

# Le Noyau liquide de la Lune: Bretagne ou Bahamas?



Jérôme Noir

Collaborators: D. Cébron, M. Calkins, Y. Charles, J. Aurnou, M. Le Bars, F. Hemmerlin, M. Lasbleis

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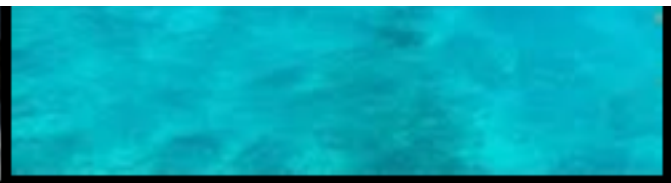
1

## Why the Earth's moon ?

**The only extraterrestrial body for which we have:**

- Samples
- Seismic data
- Rock magnetic data
- LLR (k2/dissipation/rotational dynamics)





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- Samples
- Seismic data
- Rock magnetic data
- LLR (k2/dissipation/rotational dynamics)
- Gravity
- Surface composition



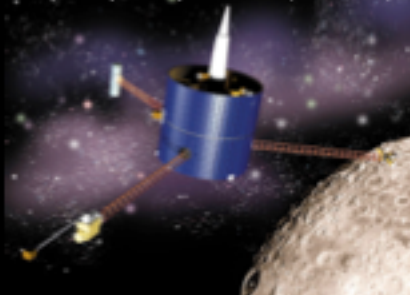
Image courtesy of NASA

Apollo (61-72)



Image courtesy of NASA

Lunar prospector (98-99)



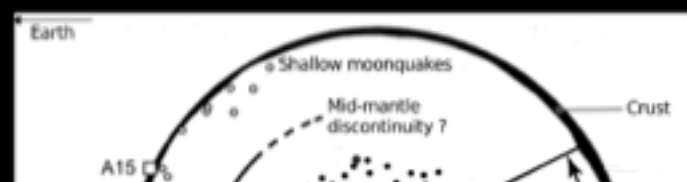
The Lunar liquid Core: Brittany or Bahamas

2

## Does the Moon possess a liquid core ?

Evidence of a liquid layer in the deep moon structure:  
Partially molten mantle versus Liquid core...

- Absence of farside deep moonquakes. (Melt/Core)



Tidal dissipation from LLR (Melt/

Lunar prospector (98-99)

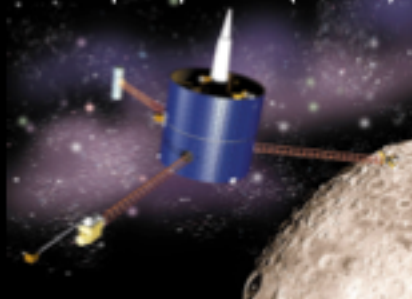


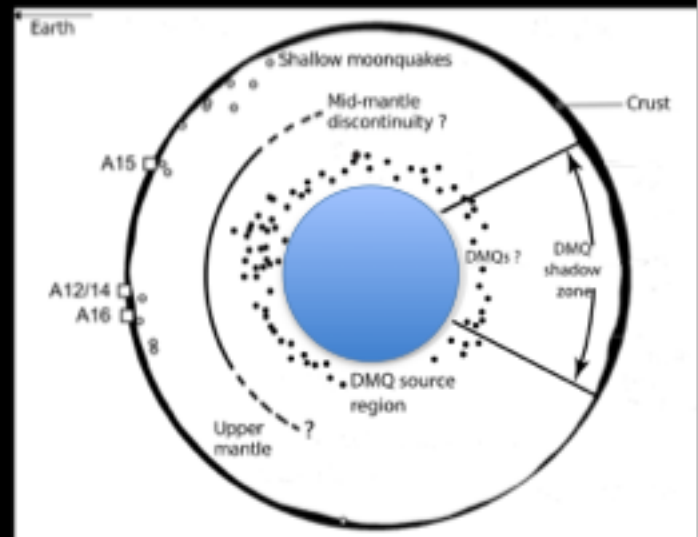
Image courtesy of NASA

ALSEP

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Evidence of a liquid layer in the deep moon structure:  
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- Absence of farside deep moonquakes. (Melt/Core)
- Tidal dissipation from LLR(Melt/Core)
- Remanent rock magnetism. (Early moon dynamo->Core)
- Moment of Inertia (Core)



## The two models of internal structure:

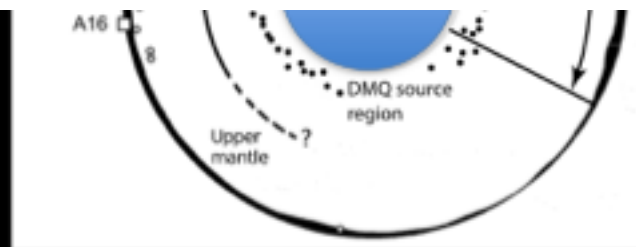
Williams et al 2001, Nimo et al. 2012

Khan et al. 2004, 2013,  
J. de Viries et al. 2010

Silicate solid Mantle

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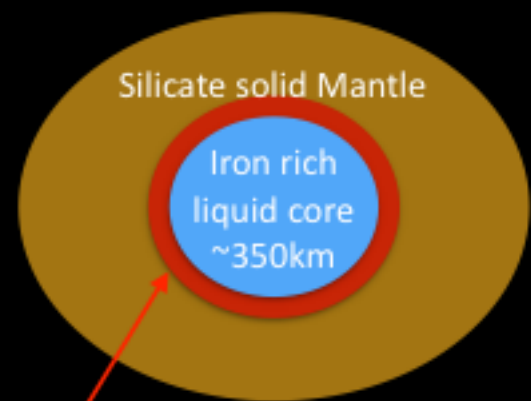
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Partial Melt Region

# The LLR observations.



Partial Melt Region

The Lunar liquid Core: Brittany or Bahamas

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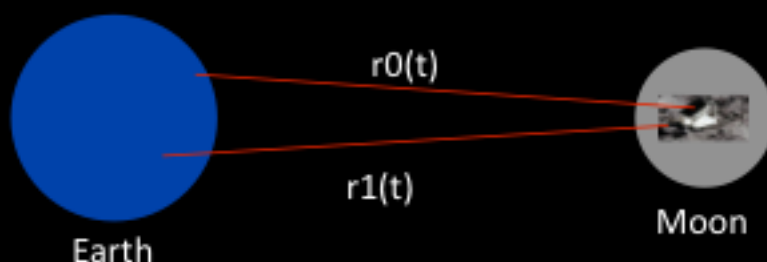
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## The dissipation from LLR observations

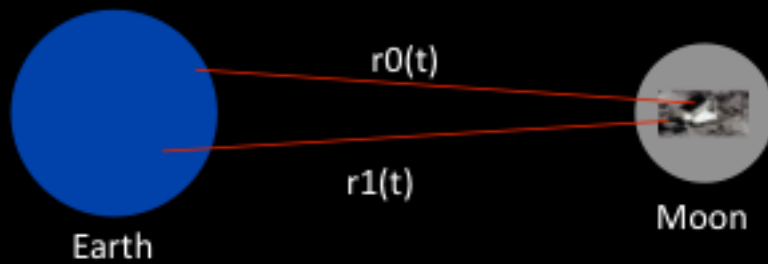
Lunar Laser Ranging (LLR)



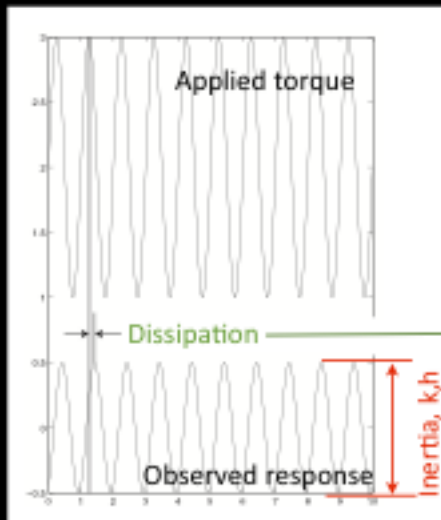
A gravitational torque acting on the moon will **deform the planet** and **change its rotation**. Both effects combine into time dependent  $r_0, r_1, \dots$  measured by LLR.

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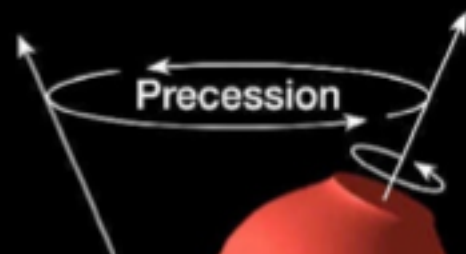
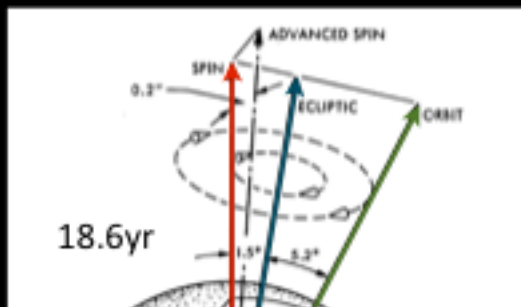


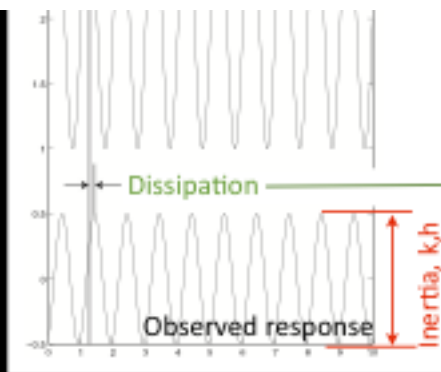
Dissipation from LLR

$$1/Q = 1/Q_{\text{tidal}} + 1/Q_{\text{core}}$$

$1/Q_{\text{tidal}}$  → Anelasticity of the mantle  
 $1/Q_{\text{core}}$  → Viscous friction at the CMB

In the absence of dissipation, the spin vector of the moon would remain in the Cassini plane.

