Pattern formation: building mazes with grains

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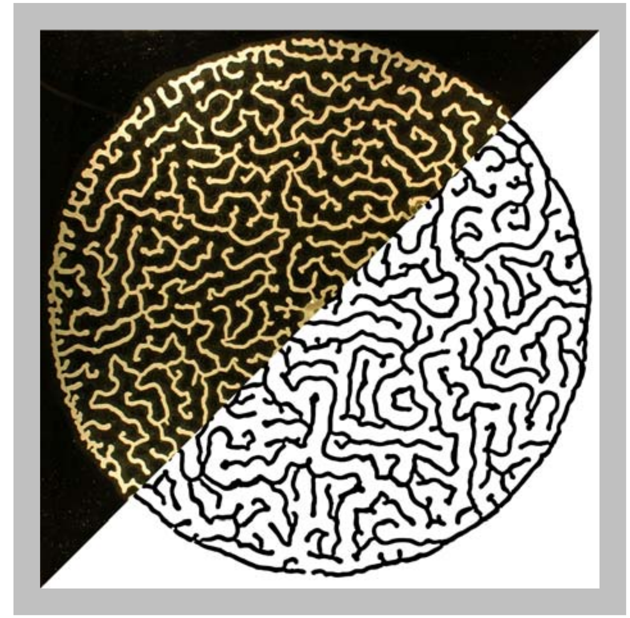
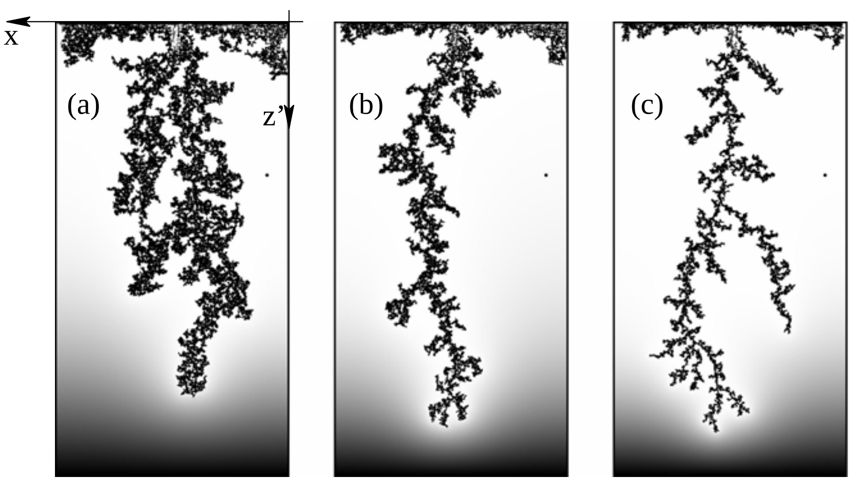
- Henning Arendt Knudsen
- Knut Jørgen Måløy
- Eirik Grude Flekkøy





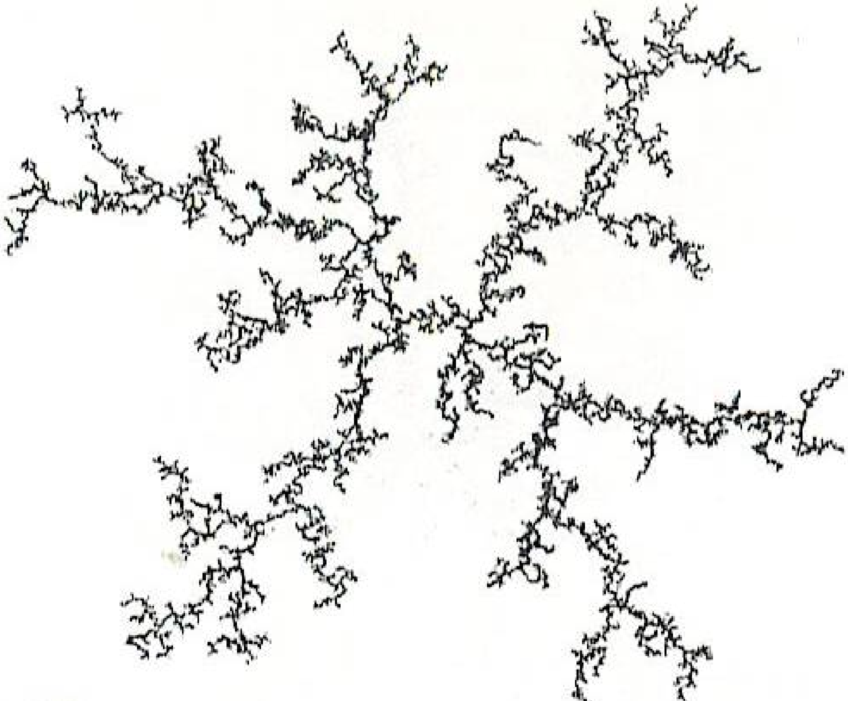
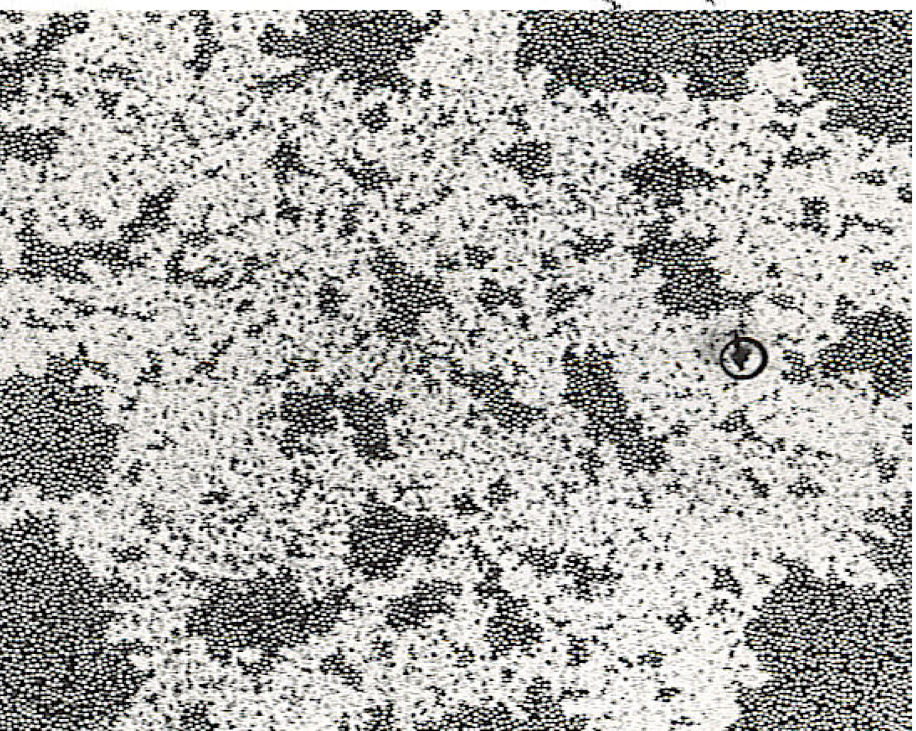
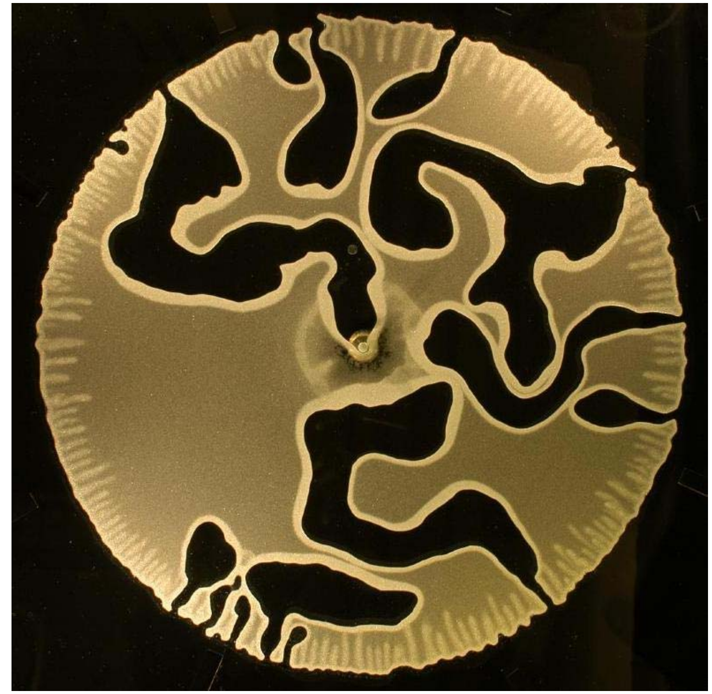
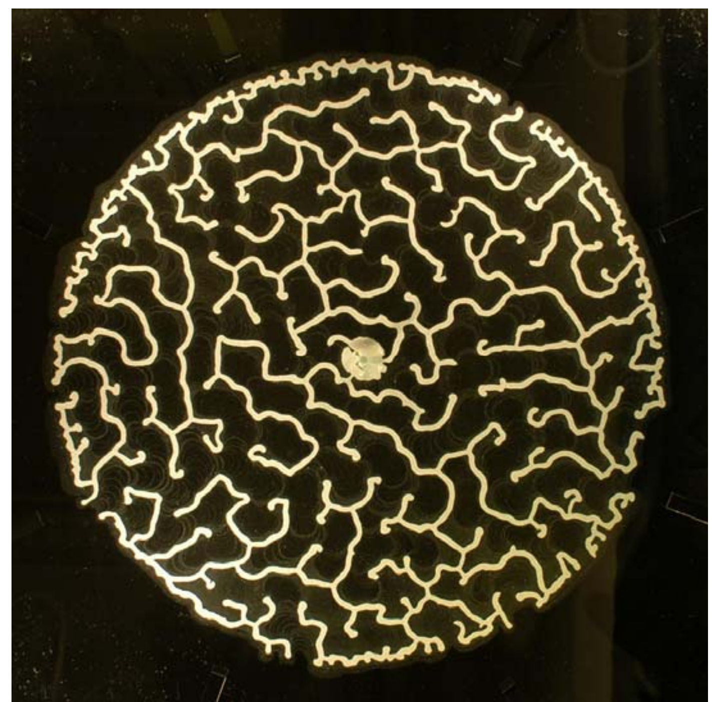
•Grunde Løvoll

