Problems with the energy budget of the Earth - constraints from geodesy and geomagnetism

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Plan

• Heat loss in the mantle

• The geodynamo
  – Background to magnetic field generation and heat transfer

• Core’s energy budget estimate
  – Use observational constraints from geomagnetism & geodesy
Age of ocean floor
44 TW Total Heat Loss
Unwary readers should take warning that ordinary language undergoes modification to a high-pressure form when applied to the interior of the Earth. A few examples of equivalents follow:

<table>
<thead>
<tr>
<th>High Pressure Form</th>
<th>Ordinary Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Certain</td>
<td>Dubious</td>
</tr>
<tr>
<td>Undoubtedly</td>
<td>Perhaps</td>
</tr>
<tr>
<td>Positive proof</td>
<td>Vague suggestion</td>
</tr>
<tr>
<td>Unanswerable argument</td>
<td>Trivial objection</td>
</tr>
<tr>
<td>Pure iron</td>
<td>Uncertain mixture of all the elements</td>
</tr>
</tbody>
</table>
Heat loss from the Earth

- Generally accepted that global heat loss through Earth’s surface is ~ 44TW
- Wide range of estimates for heat generation from mantle radioactivity and secular cooling
- Bulk Silicate Earth of McDonough & Sun (1995) based on CI chondrites generates 20TW from radioactivity of mantle [240ppm K]
- 8TW from radioactivity of crust
- Geological arguments have Earth cooling by 50-100K/Ga => 7.5-15TW lost by cooling
- Associating hotspot flux with heat loss from core gives 2-3TW
- **There is a wide range of possibilities**
44TW Total

8TW Crust

~20TW Mantle

7-15TW Cooling

??Core 1-9TW

Plumes 2-3TW ?
Temperature jump across D'' really depends on knowing core melting temperature.
Thermal boundary layer at base of mantle

- Layer ~200km thick
- Temperature drop 1000-2000K
- Heat flux drawn from core by conduction ~ 6-12TW
The inner core has been growing throughout time.
Adiabatic gradient -> Heat flux

Wiedemann-Franz Law (metals)

\[ k = LT\sigma \]

- \( k \) = thermal conductivity
- \( \sigma \) = electrical conductivity
- \( L \) = Lorentz number (very constant 2.45 E-8)

Gives \( k = 38 \text{W/m/K} \) from electronic part

Lattice contribution makes total \( \sim 45 \text{W/m/K} \)
Adiabatic gradients

- Ab initio calculations of the properties of iron give the adiabatic gradient in the core.
- Results predict that roughly 8 TW of heat flows down the adiabatic gradient (Gubbins et al 2003) and out of the core, if the whole core convects thermally.
- Roberts et al (2003) give 6 TW.
The boundary layer
Perovskite/post-Perovskite transition at ~100 GPa

- Transition in MgSiO$_3$ explains the seismic region D$''$ - discontinuities, anisotropy, maybe ultra low velocity zones
- Fe can enter readily (Caracas & Cohen)
- FeSiO$_3$ is metallic (high electrical conductivity) and stable at pPV pressures
- Enhanced thermal conductivity?
Hernlund et al (2005) use double crossing of post-Perovskite stability field to indicate temperature gradient.

- Suggest 9-13TW flowing from the core.
- Depends on thermal conductivity – Hofmeister gives half the above figure.
Recap:
Melting temperatures: 6-12TW
Core adiabat: > 6-8TW
Post perovskite: 9-13TW

Various lines suggest we might be extracting 8-12TW from core

WHAT’S THE PROBLEM?
Core Magnetic Field
Core Fluid Flow
Inner core growth

Radius ratio

Heat flux 12TW

Heat flux 8TW

Current inner core size

Time since solidification onset/Ga
The magnetic field strength through time

![Graph showing magnetic field strength over time](image)

**Graph Details:**
- **Y-axis:** VDM, $10^{22}$ A m$^2$
- **X-axis:** Age, Ma

The graph illustrates the change in magnetic field strength over time, with age measured in millions of years (Ma). The data points and error bars suggest variability in the field strength at different ages.
Some numbers

• Earth has possessed magnetic field for at least 3.5 Ga

• Timescale for field decay if no convection ~30 ka
Preamble – the core

- Pressure at Inner Core Boundary (ICB) ~ 330 GPa
- Density jump at ICB from seismology 0.59g/cm³ (0.82g/cm³, Masters & Gubbins, 2003)
- That due to solidification of iron ~ 0.24g/cm³
- Lighter element released at ICB on freezing
- Melting temperature of pure iron in range is 5000-6000K - difficult to determine experimentally
- Impurities alter melting temperature
Core Heat Budget

• How much heat $Q_{CMB}$ flows through the core-mantle boundary?

