Normal Modes of Rotating Earth Models

Habilitation à Diriger des Recherches

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Scientific activities (1/2)

- 20 authored or co-authored papers
- Co-supervisor of 2 PhD students
  1. Anthony Mémin 2007-11
     Modélisation des variations géodésiques produites par la fonte de glaciers. Séparabilité des effets des déglaciations passée et actuelle.
  2. Yann Ziegler 2012-15
     Modélisation de la rotation de la Terre et analyse conjointe des données du mouvement du pôle et de gravimétrie

Scientific activities (2/2)

- Leader of
  - Polar gravity program (IPEV 2011-15)
  - Earth rotation research projects (GRAM 2011 and INSU-PNP 2012)
- Member of
  - 2 IAG study groups 2007
  - SCAR research program and expert group 2011
  - CNFRA 2012
  - IAU (Commission 19 *Earth rotation*) 2012

Motivation

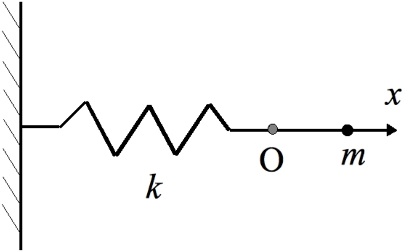
Comments from colleagues

Normal modes? Earth rotation?

- *What is it used for?*
- *It looks interesting, but it's complicated!*

Normal mode = Free oscillation

Example 1

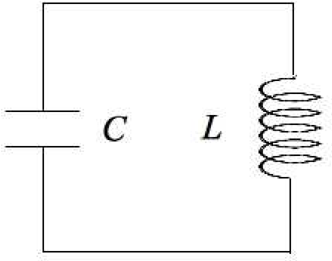


$$x = A \sin \omega t$$

$$\omega = \sqrt{\frac{k}{m}}$$

Normal mode = Free oscillation

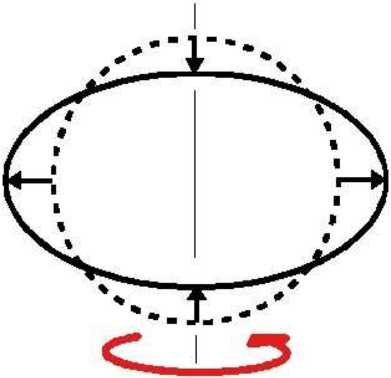
Example 2



$$i = i_0 \sin \omega t$$

$$\omega = \frac{1}{\sqrt{LC}}$$

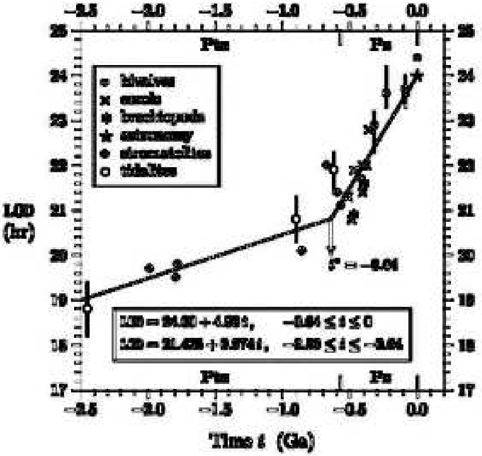
Earth rotation



$$\text{flattening} = \frac{1}{300}$$

Variations of Earth rotation (1/3)

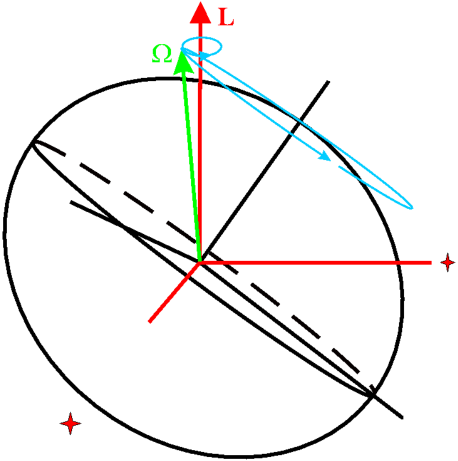
(Secular) Variation of the length of the day



Denis et al. 2011

Variations of Earth rotation (2/3)

Variations of the position of the rotation axis



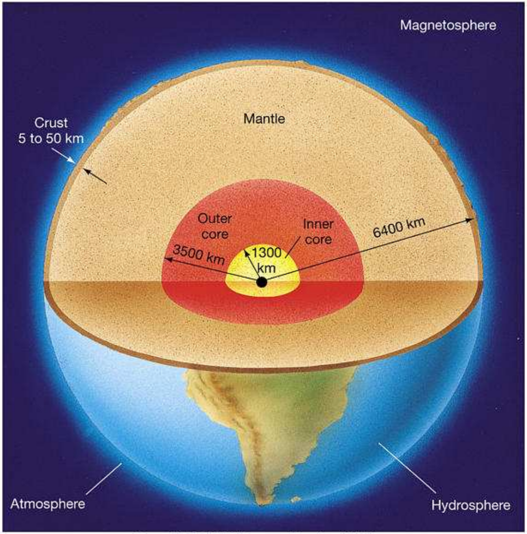
Variations of Earth rotation (3/3)

Polar motion

Phenomenon	Causes
Secular polar motion	Post-glacial rebound, melting of glaciers
Decadal variations	Global mass redistribution, core-mantle coupling
Markowitz wobble	$\sim$ 30 years
Chandler wobble	Ocean-bottom and atmosphere pressure changes
Annual wobble	Seasonal air and water mass redistributions
Diurnal motion	Ocean tides

Adapted from Höpfner 2004

Structure of the Earth



Constraints from Earth's rotation

- Departure from spherical symmetry $\Rightarrow$ *flattening* and *flattening variation*
- Deformability of the Earth $\Rightarrow$ *rheological parameters*
- Differential rotation of inner core, outer core and mantle $\Rightarrow$ *couplings: electromagnetic, viscous...*

