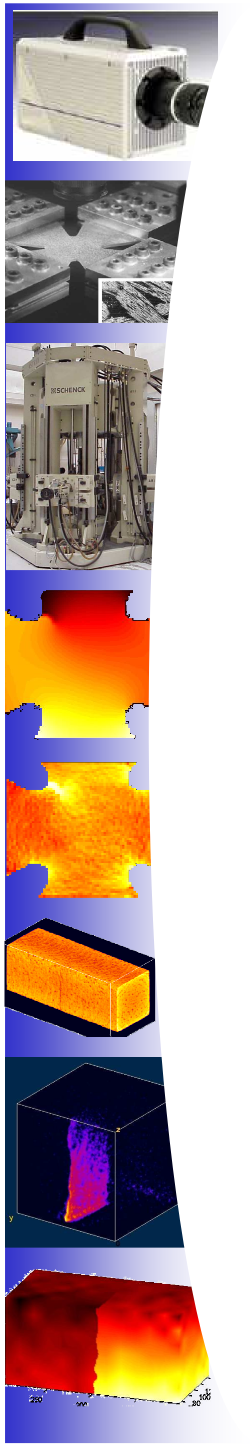
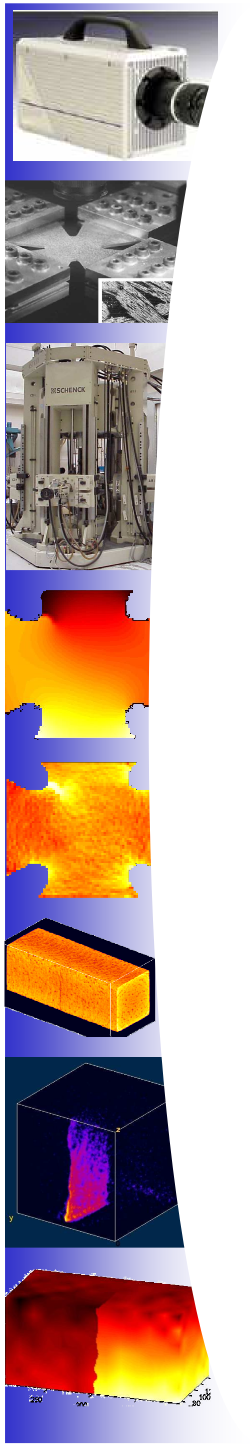


Progrès récents en corrélation d'images : de la mesure de champs à l'identification de propriétés mécaniques

