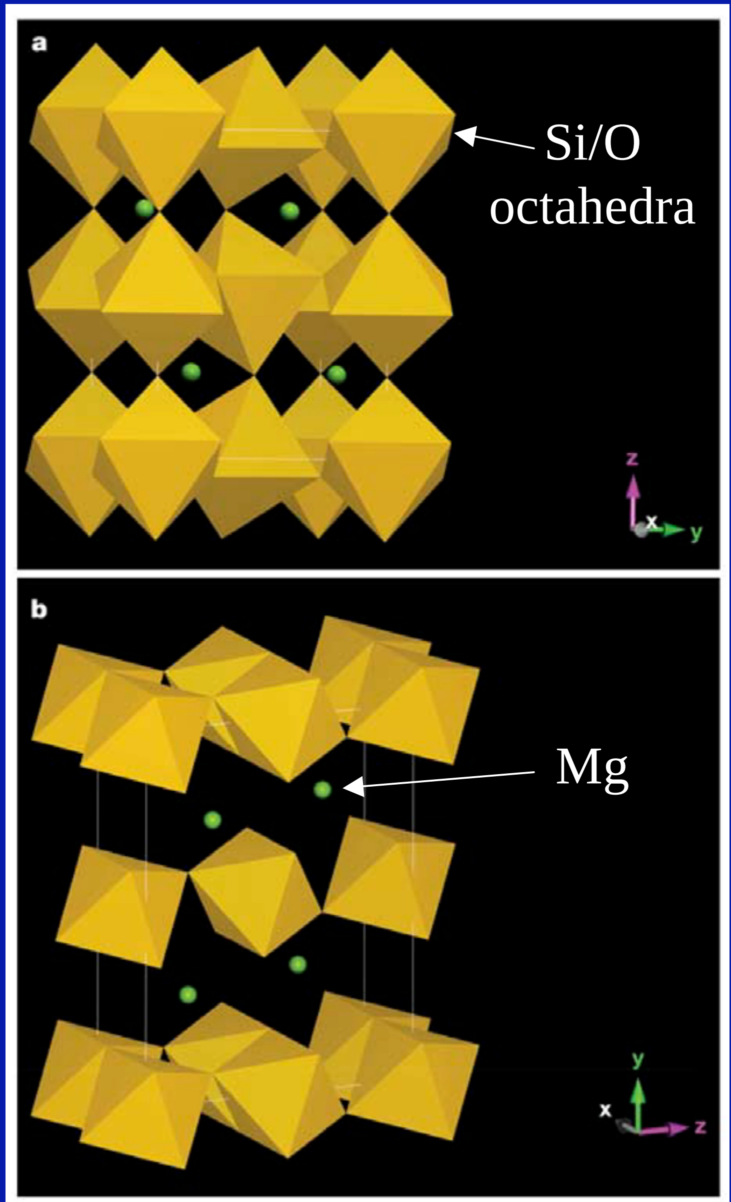
- Ab initio calculations of the properties of iron give the adiabatic gradient in the core
- Results predict that roughly 8 TW of heat flows down the adiabatic gradient (Gubbins et al 2003) and out of the core, if the whole core convects thermally
- Roberts et al (2003) give 6 TW

The boundary layer



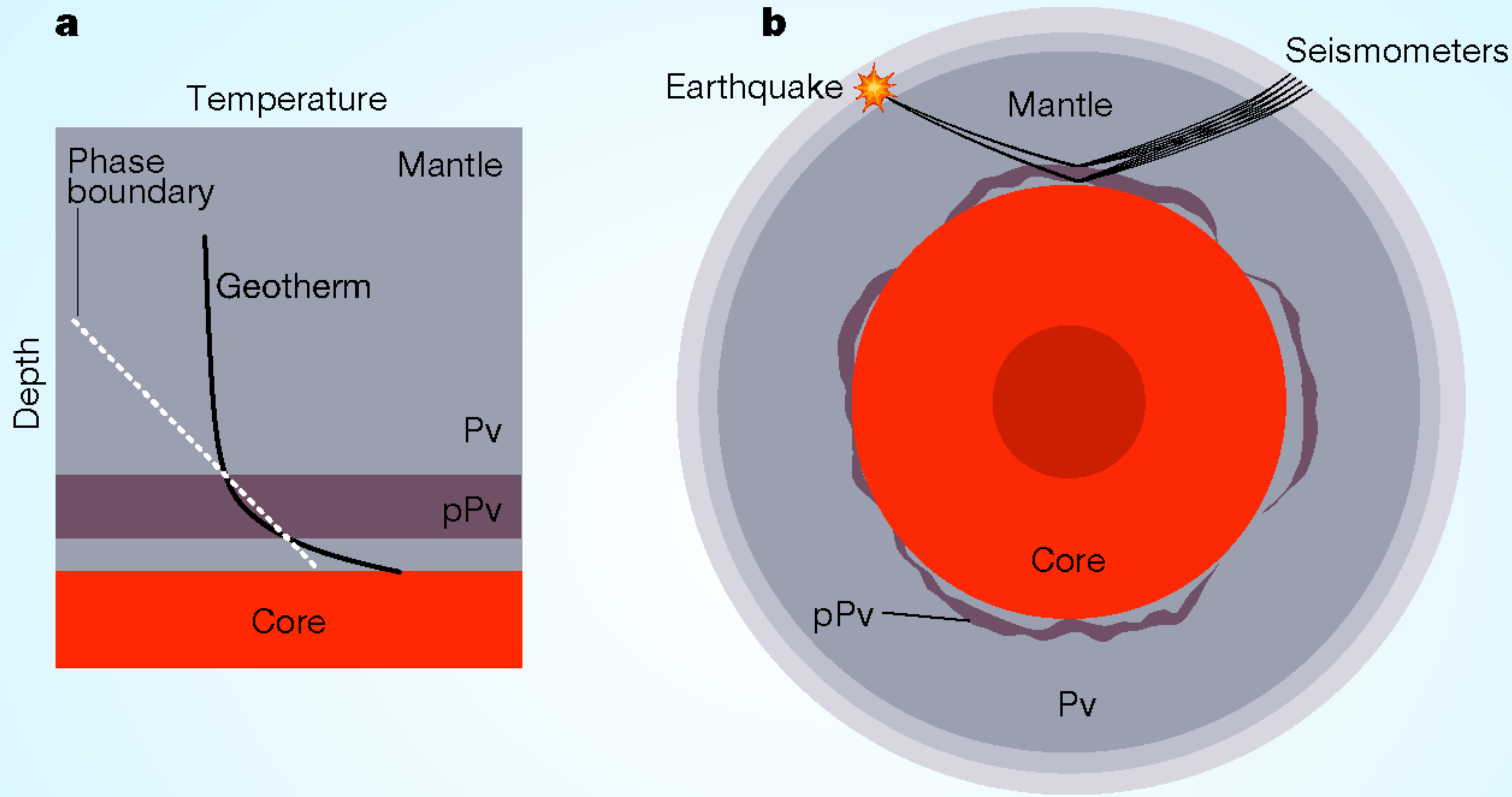
Perovskite/post-Perovskite transition at ~100 GPa

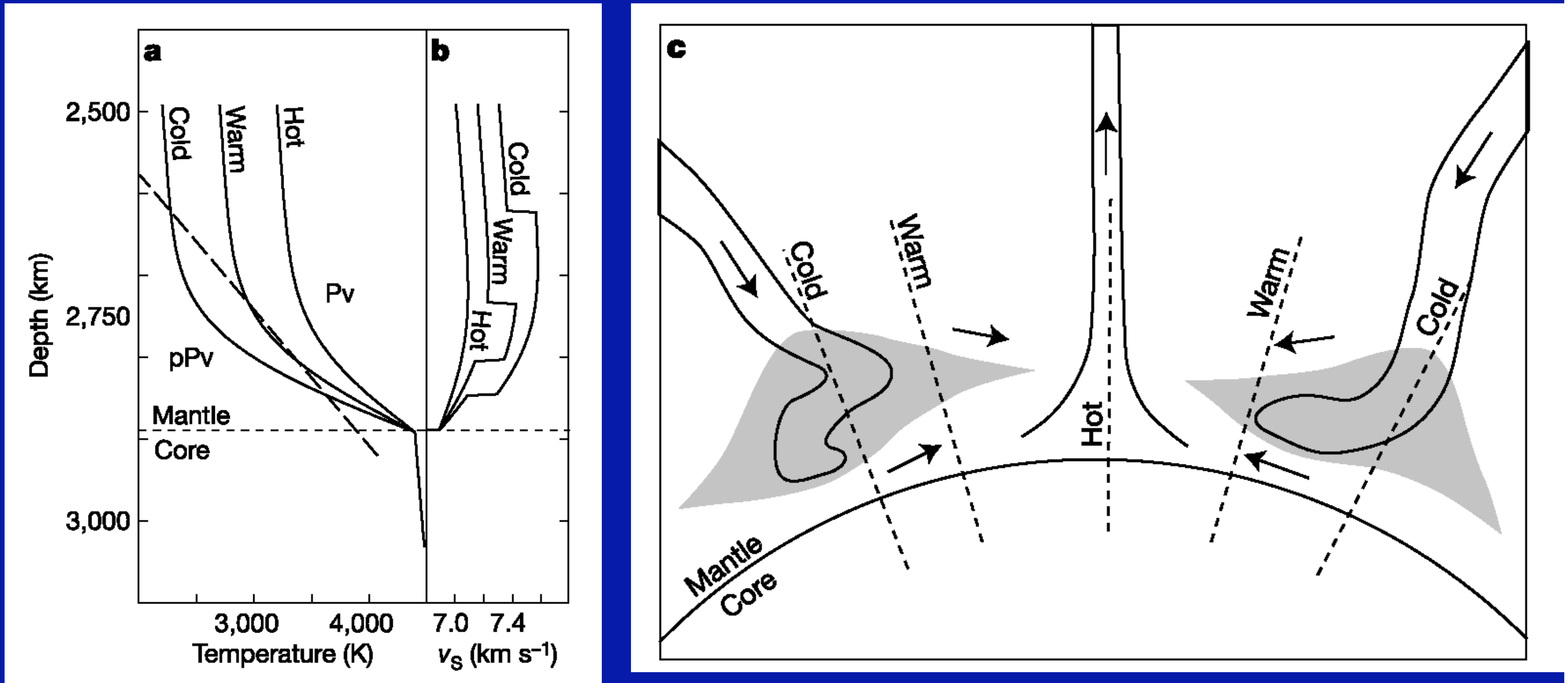
Pv



pPv

- Transition in $MgSiO_3$ explains the seismic region D´´ - discontinuities, anisotropy, maybe ultra low velocity zones
- Fe can enter readily (Caracas & Cohen)
- $FeSiO_3$ is metallic (high electrical conductivity) and stable at pPV pressures
- Enhanced thermal conductivity?


a
Temperature
Phase
boundary
Mantle
Geotherm
Depth
Pv
pPv
Core
b
Earthquake
Seismometers
Mantle
Core
pPv
Pv



- Hernlund et al (2005) use double crossing of post-Perovskite stability field to indicate temperature gradient
- Suggest 9-13TW flowing from the core
- Depends on thermal conductivity – Hofmeister gives half the above figure

Recap:

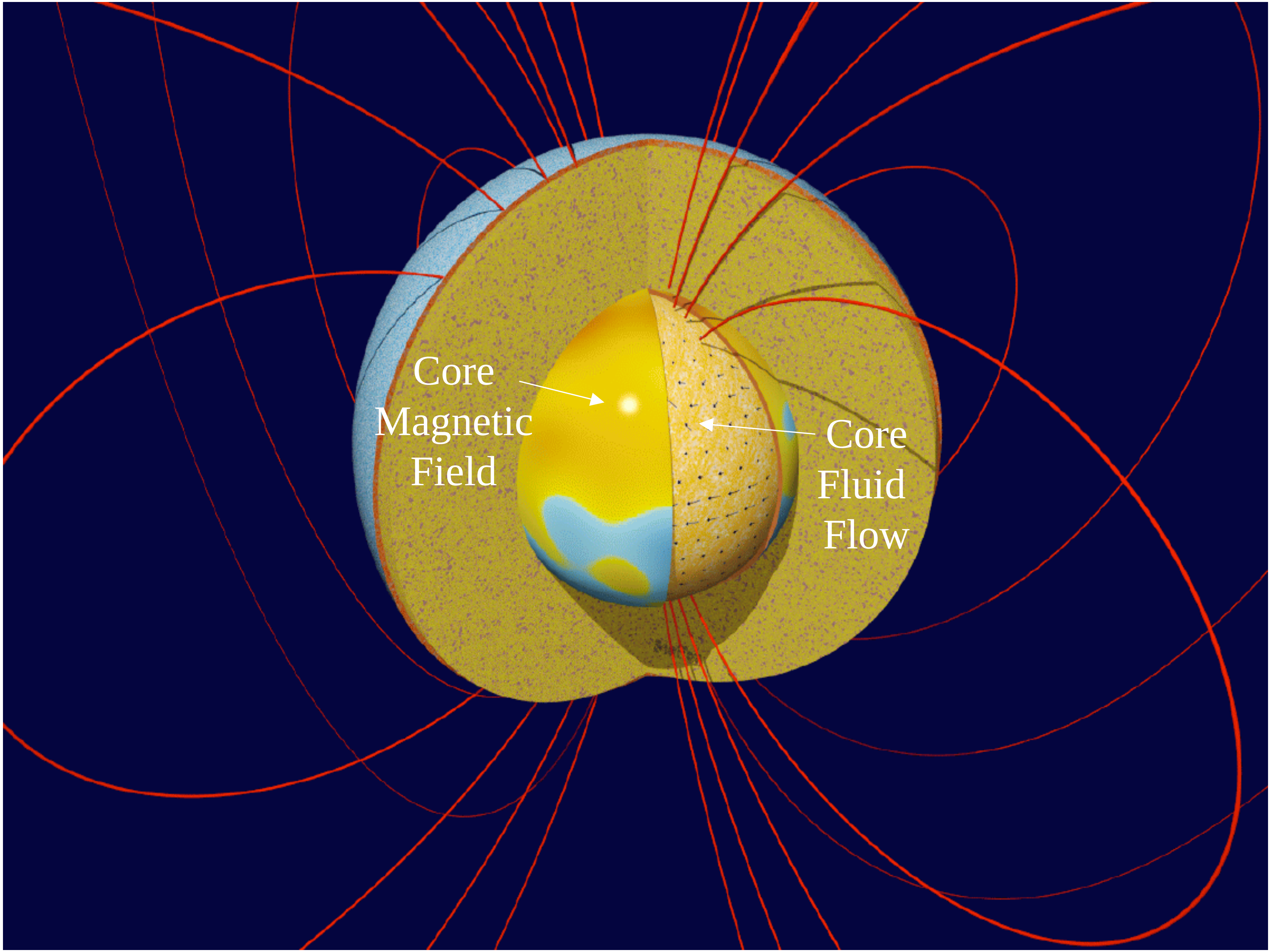
Melting temperatures: 6-12TW

Core adiabat: > 6-8TW

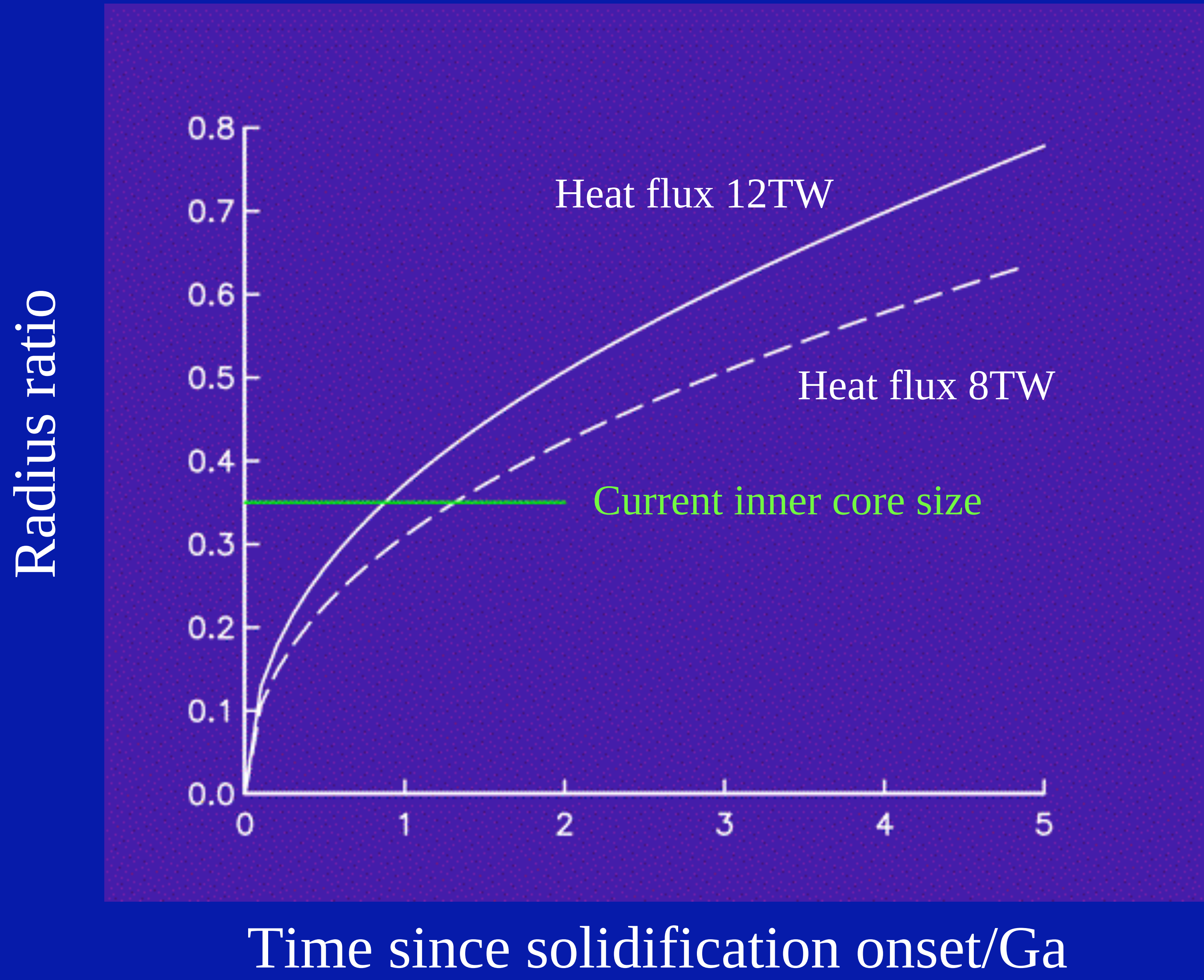
Post perovskite: 9-13TW

Various lines suggest we might be extracting 8-12TW from core

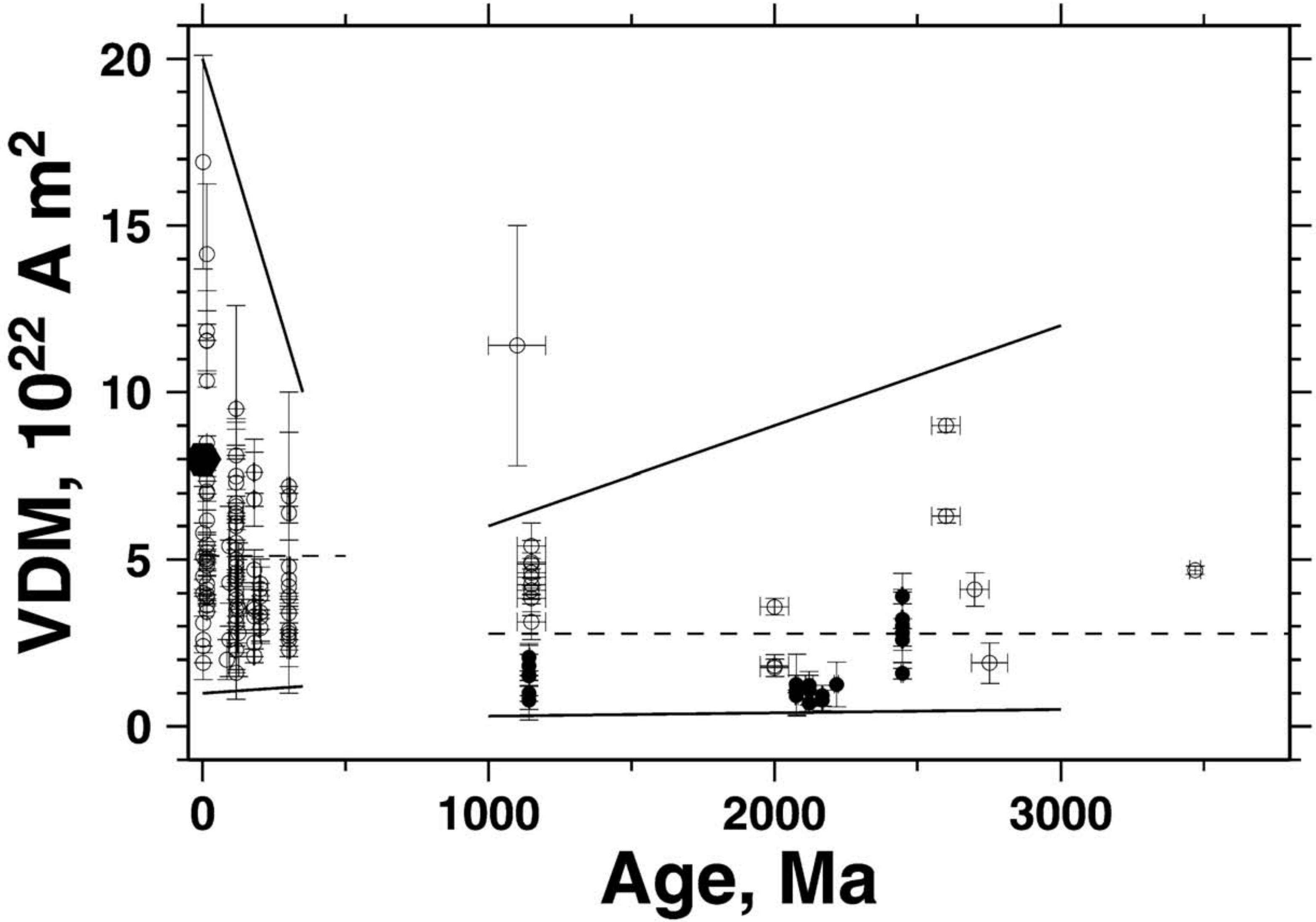
WHAT'S THE PROBLEM?


Core
Magnetic
Field
Core
Fluid
Flow

Inner core growth



The magnetic field strength through time

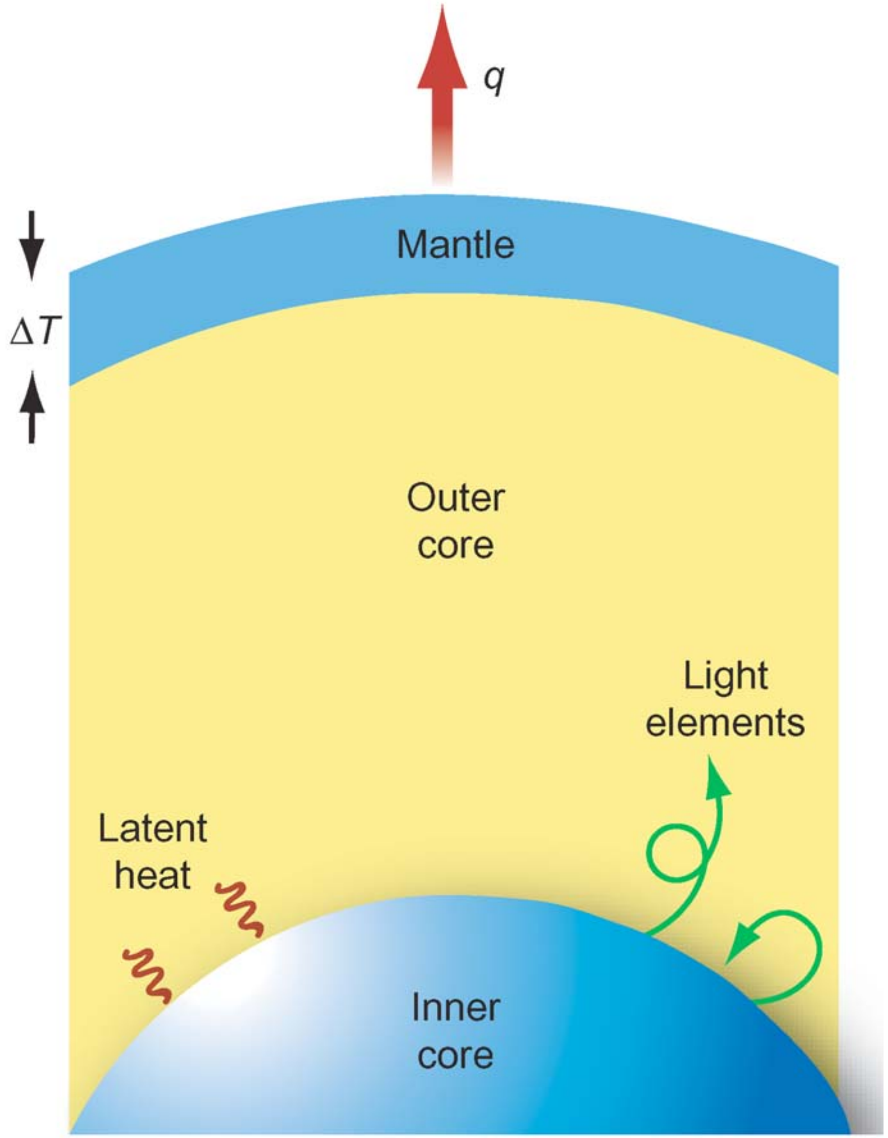


Some numbers

- Earth has possessed magnetic field for at least 3.5Ga
- Timescale for field decay if no convection ~30ka