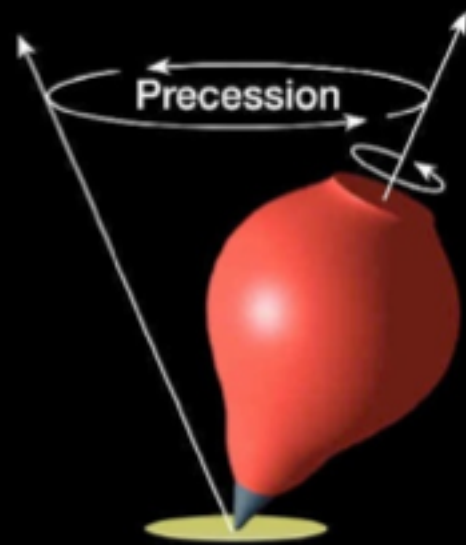
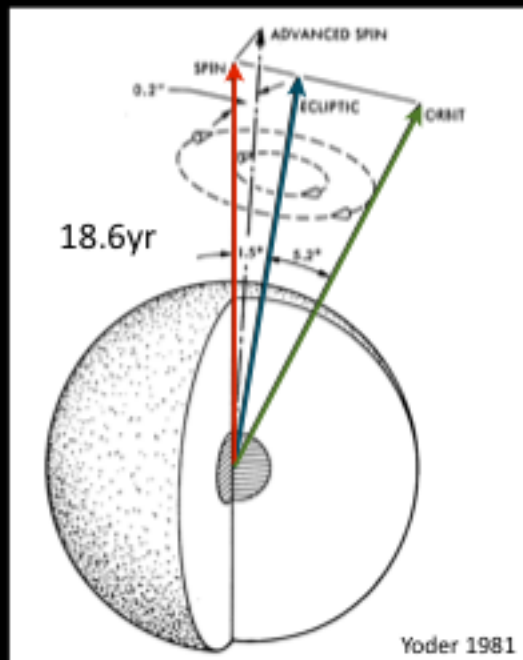




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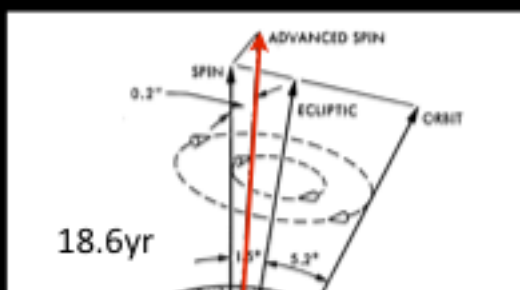
Anelasticity of the mantle (pointing to  $1/Q_{\text{tidal}}$ )  
 Viscous friction at the CMB (pointing to  $1/Q_{\text{core}}$ )

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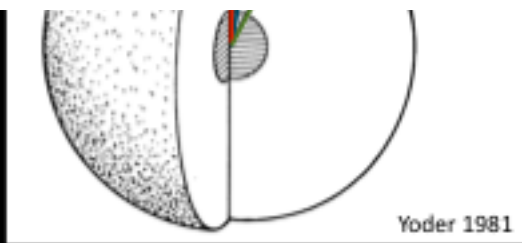
Precession of the lunar mantle exhibit -264 mas phase lag between the mantle rotation axis compare to the Cassini plane.

Yoder 1981:



If the dissipation is entirely due to solid friction:

$$\Delta\phi = -223 \frac{k_2}{Q}$$



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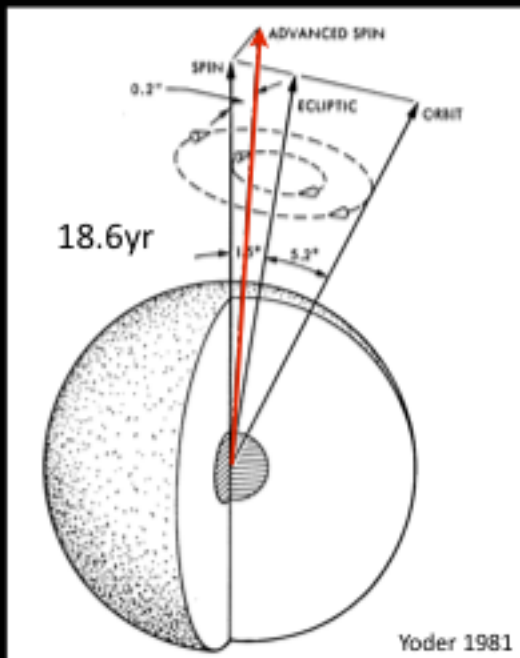
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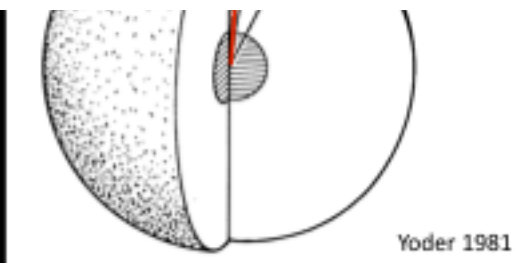
$k_2 \sim 0.02$  -  $Q \sim 35$  are obtain from observations at 1month period. Assuming ( $k_2$ ,  $Q$ ) for a silicate mantle do not vary significantly in the long period range [1month-20years]

$$Q \sim 2 \ll Q = 35$$



**A simple model of Lunar Core dynamics.**





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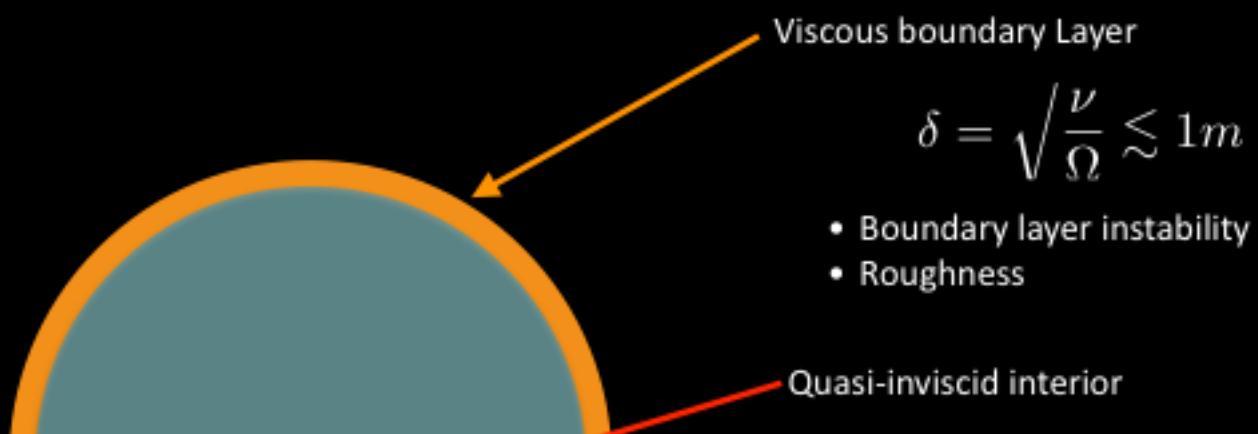
The Lunar liquid Core: Brittany or Bahamas

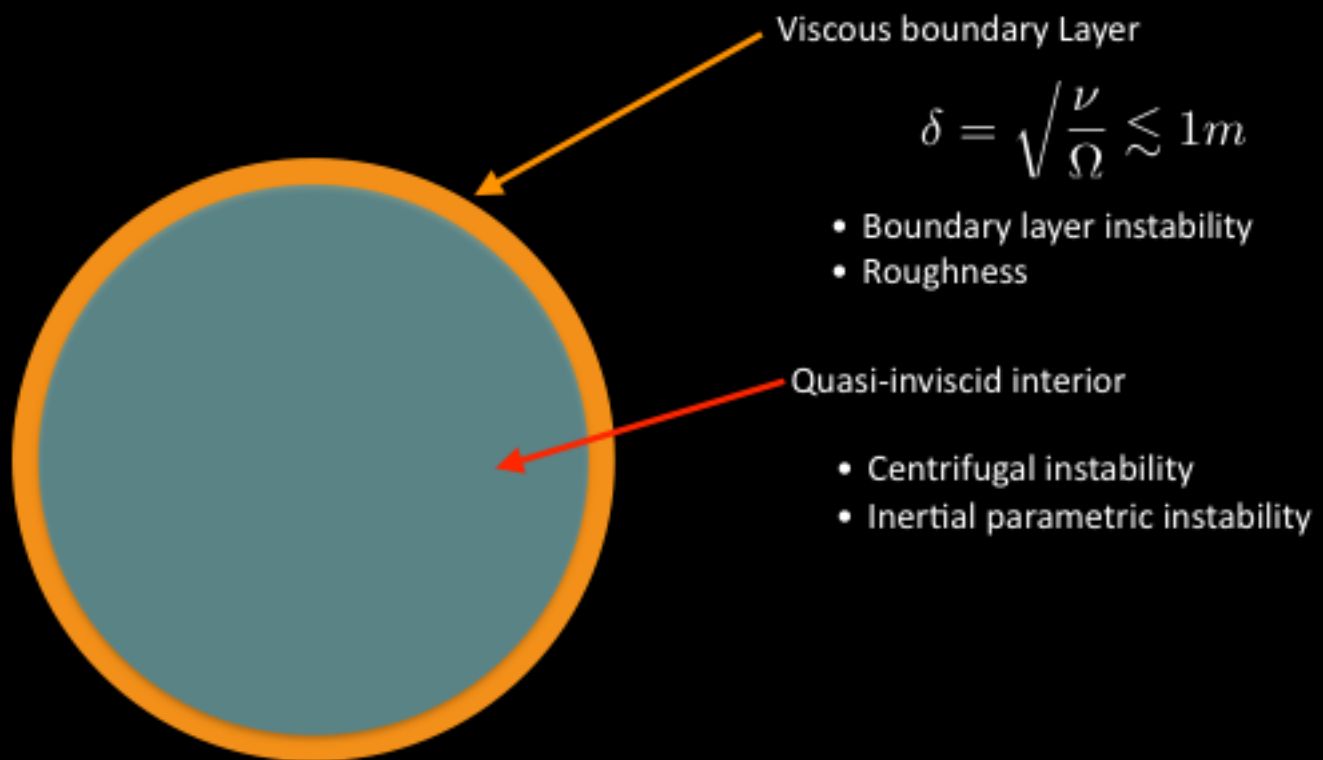
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## A simple model of Lunar Core dynamics.