Normal modes of a rotating elastic Earth model

Theory : Clairaut coordinates

Seismic modes, rotational modes, core spectrum

Chandler wobble and core spectrum

Normal Modes of Rotating Earth Models

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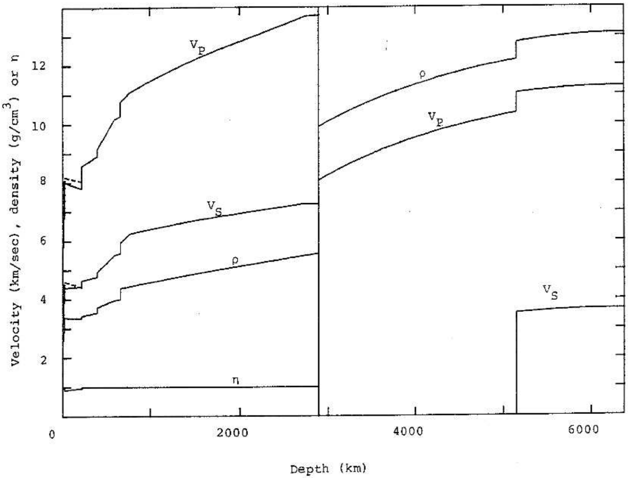
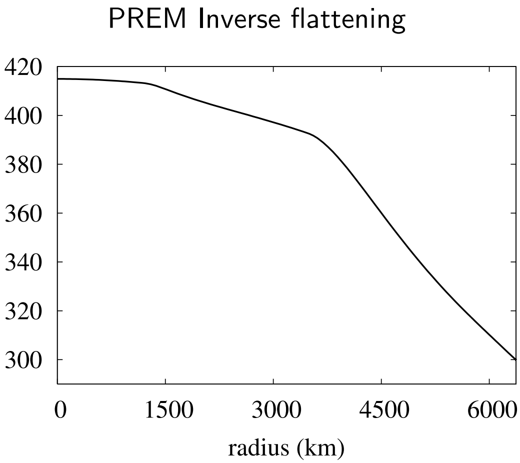
Spherical Earth model



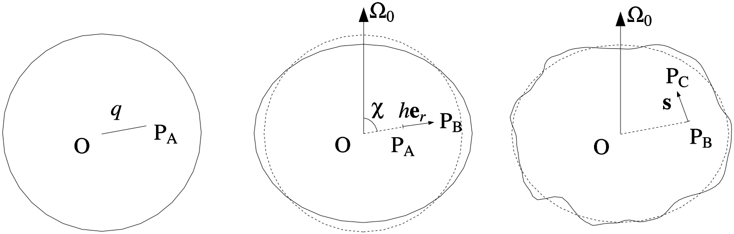
Dziewonski & Anderson 1981

Hydrostatic figures of equilibrium of rotating planets

$$\nabla p = -\rho \nabla V - \rho \boldsymbol{\Omega}_0 \times (\boldsymbol{\Omega}_0 \times \mathbf{r})$$



Clairaut coordinates q, χ, ν



A. Non-rotating spherical reference model

B. Steadily-rotating spheroidal model

C. Spheroidal model perturbed by normal mode

$$r = q + h(q, \chi)$$

$$\theta = \chi$$

$$\varphi = \nu$$

$r, \theta, \varphi =$ spherical polar coordinates

Local equations of motion

$$\rho \frac{d^2\mathbf{s}}{dt^2} = \boldsymbol{\nabla} \cdot \delta\mathbf{t} - \rho \boldsymbol{\nabla} \phi' + \rho \left(\boldsymbol{\nabla} \cdot \mathbf{s} \right) \boldsymbol{\nabla} \phi - \rho \boldsymbol{\nabla} \left(\mathbf{s} \cdot \boldsymbol{\nabla} \phi \right) - 2\rho \boldsymbol{\Omega} \times \frac{d\mathbf{s}}{dt}$$

$$\delta\mathbf{t} = \lambda \boldsymbol{\nabla} \cdot \mathbf{s}\, \mathbf{I} + \mu \left[\boldsymbol{\nabla}\mathbf{s} + \left(\boldsymbol{\nabla}\mathbf{s} \right)^T \right]$$

$$\phi = \tilde{\phi} - \frac{1}{2} |\boldsymbol{\Omega} \times \mathbf{r}|^2$$

$$\tilde{\phi}\left(\mathbf{r} \right) = -\mathrm{G} \int_{\mathcal{V}} \frac{\rho}{|\mathbf{r} - \mathbf{r}'|}\, d\mathcal{V}'$$

$$\nabla^2 \phi' = -4\pi \mathrm{G}\, \boldsymbol{\nabla} \cdot \left(\rho \mathbf{s} \right)$$

Local equations of motion in Clairaut coordinates

$$
\begin{aligned}
r &= q + h(q, \chi) \\
\theta &= \chi \\
\varphi &= \nu
\end{aligned}
$$

Method

1. Spherical harmonics expansion of displacement field **s**
2. Equations developed up to 2nd order in h

Therefore, the following functions of q must be continuous:

$$U_\ell^m - \sum_{\ell'=|\ell-2|}^{\ell+2} \begin{bmatrix} \ell & 2 & \ell' \\ 0 & 1 & -1 \\ m & 0 & m \end{bmatrix} 2\sqrt{3} L_0^{\ell'} \frac{h_2}{q} \begin{Bmatrix} -V_{\ell'}^m \\ i W_{\ell'}^m \end{Bmatrix} \tag{141}$$

$$P_\ell^m - \sum_{\ell'=|\ell-2|}^{\ell+2} \begin{bmatrix} \ell & 2 & \ell' \\ 0 & 1 & -1 \\ m & 0 & m \end{bmatrix} 2\sqrt{3} L_0^{\ell'} \frac{h_2}{q} \begin{Bmatrix} -Q_{\ell'}^m \\ i R_{\ell'}^m \end{Bmatrix} \tag{142}$$