•Yves Meheust NTNU

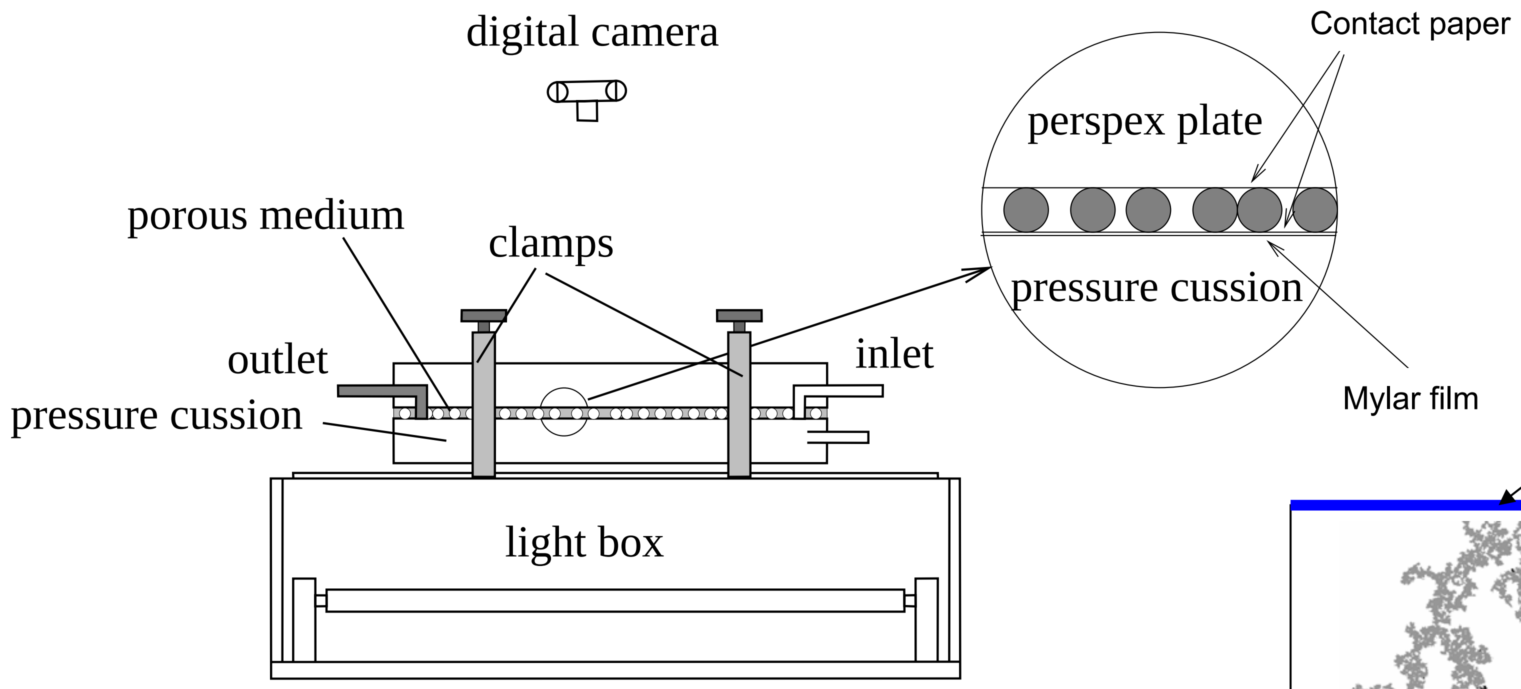
•Renaud Toussaint Univ. Strasbourg

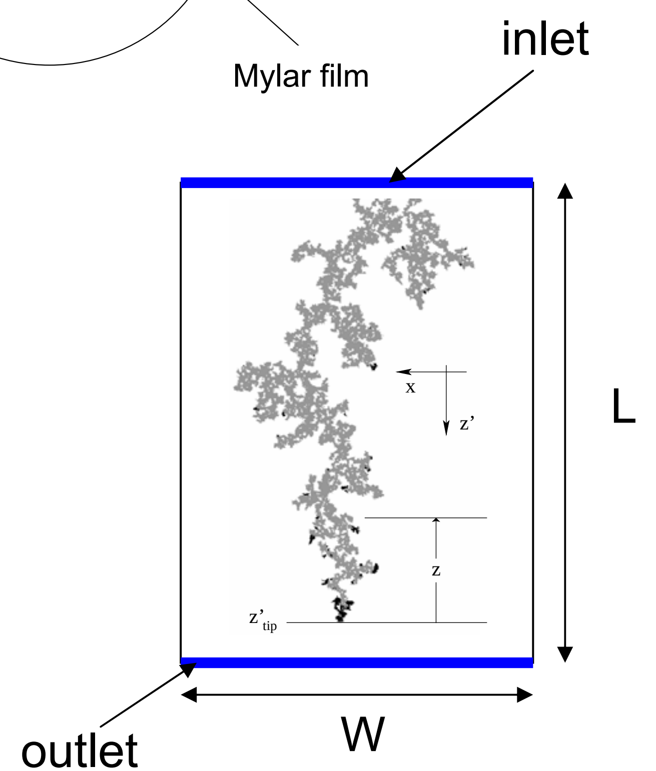
•Jean Schmittbuhl Univ. Strasbourg

UNIVERSITY OF OSLO

Experimental setup

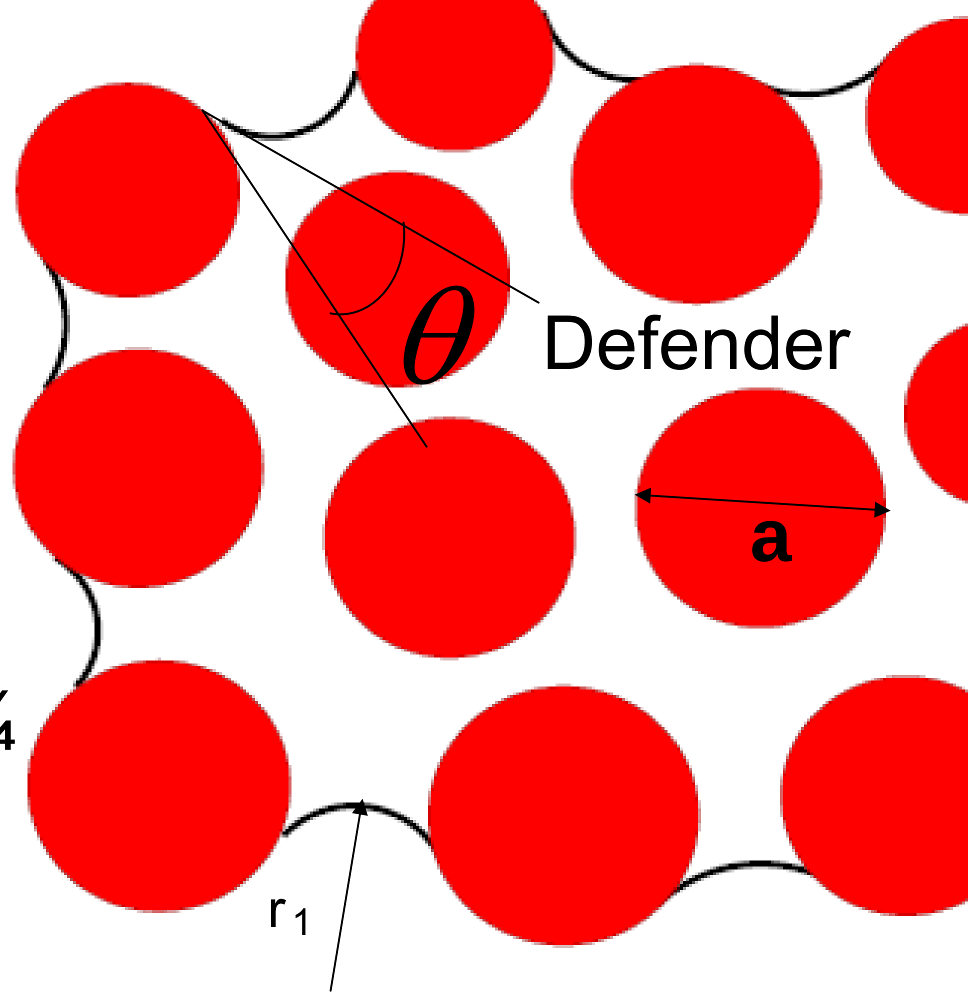




Fluid:
90% by weight Glycerol - water solution
Viscosity: 0.165 Pa s
Interface tension: 0.064 N/m
Viscosity ratio M=0.0001

Drainage in porous media

Invader



Interface boundary condition:

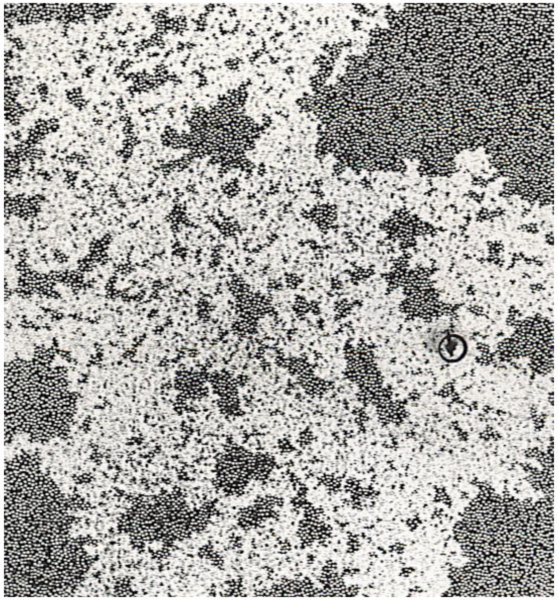
$$¢P = °(1=r_1 + 1=r_2) + ¾$$

Darcy's law:

$$v = ¡ \frac{·}{1} r P$$

Capillary number:

$$Ca = \frac{1 v=a}{° =a} = \frac{1 v}{°}$$

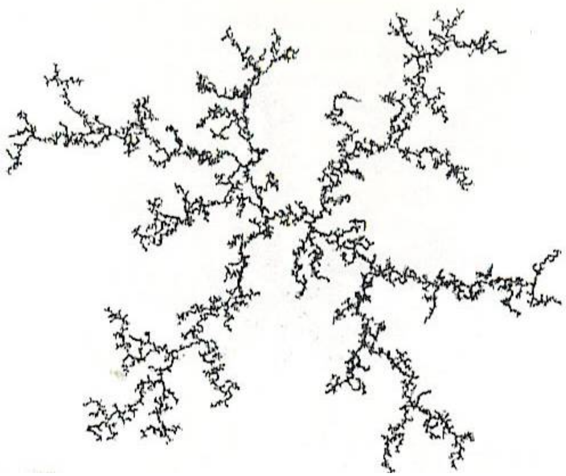


Capillary fingering: low Ca

Lenormand and Zarconne, Phys. Rev. Lett **54,** 2226, (1985).

D = 1:82

- Structure controlled by capillary threshold fluctuations
- Structured well described by Invasion Percolation model. Wilkinson and Willemsen J. Phys. A **16**, 3365, (1983),

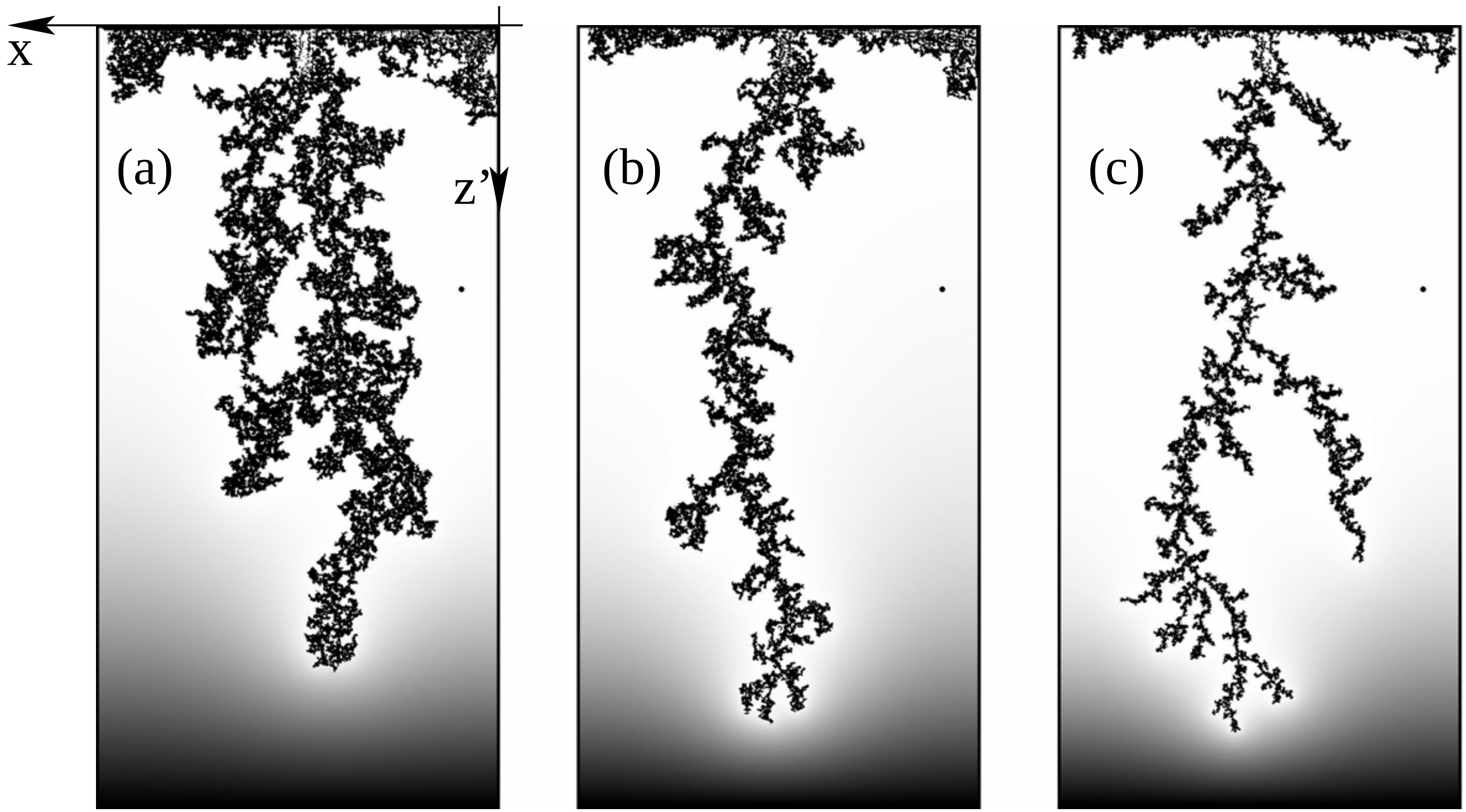


K. J. Måløy, Jens. Feder and T. Jøssang. *Viscous Fingering Fractals in Porous Media.* Phys. Rev. Lett. **55,** 2688, (1985).