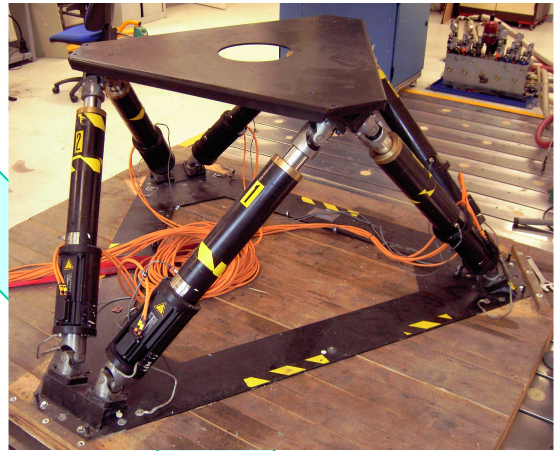
F. Hild, J. Réthoré & S. Roux

Séminaire IPGS 29/04/08



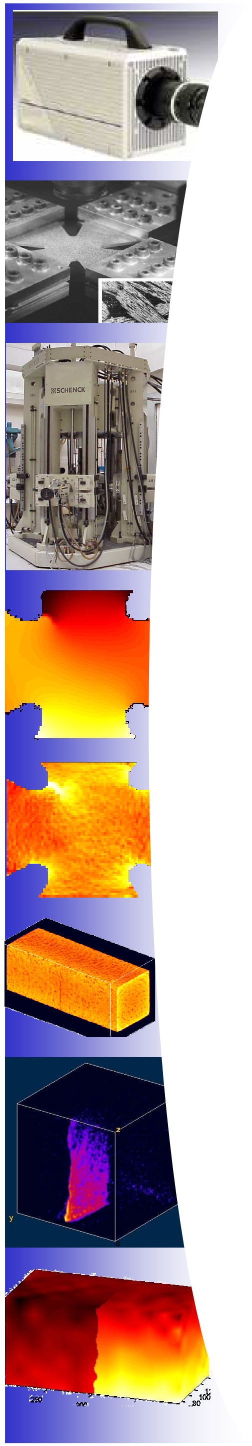
Needs for Identification

From experiments…



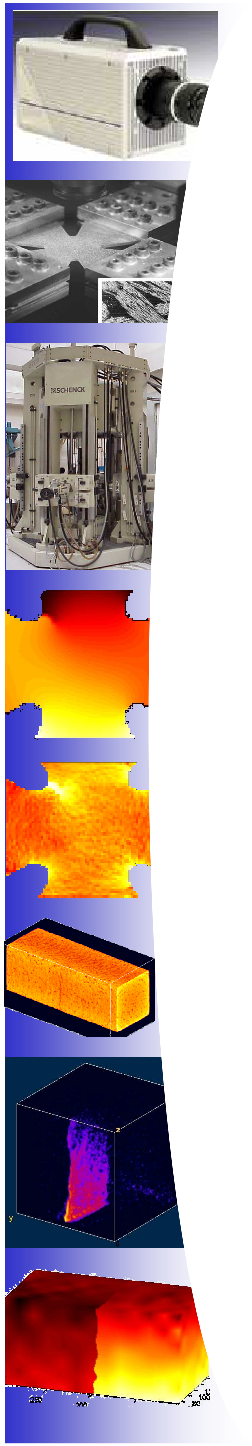
…to relevant mechanical properties

$$E, \nu, K, D(Y), \ldots$$



Needs for Identification

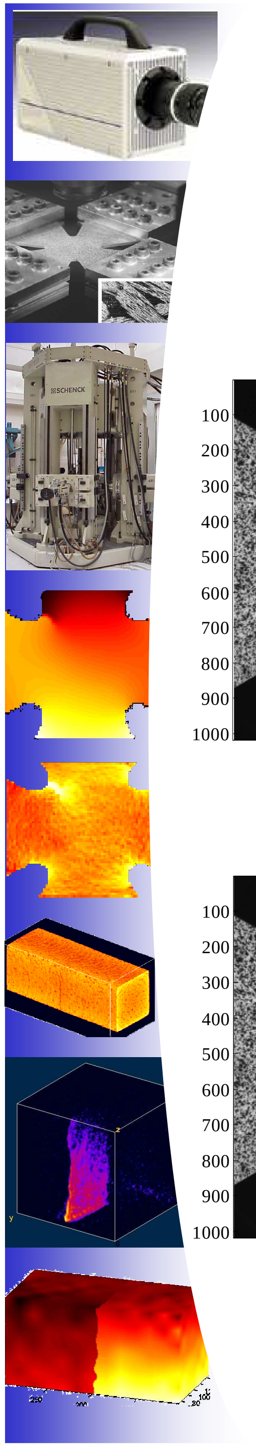
- Well-designed and executed experiment(s)
- Accurate kinematic measurements
- Accurate 'static' measurements
- Direct numerical modeling
- Adapted identification strategies with well chosen observables
- Validation(s) of the result