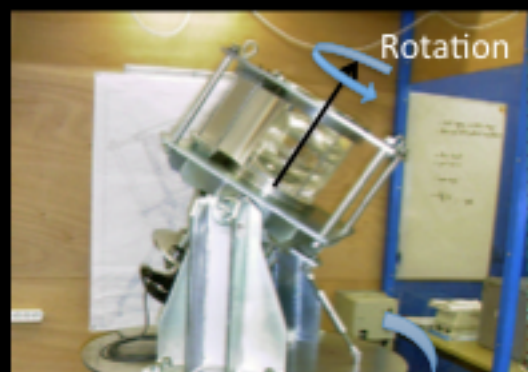
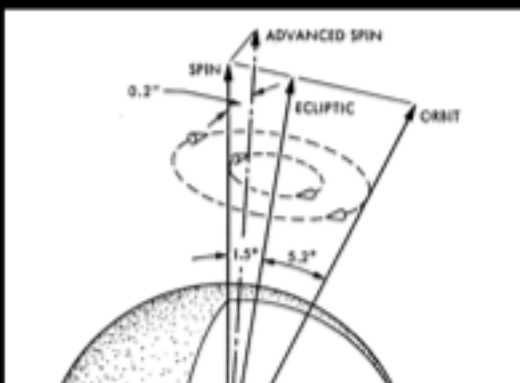
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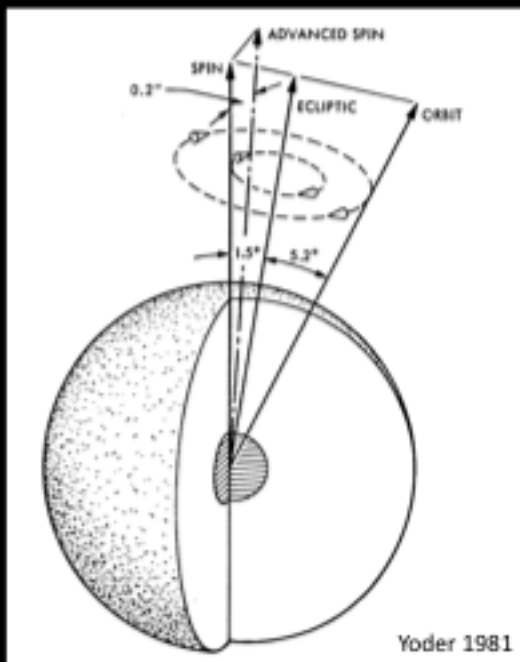




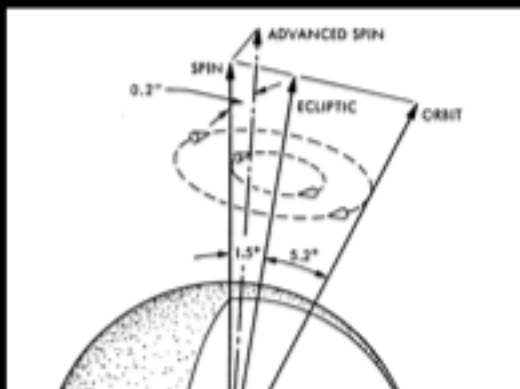
# Precession driven flows



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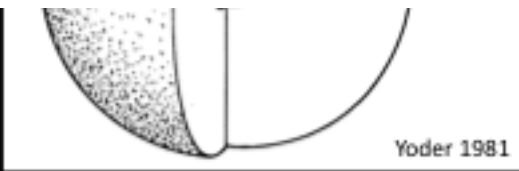
$\Omega_c = \text{core rotation rate}$

$\Omega_m = \text{mantle rotation rate}$

$\Omega_p = \text{precession rate}$

$\Delta\Omega = \Omega_m - \Omega_c = \text{differential rotation}$

$$Po = \frac{\Omega_p}{\Omega_m} \sim 4e - 3$$



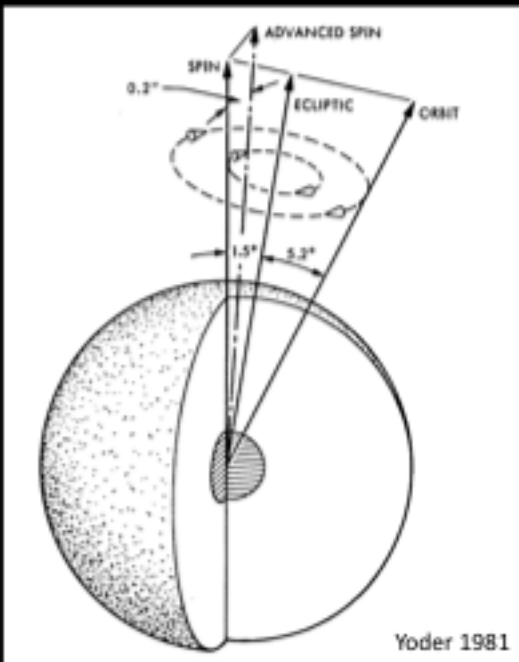
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$$Po = \frac{\Omega_P}{\Omega_m} \sim 4e - 3$$

$$E = \frac{\nu}{\Omega_m R_c^2} \sim 10^{-12}$$

$$f_c = \frac{a - c}{a} \sim 10^{-5}$$

# Precession driven primary flow.

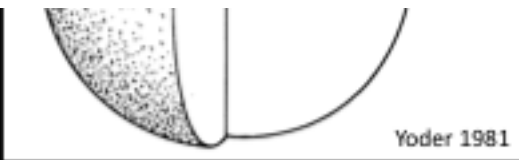
Poincaré 1910:

A first order the flow remains in quasi solid body rotation along an axis that is tilted compare to the surrounding solid shell



$$U_c = \Omega_c \times r + \nabla\psi$$





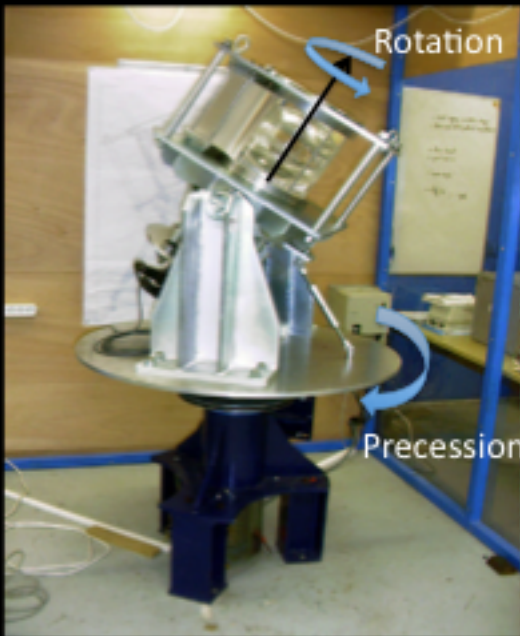
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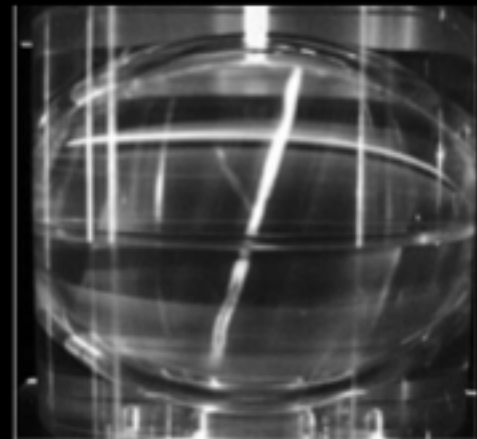
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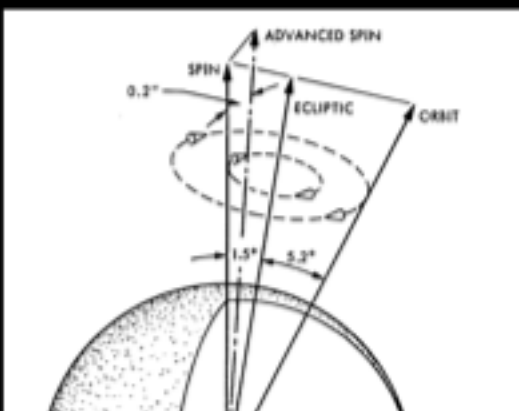


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# Uniform vorticity assumption

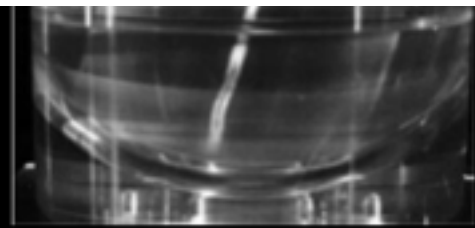
In a steady state the direction and amplitude of the rotation vector of the liquid is fully determined by the balance between the gyroscopic, pressure and viscous torque (Busse 1968, Noir et al. 2003):



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Necessary to satisfy the non penetration condition:

$$U_c \cdot \hat{n} = 0$$



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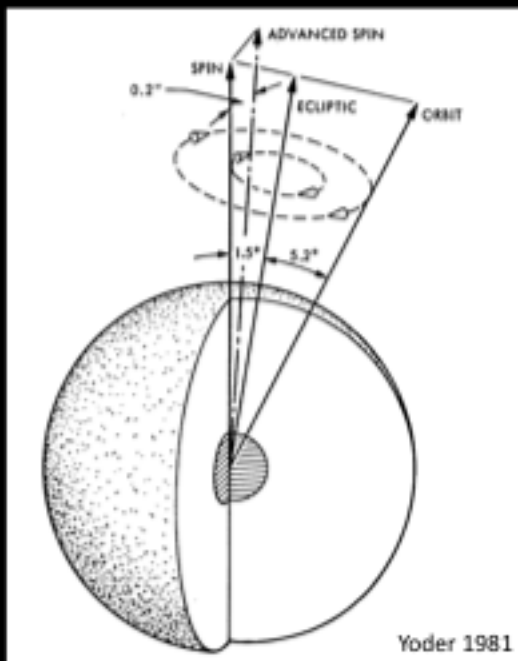
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$$\Gamma_{Precession} \propto \boldsymbol{\Omega}_p \times \boldsymbol{\Omega}_m$$

$$\Gamma_P \propto f_c \Delta \Omega$$

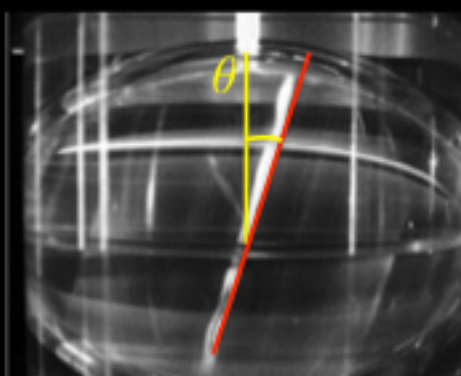
$$\Gamma_\nu \propto K \Delta \Omega$$



# Precession driven flows

At first order the fluid rotates around an axis different from the mantle spin axis