Viscous fingering: high Ca.

- Structure controlled by viscous pressure Field.
- Fractal structure D = 1:62

Two phase flow in porous Hele Shaw cell



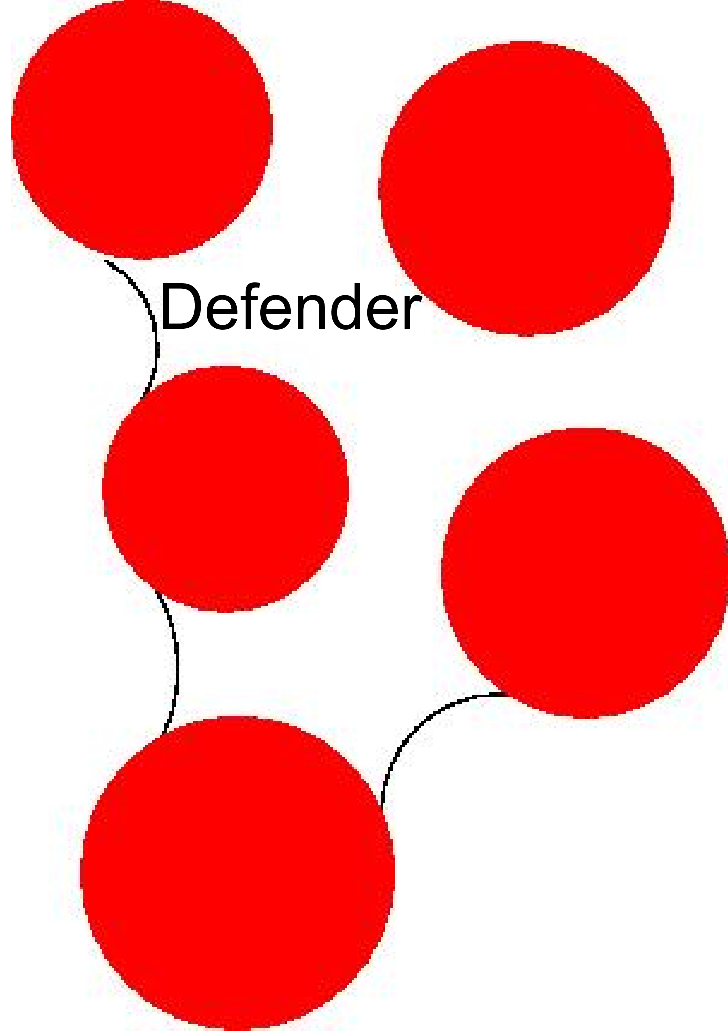
Toussaint, Løvoll, Meheust, Måløy and Schmittbuhl, Europhysics letter **71**, 583, (2005).

Løvoll, Meheust, Toussaint, Schmittbuhl and Måløy, Phys. Rev. E. **70**,026301, (2004).

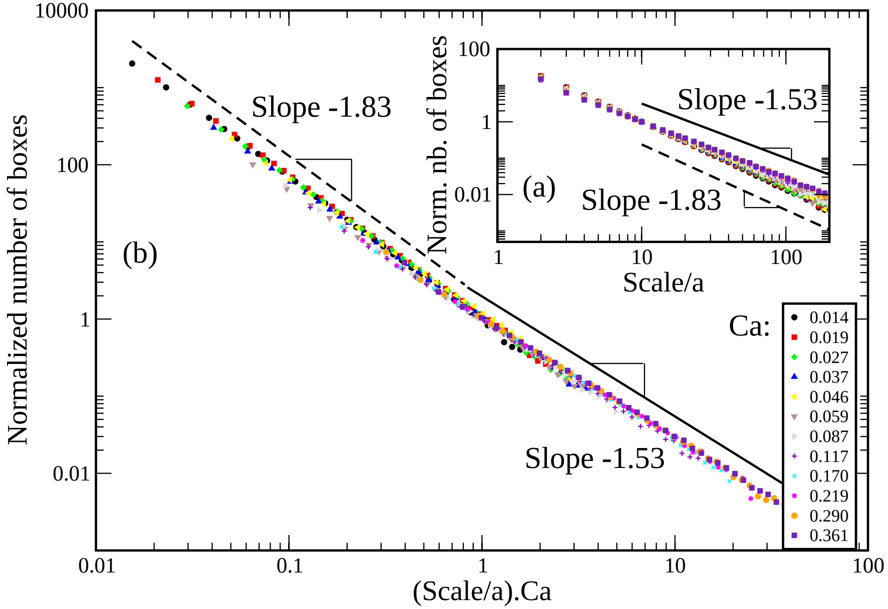
Necessary condition for invasion:

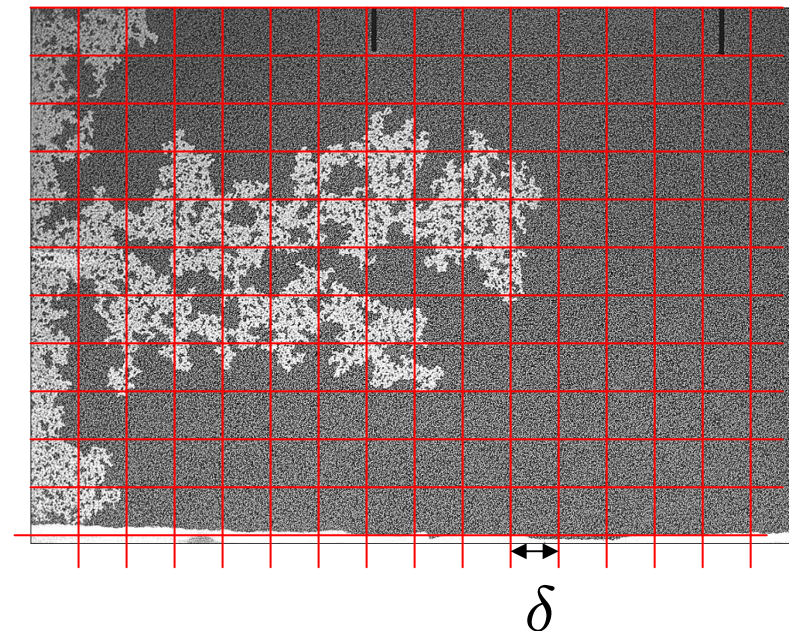
¢ $P = P_2$ ¡ $P_1 > P_t$

P_t is capillary threshold pressure



Box counting and Fractal dimension

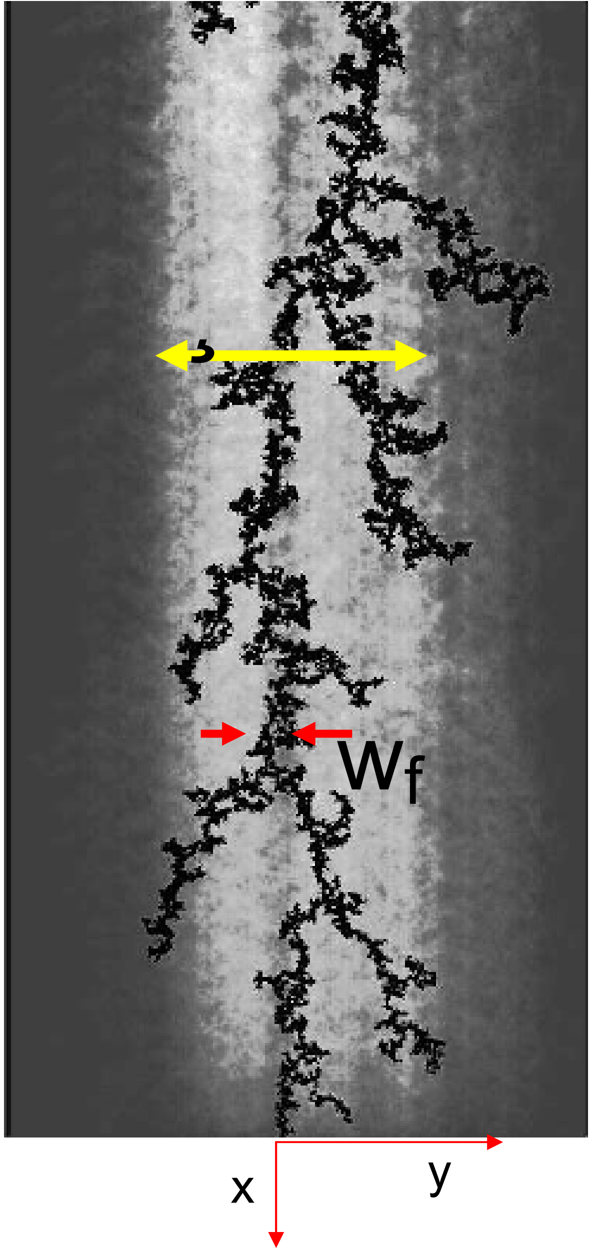




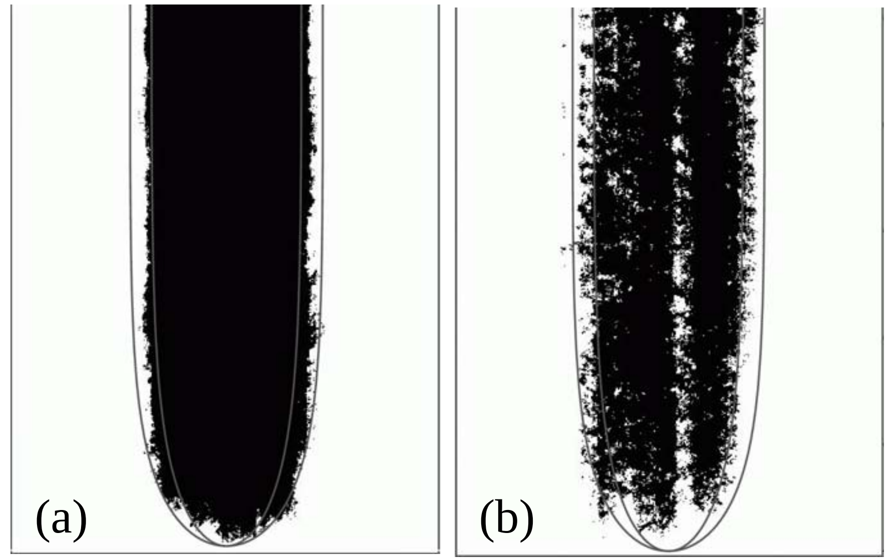
$D_b = 1.83$ on scales less than $w_f = a/Ca$ corresponding to capillary fingering.