• $Q_{CMB} = Q_{ICB} + Q_{S} + Q_{L} + Q_{G} + Q_{R}$

• Just a heat balance

• $Q_{ICB}$ flux from inner core $\sim 0.3$TW

• $Q_{S}$ secular cooling $\sim 2.3$TW

• $Q_{L}$ latent heat release $\sim 4$TW

• $Q_{G}$ gravitational energy $0.5-2$TW

• $Q_{R}$ radioactivity $0-??$

The Geodynamo

- Energy Source
  - Buoyancy
    - Drives V
      - Induce B
        - Heat Lost To Lower Mantle
      - Body Forces
        - Magnetic Forces
      - Dissipation/Energy < 1/5

- Resistive Heating
Role of the magnetic field

- Magnetic field doesn’t enter the energy balance
- Does provide a mechanism of increasing the entropy, by changing mechanical energy to heat
- Numerical models of the geodynamo require energy input at a rate much greater than that due to Ohmic decay (heating due to finite conductivity)

\[
\text{Dissipation } \Phi = \int \frac{j^2}{\sigma} \, dV \sim \frac{B^2 V}{L^2 \sigma}
\]

- \( B \) = magnetic field strength
- \( j \) = current density
- \( \sigma \) = core electrical conductivity
Thermodynamic Efficiency

• Analysis of entropy shows that heat dissipated by the magnetic field is ~1/5 of the total heat flow
• Powering with latent heat and light element release is most efficient
• Using buoyancy from latent heat and light element release at the ICB, can power dynamo with heat flow of ~6-9TW
• But this results in inner core only being ~1Ga old
• How was the dynamo powered before it existed?
No inner core

- Only secular cooling is available – this is even more inefficient
- Can drive dynamo with 12-20TW of heat flux, but results in extremely high core temperatures at CMB (>4300K) for early Earth (~2.5Ga), enough to melt mantle
Radioactivity - a way out?

- Heat flux \( = A \ h + B \ \frac{dT}{dt} \)

- Dissipation \( = C \ h + D \ \frac{dT}{dt} \)

\( (= \text{Magnetic energy}) \)

Heat \hspace{2cm} \text{Cooling Rate}

Generation Rate
Recap

Need to define the energy (entropy) budget of the core, to determine:

- The heat flow into the mantle from the core, and its repercussions for realistic mantle convection models
- Whether the heat flow is below the adiabatic value for the core, and its repercussions for the style of magnetic field generation
- Whether the inner core is an old or young feature, and its repercussions for the history of the Earth and implications for the presence of radioactive elements in the core
Some new estimates of core power requirements
Christensen & Tilgner (2004)
Lots of numerical dynamo results lead to $0.2 < \Phi < 0.5 \text{TW}$

Rm is the strength of the convective driving

No rotation
Rotation very slow $E_k \approx 10^{-6}$
Rotation extremely slow $E_k \approx 10^{-4}$

Earth

$R_m$ is the strength of the convective driving
The Karlsruhe Dynamo Experiment
Estimates of the energy budget

- There are varying estimates from numerical dynamos in the range a few tenths to several TW.

- A different approach is to use observations of the magnetic field ($1 \leq l \leq 14$) and constraints from an interpretation of the Earth’s nutations in terms of magnetic dissipation to place lower bounds on the power, through the solution of variational problems.

- The lower bounds arise from idealised solutions that will not in general be realised in nature. However, when the lower bounds approach or exceed other estimates, they become interesting.
Heating

Power dissipated $\Phi$ as heat in a piece of wire is Voltage $\times$ Current, or

$$\Phi = I^2 R$$

In terms of resistivity $\rho$, conductivity $\sigma$ and current density $J$, heating per unit volume $\phi$ is

$$\phi = |J|^2 \rho = |J|^2 / \sigma$$

Note that $\nabla \wedge B = \mu_0 J$ by Maxwell, so can cast entirely in terms of magnetic field:

$$\Phi = \int_V \frac{|\nabla \wedge B|^2}{\mu_0 \sigma} dV \quad \text{← The key quantity}$$
Oersted satellite data (2000): field at CMB has \((B_r)_{\text{rms}} \sim 0.32\text{mT}\)
Parker (1972)

Lower Bound on $\phi / w$

- Dipole
- Christensen & Tilgner Estimate
Gubbins (1975)
Observed nutations (VLBI)

Herring et al (2002)
Buffett et al (2002), assuming a highly conducting D’’
The information from nutations