$$\begin{aligned} &\ell(\ell+1) Q_\ell^m + \left[2\ell(\ell+1) - 6m^2\right] \frac{h_2}{q} \mu_0 \frac{V_\ell^m}{q} \\ &+ \sum_{\ell'=|\ell-2|}^{\ell+2} \begin{bmatrix} \ell & 2 & \ell' \\ 0 & 0 & 0 \\ m & 0 & m \end{bmatrix} \left[-\frac{dh_2}{dq} \begin{Bmatrix} \ell'(\ell'+1) Q_{\ell'}^m \\ 0 \end{Bmatrix} \right. \\ &\left. + 6\frac{h_2}{q} \begin{Bmatrix} P_{\ell'}^m - q\frac{dQ_{\ell'}^m}{dq} - 2(\lambda_0+\mu_0)\frac{U_{\ell'}^m}{q} - \lambda_0 \frac{dU_{\ell'}^m}{dq} + \ell'(\ell'+1)\left(\lambda_0 + \frac{4}{3}\mu_0\right)\frac{V_{\ell'}^m}{q} \\ 0 \end{Bmatrix} \right] \\ &+ \sum_{\ell'=|\ell-2|}^{\ell+2} \begin{bmatrix} \ell & 2 & \ell' \\ 0 & 1 & -1 \\ m & 0 & m \end{bmatrix} 2\sqrt{3} L_0^{\ell'} \left[\frac{dh_2}{dq} \begin{Bmatrix} -Q_{\ell'}^m \\ i R_{\ell'}^m \end{Bmatrix} - \frac{h_2}{q} \begin{Bmatrix} -P_{\ell'}^m + q\frac{dQ_{\ell'}^m}{dq} + 2(\lambda_0+\mu_0)\frac{U_{\ell'}^m}{q} + \lambda_0\frac{dU_{\ell'}^m}{dq} \\ 0 \end{Bmatrix} \right. \\ &\left. - \frac{h_2}{q} \begin{Bmatrix} \left[-\ell'(\ell'+1)\lambda_0 + (8 - 2\ell(\ell+1))\mu_0\right] \frac{V_{\ell'}^m}{q} \\ i\left[-\ell'(\ell'+1) - 8 + 2\ell(\ell+1)\right]\mu_0 \frac{W_{\ell'}^m}{q} \end{Bmatrix} \right] \\ &- \sum_{\ell'=|\ell-2|}^{\ell+2} \sum_{\ell''=|\ell'-1|}^{\ell'+1} \begin{bmatrix} \ell & 2 & \ell' \\ 0 & 0 & 0 \\ m & 0 & m \end{bmatrix} \begin{bmatrix} \ell' & 1 & \ell'' \\ 0 & 1 & -1 \\ m & 0 & m \end{bmatrix} 12 L_0^{\ell''} m \frac{h_2}{q} \mu_0 \begin{Bmatrix} 0 \\ i\frac{V_{\ell''}^m}{q} \end{Bmatrix} \end{aligned} \tag{143}$$

$$\begin{aligned} &\ell(\ell+1) R_\ell^m + \sum_{\ell'=|\ell-2|}^{\ell+2} \begin{bmatrix} \ell & 2 & \ell' \\ 0 & 0 & 0 \\ m & 0 & m \end{bmatrix} \left[-\frac{dh_2}{dq} \begin{Bmatrix} \ell'(\ell'+1) Q_{\ell'}^m \\ 0 \end{Bmatrix} - 6\frac{h_2}{q} \begin{Bmatrix} q\frac{dQ_{\ell'}^m}{dq} - \ell'(\ell'+1)\mu_0 \frac{W_{\ell'}^m}{q} \\ 0 \end{Bmatrix} \right] \\ &+ \sum_{\ell'=|\ell-2|}^{\ell+2} \begin{bmatrix} \ell & 2 & \ell' \\ 0 & 1 & -1 \\ m & 0 & m \end{bmatrix} 2\sqrt{3} L_0^{\ell'} \left[\frac{dh_2}{dq} \begin{Bmatrix} -R_{\ell'}^m \\ -i Q_{\ell'}^m \end{Bmatrix} - \frac{h_2}{q} \begin{Bmatrix} q\frac{dR_{\ell'}^m}{dq} \\ i\left(P_{\ell'}^m - q\frac{dQ_{\ell'}^m}{dq}\right) \end{Bmatrix} \right. \\ &\left. + \frac{h_2}{q} \begin{Bmatrix} \left[\ell'(\ell'+1) - 6\right]\mu_0 \frac{W_{\ell'}^m}{q} \\ i\left[2(\lambda_0+\mu_0)\frac{U_{\ell'}^m}{q} + \lambda_0\frac{dU_{\ell'}^m}{dq} + \left[-\ell'(\ell'+1)\lambda_0 + 2\mu_0\right]\frac{V_{\ell'}^m}{q}\right] \end{Bmatrix} \right] \\ &- \sum_{\ell'=|\ell-2|}^{\ell+2} \sum_{\ell''=|\ell'-1|}^{\ell'+1} \begin{bmatrix} \ell & 2 & \ell' \\ 0 & 0 & 0 \\ m & 0 & m \end{bmatrix} \begin{bmatrix} \ell' & 1 & \ell'' \\ 0 & 1 & -1 \\ m & 0 & m \end{bmatrix} 12 L_0^{\ell''} m \frac{h_2}{q} \mu_0 \begin{Bmatrix} 0 \\ i\frac{V_{\ell''}^m}{q} \end{Bmatrix} \end{aligned} \tag{144}$$

$$\phi_\ell^m \tag{145}$$

$$g_\ell^m \tag{146}$$

If the boundary is welded, s is continuous. Therefore, instead of quantity (141), the quantities

$$U_\ell^m + \sum_{\ell'=|\ell-2|}^{\ell+2} \begin{bmatrix} \ell & 2 & \ell' \\ 0 & 0 & 0 \\ m & 0 & m \end{bmatrix} \frac{dh_2}{dq} \begin{Bmatrix} U_{\ell'}^m \\ 0 \end{Bmatrix} \tag{147}$$

$$\ell(\ell+1) V_\ell^m + \sum_{\ell'=|\ell-2|}^{\ell+2} \begin{bmatrix} \ell & 2 & \ell' \\ 0 & 0 & 0 \\ m & 0 & m \end{bmatrix} 6h_2 \begin{Bmatrix} \frac{U_{\ell'}^m}{q} - \frac{dV_{\ell'}^m}{dq} \\ 0 \end{Bmatrix} - \sum_{\ell'=|\ell-2|}^{\ell+2} \begin{bmatrix} \ell & 2 & \ell' \\ 0 & 1 & -1 \\ m & 0 & m \end{bmatrix} h_2 \begin{Bmatrix} -\left(\frac{U_{\ell'}^m}{q} - \frac{dV_{\ell'}^m}{dq}\right) \\ i\frac{dW_{\ell'}^m}{dq} \end{Bmatrix} \tag{148}$$

$$\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\left[-\cos\chi\frac{\partial^2\mathcal{D}_\ell^m}{\partial\chi^2}+\left(\sin\chi-\frac{\cos^2\chi}{\sin\chi}\right)\frac{\partial\mathcal{D}_\ell^m}{\partial\chi}-\frac{\cos\chi}{\sin^2\chi}\frac{\partial^2\mathcal{D}_\ell^m}{\partial v^2}\right]f_{1\ell}^m+\frac{\partial\mathcal{D}_\ell^m}{\partial v}f_{2\ell}^m$$
$$=\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}L_0^\ell\left\{\sum_{\ell'=|\ell-1|}^{\ell+1}\begin{bmatrix}\ell & 1 & \ell'\\ -1 & 0 & -1\\ m & 0 & m\end{bmatrix}\sqrt{2\ell'(\ell'+1)}\begin{Bmatrix}-i f_{2\ell'}^m\\ f_{1\ell'}^m\end{Bmatrix}\right\}\mathcal{D}_\ell^m, \tag{A.42}$$