The crossover w_f scale scales as $w_f \propto a/Ca$

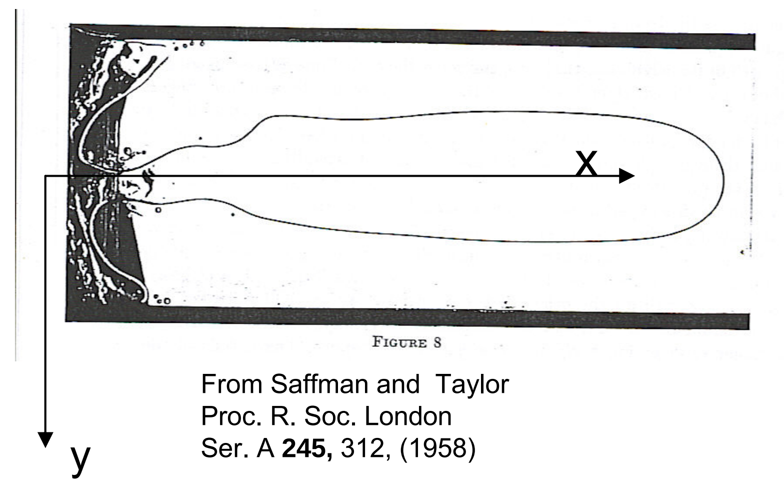
$D_b = 1.53$ on scales larger than $w_f = a/Ca$ corresponding to viscous fingering.



$$x(y) = \frac{W(1-\lambda)}{2\pi} \ln\left[\frac{1}{2}\left(1+\cos\frac{2\pi y}{\lambda W}\right)\right]$$

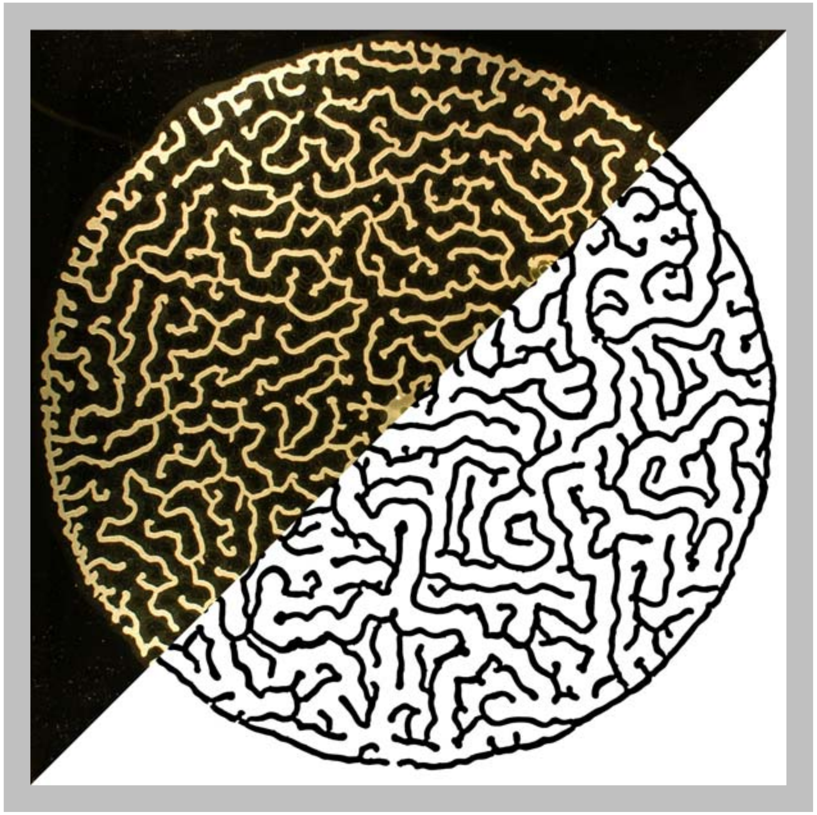


Ca = 0:06 Ca = 0:22

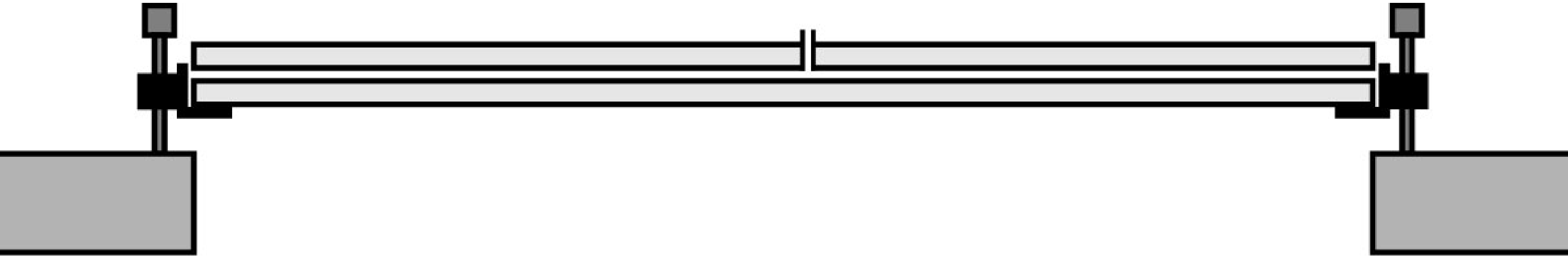


From Saffman and Taylor
Proc. R. Soc. London
Ser. A **245,** 312, (1958)

Maze pattern from drainage of granular suspension .

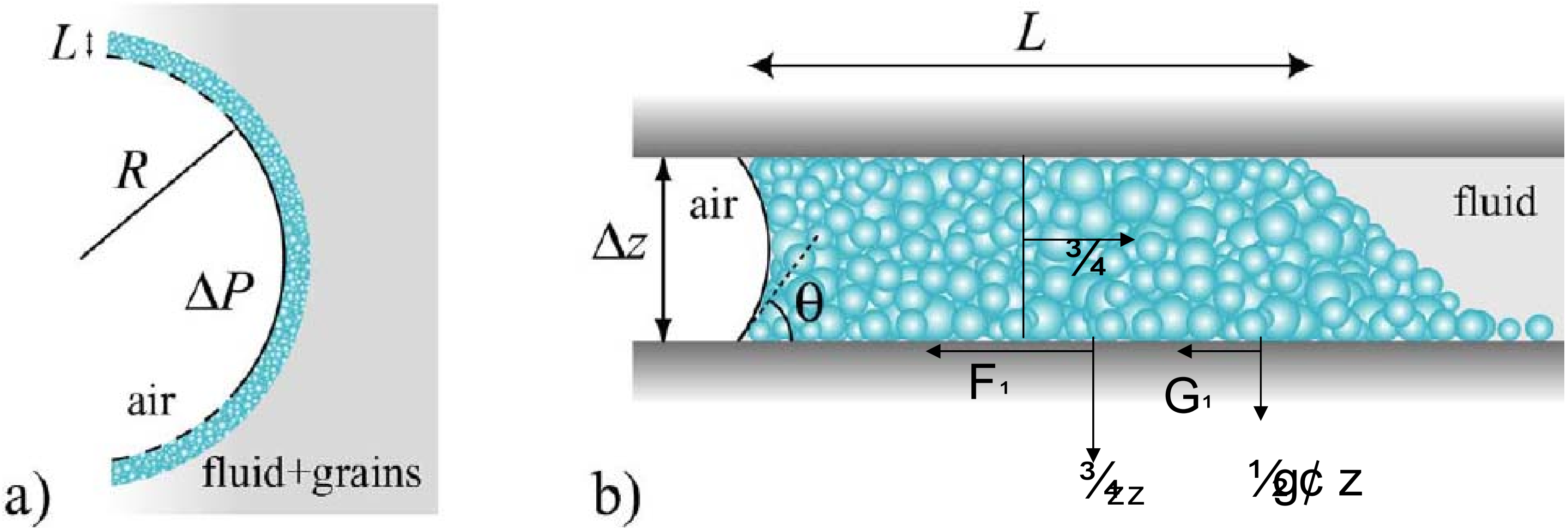


Experimental setup



Diameter og glass
Plate D=35cm

50% volume glycerine/ water solution
mixed with Glass beads d= 50-100 µm.



Pressure boundary condition:

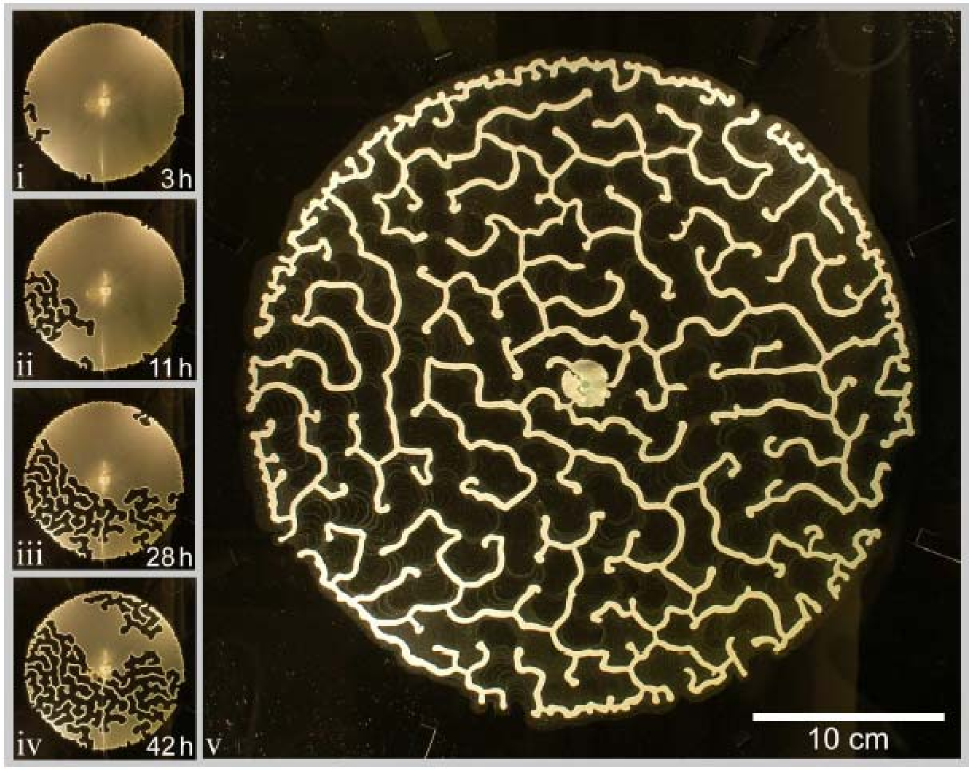
$$\Delta P = \gamma\left(\frac{2\cos(\theta)}{\Delta z} + \frac{1}{R}\right) + \sigma$$

Coloumb friction and Janssen’s assumption:

$$F_{\mu} = \mu\,\sigma_{zz} = \mu\,\kappa\,\sigma, \qquad G_{\mu} = \mu\,\rho g \Delta z$$

Granular stress due to friction:

$$\sigma = \frac{\rho g \Delta z}{2\kappa}\left((1+\kappa\mu)\exp\left(\frac{2\kappa\mu L}{\Delta z}\right) - 1\right)$$



i
3 h
ii
11 h
iii
28 h
iv
42 h
v
10 cm

•Space-filling pattern with a Characteristic length scale.

•One connected cluster, a simply-connected maze.

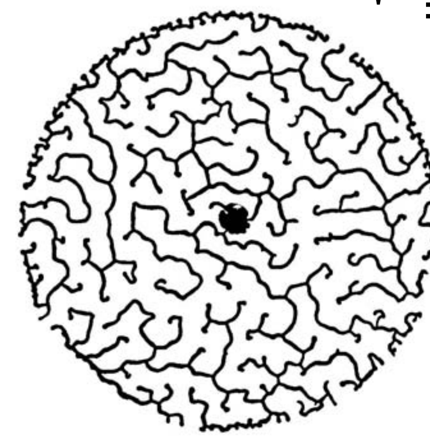
•Measure area and circumference of the cluster.