Sphere / Spheroid



The uniform vorticity theory:

- Sphere/Spheroid (Earth): Poincare 1910, Busse 1968, Noir et al. 2003.
- Triaxial ellipsoid (Moon): Noir and Cebon 2013.

$$\theta \sim 1.55^\circ, \quad \frac{|\boldsymbol{\Omega}_c - \boldsymbol{\Omega}_m|}{\boldsymbol{\Omega}_m} \sim 3\%$$



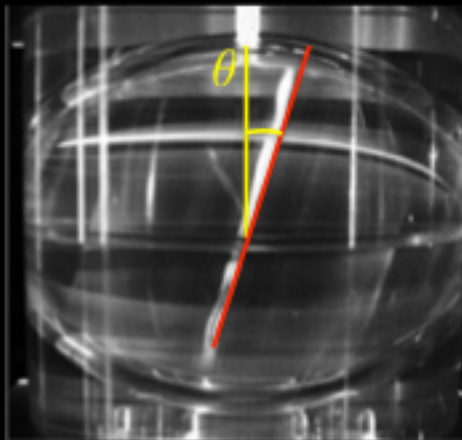
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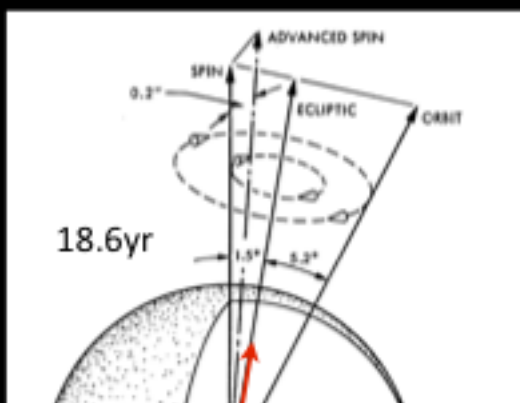
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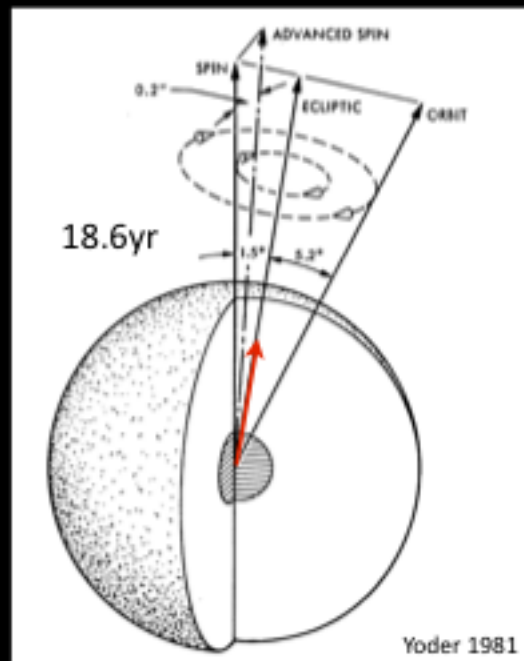
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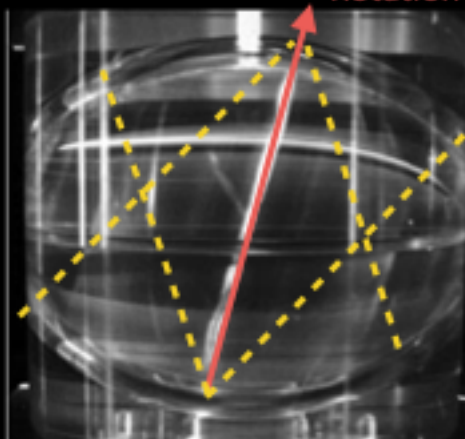
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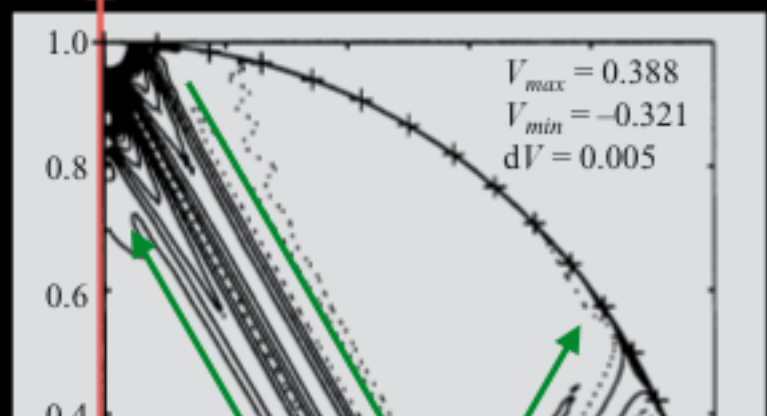
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## Precession driven secondary flows

Rotation axis of the fluid



$$u = \Omega_c \times r + \nabla \psi + \delta u$$



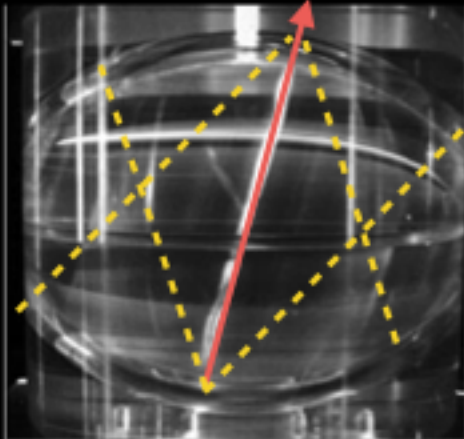




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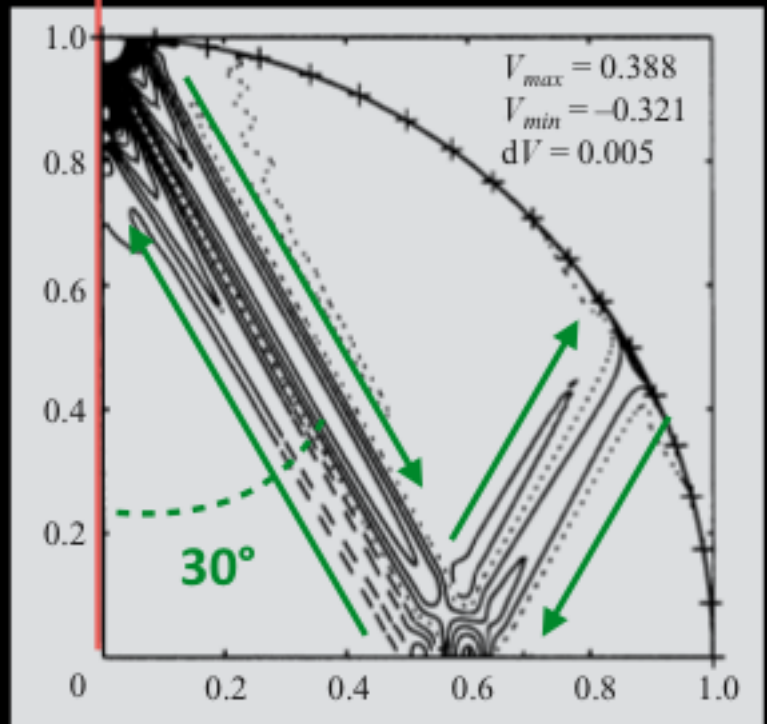
# Precession driven secondary flows

Rotation axis of the fluid



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Due to the viscosity in the boundary layer, there exists a secondary flow spawned from the critical latitudes along conical surfaces.



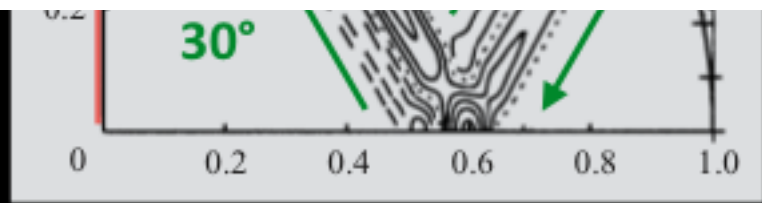
Roberts and Stewartson 1963

Noir et al. 2001

**Can the differential rotation account for the observed dissipation ?**

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The Lunar liquid Core: Brittany or Bahamas

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## Can the differential rotation account for the observed dissipation ?

The Lunar liquid Core: Brittany or Bahamas

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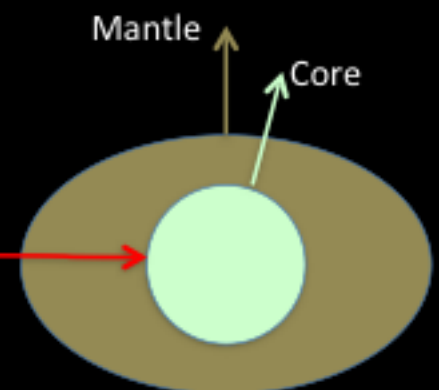
## Estimated dissipation from LLR measurements

Yoder 1981

$$0.260'' = 1.3 \frac{I_c}{I - m} \frac{\chi \sin 1.5^\circ}{1 + \chi^2}$$

$$\chi = \frac{\Gamma_\nu}{\Delta\Omega I_c \Omega_p}$$

Viscous friction at the CMB.



$$U_c = \Omega_c \times r$$

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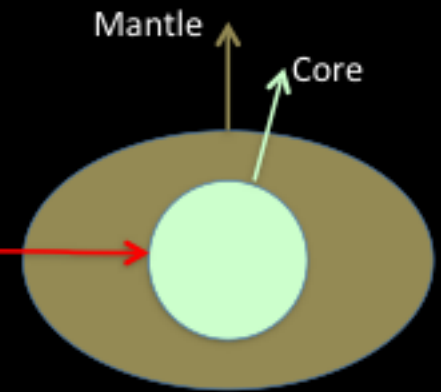
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$$\Gamma_\nu \sim 2 \times 10^{15} \text{ Nm}$$

Viscous friction  
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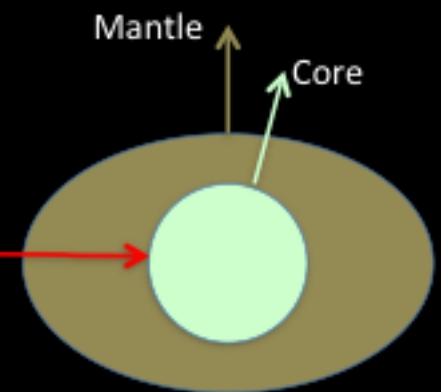