Preamble – the core

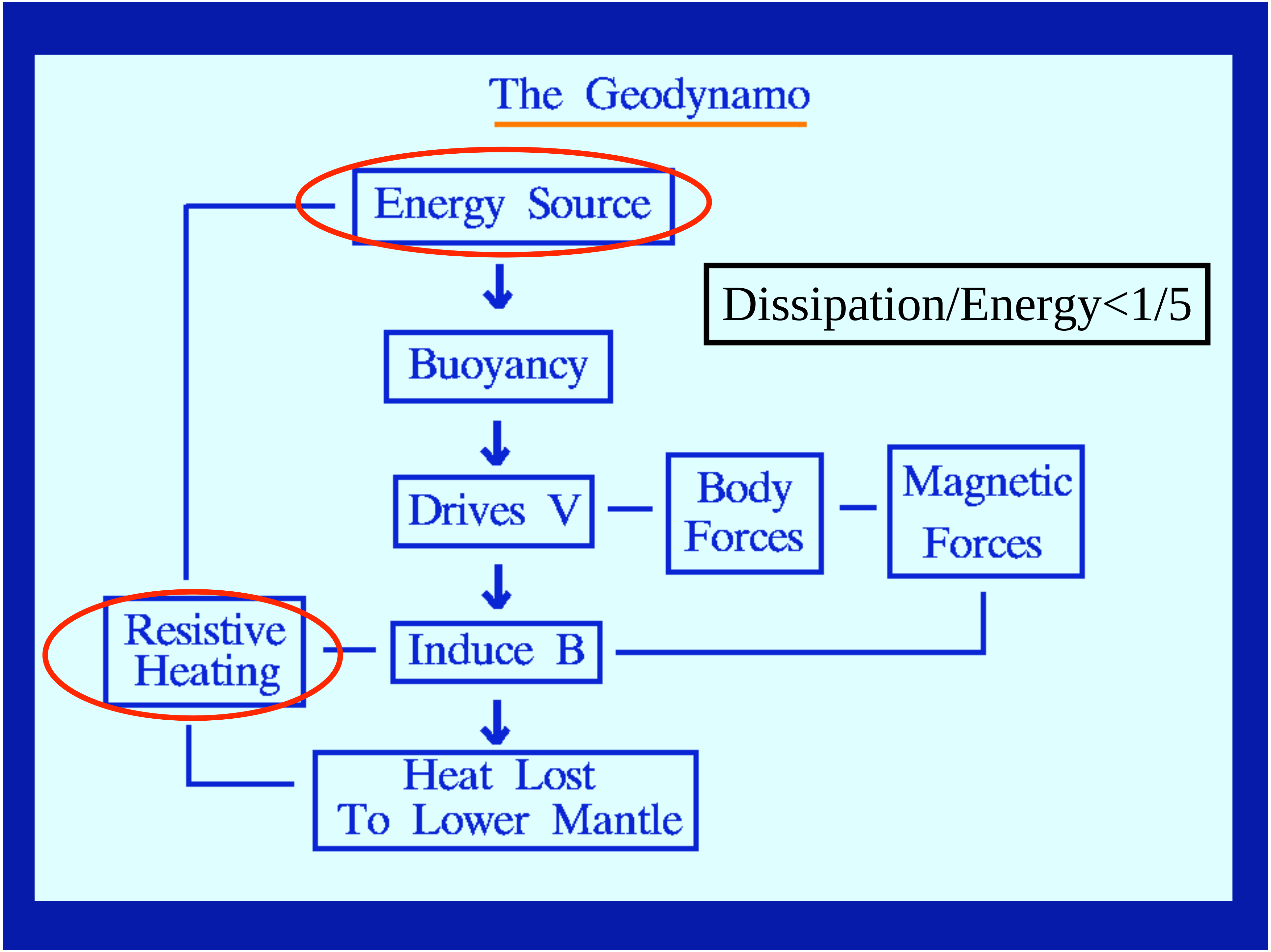
- Pressure at Inner Core Boundary (ICB) ~ 330 GPa
- Density jump at ICB from seismology 0.59g/cm^3 (0.82g/cm^3, Masters & Gubbins, 2003)
- That due to solidification of iron ~ 0.24g/cm^3
- Lighter element released at ICB on freezing
- Melting temperature of pure iron in range is 5000-6000K - difficult to determine experimentally
- Impurities alter melting temperature



Core Heat Budget

- How much heat Q_{CMB} flows through the core-mantle boundary?
- $Q_{CMB} = Q_{ICB} + Q_S + Q_L + Q_G + Q_R$
- Just a heat balance
- Q_{ICB} flux from inner core ~0.3TW
- Q_S secular cooling ~2.3TW
- Q_L latent heat release ~4 TW
- Q_G gravitational energy 0.5-2TW
- Q_R radioactivity 0-??

Roberts et al (2003)


The Geodynamo
Energy Source
Dissipation/Energy<1/5
Buoyancy
Drives V
Body Forces
Magnetic Forces
Resistive Heating
Induce B
Heat Lost To Lower Mantle

Role of the magnetic field

- Magnetic field doesn't enter the energy balance
- Does provide a mechanism of increasing the entropy, by changing mechanical energy to heat
- Numerical models of the geodynamo require energy input at a rate much greater than that due to Ohmic decay (heating due to finite conductivity)
- Dissipation $\Phi = \int \frac{j^2}{\sigma} dV \sim \frac{B^2 V}{L^2 \sigma}$
- B = magnetic field strength
- j = current density
- σ = core electrical conductivity

Thermodynamic Efficiency

- Analysis of entropy shows that heat dissipated by the magnetic field is ~1/5 of the total heat flow
- Powering with latent heat and light element release is most efficient
- Using buoyancy from latent heat and light element release at the ICB, can power dynamo with heat flow of ~6-9TW
- But this results in inner core only being ~1Ga old
- How was the dynamo powered before it existed?

No inner core

- Only secular cooling is available – this is even more inefficient
- Can drive dynamo with 12-20TW of heat flux, but results in extremely high core temperatures at CMB (>4300K) for early Earth (~2.5Ga), enough to melt mantle

Radioactivity - a way out?

- Heat flux = A h + B dT/dt

- Dissipation = C h + D dT/dt

  (=Magnetic energy)

Heat Generation Rate

Cooling Rate

Recap

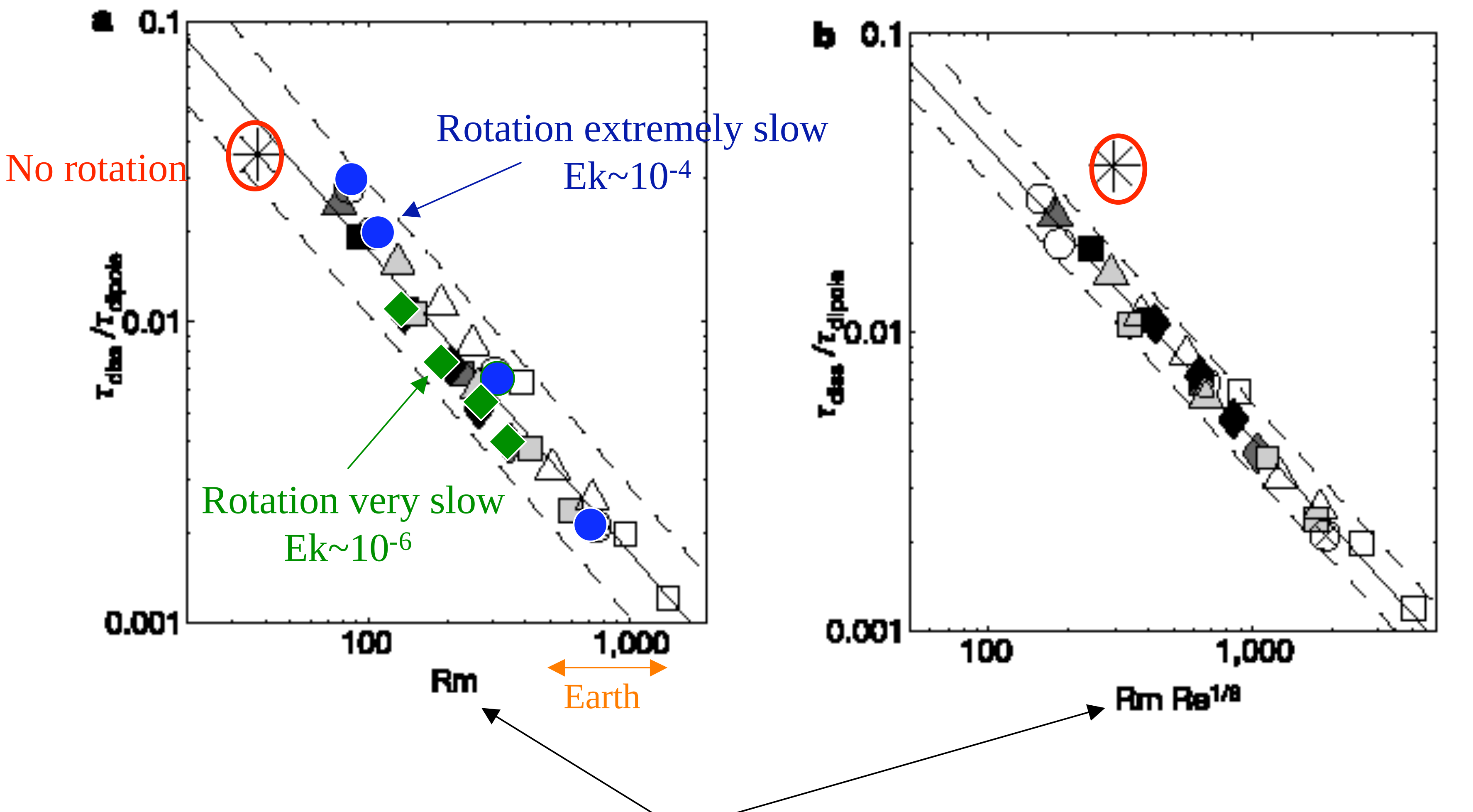
Need to define the energy (entropy) budget of the core, to determine:

- The heat flow into the mantle from the core, and its repercussions for realistic mantle convection models
- Whether the heat flow is below the adiabatic value for the core, and its repercussions for the style of magnetic field generation
- Whether the inner core is an old or young feature, and its repercussions for the history of the Earth and implications for the presence of radioactive elements in the core

Some new estimates of core power requirements

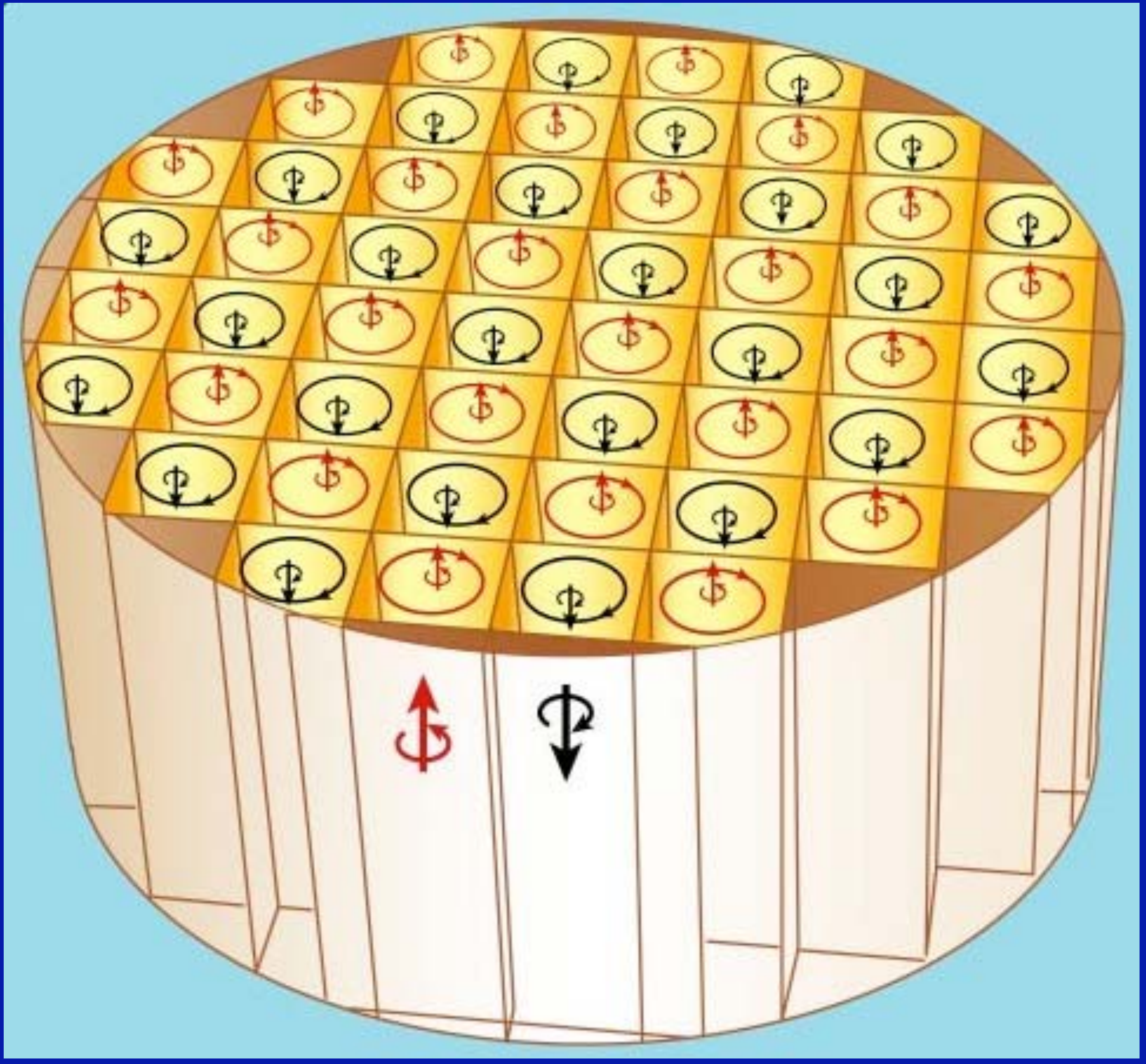
Christensen & Tilgner (2004)