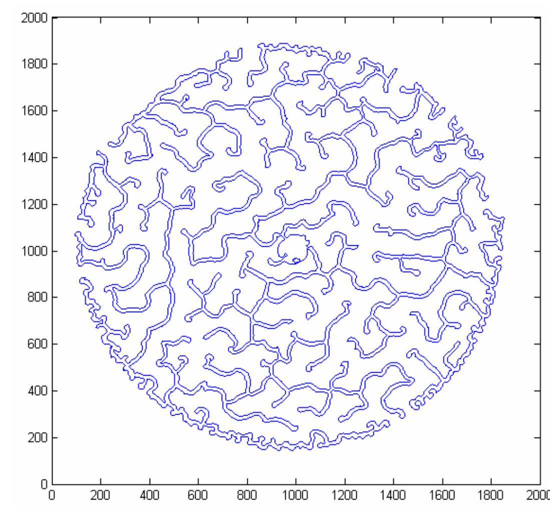
$$¸ = 2A_{disc}=O$$

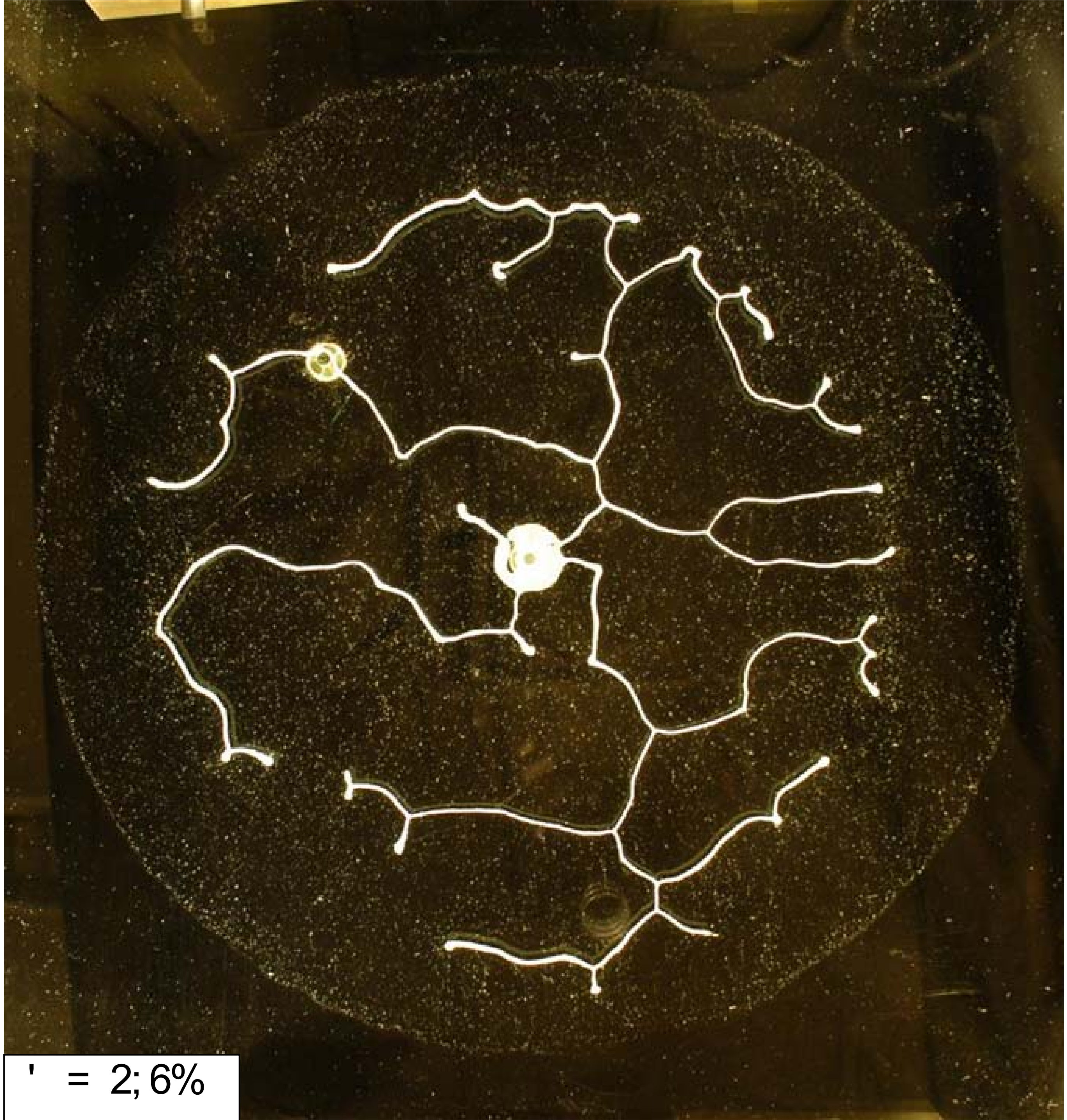
Results:

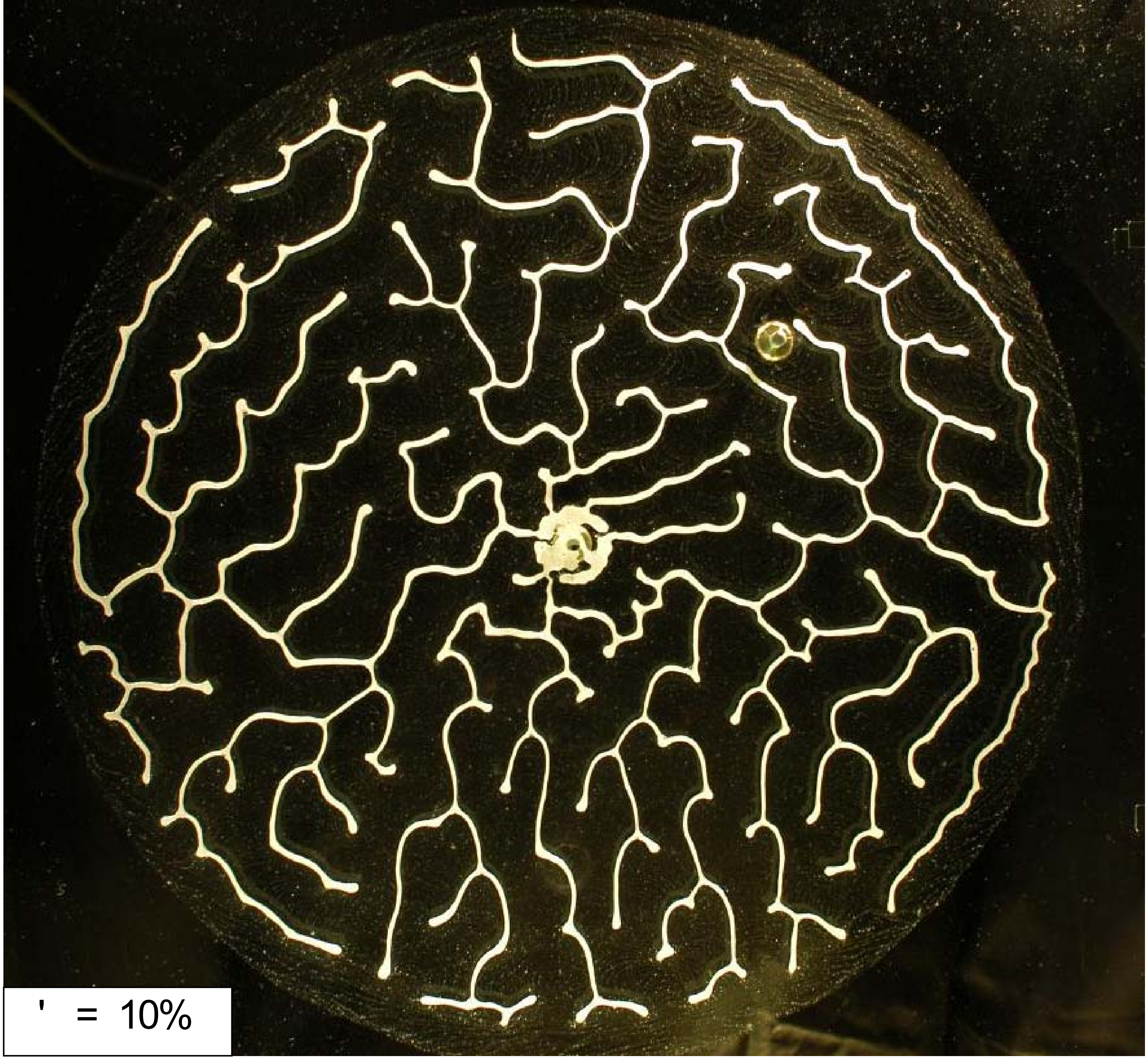
Initial diameter = 35 cm
Circumference = 13 m
Branch width = 3 mm
Wavelength = 14 mm

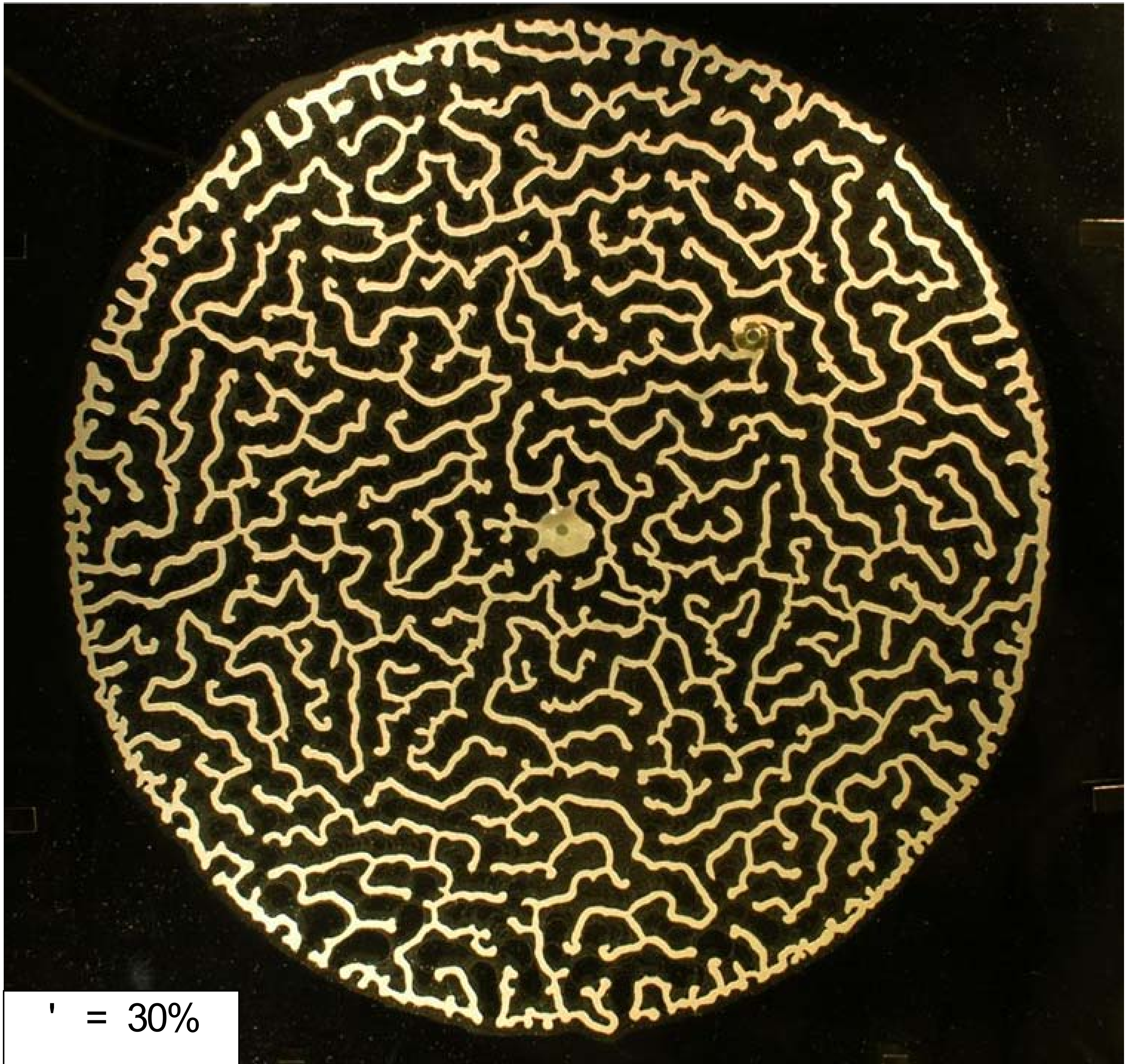
= 30%






' = 2; 6%


' = 10%



' = 30%

Simulations

•One dimensional model where tousands of points discretize the front.