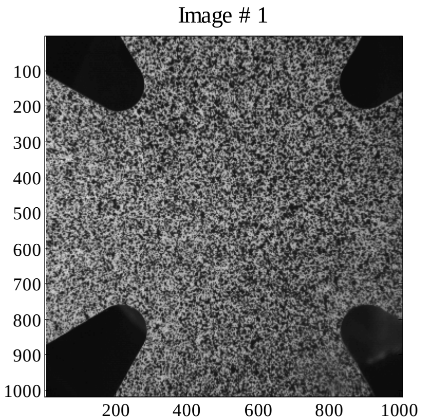


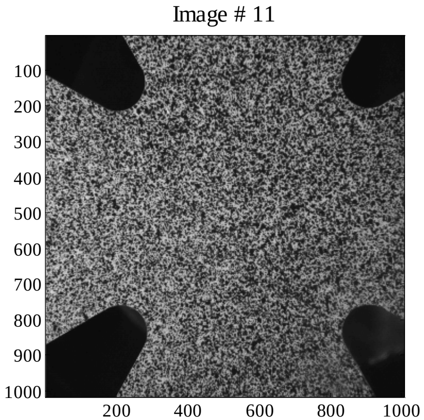
Proposal for Identification

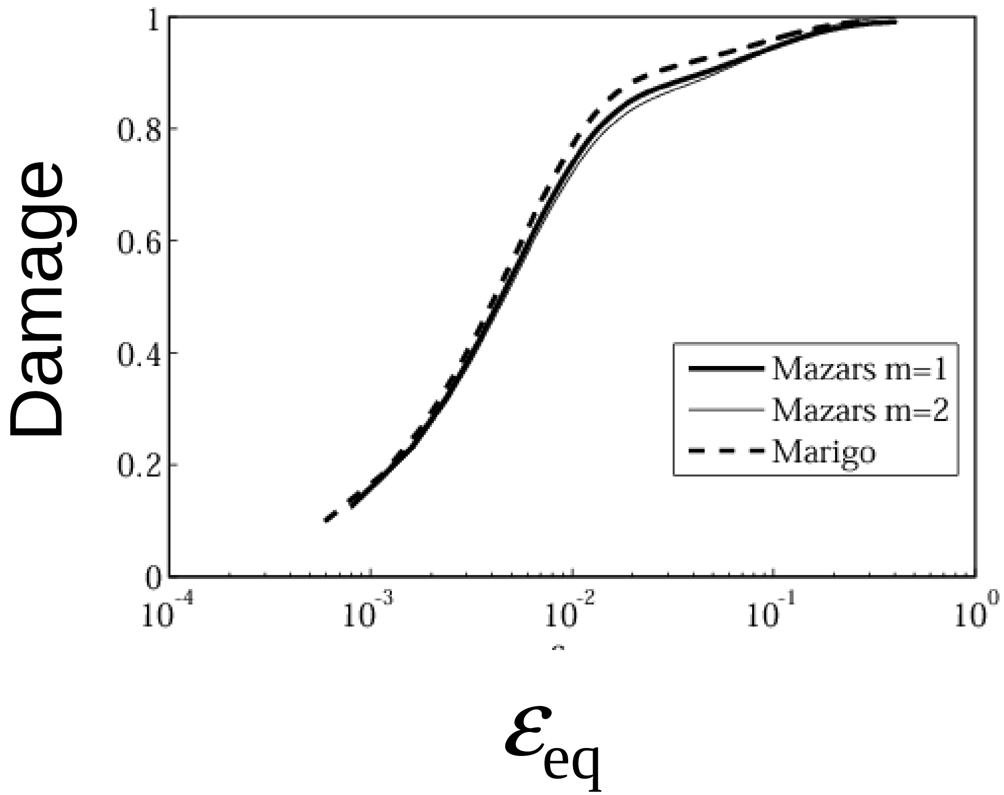
- Full-field kinematic measurements
  - Thousands of measurement points
  - Accuracy
  - Adjustable and versatile tool
- Identification procedure
  - Based on a consistent description
  - “Full-field” validation