Lots of numerical dynamo results lead to 0.2<Φ<0.5TW



Rm is the strength of the convective driving

The Karlsruhe Dynamo Experiment



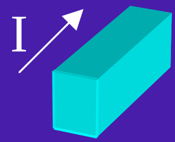
Estimates of the energy budget

- There are varying estimates from numerical dynamos in the range a few tenths to several TW

- A different approach is to use observations of the magnetic field ($1 \leq l \leq 14$) and constraints from an interpretation of the Earth's nutations in terms of magnetic dissipation to place lower bounds on the power, through the solution of variational problems
- The lower bounds arise from idealised solutions that will not in general be realised in nature. However, when the lower bounds approach or exceed other estimates, they become interesting

Heating

Power dissipated Φ as heat in a piece of wire is Voltage$\times$Current, or



$$\Phi = I^2 R$$

In terms of resistivity ρ, conductivity σ and current density $\mathbf{J}$, heating per unit volume ϕ is

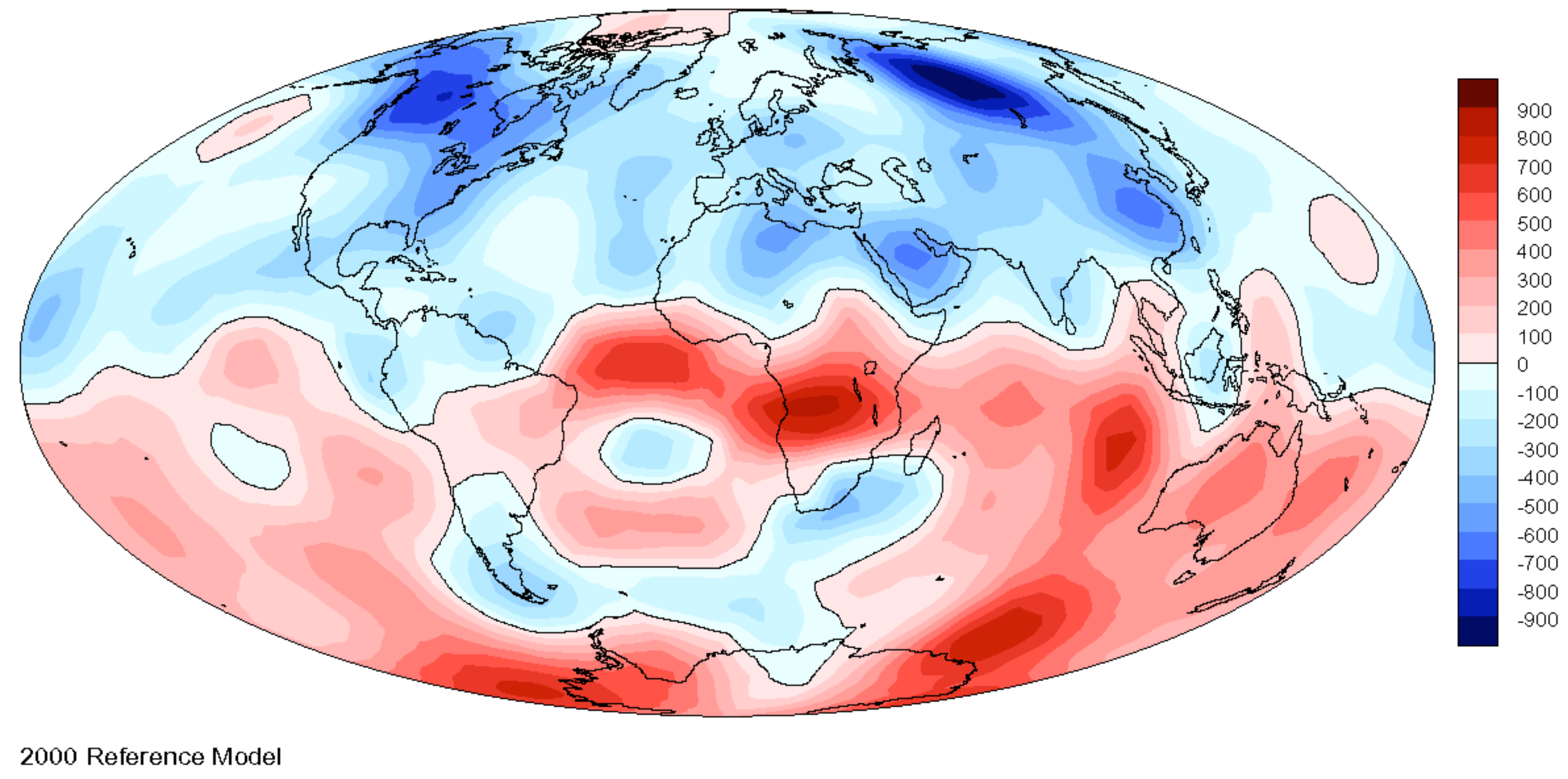
$$\phi = |\mathbf{J}|^2 \rho = |\mathbf{J}|^2 / \sigma$$

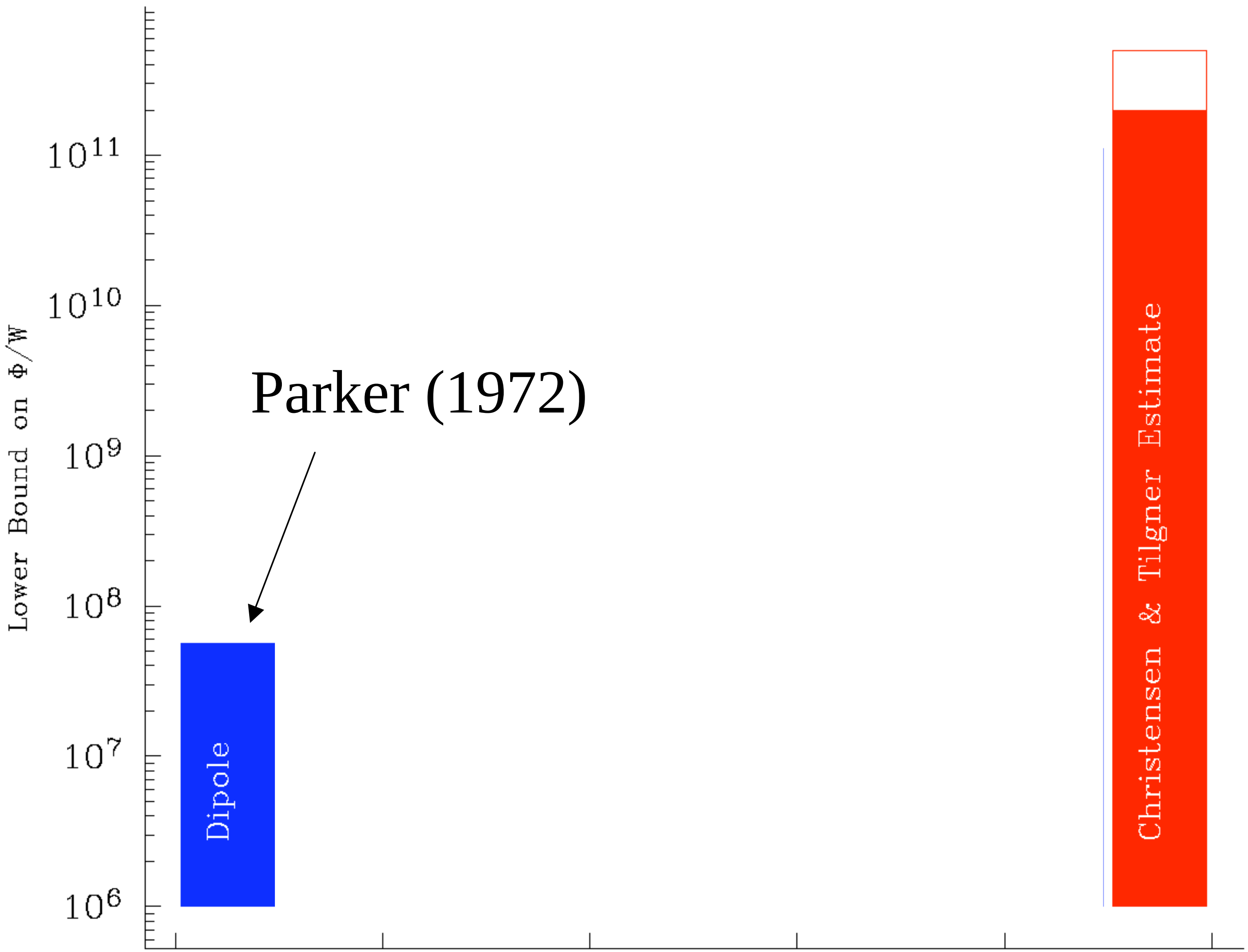
Note that $\nabla \wedge \mathbf{B} = \mu_0 \mathbf{J}$ by Maxwell, so can cast entirely in terms of magnetic field:

$$\Phi = \int_V \frac{|\nabla \wedge \mathbf{B}|^2}{\mu_0^2 \sigma} dV$$

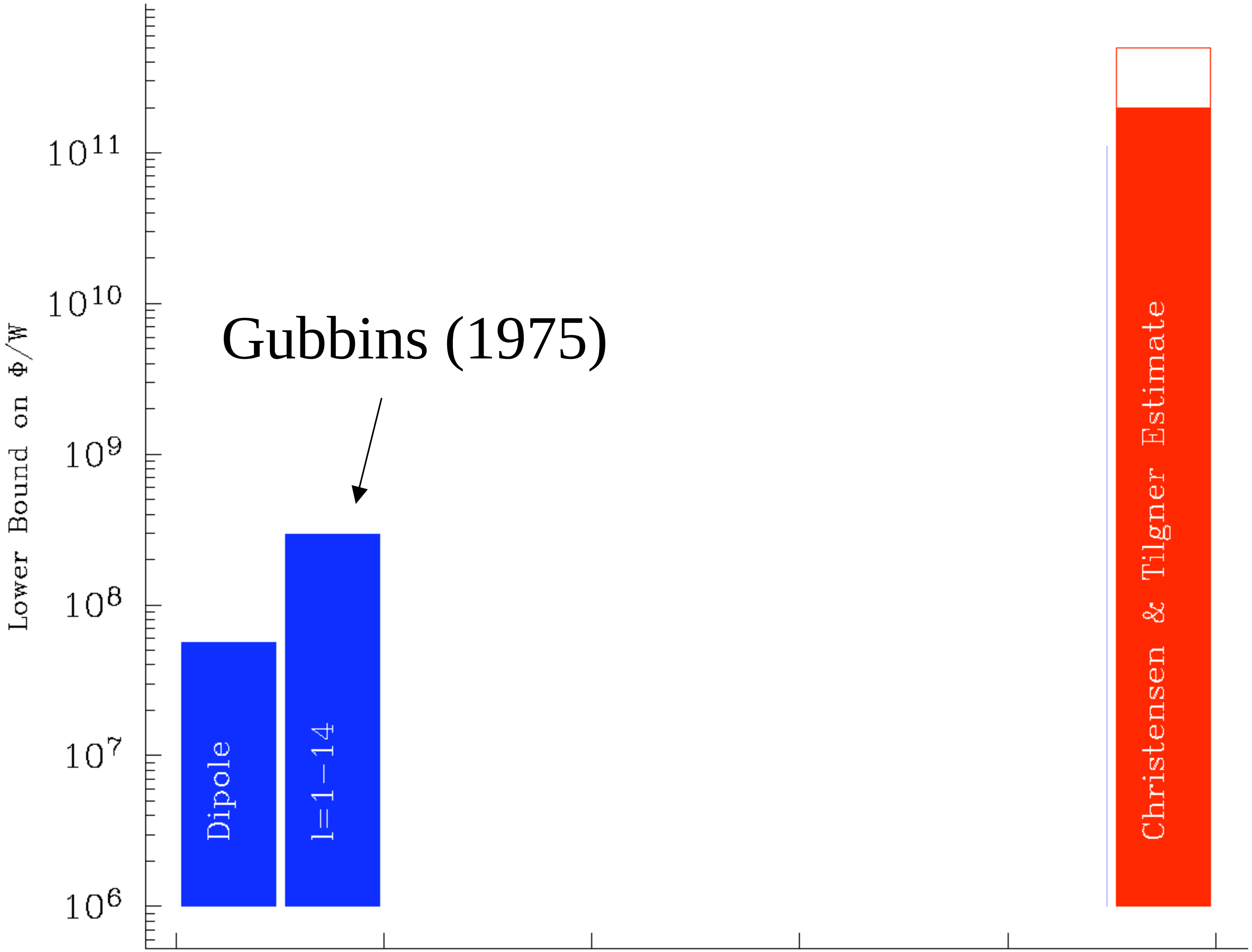
← The key quantity

Oersted satellite data (2000): field at CMB has $(B_r)_{rms} \sim 0.32\text{mT}$

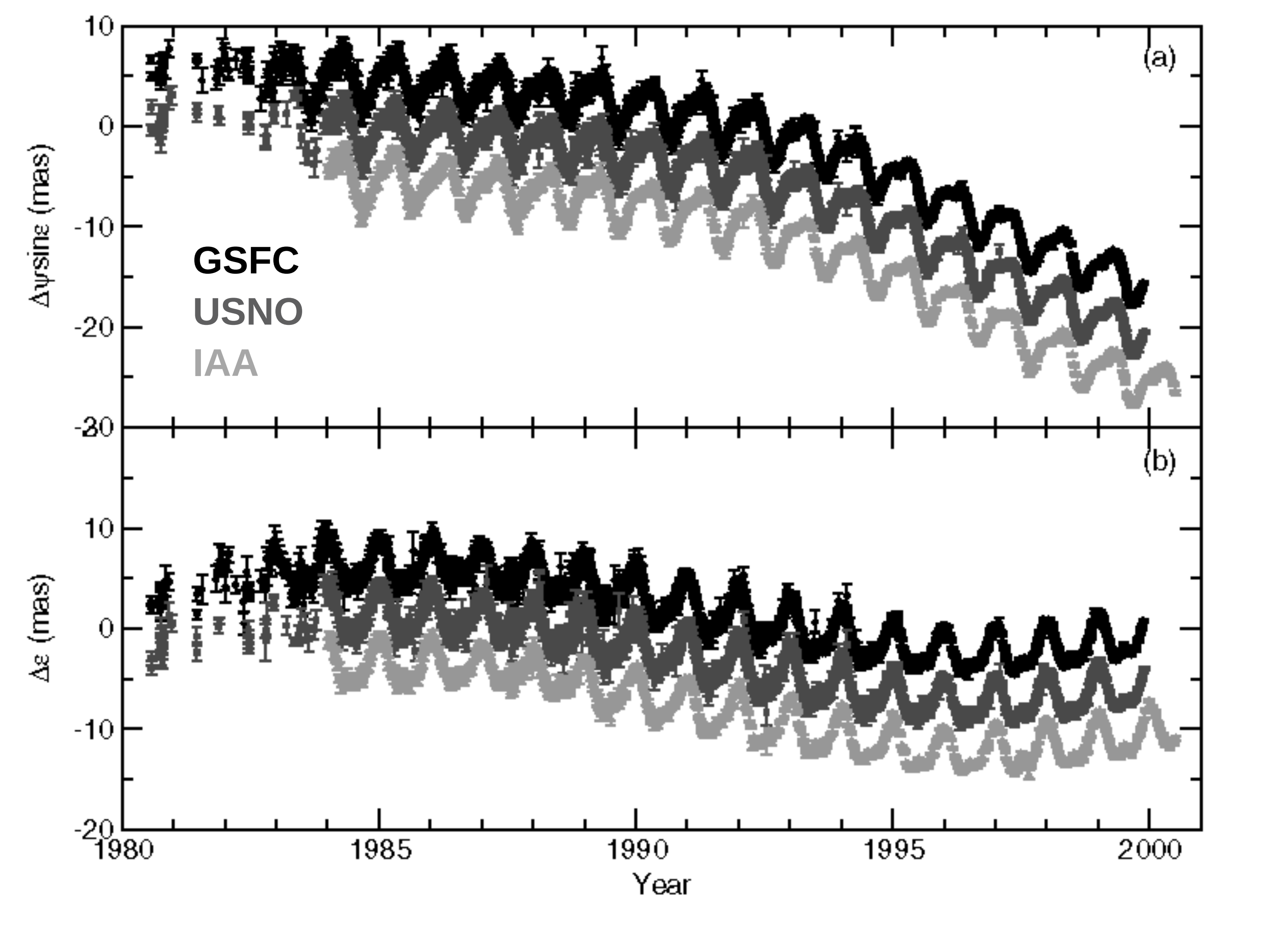




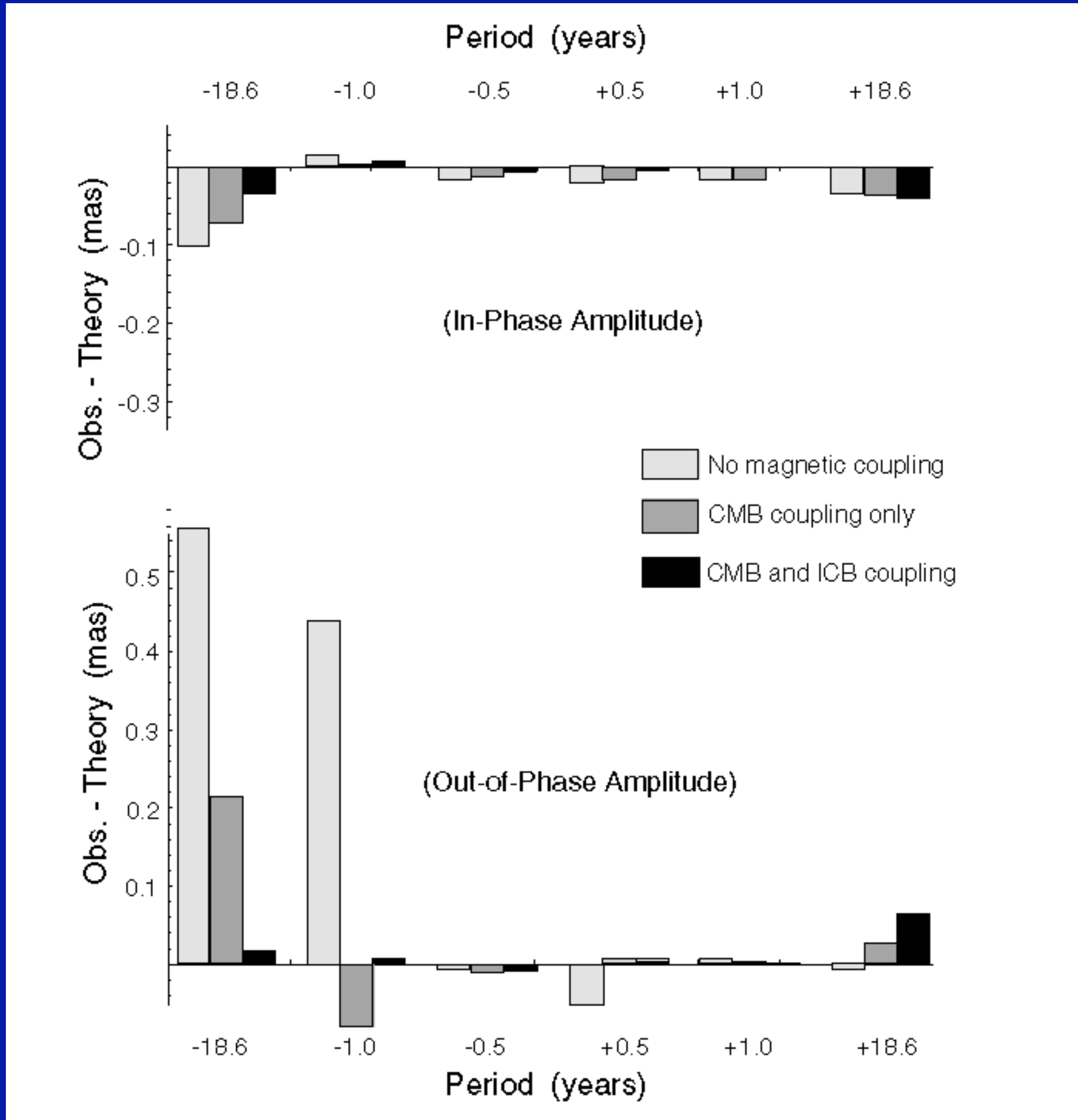
Lower Bound on Φ/W
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
Parker (1972)
Dipole
Christensen & Tilgner Estimate


Gubbins (1975)
Lower Bound on Φ/W
10^11
10^10
10^9
10^8
10^7
10^6
Dipole
l=1−14
Christensen & Tilgner Estimate

Observed nutations (VLBI)



Herring et al (2002)



Buffett et al (2002), assuming a highly conducting D’’

The information from nutations

Nutations give us

- the root-mean-square radial component of magnetic field on the CMB $(B_r)_{\rm rms}(c) \sim 0.69$mT
- the root-mean-square radial component of magnetic field on the ICB $(B_r)_{\rm rms}(b) \sim 7.17$mT
- Compare this with the root-mean-square radial component of the visible magnetic field $0 \le l \le 14 \sim 0.32$mT

These observations contain sensitivity to smaller scale magnetic fields than are visible