•Simulation carried out with a small amount of disorder in background mass density

•At each point a local curvature is calculated from position of neighbour points.

•The yield pressure given by the curvature and friction is calculated from each point. The point with smallest yield pressure is moved a small step for each time step.

•As front advances, the width of the compacted layer is recalculated based on the invaded area and the background volume density of grains.

•As the front moves the interface stretches. When the distance between neighboring exceeds a set limit a new point is inserted between the two points.

•We have used the values : $^1 = 0;47, \cdot = 0;80$
Which is within a realistic range and fits the experiments.

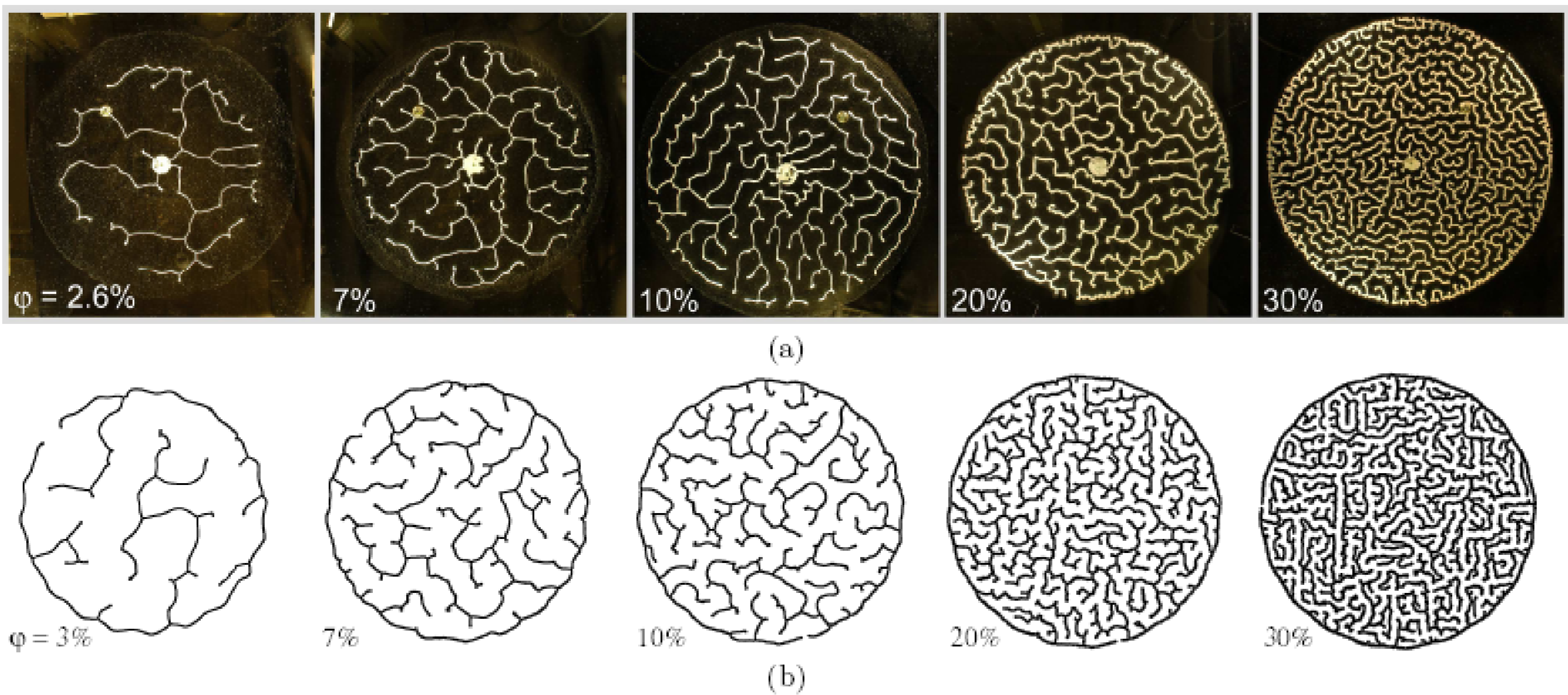
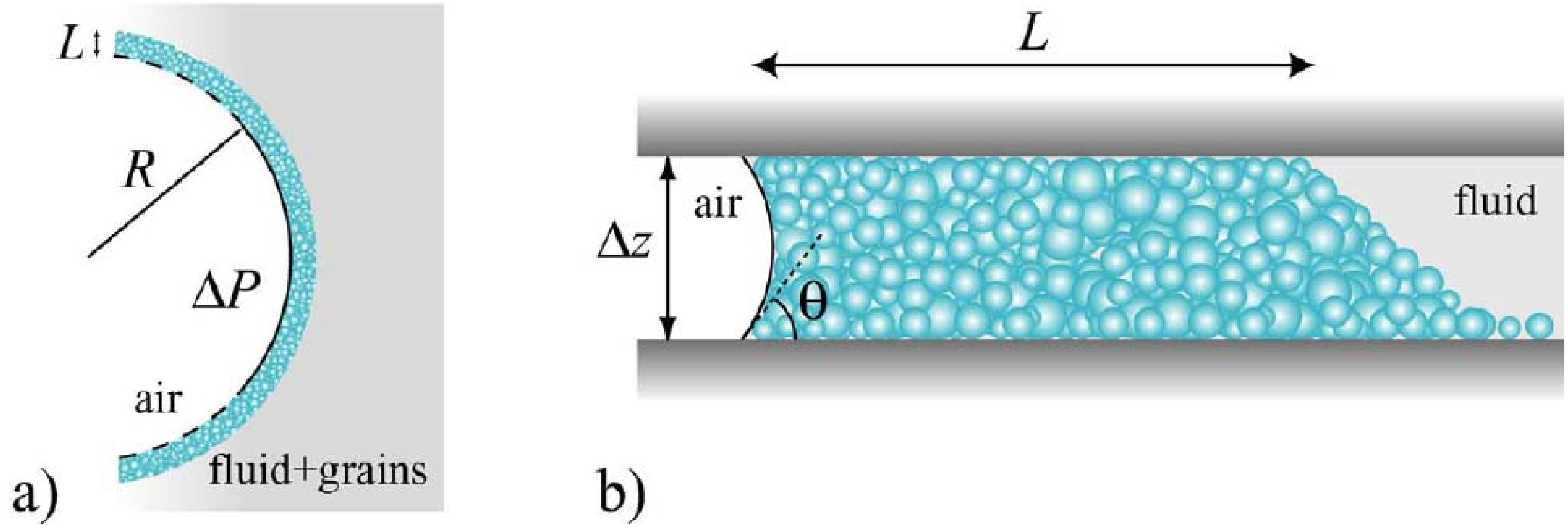


FIG. 2: Characteristic length-scale as a function of normalized volume fraction. (a) Experiments. The volume fraction of grains φ in the mixture increases from left to right as labeled in the pictures. All experiments were conducted with a plate spacing of 0.4 mm. The dimensions of each picture frame are 40×40 cm. (b) Simulations. The observed decrease in the length-scale of the pattern with increased volume fraction is reproduced in the simulations.

B. Sandnes, H. A. Knudsen, K. J. Måløy and E. G. Flekkøy. To appear in Phys. Rev Lett. July 2007.



Pressure boundary condition:

$$\Delta P = \gamma\left(\frac{2\cos(\theta)}{\Delta z} + \frac{1}{R}\right) + \sigma$$

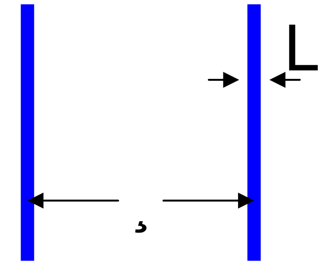
Coloumb friction and Janssen's assumption:

$$\mu\sigma_{zz} + \mu\rho_2 g\Delta z = \mu\kappa\sigma + \mu\rho_2 g\Delta z$$

Granular stress due to friction:

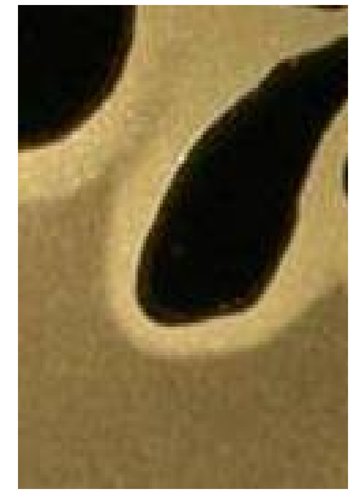
$$\sigma = \frac{\rho_2 g\Delta z}{2\kappa}\left[(1+\kappa\mu)\exp\left(\frac{2\kappa\mu L}{\Delta z}\right) - 1\right]$$

Mass conservation



$L = \varphi\lambda/2$ $\qquad$ $L_{tip} = R\varphi/(1-\varphi)$

Finger moves where it is most easy to move. Minimize the pressure with respect to R at the tip.



$$\delta\Delta P/\delta R = 0$$

Must minimize

$$\Delta P = \gamma\left(\frac{2\cos(\theta)}{\Delta z} + \frac{1}{R}\right) + \frac{\rho g \Delta z}{2\kappa}\left((1+\kappa\mu)\exp\left(\frac{2\kappa\mu R\varphi}{\Delta z(1-\varphi)}\right) - 1\right)$$

Gives a critical radius:

$$R_c = \frac{\Delta z(1-\varphi)}{\kappa\mu\varphi}\mathrm{W}\left(\left[\frac{\gamma\mu\kappa^2\varphi}{\rho g(1-\varphi)(1+\kappa\mu)(\Delta z)^2}\right]^{\frac{1}{2}}\right)$$

Where W is the Weibul function (x=yexp(y))