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# Estimated dissipation from LLR measurements

For small precession rate, as  
for the moon, the  
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Viscous friction  
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$$P = \Gamma_\nu \cdot (\Omega_m + \Delta\Omega) \sim \Gamma_\nu \cdot \Delta\Omega$$

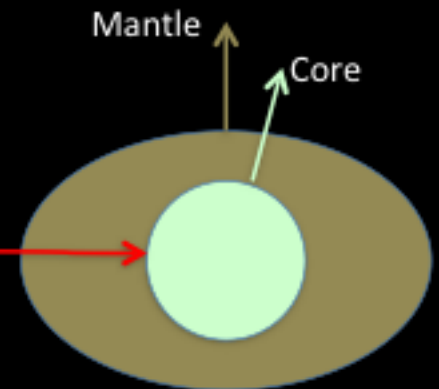
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$$U_c = \Omega_c \times r$$

$$P \sim 1.4 \times 10^8 W$$

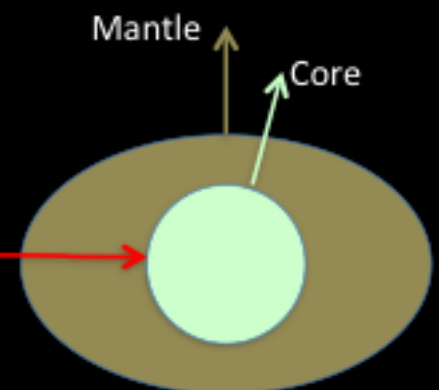
## The classical core dissipation models

Classical Core flow models for the dissipation:

- Assume spherical shape
- Assume the fluid to be in solid body rotation along an axis tilted compare to the mantle (Poincaré 1910)
- The viscous torque at the CMB (dissipation) is given by:

$$\Gamma_\nu = K(\Omega_m - \Omega_c)$$

Viscous friction at the CMB.



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# The classical core dissipation models

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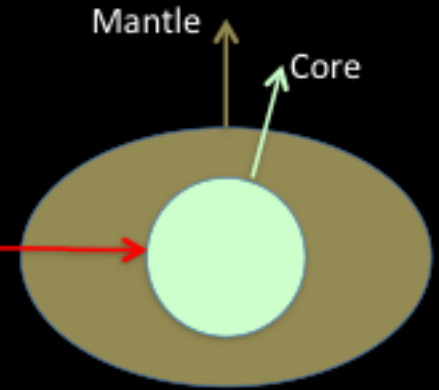
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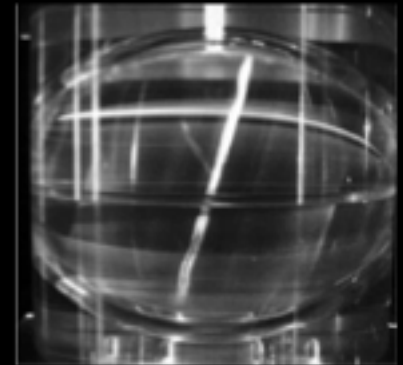
Laminar coupling

$$K \propto (I_c \Omega_c) \sqrt{E}$$

Viscous friction at the CMB.



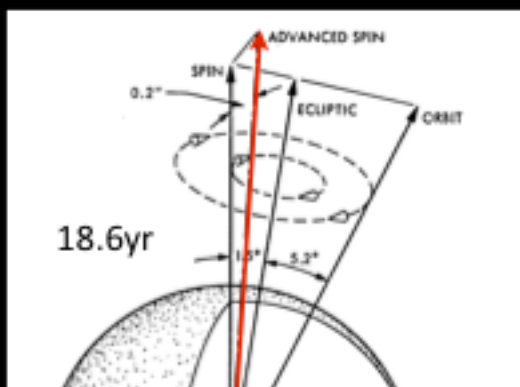
$$U_c = \Omega_c \times r$$



Noir 2000

# The classical core dissipation models

Precession of the lunar mantle exhibit -264 mas phase lag between the mantle rotation axis compare to the Cassini plane, which can not be explain with realistic model of mantle tidal dissipation (Williams 2001)

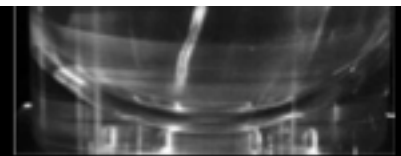


Assuming a large fraction of the dissipation at 18.6yr comes from the liquid core and a simple model of dissipation at the CMB with a laminar coupling:

$$10^{-7} m^2 \cdot s^{-1} \lesssim \nu \lesssim 10^{-6} m^2 \cdot s^{-1}$$

$$R \sim 900km \quad (\text{Williams et al. 2001})$$

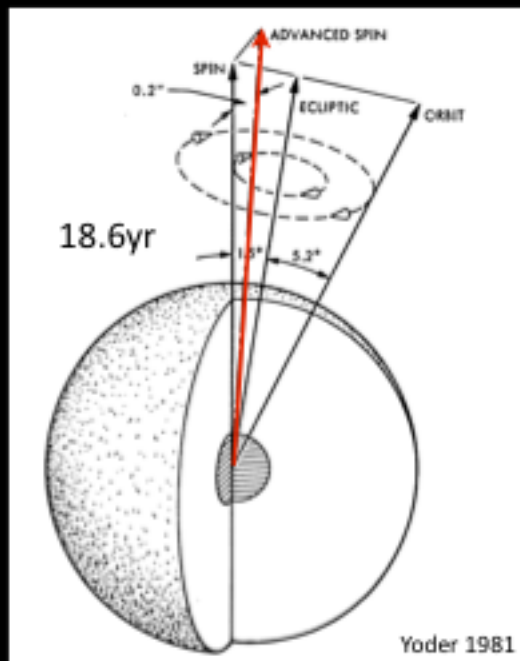
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Noir 2000

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Precession of the lunar mantle exhibit -264 mas phase lag between the mantle rotation axis compare to the Cassini plane, which can not be explain with realistic model of mantle tidal dissipation (Williams 2001)



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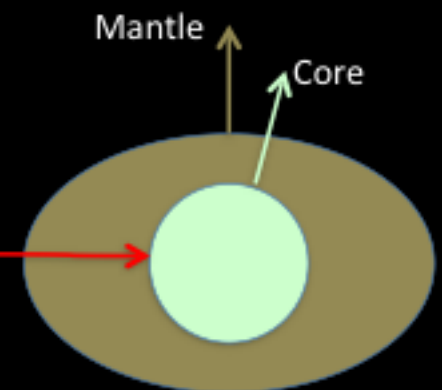
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Classical Core flow models for the dissipation:

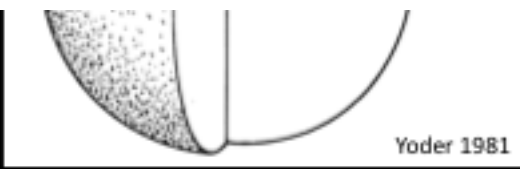
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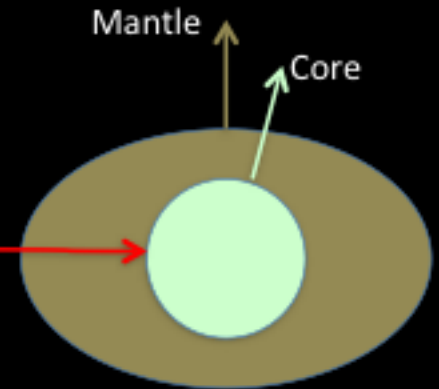
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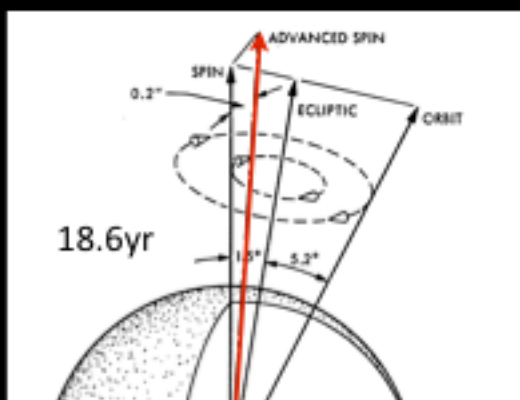
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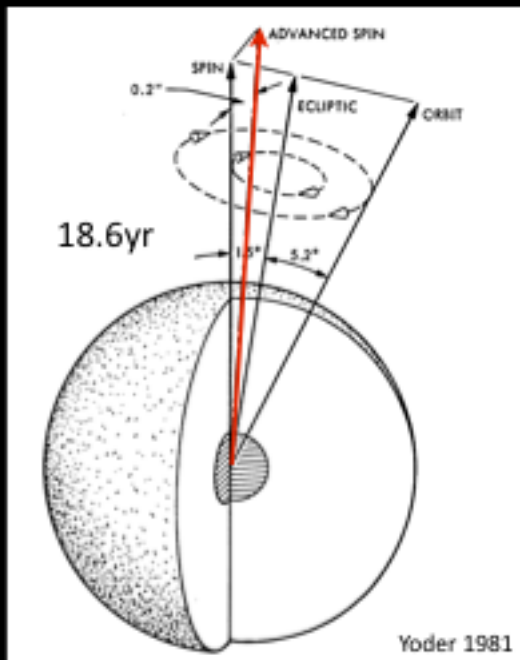
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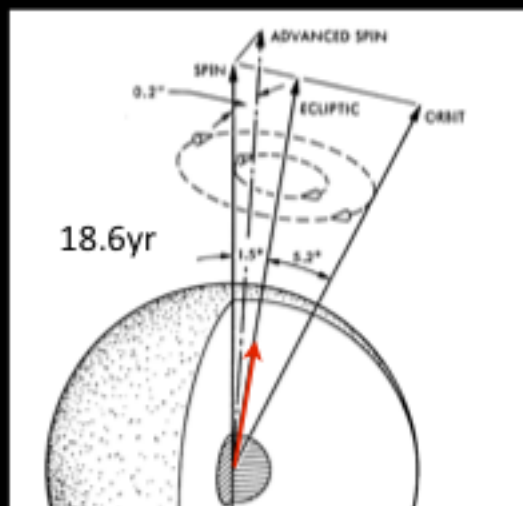