A new variational problem

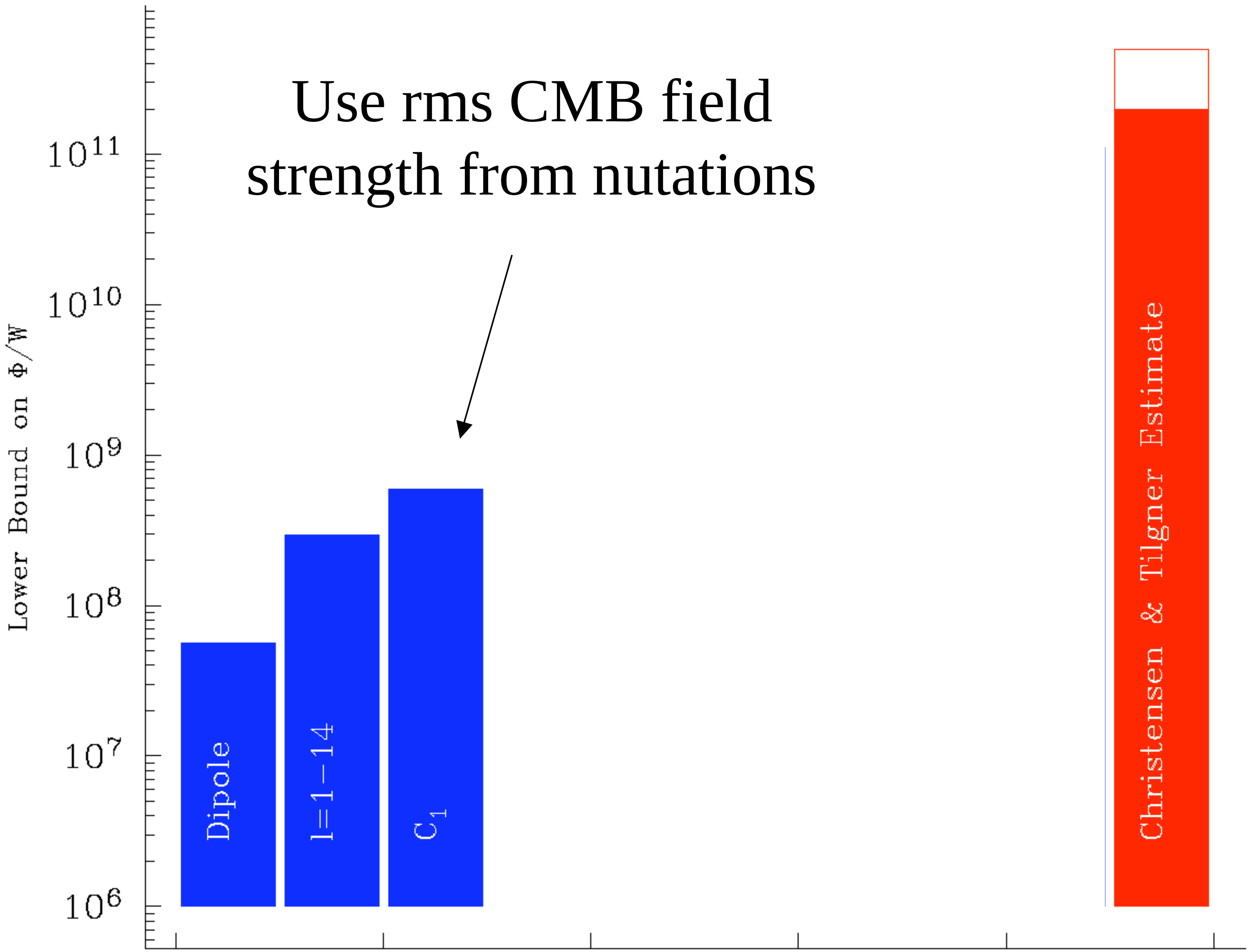
Minimise

$$\text{Ohmic heating} = \int_V \frac{|\mathbf{J}|^2}{\sigma}\, dV$$

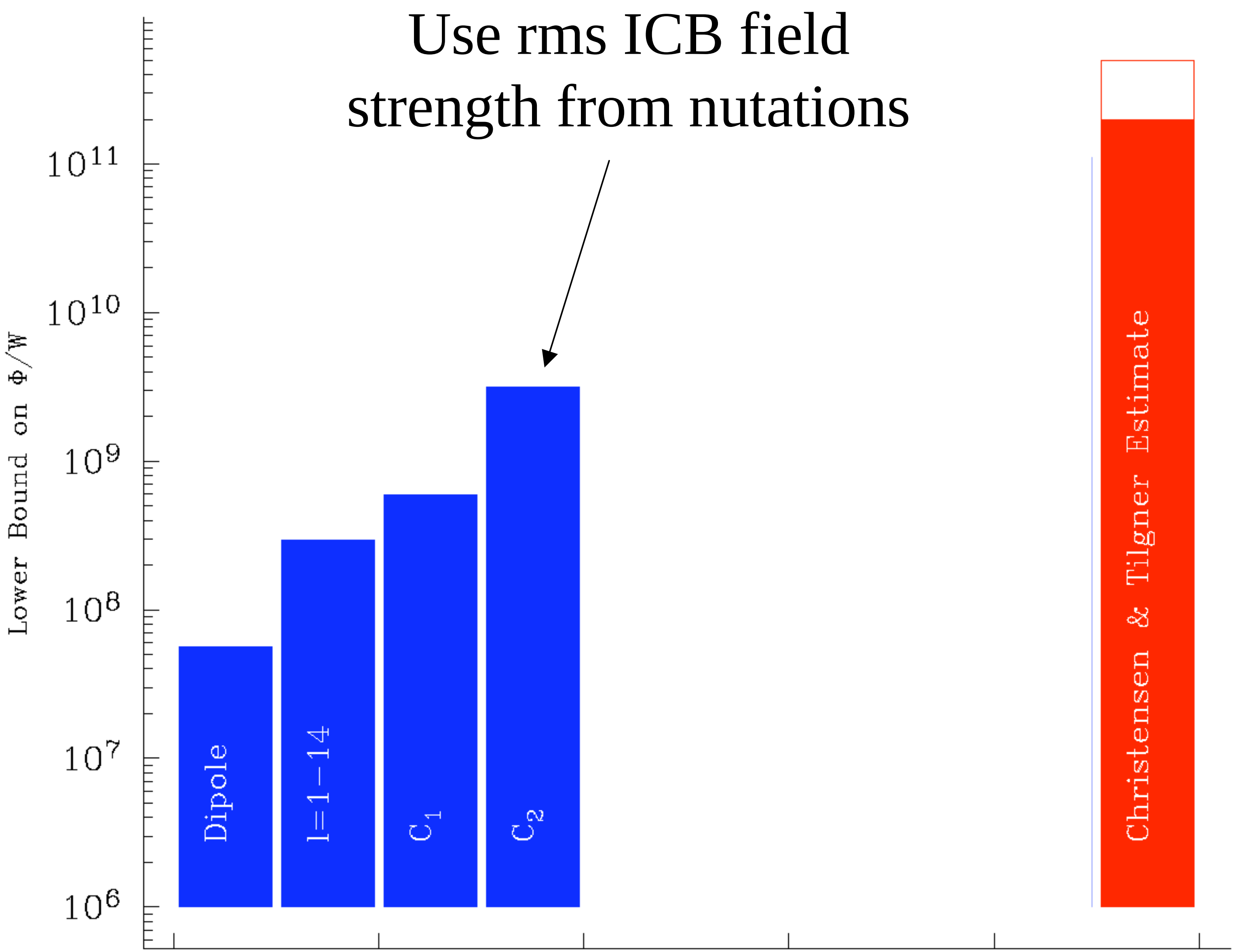
subject to

$$\text{Mean square radial field} = \int_S |B_r|^2\, ds = \text{target}$$

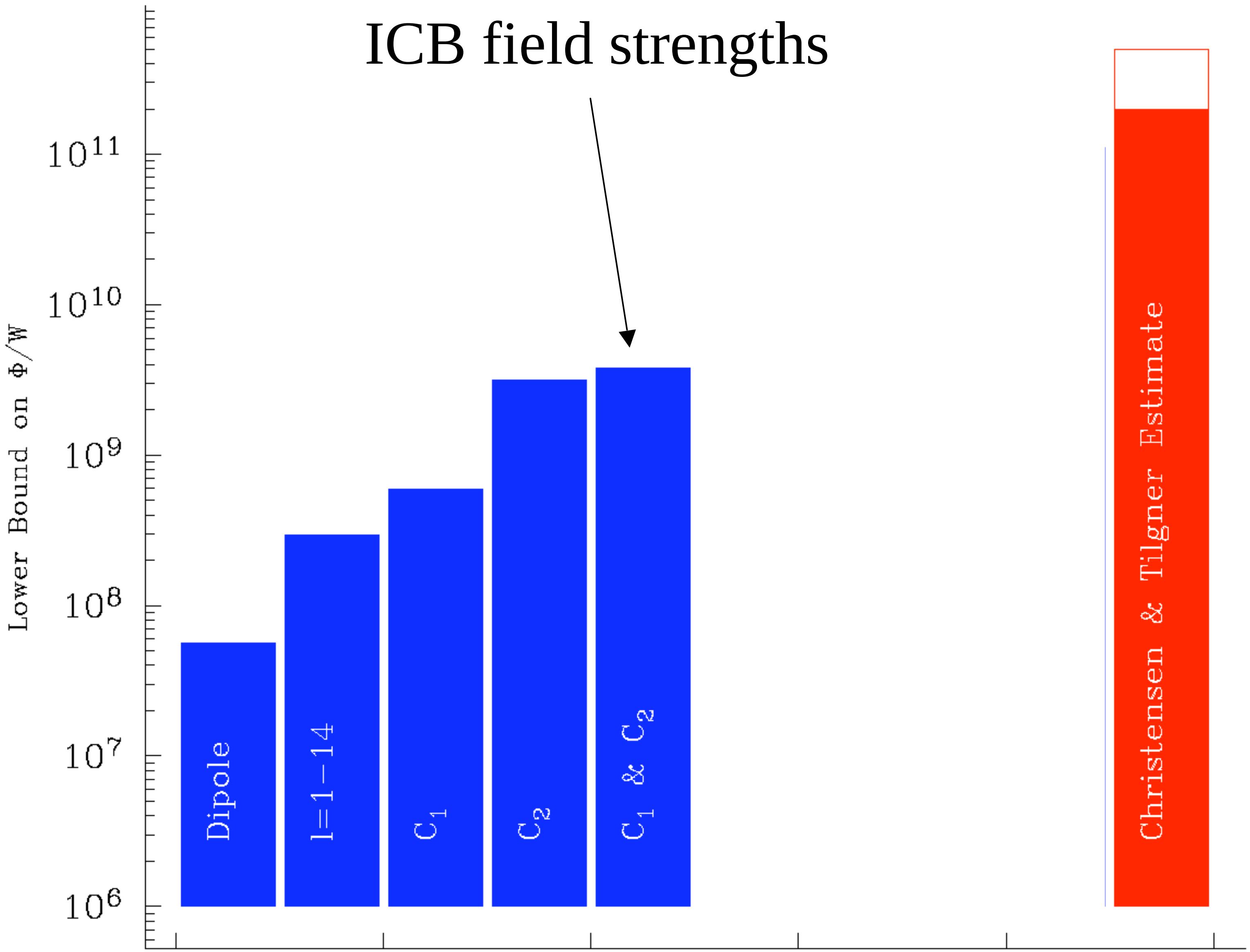
on one or both of CMB and ICB



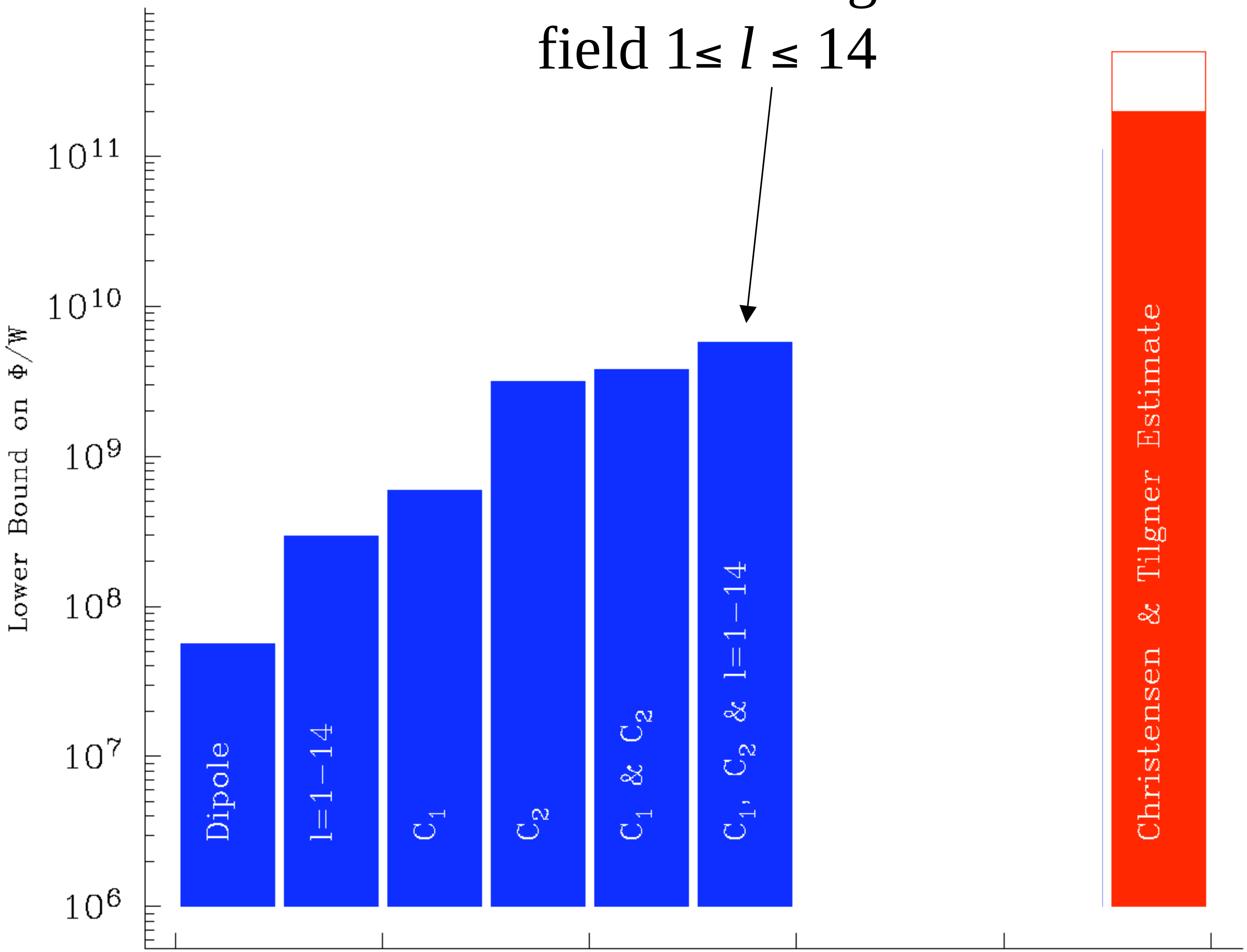
Use rms CMB field
strength from nutations
Lower Bound on Φ/W
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
Dipole
l=1−14
C_1
Christensen & Tilgner Estimate