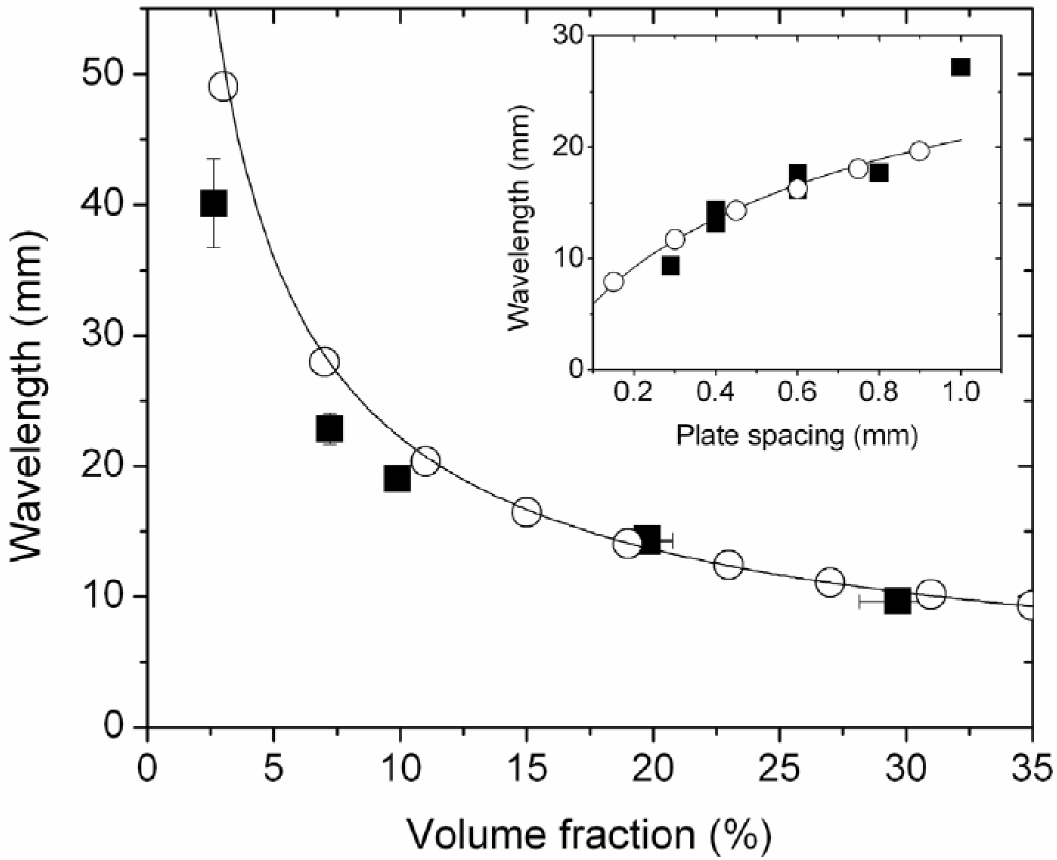
The pressure difference is the same all along the interface hence:

$$\Delta P(R_c) = \Delta P(L) = \frac{\rho g \Delta z}{2\kappa}\left((1+\kappa\mu)\exp\left(\frac{2\kappa\mu L}{\Delta z}\right) - 1\right)$$

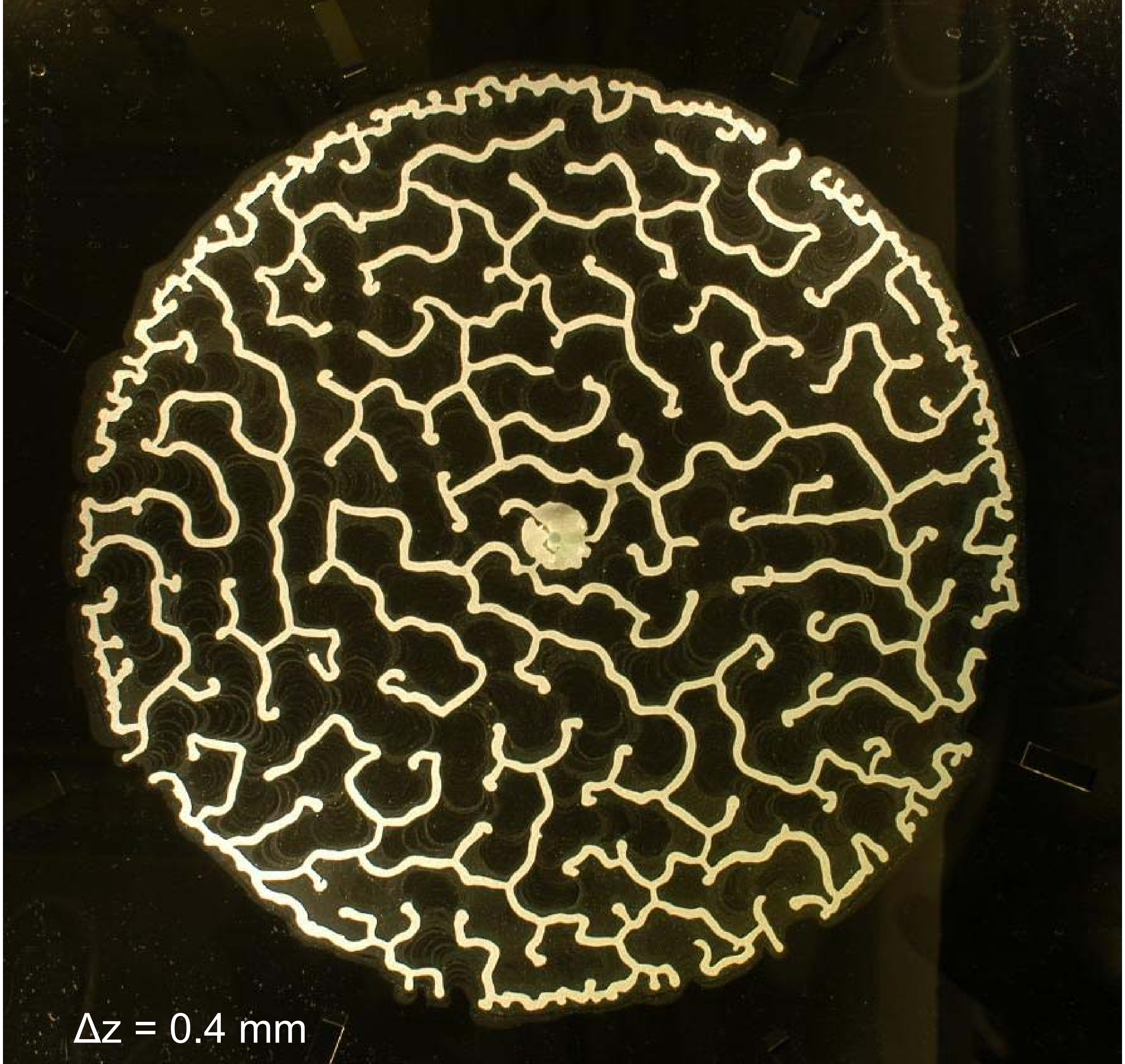
From this equation we can find L and therefor λ because:

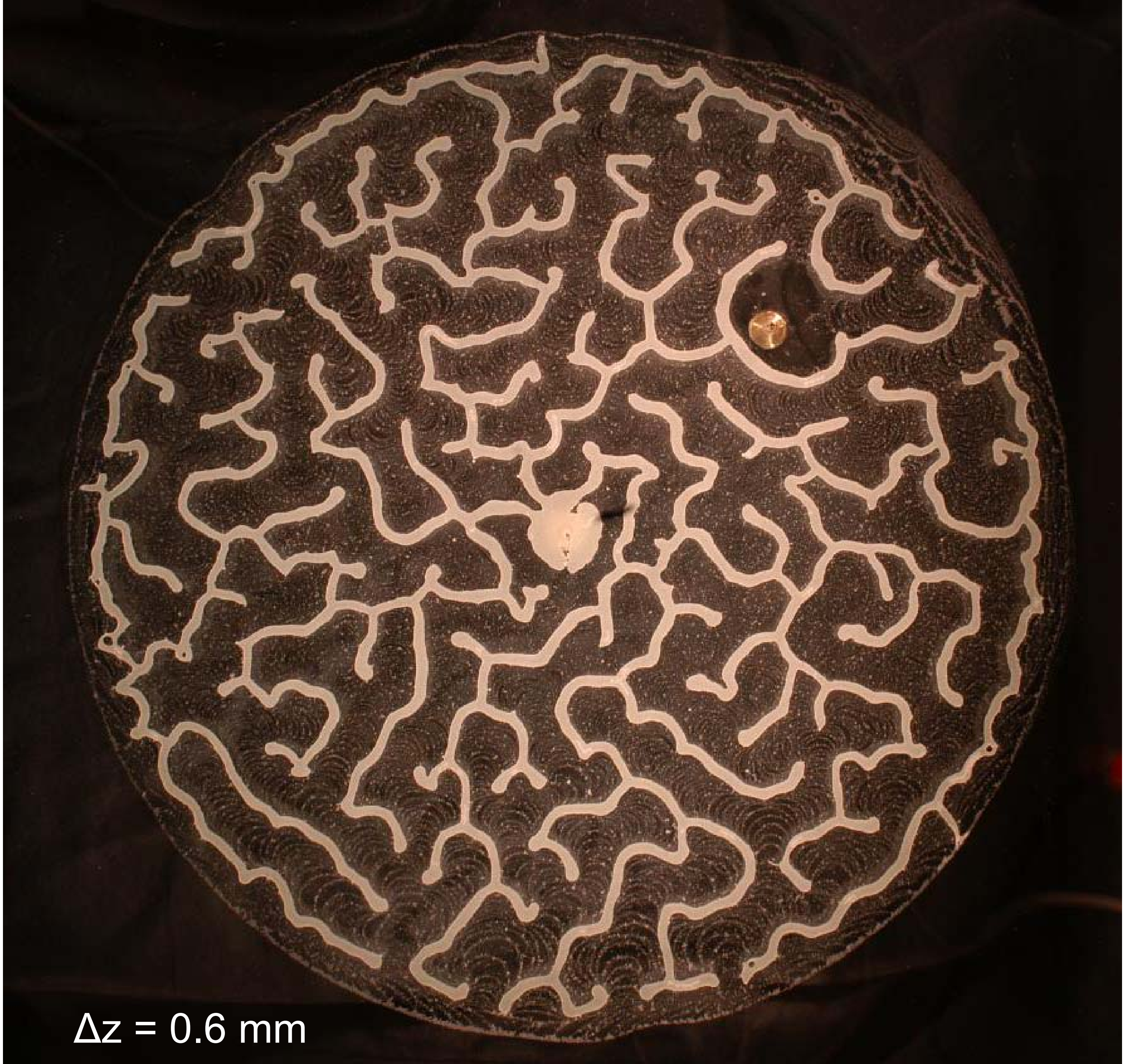
$$\lambda\varphi = 2L$$

from mass conservation



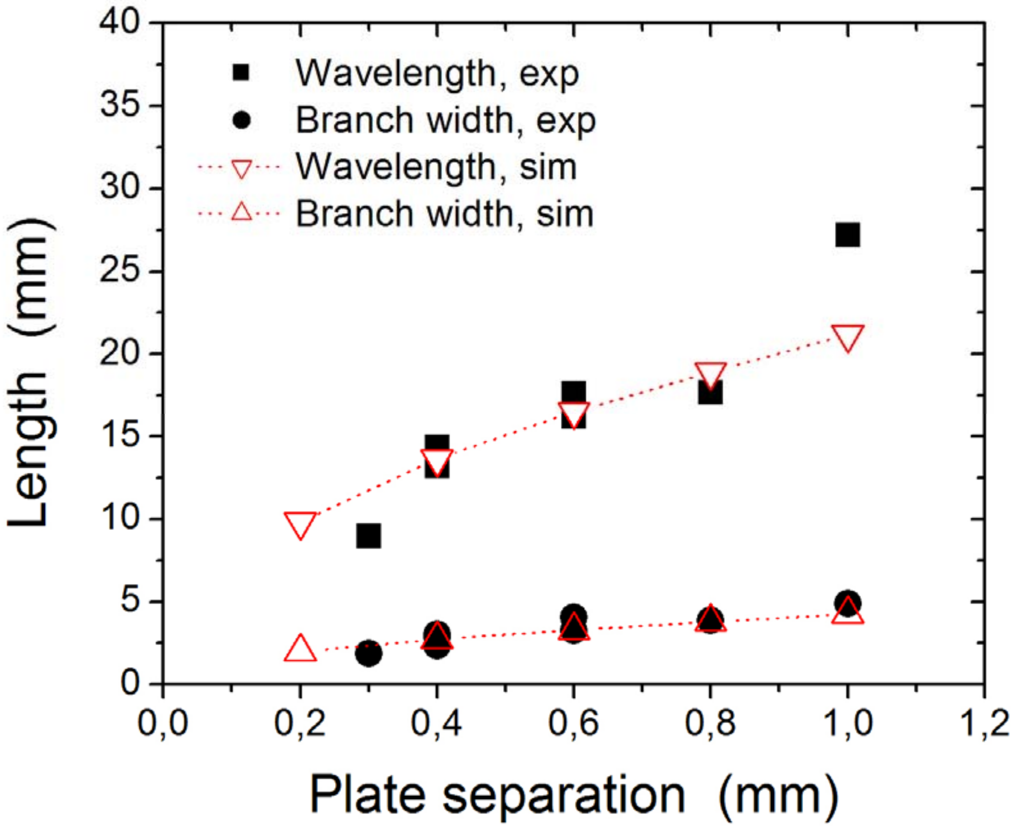
Wavelength (mm)
Volume fraction (%)
0
10
20
30
40
50
0
5
10
15
20
25
30
35
Wavelength (mm)
Plate spacing (mm)
0
10
20
30
0.2
0.4
0.6
0.8
1.0