# Can turbulence be driven in the Lunar Core ?

## Ekman Layer stability

At first order the fluid rotates around an axis different from the mantle spin axis



$$\theta \sim 1.55^\circ, \frac{|\Omega_c - \Omega_m|}{\Omega_m} \sim 3\%$$

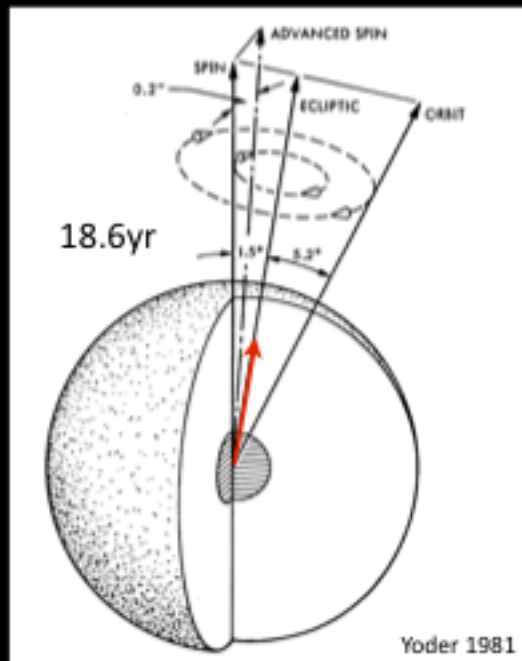
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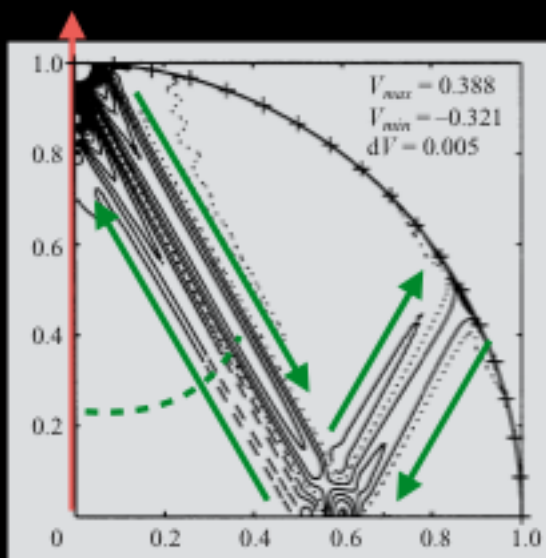
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The Lunar liquid Core: Brittany or Bahamas

# Conical shear layer stability



$$\mathbf{u} = \Omega_c \times \mathbf{r} + \nabla \psi + \delta \mathbf{u}$$

Roberts and Stewartson 1963

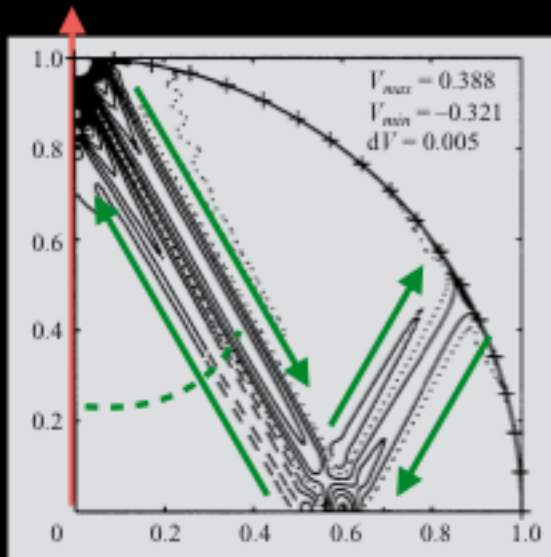
$$\delta \mathbf{u} \propto \Delta \Omega E^{1/5}$$

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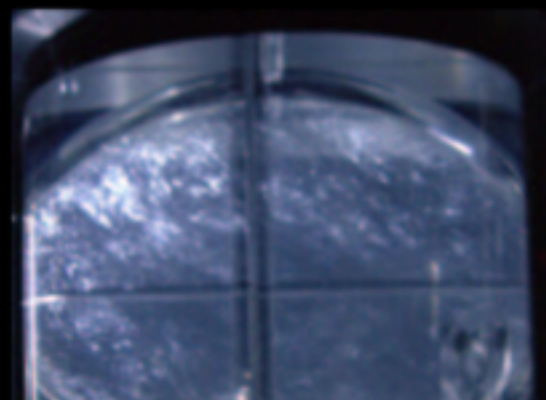
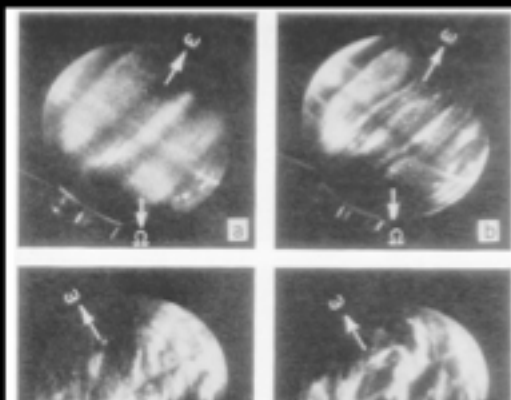
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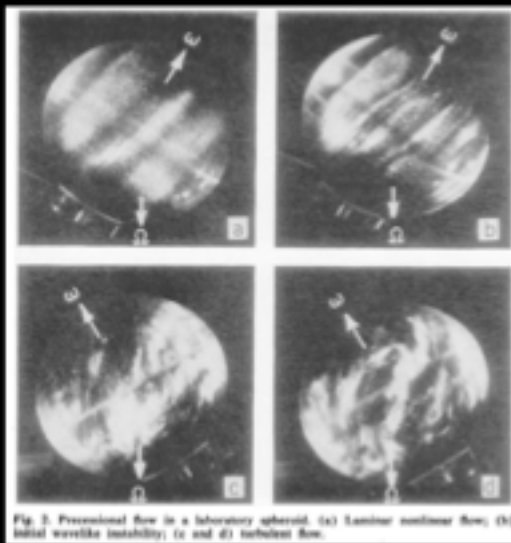
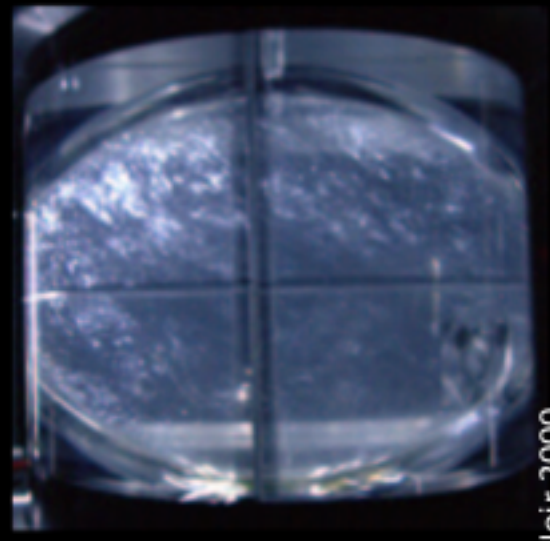


Fig. 3. Precessional flow in a laboratory spheroid. (a) Laminar nonlinear flow; (b) initial wavelike instability; (c and d) turbulent flow.

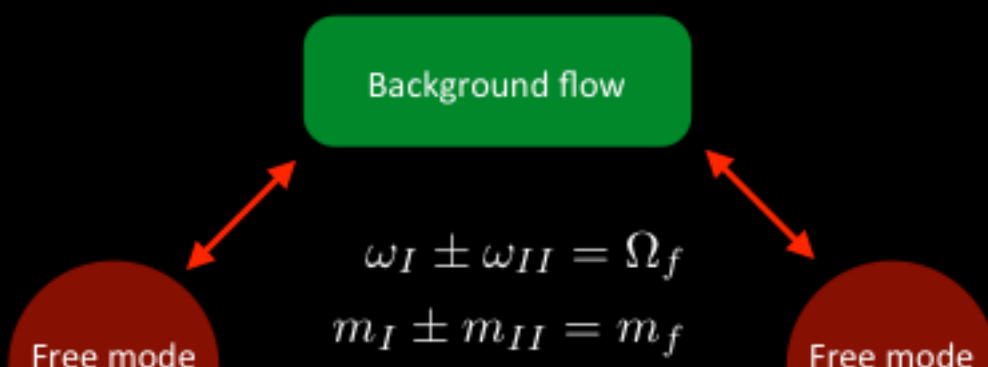
Malkus 1968



Noir 2000

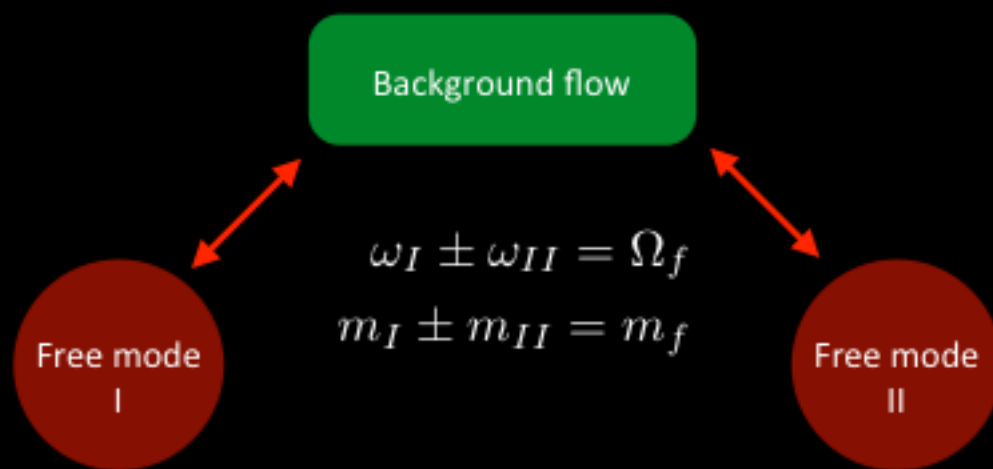
## Stability of Precession driven flows

- Inertial modes = Resonant modes of a rotating fluid cavity (Greenspan 1968)
- The solid body rotation induced by precession is certainly the simplest inertial mode, also known as the Poincaré mode.