$$\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}h\left\{\left[-\cos\chi\frac{\partial^2\mathcal{D}_\ell^m}{\partial\chi^2}+\left(\sin\chi-\frac{\cos^2\chi}{\sin\chi}\right)\frac{\partial\mathcal{D}_\ell^m}{\partial\chi}-\frac{\cos\chi}{\sin^2\chi}\frac{\partial^2\mathcal{D}_\ell^m}{\partial v^2}\right]f_{1\ell}^m+\frac{\partial\mathcal{D}_\ell^m}{\partial v}f_{2\ell}^m\right\}$$
$$=\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\left\{\sum_{\ell'=|\ell-2|}^{\ell+2}\sum_{\ell''=|\ell'-1|}^{\ell'+1}\begin{bmatrix}\ell & 2 & \ell'\\ 0 & 0 & 0\\ m & 0 & m\end{bmatrix}\begin{bmatrix}\ell' & 1 & \ell''\\ -1 & 0 & -1\\ m & 0 & m\end{bmatrix}h_2L_0^{\ell'}\sqrt{2\ell''(\ell''+1)}\begin{Bmatrix}-i f_{2\ell''}^m\\ f_{1\ell''}^m\end{Bmatrix}\right\}\mathcal{D}_\ell^m, \tag{A.43}$$

$$\frac{\cos\chi}{\sin\chi}\left(\frac{\partial h}{\partial\chi}\right)^2\frac{\partial f}{\partial\chi}=$$
$$\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}h_2^2 2\sqrt{3}\left\{\sum_{\ell'=|\ell-2|}^{\ell+2}L_0^{\ell'}\begin{bmatrix}\ell & 2 & \ell'\\ 0 & 1 & -1\\ m & 0 & m\end{bmatrix}\begin{Bmatrix}f_{\ell'}^m\\ 0\end{Bmatrix}\right.$$
$$\left.+\sum_{\ell'=|\ell-2|}^{\ell+2}\sum_{\ell''=|\ell'-2|}^{\ell'+2}L_0^{\ell''}\begin{bmatrix}\ell & 2 & \ell'\\ 0 & 0 & 0\\ m & 0 & m\end{bmatrix}\begin{bmatrix}\ell' & 2 & \ell''\\ 0 & 1 & -1\\ m & 0 & m\end{bmatrix}\begin{Bmatrix}f_{\ell''}^m\\ 0\end{Bmatrix}\right\}\mathcal{D}_\ell^m, \tag{A.44}$$

$$\frac{1}{\sin^2\chi}\left(\frac{\partial h}{\partial\chi}\right)^2\frac{\partial^2 f}{\partial v^2}=$$
$$\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}h_2^2\left\{-3m^2f_\ell^m-\sum_{\ell'=|\ell-2|}^{\ell+2}6m^2\begin{bmatrix}\ell & 2 & \ell'\\ 0 & 0 & 0\\ m & 0 & m\end{bmatrix}\begin{Bmatrix}f_{\ell'}^m\\ 0\end{Bmatrix}\right\}\mathcal{D}_\ell^m, \tag{A.45}$$

and

$$\frac{1}{\sin\chi}\left(\frac{\partial h}{\partial\chi}\right)^2\frac{\partial^2 f}{\partial\chi\partial v}=$$
$$\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\sum_{\ell'=|\ell-2|}^{\ell+2}\sum_{\ell''=|\ell'-2|}^{\ell'+2}\left\{12L_0^\ell L_0^{\ell''}h_2^2\begin{bmatrix}\ell & 2 & \ell'\\ 0 & 1 & -1\\ m & 0 & m\end{bmatrix}\begin{bmatrix}\ell' & 2 & \ell''\\ 0 & 1 & -1\\ m & 0 & m\end{bmatrix}\begin{Bmatrix}0\\ -i f_{\ell''}^m\end{Bmatrix}\right\}\mathcal{D}_\ell^m. \tag{A.46}$$

Normal Modes of Rotating Earth Models

Theory : Clairaut coordinates

Seismic modes, rotational modes, core spectrum

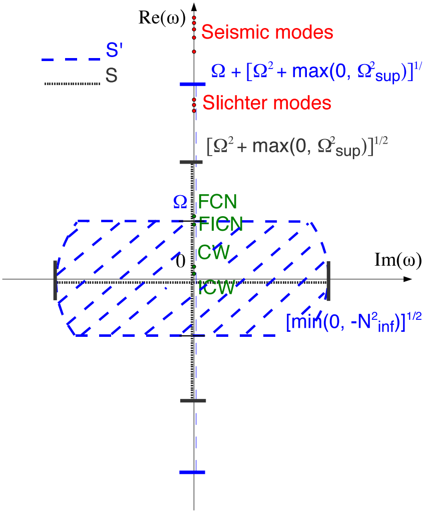
Chandler wobble and core spectrum

Normal modes of a rotating elastic Earth model (1/2)

3 families

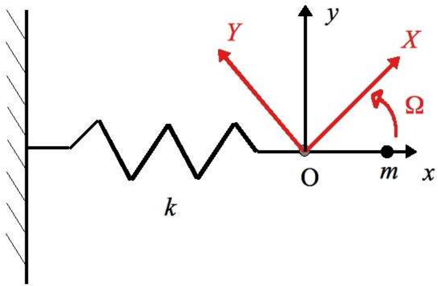
1. Seismic modes
2. Rotational modes
3. Spectrum of the liquid outer core

Normal modes of a rotating elastic Earth model (2/2)



Effect of rotation on seismic modes

Example 1 viewed in a rotating frame

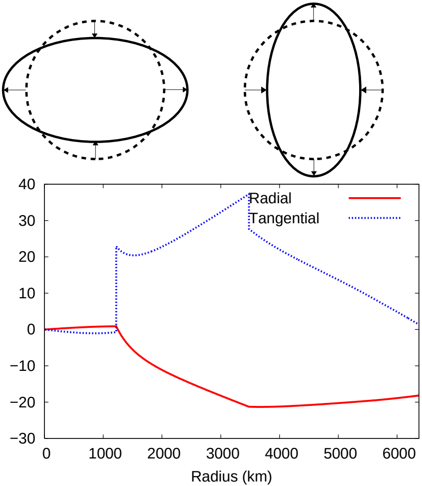


$$X = a_1 \sin(\omega + \Omega)t + a_2 \sin(\omega - \Omega)t$$
$$Y = a_1 \cos(\omega + \Omega)t - a_2 \cos(\omega - \Omega)t$$