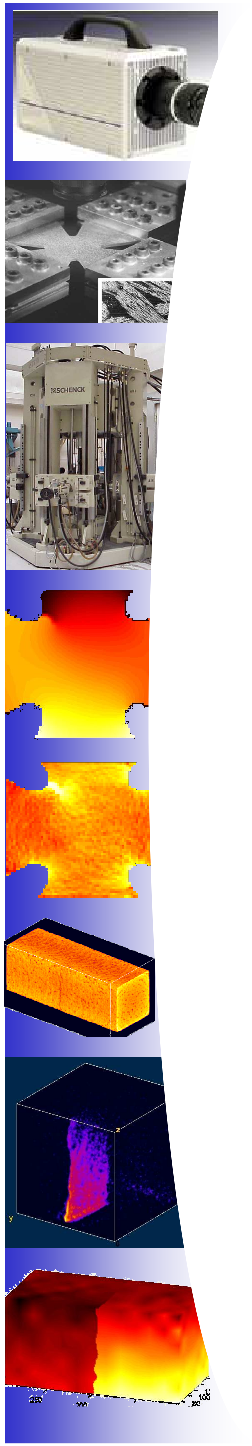
Goal: From pictures to model









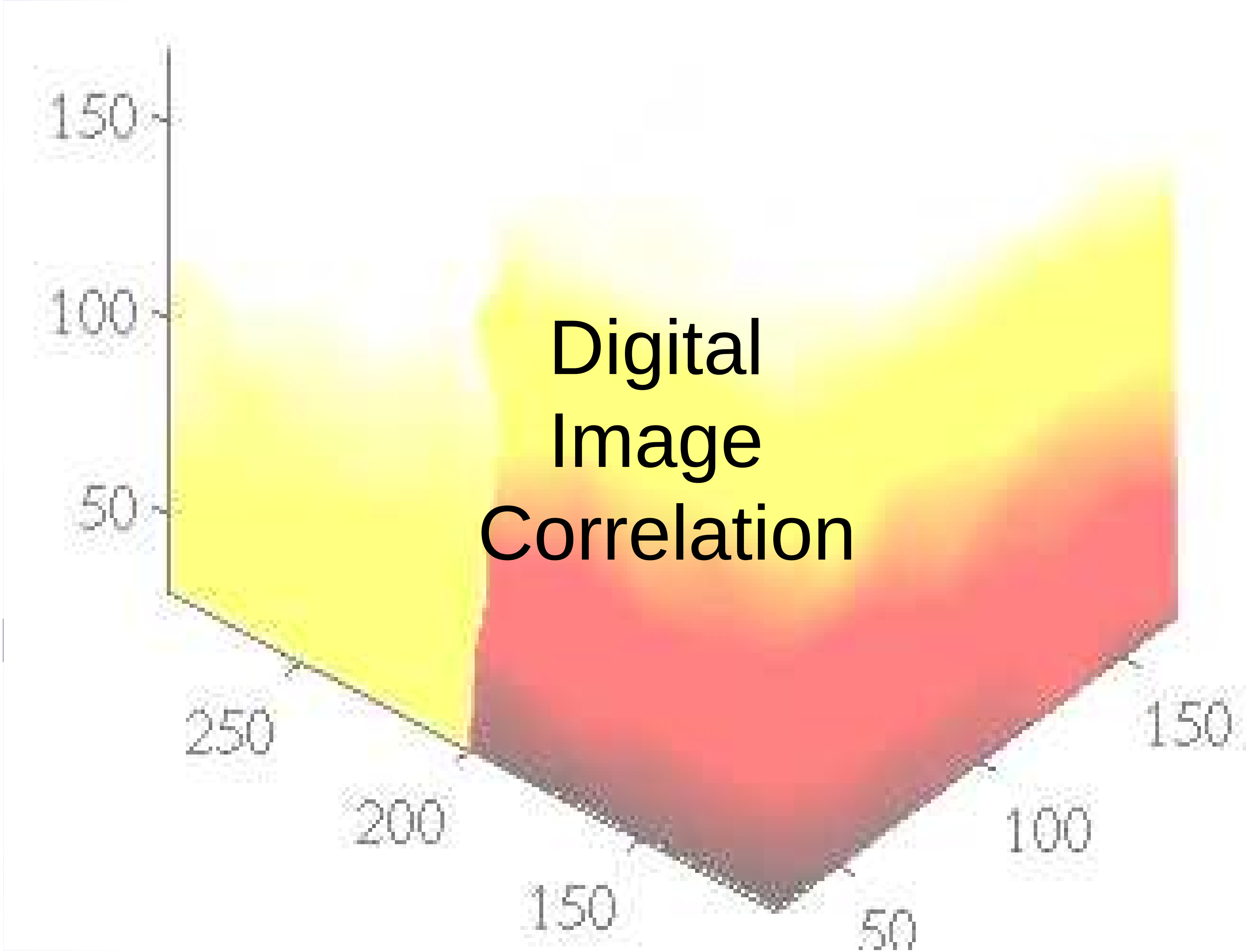


Outline

- Digital Image Correlation
  - General remarks
  - Some examples

- Identification
  - Elasticity / Static b.c.s
  - Cracks
  - Heterogeneous elastic properties
  - Damage law
  - Plastic buckling