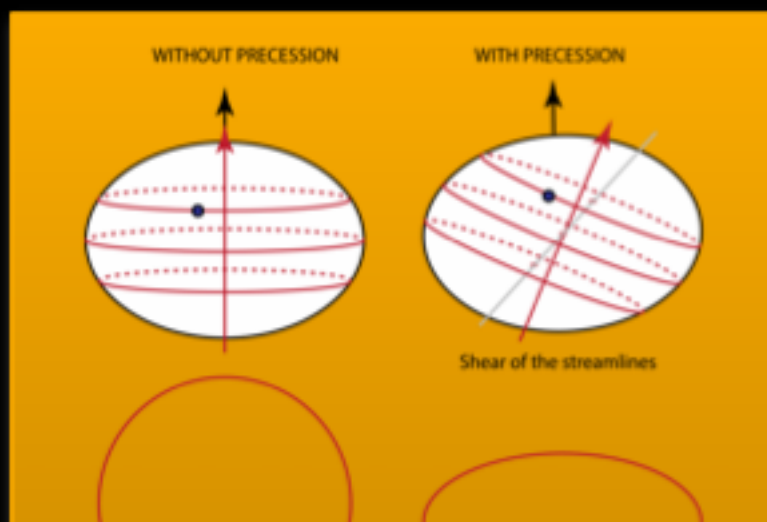
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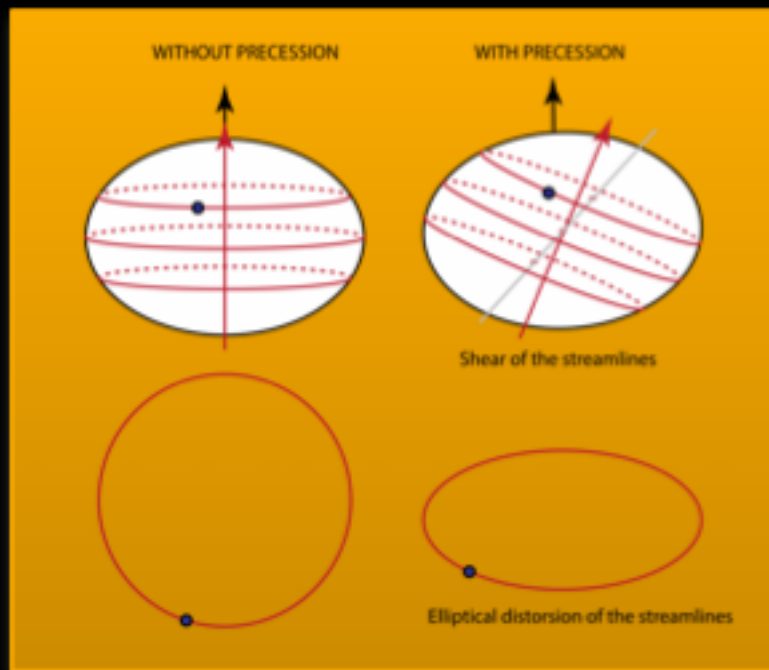


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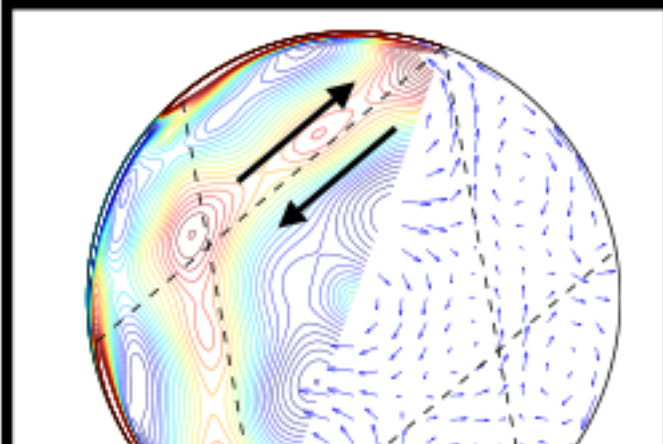
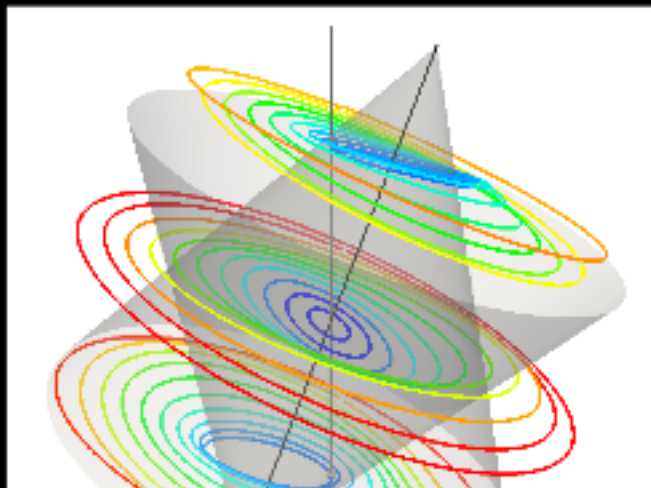