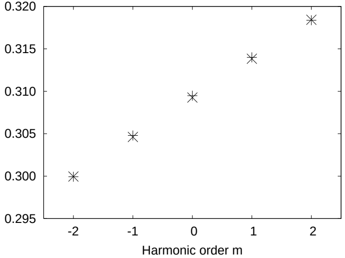
$$\omega = \sqrt{\frac{k}{m}}$$

Splitting of the seismic mode ${}_0S_2$

Eigenfunctions



Eigenfrequencies (mHz)

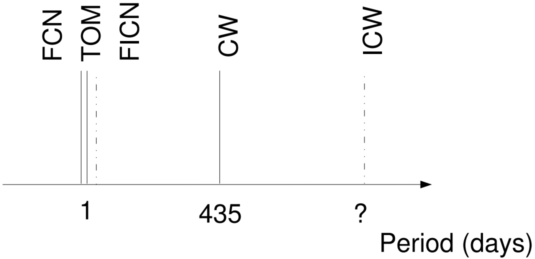


× observed

+ computed

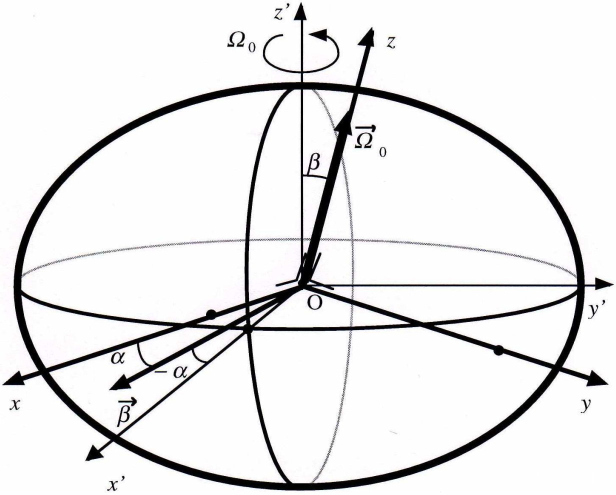
Rotational modes

Feature : motion of rotation axis



i. Tilt-over Mode (TOM)
ii. Free Core Nutation (FCN)
ii. Free Inner Core Nutation (FICN)
iv. Chandler Wobble (CW)
v. Inner Core Wobble (ICW)

Tilt-over mode

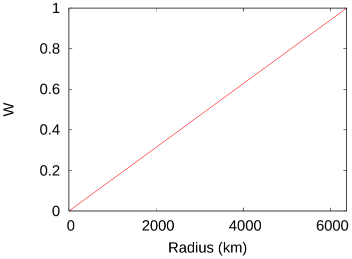


Tilt-over mode

$$\mathbf{s} = \boldsymbol{\beta} \times \mathbf{r} = \boldsymbol{\nabla} \times (W\mathbf{r}) = \boldsymbol{\tau}_1^1$$

$$\boldsymbol{\beta} = \beta \left[\cos\left(\Omega_0 t\right) \mathbf{e}_x - \sin\left(\Omega_0 t\right) \mathbf{e}_y\right]$$

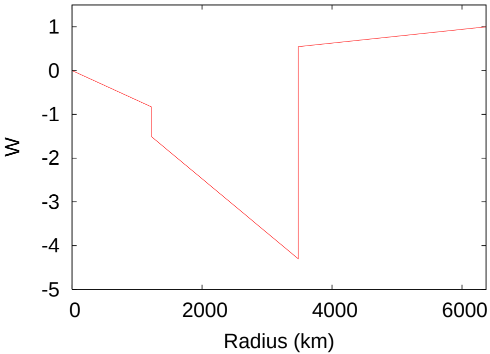
$$W = r\beta \sin\theta \cos\left(\Omega_0 t + \varphi\right)$$



$$T_{\mathrm{comp}} = 86164.22\ \mathrm{s}$$
$$T_{\mathrm{TOM}} = 86164.10\ \mathrm{s}$$

Free Core Nutation

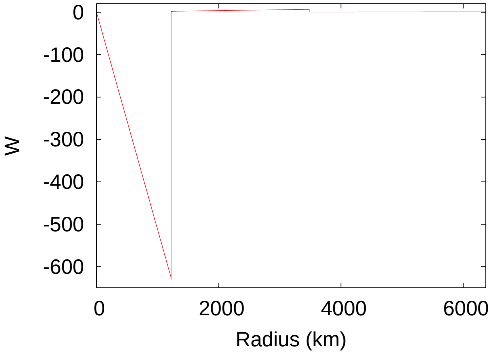
$$\mathbf{s} = \boldsymbol{\tau}_1^1 + \boldsymbol{\sigma}_2^1 + \boldsymbol{\tau}_3^1$$



$$T_{\rm FCN} = 1 \;/\; (1 + 1/458.4) \text{ sidereal day}$$
$$T_{\rm VLBI} = 1 \;/\; [1 + 1/(429.0 \pm 0.3)] \text{ sd}$$
$$T_{\rm GGP} = 1 \;/\; [1 + 1/(429.7 \pm 2.9)] \text{ sd}$$

Free Inner Core Nutation

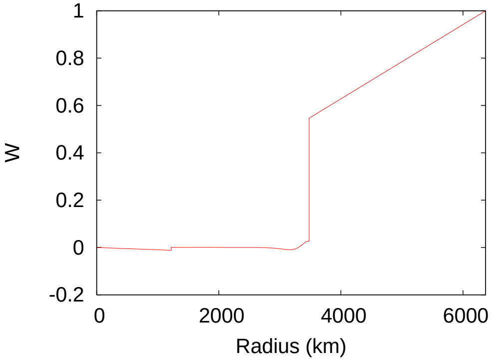
$$\mathbf{s} = \boldsymbol{\tau}_1^1 + \boldsymbol{\sigma}_2^1 + \boldsymbol{\tau}_3^1$$



$T_{\mathrm{FICN}} = 1 \ / \ (1 \text{ - } 1/472.9)$ sidereal day

Chandler Wobble

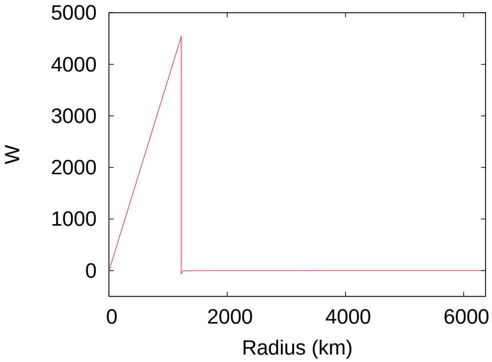
$$\mathbf{s} = \boldsymbol{\tau}_1^{-1} + \boldsymbol{\sigma}_2^{-1} + \boldsymbol{\tau}_3^{-1}$$



$$T_{\mathrm{CW}} = 404.7 \text{ solar days}$$
$$T_{\mathrm{obs}} = 434 \text{ solar days}$$