Digital Image Correlation



Digital Image Correlation

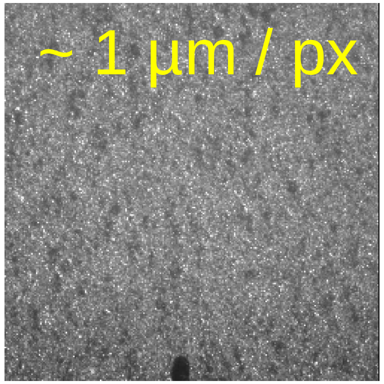
- Two gray level images:

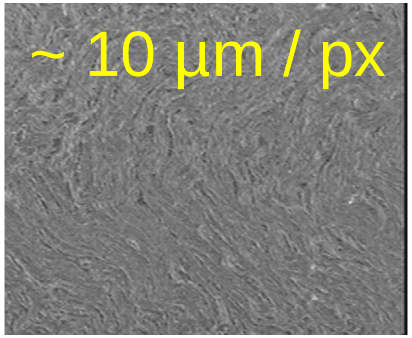
$$f(\underline{x}) \qquad g(\underline{x})$$

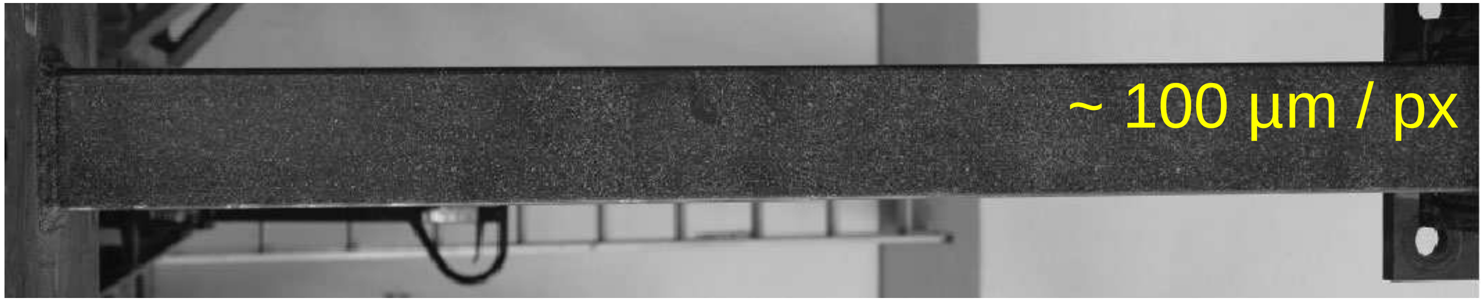
- Passive advection :

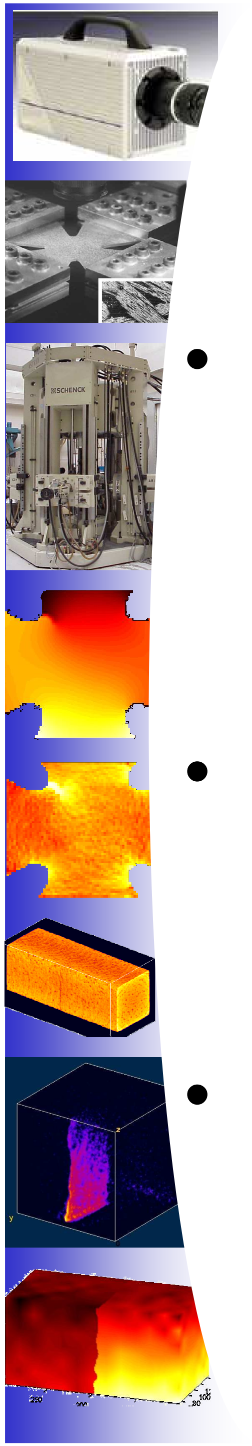
$$g(\underline{x}) = f(\underline{x} + \underline{u}(\underline{x}))$$

- Question: Identify $\underline{u}(\underline{x})$









Digital Image Correlation

- Select a specific displacement basis $\underline{\varphi}_i(\underline{x})$ such that