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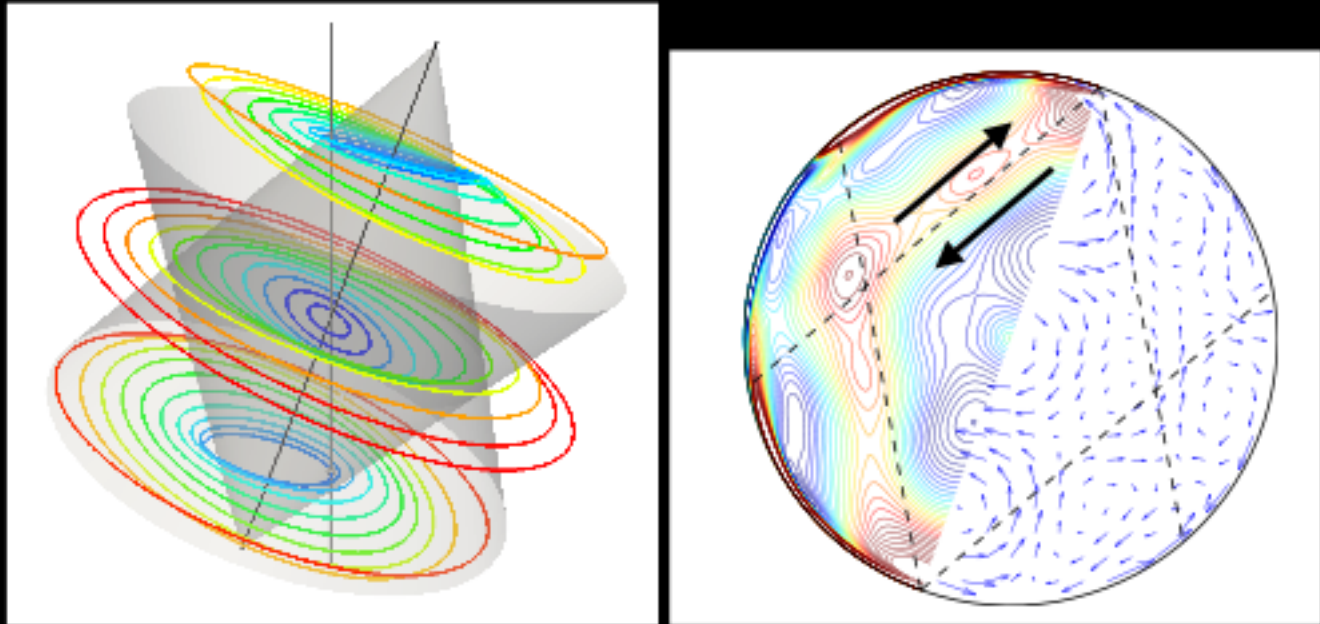
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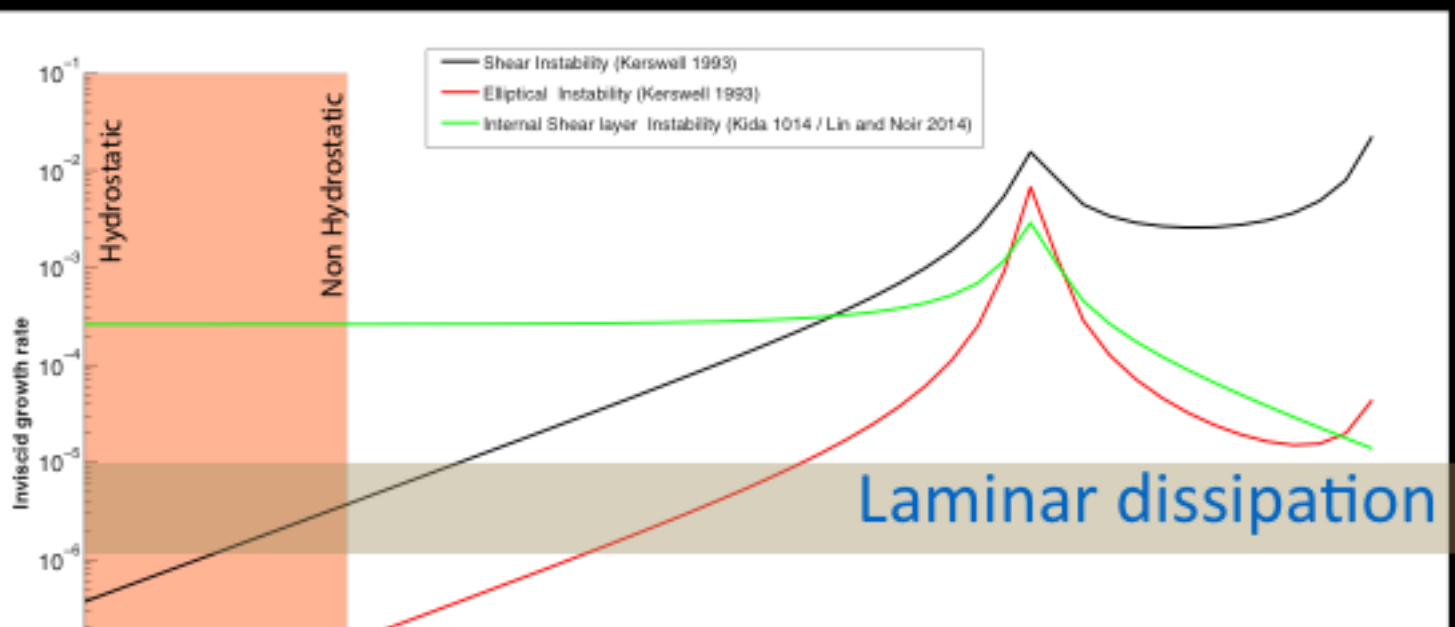
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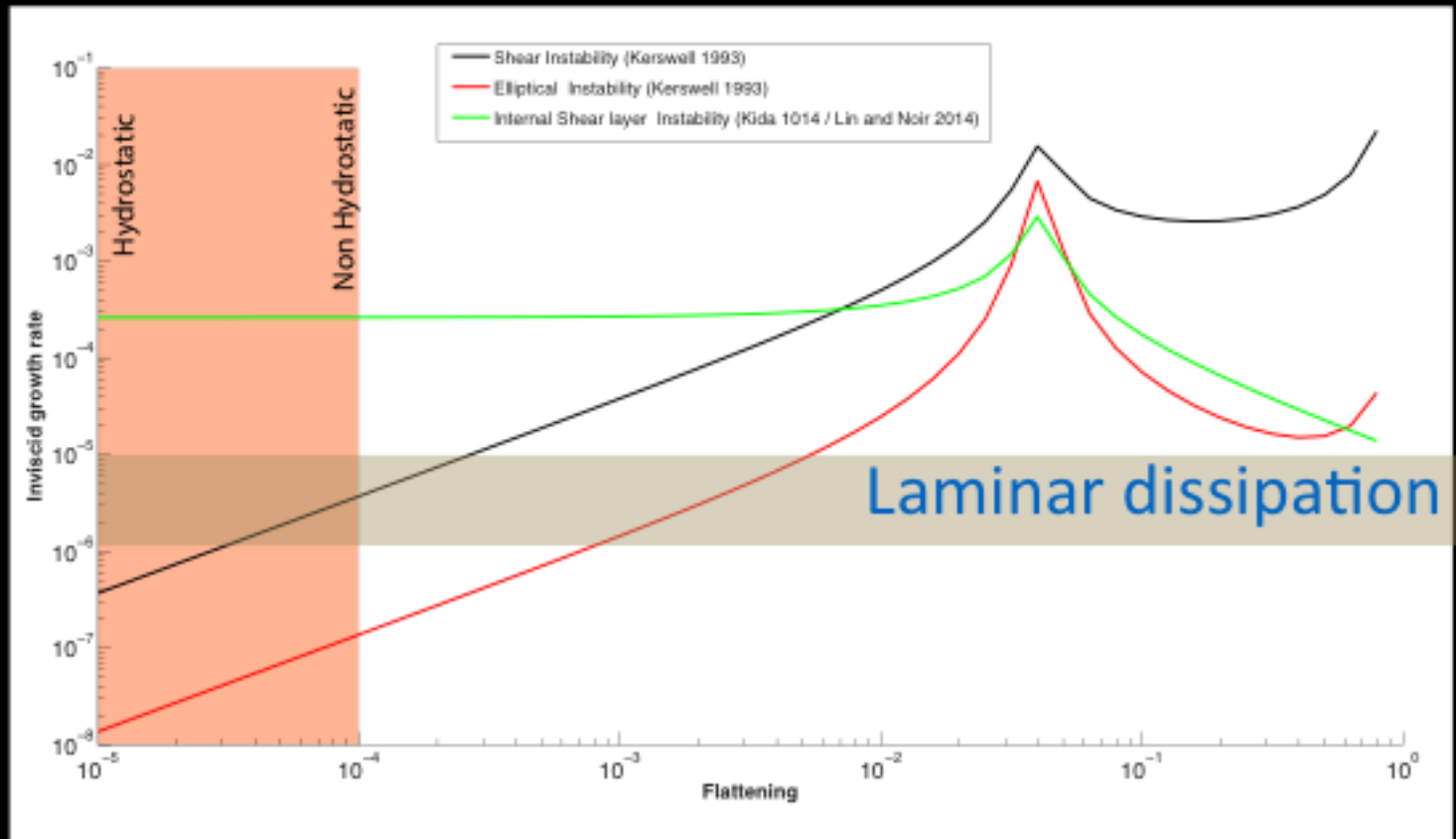
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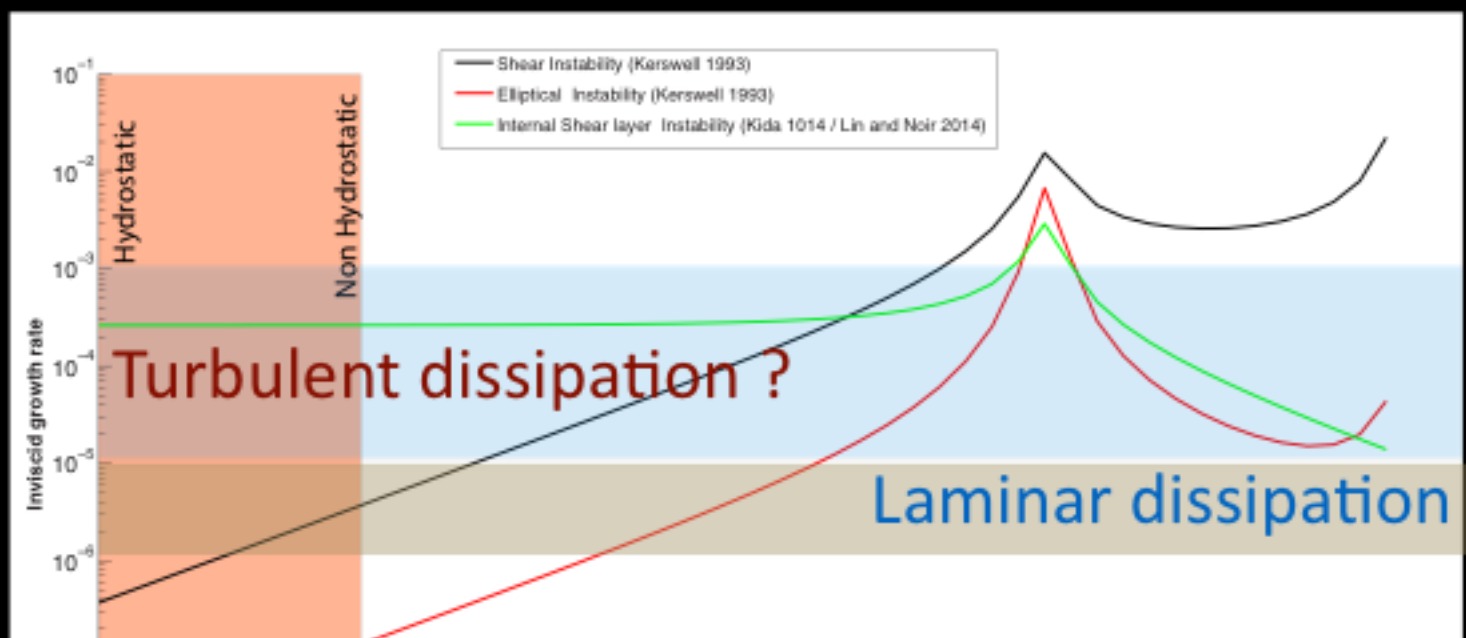
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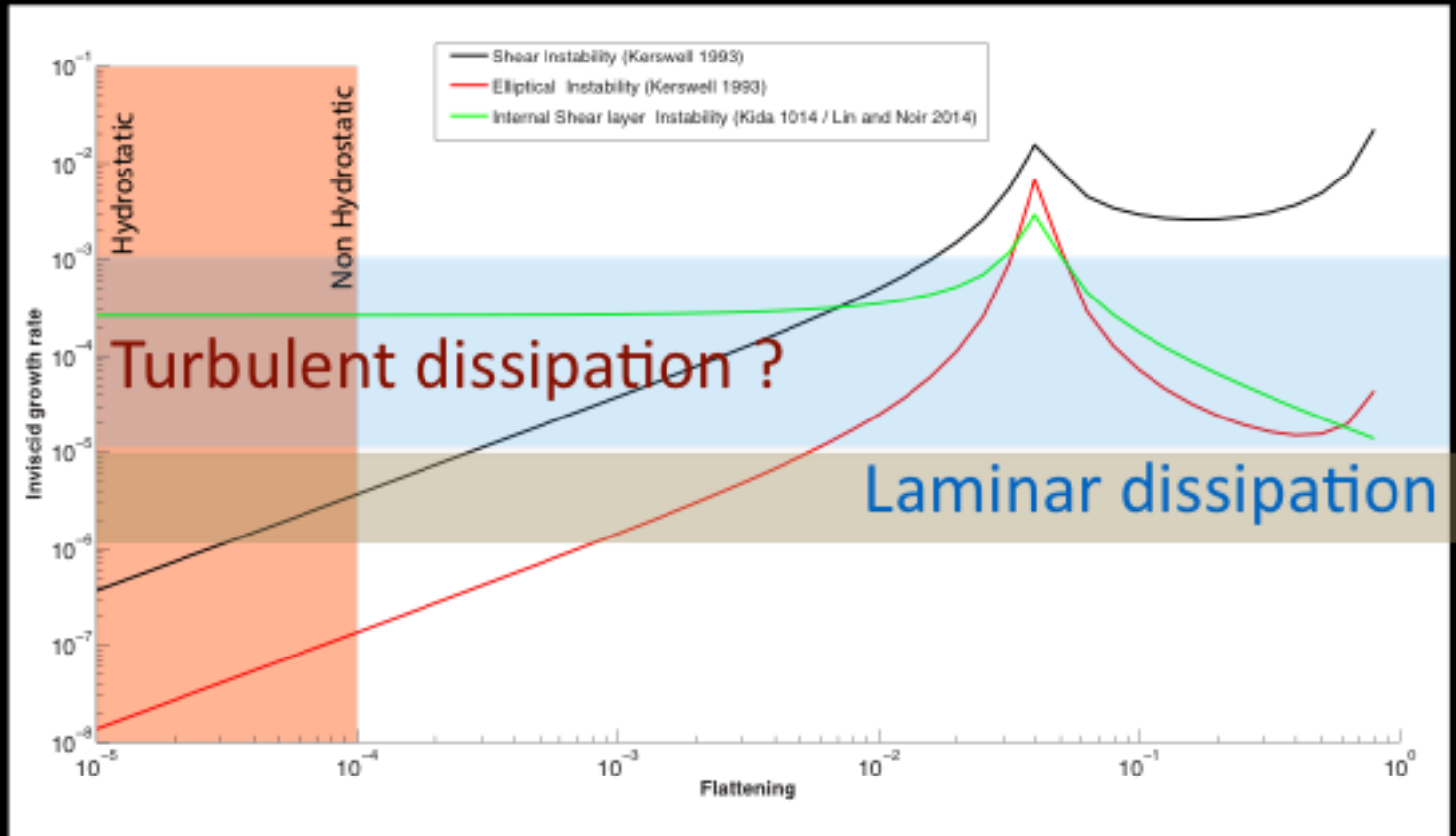
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## Take away message

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