Inner Core Wobble

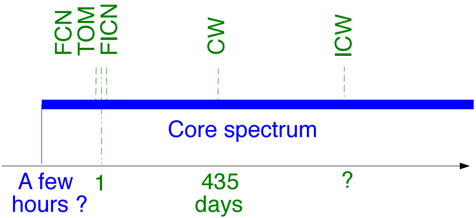
$$\mathbf{s} = \boldsymbol{\tau}_1^{-1} + \boldsymbol{\sigma}_2^{-1} + \boldsymbol{\tau}_3^{-1}$$



$$T_{\mathrm{ICW}} = 4008 \text{ solar days}$$
$$T_{\mathrm{AM}} = 2750 \text{ solar days}$$

for PREM (Rochester & Crossley 2009)

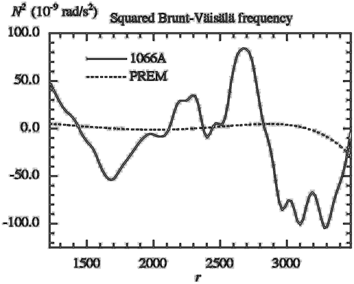
Core Spectrum



- Confined in the liquid outer core
- Restoring forces:
  gravity (buoyancy) + inertia (Coriolis)
- Unobserved

Non-rotating spherical model

Key parameter: $N^2 = -g\left(\frac{1}{\rho}\frac{d\rho}{dr} + \frac{g}{v_p^2}\right)$

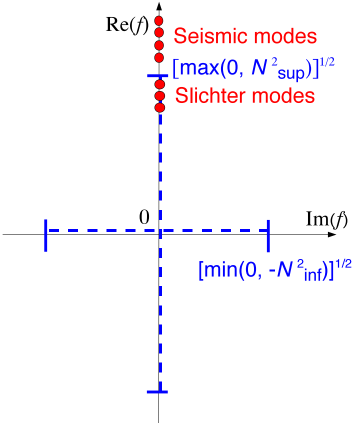


$$N^2_{\mathrm{sup}} = 6 \cdot 10^{-8}\ \mathrm{rad}^2\ \mathrm{s}^{-2}$$

$$N^2_{\mathrm{inf}} = -8 \cdot 10^{-8}\ \mathrm{rad}^2\ \mathrm{s}^{-2}$$

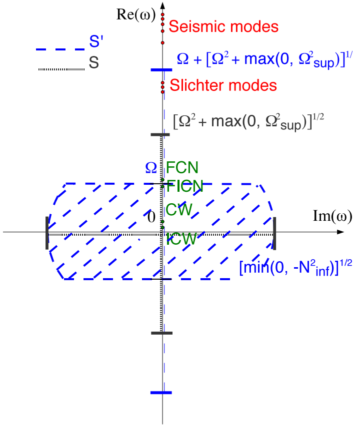
(Valette & Lesage 2007)

Schwarzschild criterion



N^2	ω^2
> 0	$0 \leq \omega^2 \leq N^2_{\text{sup}}$
< 0	$N^2_{\text{inf}} \leq \omega^2 \leq 0$

Rotating model



S ⊆ Essential spectrum ⊆ S'
(Valette 1989)

If $N^2 < 0$, *stable* modes

Planetary (Rossby) modes

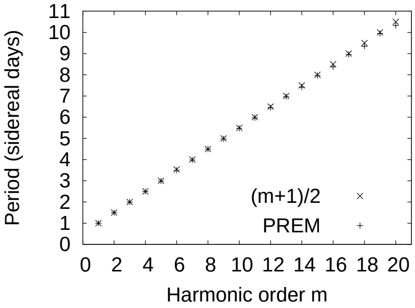
Incompressible liquid shell

$$\mathbf{s} = \boldsymbol{\tau}_m^m = \boldsymbol{\nabla} \times (W_m^m \mathbf{r})$$

$$W_m^m = Ar^m$$

$$\omega_m = \frac{2\Omega}{m+1}$$

$$m = 1 \longrightarrow \text{tilt-over mode}$$



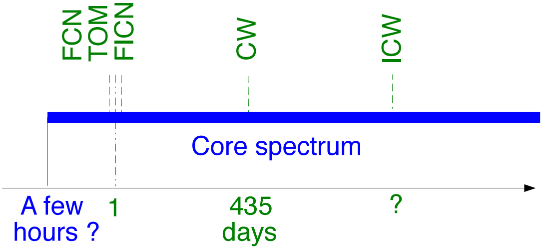
Normal Modes of Rotating Earth Models

Theory : Clairaut coordinates

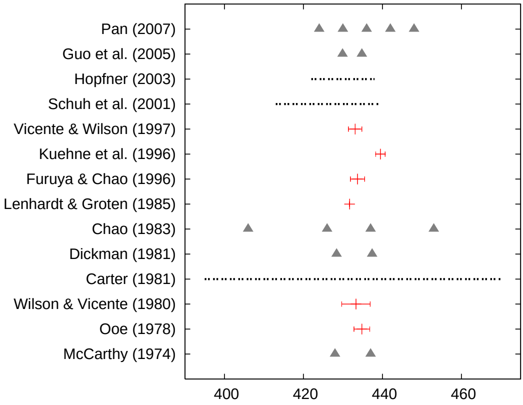
Seismic modes, rotational modes, core spectrum

Chandler wobble and core spectrum

Rotational modes and core spectrum



Observed Chandler wobble (solar days)