Nutations give us

- the root-mean-square radial component of magnetic field on the CMB \((B_r)_{\text{rms}}(c) \sim 0.69\text{mT}\)
- the root-mean-square radial component of magnetic field on the ICB \((B_r)_{\text{rms}}(b) \sim 7.17\text{mT}\)
- Compare this with the root-mean-square radial component of the visible magnetic field \(0 \leq l \leq 14 \sim 0.32\text{mT}\)

These observations contain sensitivity to smaller scale magnetic fields than are visible
A new variational problem

Minimise

Ohmic heating = \int_{V} \frac{|J|^2}{\sigma} dV

subject to

Mean square radial field = \int_{S} |B_r|^2 ds = target

on one or both of CMB and ICB
Use rms CMB field strength from nutations
Use rms ICB field strength from nutations
Use both rms CMB & ICB field strengths
Add observed magnetic field $1 \leq l \leq 14$
Use upper bounds on CMB field at short wavelengths.
CMB and ICB spectra same shape
Everything so far has simply used classical electromagnetism

Improved estimates – add the dynamics

• Work in progress...
Expected force balance:

Treat core with following simplifications:

- Rossby number \( \sim 0 \) (inertial terms unimportant)
- Ekman number \( \sim 0 \) (viscosity unimportant)

Navier-Stokes equation:

\[
2\rho \Omega \wedge \mathbf{v} = -\nabla p + \mathbf{J} \wedge \mathbf{B} + \rho' g
\]

Coriolis force = Pressure Gradient + Lorentz Force + Buoyancy

Leads to estimate of magnetic field strength:

\[
B \sim \sqrt{2\rho \Omega \mu_0 V L} \sim 20 \text{mT}
\]

Considerably stronger than observed poloidal magnetic field at core surface
Poloidal magnetic fields are visible at Earth’s surface

Toroidal magnetic fields are not – live entirely within core
The Taylor State

This truly gets us to the Ekman number $= 0$ limit

Integrate

$$2 \rho \Omega \wedge \mathbf{v} = - \nabla p + \mathbf{J} \wedge \mathbf{B} + \rho' \mathbf{g}$$

Coriolis force $=$ Pressure Gradient + Lorentz Force + Buoyancy

over cylinders coaxial with rotation axis; find

$$T = \int_{C(s)} [\mathbf{J} \wedge \mathbf{B}]_\phi \, d\phi \, dz = 0 \quad \forall s$$

$$T = \int_{C(s)} [\nabla \wedge \mathbf{B} \wedge \mathbf{B}]_\phi \, d\phi \, dz = 0 \quad \forall s$$

This can be imposed on a model for $\mathbf{B}$, but note it involves the toroidal as well as the poloidal field.
Realistic estimate from magnetic observations, nutations, dynamics ~0.3TW
Summary of optimization calculations

Dynamo with sensible dynamics that meets the constraints from nutations and geomagnetic observations plausibly dissipates ~ 0.5TW, with heat flow out of core into mantle of 2-3TW

Agrees with plume flux estimates

<table>
<thead>
<tr>
<th>Category</th>
<th>Power (TW)</th>
</tr>
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<tbody>
<tr>
<td>Core</td>
<td>2-3</td>
</tr>
<tr>
<td>Mantle</td>
<td>~20</td>
</tr>
<tr>
<td>Cooling</td>
<td>14</td>
</tr>
<tr>
<td>Crust</td>
<td>8</td>
</tr>
</tbody>
</table>

44TW Total
The Great Escape

Melting temperatures: 6-12TW
Iron melting poorly known, impurities depress temps

Core adiabat: > 6-8TW
Core subadiabatic

Post perovskite: 9-13TW
Clapeyron slope poorly known, also thermal conductivity
Summary

- Of 44TW heat flowing out of Earth, still unclear where a large fraction originates.
- Drawing large amounts from core generates difficulties.
- Dynamos are inefficient – especially those powered by cooling rather than crystallization of the inner core.
- Easiest to have inner core throughout age of Earth providing compositional buoyancy.
- I prefer a dynamo requiring little energy to run, say 0.1-0.8TW (in line with dynamo simulations), and sensible heat flow through the CMB in line with the plume flux (2-4TW).
- This avoids all the horrors of hot thermal histories.
- This avoids the need for potassium in the core.
- Such low values imply an old inner core (4Ga) and likely subadiabatic heat flow in the core (~3.5TW through CMB), in agreement with the plume heat flux.
- Has Earth cooling at around 100K/Ga, compatible with (dubious) observations but at the high end of the range.
Summary II

• Post perovskite – major breakthrough. Mechanism for high electrical conductivity in D’’ – helps to explain several problems in geodesy
• Need accurate thermal conductivity of pPv to reconcile apparent high temperature gradient
• Need accurate core melting temperatures – difficult due to impurities
• New generation of ab initio calculations may well address these issues