Use rms ICB field
strength from nutations
Lower Bound on Φ/W
10^11
10^10
10^9
10^8
10^7
10^6
Dipole
l=1−14
C1
C2
Christensen & Tilgner Estimate

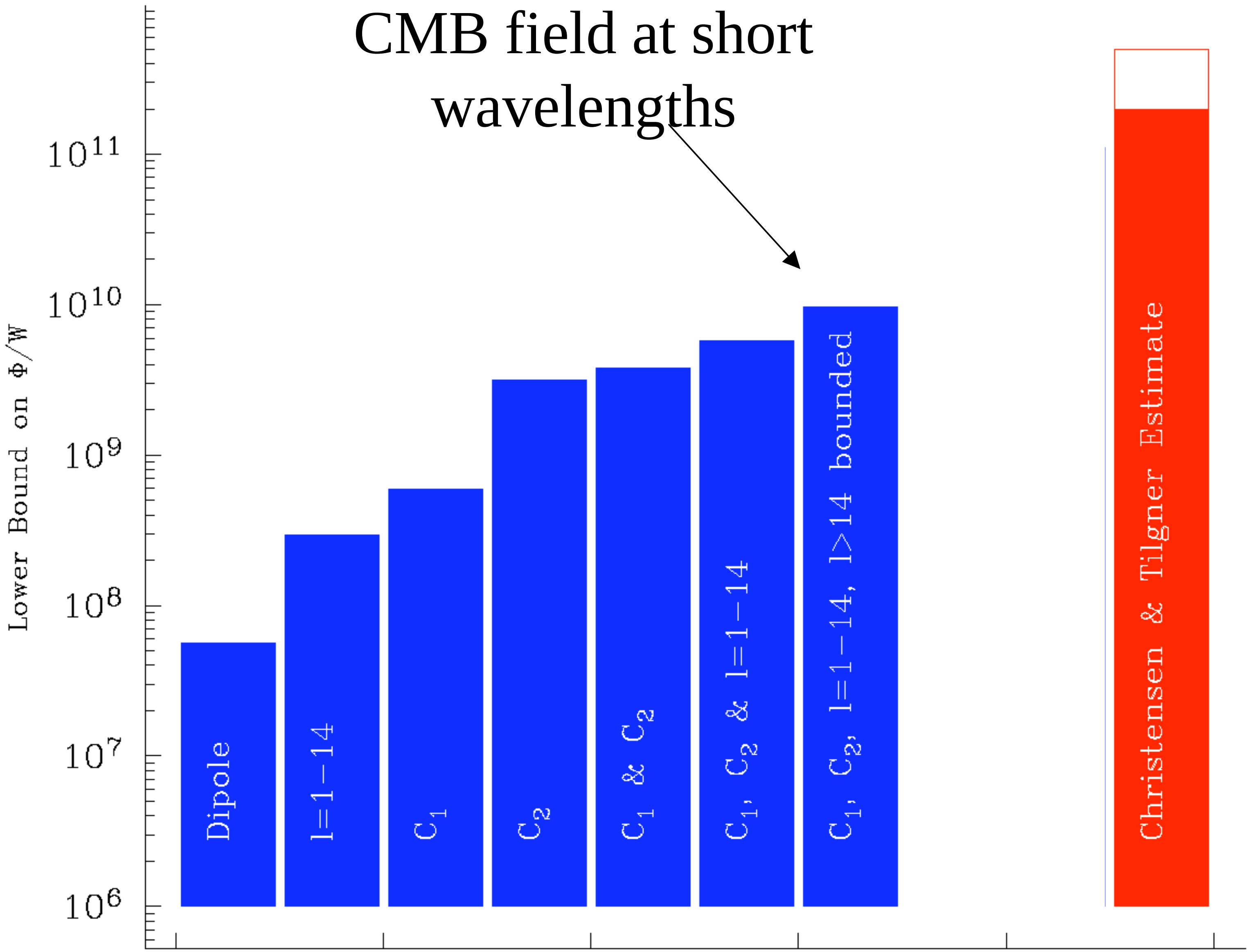
Use both rms CMB & ICB field strengths

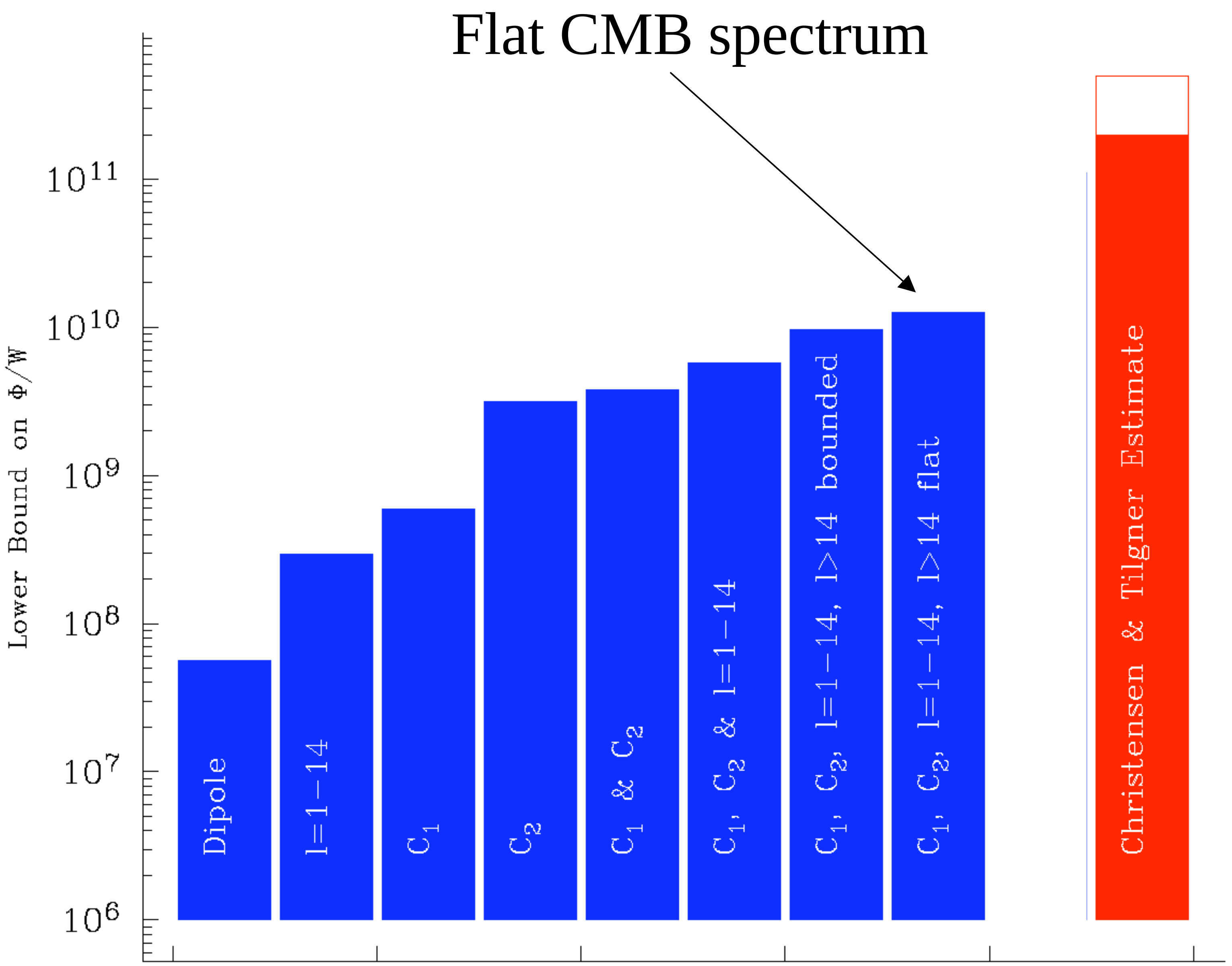


Add observed magnetic field $1 \leq l \leq 14$

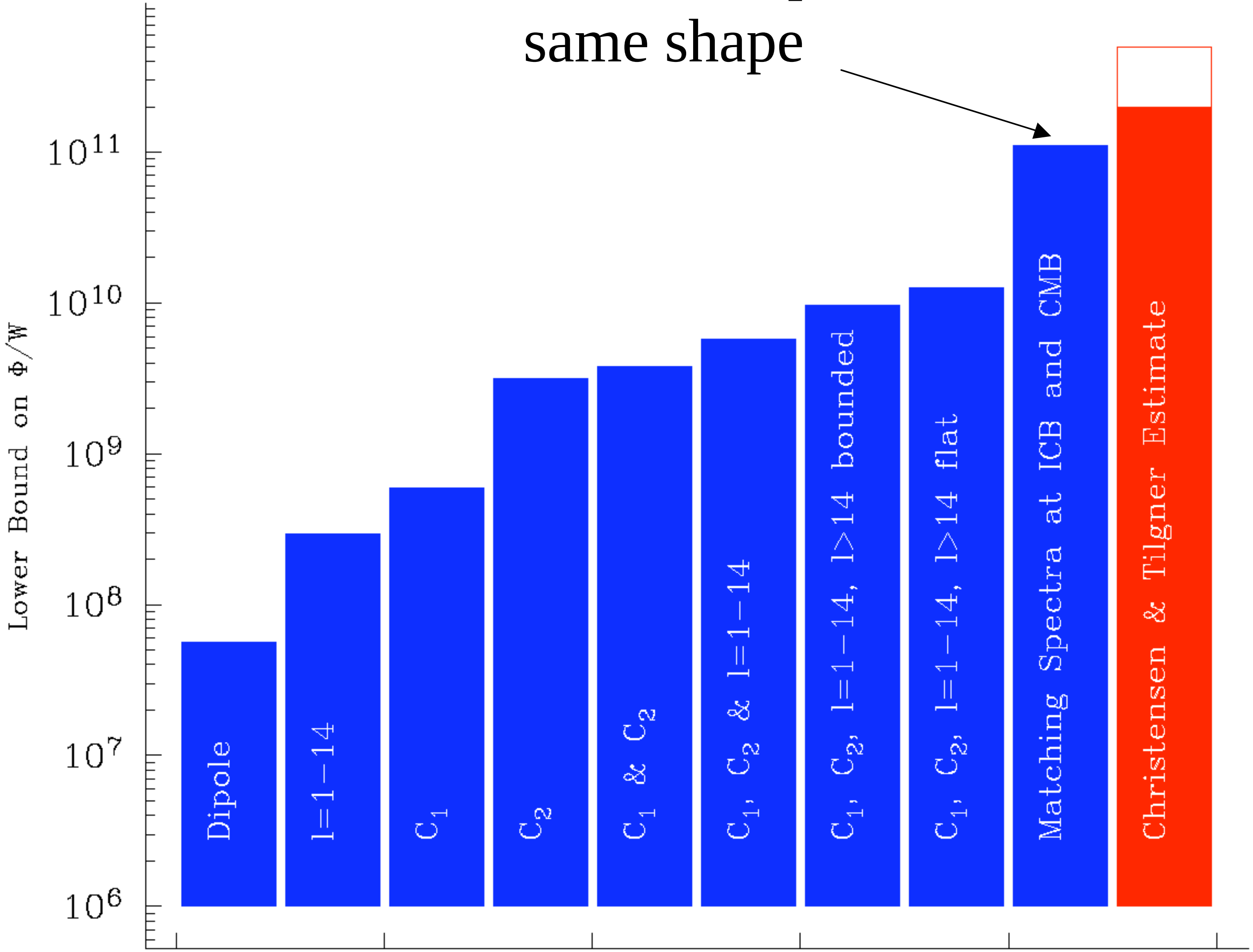


Use upper bounds on CMB field at short wavelengths




Flat CMB spectrum
Lower Bound on Φ/W
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
Dipole
l=1−14
C_1
C_2
C_1 & C_2
C_1, C_2 & l=1−14
C_1, C_2, l=1−14, l>14 bounded
C_1, C_2, l=1−14, l>14 flat
Christensen & Tilgner Estimate

CMB and ICB spectra same shape



Everything so far has simply used classical electromagnetism

Improved estimates – add the dynamics

- Work in progress...