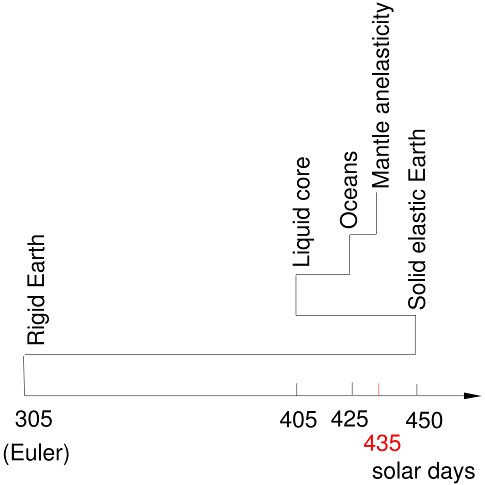


+ single period

▲ multiple period

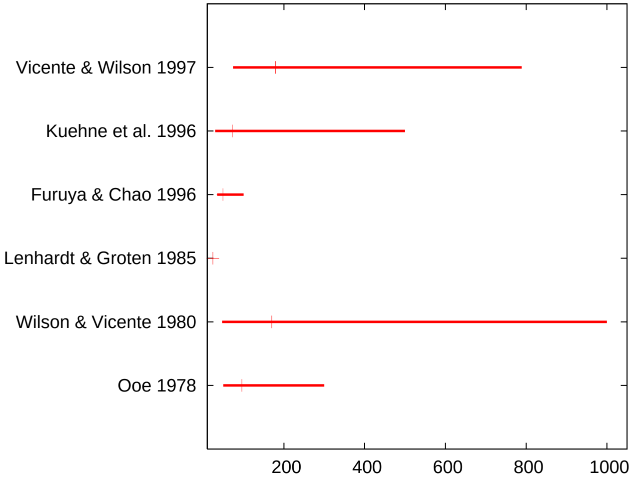
- - variable period

Theoretical Chandler wobble (solar days)



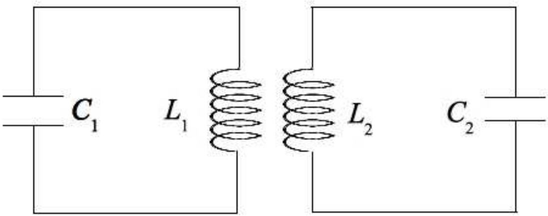
(Smith & Dahlen 1981)

Observed Chandler Wobble Q



Search for a double Chandler wobble

Coupled LC circuits

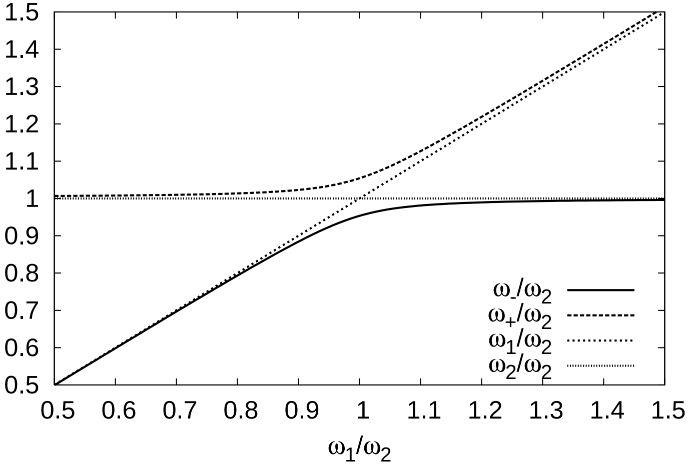


$$\omega_{\pm}^2 = \frac{\omega_1^2 + \omega_2^2 \pm \sqrt{(\omega_1^2+\omega_2^2)^2 - 4(1-\alpha_1\alpha_2)\omega_1^2\omega_2^2}}{2(1-\alpha_1\alpha_2)}$$

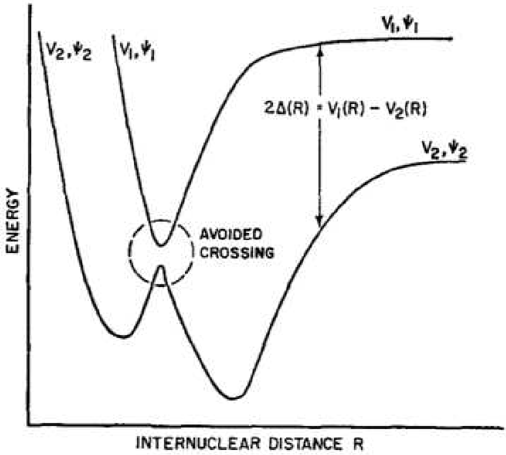
$$\omega_1 = \frac{1}{\sqrt{L_1 C_1}} \qquad \alpha_1 = \frac{M}{L_1}$$

$$\omega_2 = \frac{1}{\sqrt{L_2 C_2}} \qquad \alpha_2 = \frac{M}{L_2}$$

Avoided crossing



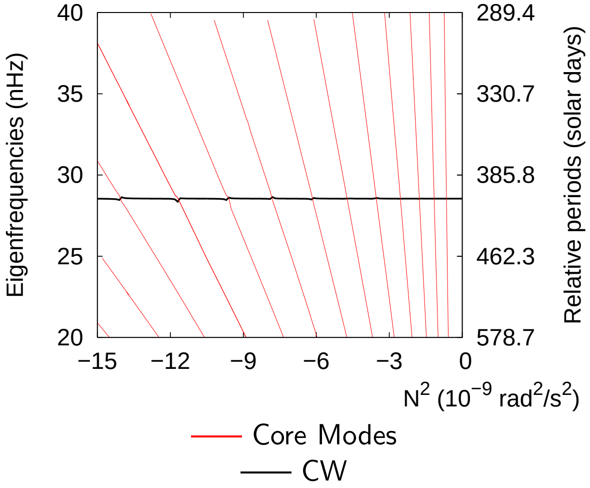
Diatomic molecule



Lewis and Hougen 1968

Chandler Wobble and core modes

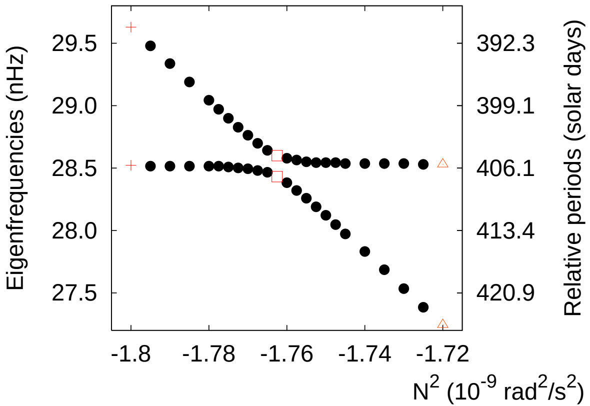
$$N^2 = \text{constant in the liquid core}$$
$$\mathbf{s} = \boldsymbol{\tau}_1^{-1} + \boldsymbol{\sigma}_2^{-1} + \boldsymbol{\tau}_3^{-1}$$



Chandler Wobble and core modes

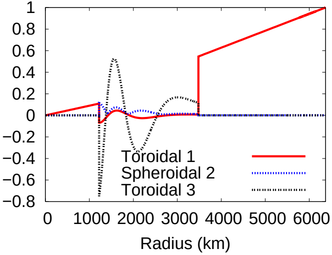
N^2 = constant in the liquid core

$$\mathbf{s} = \boldsymbol{\tau}_1^{-1} + \boldsymbol{\sigma}_2^{-1} + \boldsymbol{\tau}_3^{-1}$$

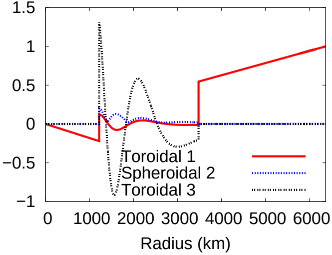


Eigenfunctions at avoided crossing

$$N^2 = -1.7625\ 10^{-9}\ \mathrm{rad}^2\ \mathrm{s}^{-2}$$



$T = 404.6$ solar days



$T = 407.1$ solar days

Two Chandler wobbles!