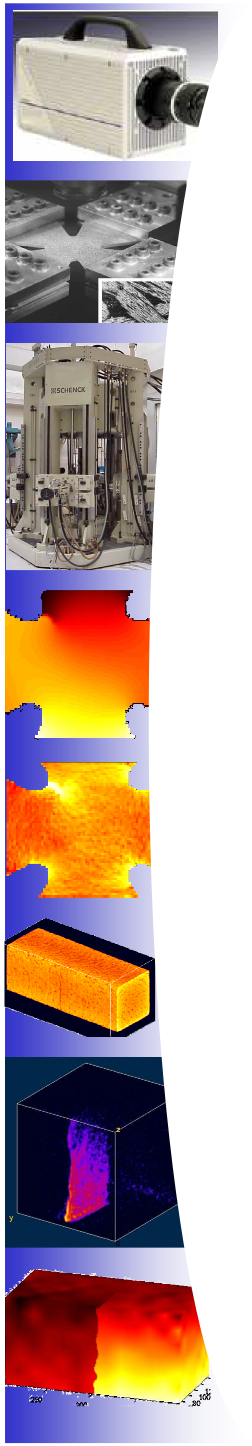
$$\underline{u}(\underline{x}) = \sum_i a_i \underline{\varphi}_i(\underline{x})$$

- Weak formulation: Minimize

$$\Phi\{a_i\} = \iint_D \left[g(\underline{x}) - f(\underline{x} + a_i \underline{\varphi}_i(\underline{x})) \right]^2 d\underline{x}$$

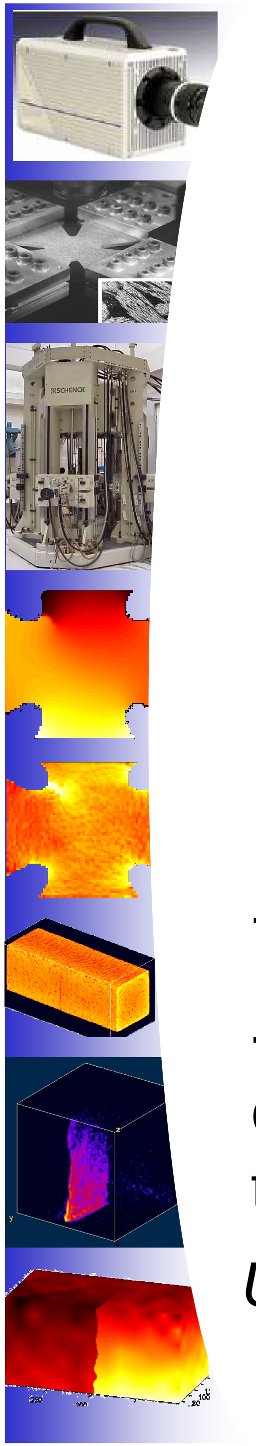
- Successive linearizations/corrections

$$M_{ij} a_j = b_i$$



Digital Image Correlation

- Which displacement basis?
  - The more closely tailored to the experimental test, the least number of parameters a_i to estimate, the more accurate the measurement
  - Examples:
    - elastic solutions
    - FE shape functions & extensions thereof
    - any other mechanically motivated fields



Uncertainty/Error

- Key ratio

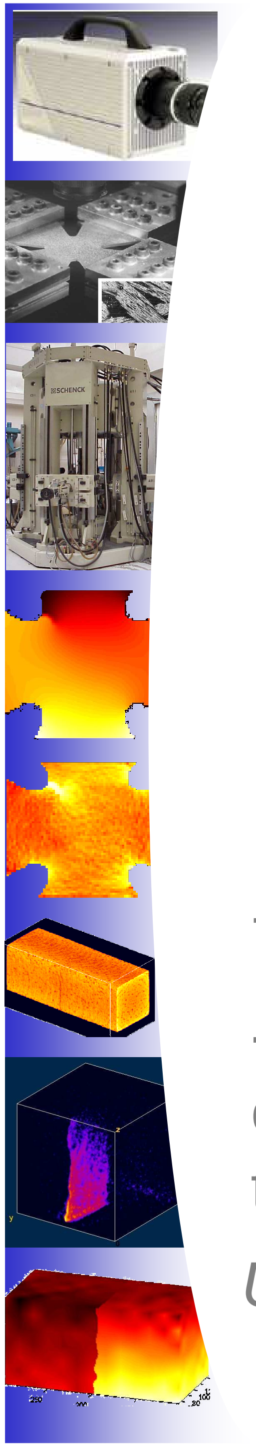
$$\alpha = \frac{\#\,\mathrm{dof}}{\#\,\mathrm{pixels}}$$