Expected force balance:

Treat core with following simplifications:

- Rossby number $\rightsquigarrow 0$ (inertial terms unimportant)
- Ekman number $\rightsquigarrow 0$ (viscosity unimportant)

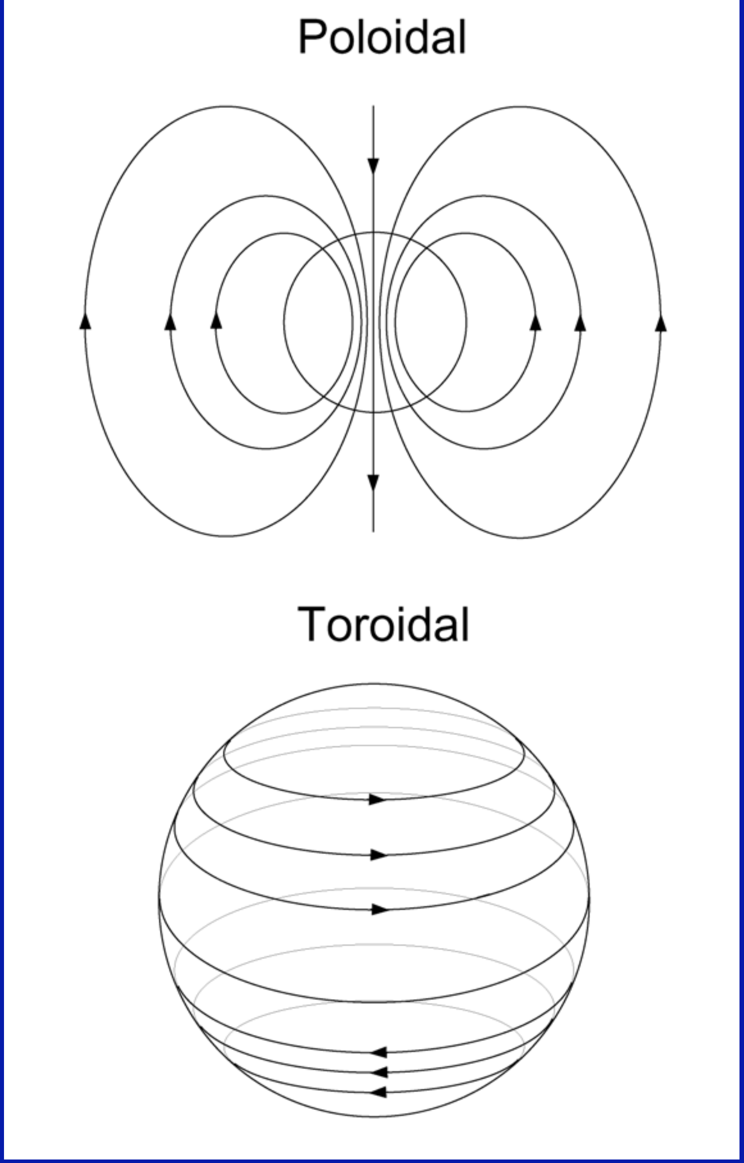
Navier-Stokes equation:

$$\begin{array}{ccccccc} 2\rho\mathbf{\Omega}\wedge\mathbf{v} & = & -\nabla p & + & \mathbf{J}\wedge\mathbf{B} & + & \rho'\mathbf{g} \\ \text{Coriolis force} & = & -\text{Pressure Gradient} & + & \text{Lorentz Force} & + & \text{Buoyancy} \end{array}$$

Leads to estimate of magnetic field strength:

$$B \sim \sqrt{2\rho\Omega\mu_0 V L} \sim 20\text{mT}$$

Considerably stronger than observed poloidal magnetic field at core surface



Poloidal magnetic fields are visible at Earth's surface

Toroidal magnetic fields are not – live entirely within core

The Taylor State

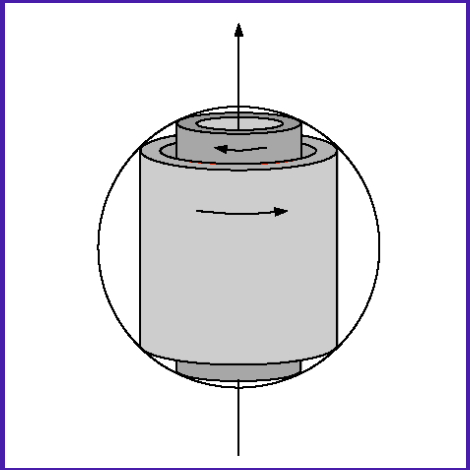
This truly gets us to the Ekman number = 0 limit

Viscosity unimportant

Integrate

$$\begin{array}{ccccccccc} 2\rho\boldsymbol{\Omega} \wedge \mathbf{v} & = & -\nabla p & + & \mathbf{J} \wedge \mathbf{B} & + & \rho' \mathbf{g} \\ \text{Coriolis force} & = & -\text{Pressure Gradient} & + & \text{Lorentz Force} & + & \text{Buoyancy} \end{array}$$

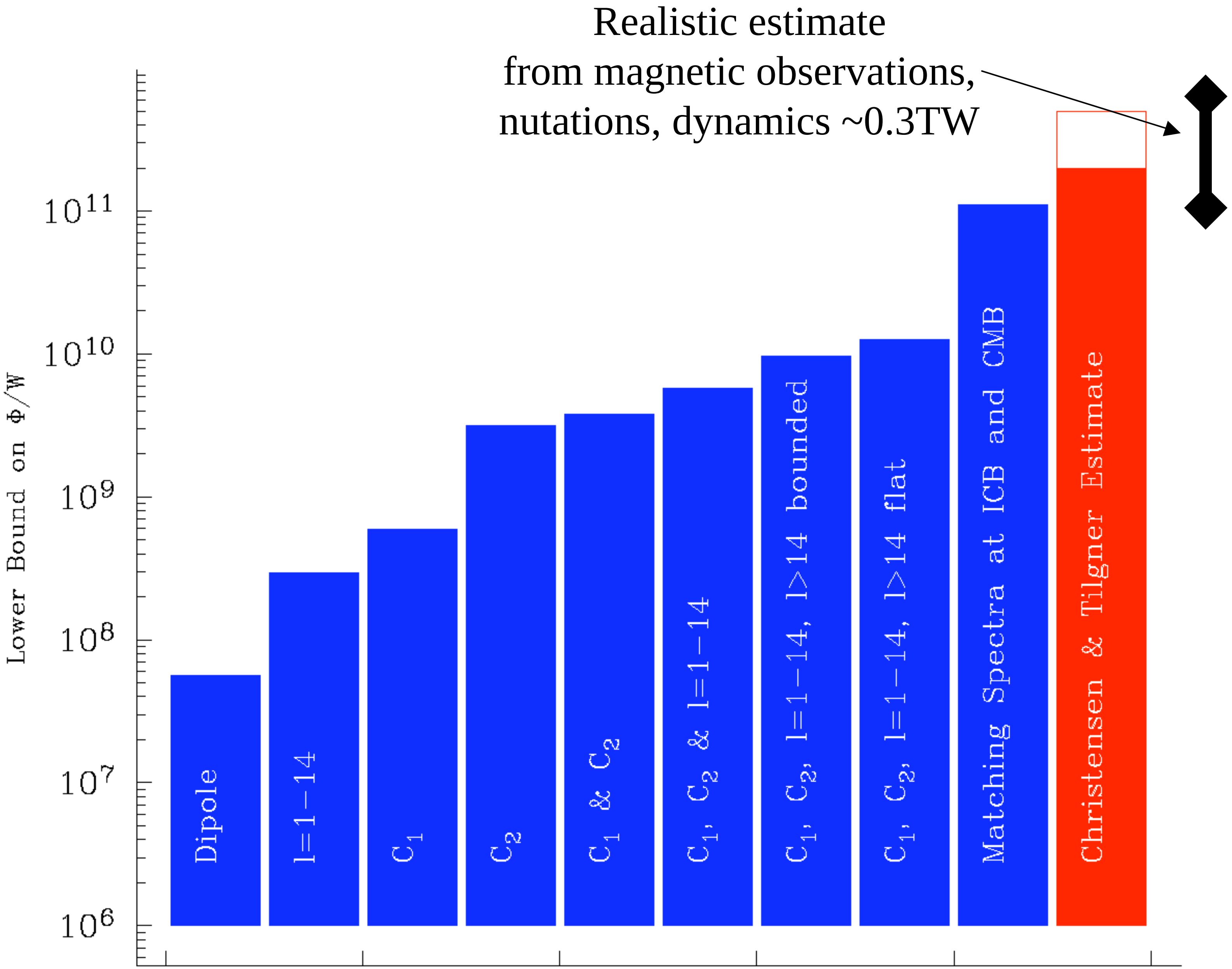
over cylinders coaxial with rotation axis; find



$$T = \int_{C(s)} [\mathbf{J} \wedge \mathbf{B}]_\phi \, d\phi \, dz = 0 \quad \forall s$$

$$T = \int_{C(s)} [\nabla \wedge \mathbf{B} \wedge \mathbf{B}]_\phi \, d\phi \, dz = 0 \quad \forall s$$

This can be imposed on a model for **B**, but note *it involves the toroidal as well as the poloidal field.*

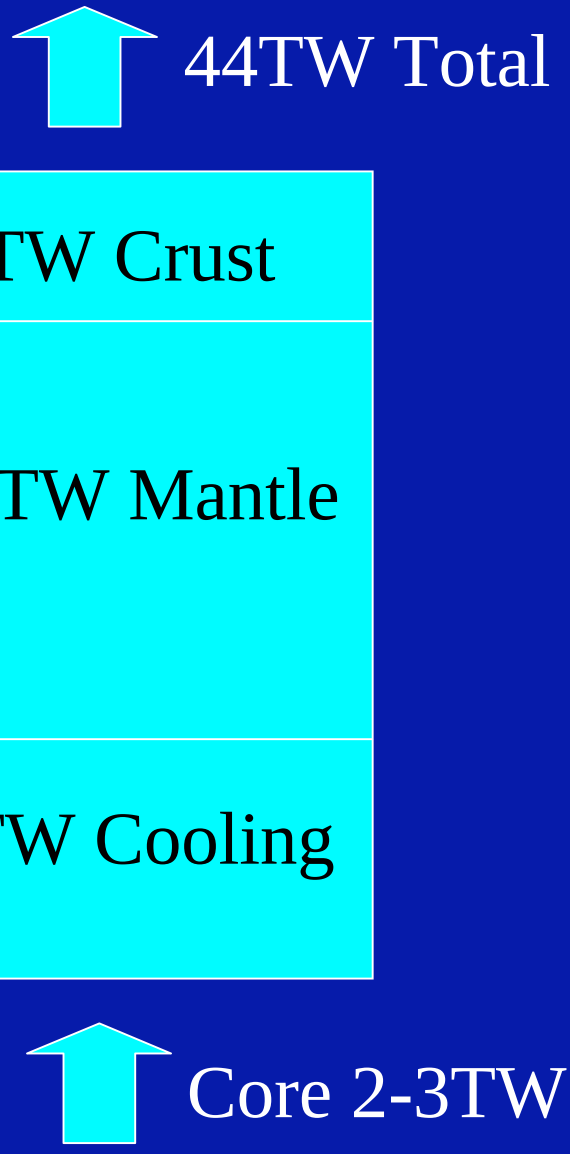


Realistic estimate
from magnetic observations,
nutations, dynamics ~0.3TW
Lower Bound on Φ/W
10^11
10^10
10^9
10^8
10^7
10^6
Dipole
l=1−14
C1
C2
C1 & C2
C1, C2 & l=1−14
C1, C2, l=1−14, l>14 bounded
C1, C2, l=1−14, l>14 flat
Matching Spectra at ICB and CMB
Christensen & Tilgner Estimate

Summary of optimization calculations

Dynamo with sensible dynamics that meets the constraints from nutations and geomagnetic observations plausibly dissipates ~ 0.5TW, with heat flow out of core into mantle of 2-3TW

Agrees with plume flux estimates



The Great Escape

Melting temperatures: 6-12TW

Iron melting poorly known, impurities depress temps

Core adiabat: > 6-8TW

Core subadiabatic

Post perovskite: 9-13TW

Clapeyron slope poorly known, also thermal conductivity

Summary

- Of 44TW heat flowing out of Earth, still unclear where a large fraction originates
- Drawing large amounts from core generates difficulties
- Dynamos are inefficient – especially those powered by cooling rather than crystallization of the inner core
- Easiest to have inner core throughout age of Earth providing compositional buoyancy
- I prefer a dynamo requiring little energy to run, say 0.1-0.8TW (in line with dynamo simulations), and sensible heat flow through the CMB in line with the plume flux (2-4TW)
- This avoids all the horrors of hot thermal histories
- This avoids the need for potassium in the core
- Such low values imply an old inner core (4Ga) and likely subadiabatic heat flow in the core (~3.5TW through CMB), in agreement with the plume heat flux
- Has Earth cooling at around 100K/Ga, compatible with (dubious) observations but at the high end of the range

Summary II

- Post perovskite – major breakthrough. Mechanism for high electrical conductivity in D'' – helps to explain several problems in geodesy
- Need accurate thermal conductivity of pPv to reconcile apparent high temperature gradient
- Need accurate core melting temperatures – difficult due to impurities
- New generation of ab initio calculations may well address these issues