Summary

- Unified approach of Earth rotation and long-period seismology
- Three families of normal modes
- Splitting of seismic modes by rotation
- Influence of core spectrum on rotational modes

Projects

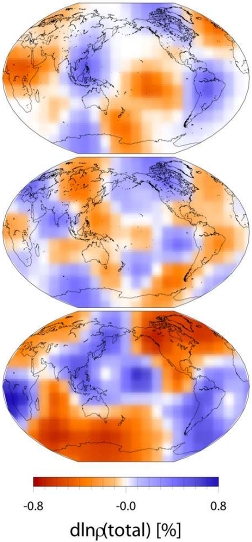
- Influence of thermo-chemical structure in the lower mantle on Earth normal modes
- Polar gravimetry

Influence of thermo-chemical structure in the lower mantle on Earth normal modes

Collaborative research project

- Taipei
  - Frédéric Deschamps (Academia Sinica)
  - Cheng-Yin Chu (PhD. student, National Central University)
- IPGS
  - Séverine Rosat
  - Yann Ziegler

How do density anomalies affect rotational modes?



Trampert et al. 2004

Depth (km)

670 - 1200

1200 - 2000

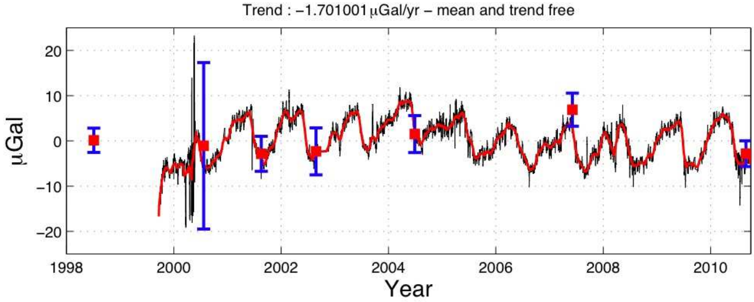
2000 - 2891

Polar gravimetry

- Reference values for gravity
  (geoid, airbourne and terrestrial surveys)
- Gravity variations due to
  - post-glacial rebound
  - current ice melting

Arctic

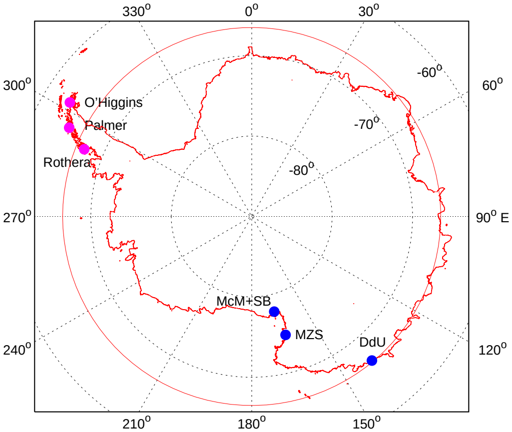
Ny-Ålesund (Svalbard)



Courtesy of A. Mémin

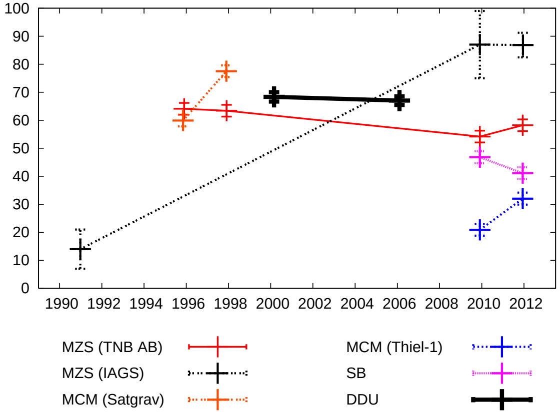
Antarctica

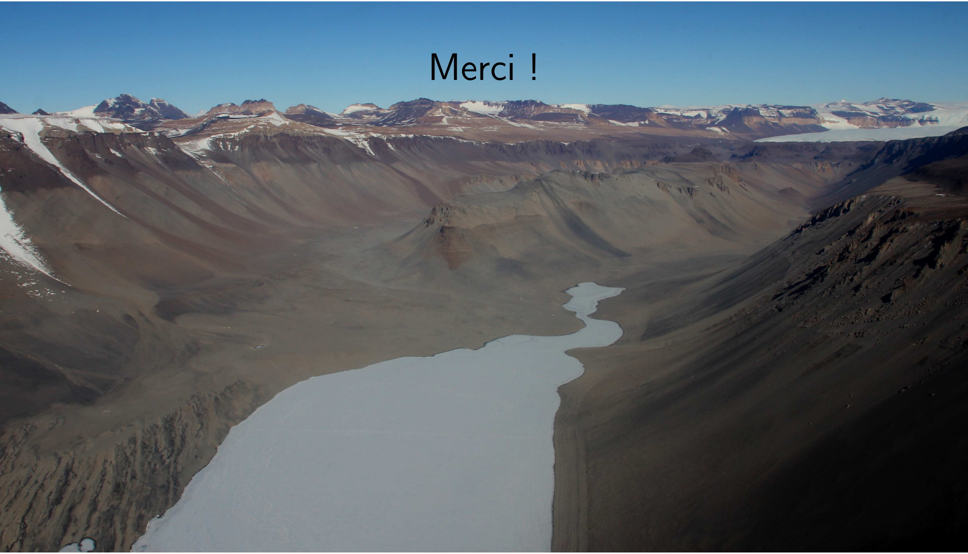
Collaborative project (F, US, I, NZ, DK, *GB, AR, CL, D*)



Antarctica

Gravity variations (μGal)




Merci !