Δz = 0.4 mm

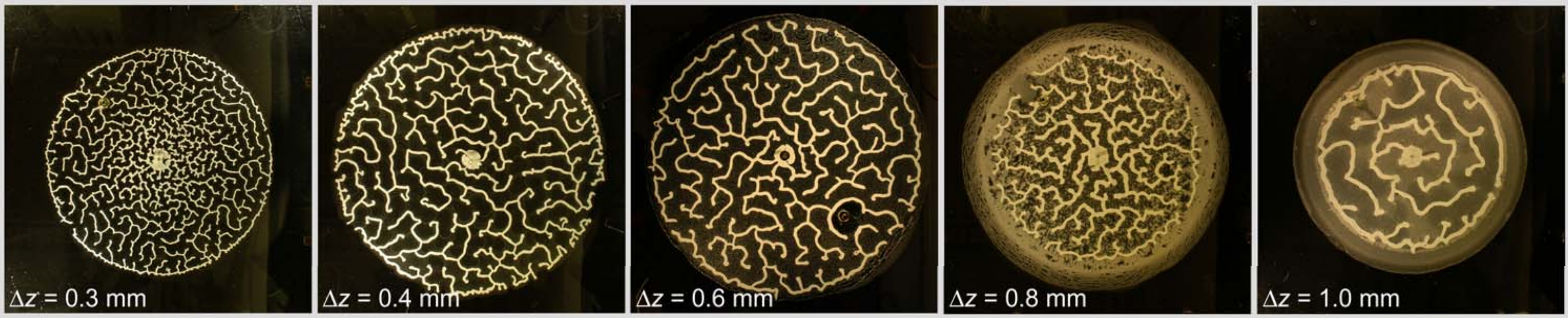


Δz = 0.6 mm

Δz = 1.0 mm



Wavelength, exp
Branch width, exp
Wavelength, sim
Branch width, sim
Length (mm)
Plate separation (mm)
0
5
10
15
20
25
30
35
40
0,0
0,2
0,4
0,6
0,8
1,0
1,2



Δz = 0.3 mm
Δz = 0.4 mm
Δz = 0.6 mm
Δz = 0.8 mm
Δz = 1.0 mm

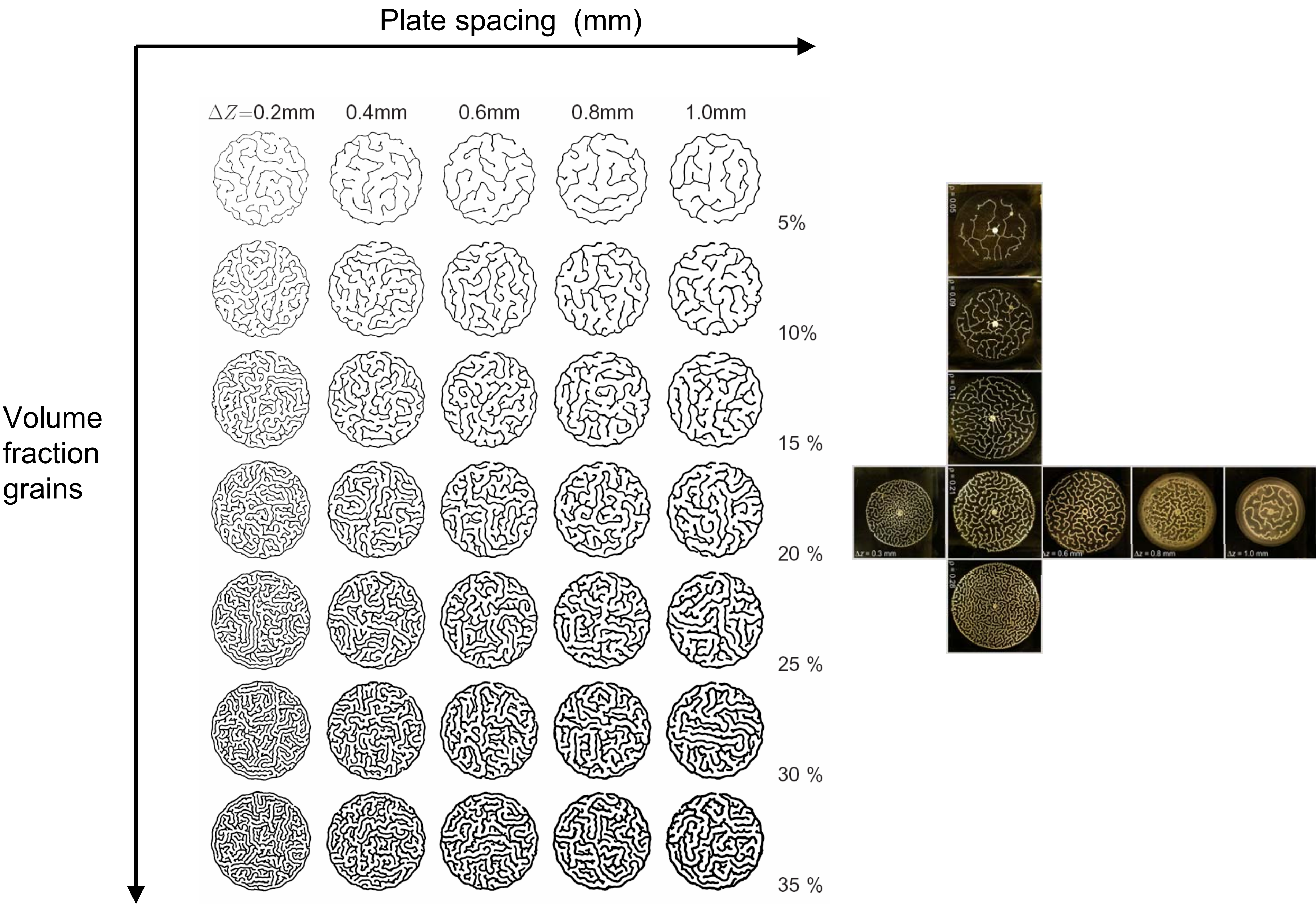


Plate spacing (mm)
ΔZ=0.2mm
0.4mm
0.6mm
0.8mm
1.0mm
5%
10%
15 %
20 %
25 %
30 %
35 %
Volume fraction grains

Similar labyrinthine patterns have been shown to form in two phase systems such as magnetic And dielectric fluids under external fields and some reaction diffusion processes.

Rosensweig, Zahn and Sumovich, J. Magn. Magn. Matter, **39,** 127, (1983).

Lee, McCormick, Quyang and Swinney, Science **261**, 193 (1993)
Petric and Goldstein, Phys. Rev. Lett. **72**, 1120, (1994).

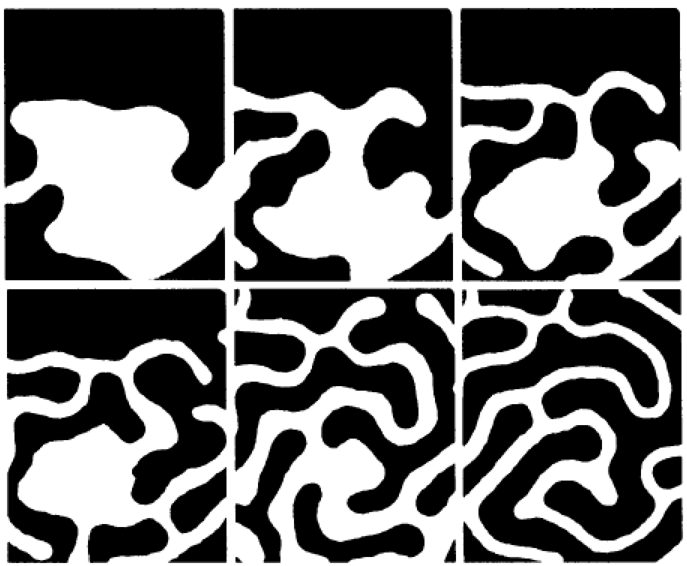
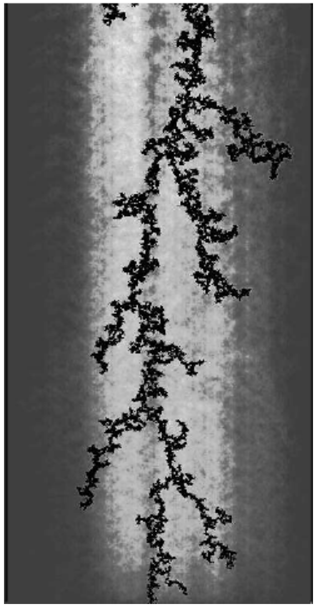


FIG. 1. Pattern formation in the chemical system of Lee, McCormick, Ouyang, and Swinney [6]. White and black regions correspond, respectively, to low and high pH, made visible with a pH indicator. Figure courtesy of Lee *et al.* [6].


' = 40%

Conclusion



Porous medium: Fixed beads.

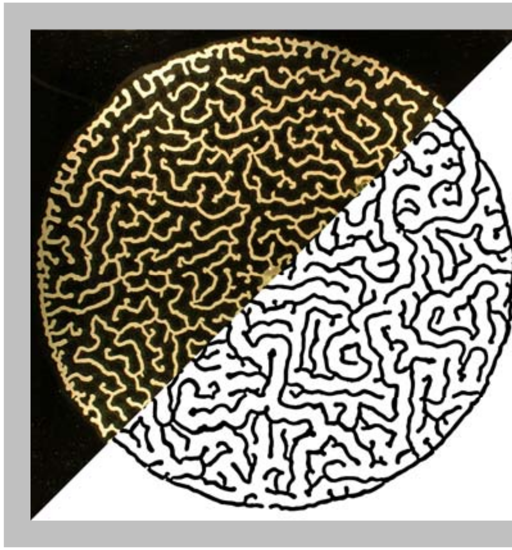
•"Maze pattern" is fractal.

•On small scale the structure is similar to the IP structure (D=1,82) with a crossover to an Viscous Fingering (D=1,62) on large scale.

•The hole structure will be within an envelope structure.

Granular suspension: Loose beads.

•A non fractal maze pattern is observed ..



•The maze pattern results from capillary pressure that acts at the air—liquid meniscus to advance the finger and from a retarding frictional force.

• The maze pattern has a characteristic wavelength of the fingers which decrease with the volume fraction of grains and increase with the plate separation.

•A simple simulation have been performed which gives a perfect match with experiments.

•A theoretical estimate of the characteristic wavelength gives a very good agreement with both simulations and experiments.

•Work in progress on Baloon like phase and on experiments on a tilted model where gravitational effects are important.


Image © 2006 DigitalGlobe
Pointer 26°25'49.09" S 15°06'38.23" E elev 37 ft
Streaming 100%
Image © 2006 TerraMetrics
Google
N 48°12'40.68" E elev 3310 ft
Streaming 100%
Eye alt 39.68 mi
J. Todd
S. Camazine
Image © 2006 TerraMetrics
Google
Pointer 4°30'57.87" S 67°59'46.81" W elev 262 ft
Streaming 100%
Eye alt 11.76 mi

Questions?