Small α:

+ A lot of information

- May not be rich enough to capture the real kinematics

Uncertainty ➦ Error ➤



Uncertainty/Error

- Key ratio

$$\alpha = \frac{\#\,\mathrm{dof}}{\#\,\mathrm{pixels}}$$

Small α:

+ A lot of information

- May not be rich enough to capture the real kinematics

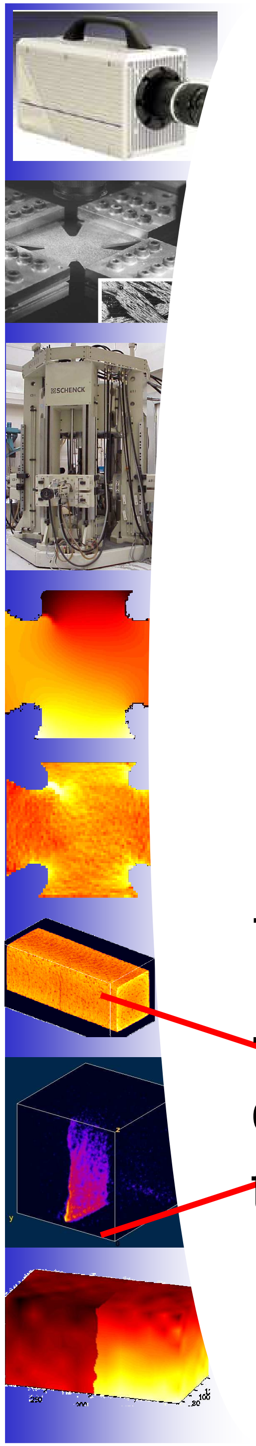
Uncertainty ➦ Error ➤

Large α:

- Few information

+ Few restrictions

Uncertainty ➤ Error ➦



Uncertainty/Error

- “Smart choice” of a (mechanically relevant) displacement basis

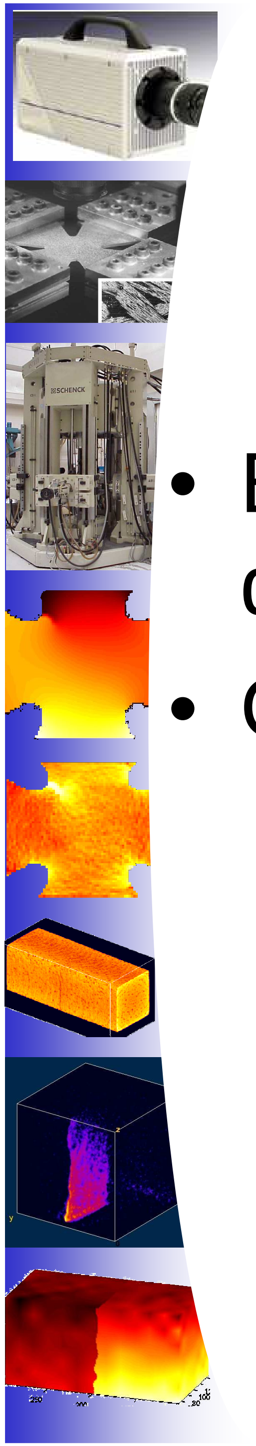
+ A lot of information

~~- May not be rich enough to capture the real kinematics~~

~~- Few information~~

+ Few restrictions

Uncertainty ➦ Error ➦



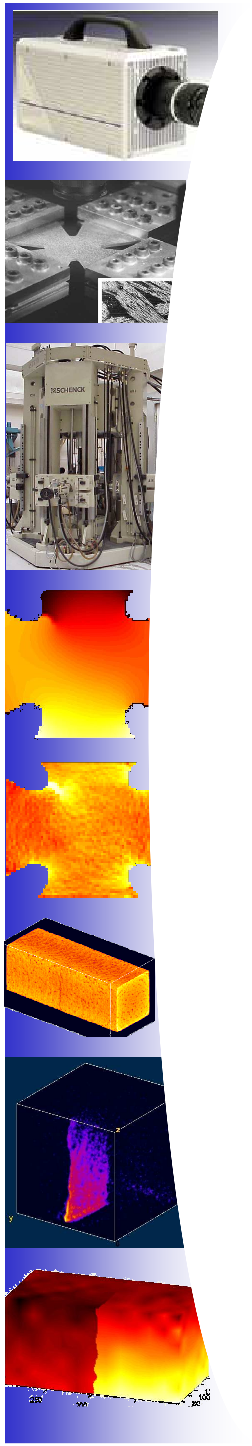
Noise sensitivity analysis

- Effect of random (white) noise, η, on displacement estimates
- Covariance

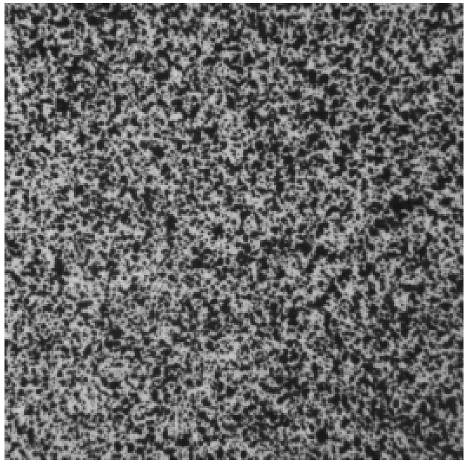
$$Cov(a_i, a_j) = 2\left\langle \eta^2 \right\rangle (M^{-1})_{ij}$$

➔Selection of fields

➔Uncertainty on further measurements

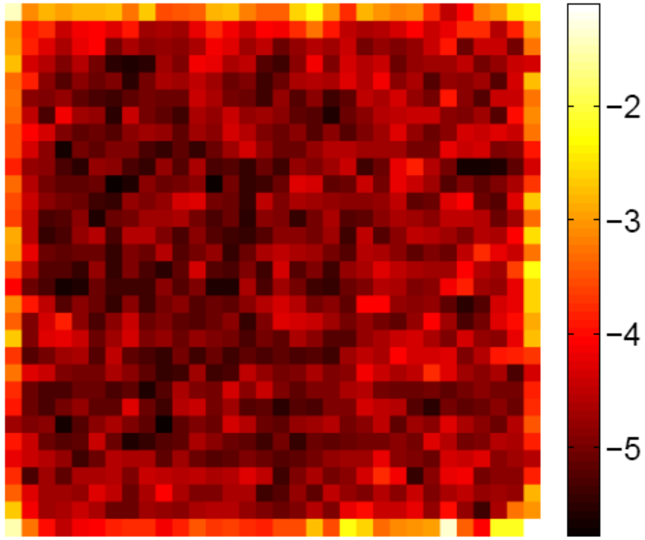


Noise sensitivity analysis

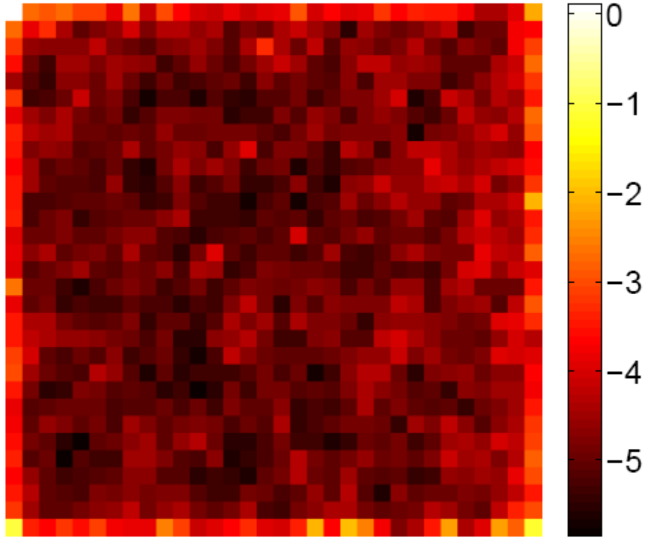


$$[M]\{a\} = \{b\}$$

$$\left\langle \{\delta a\}\{\delta a\}^t \right\rangle = 2\sigma^2 [M]^{-1}$$

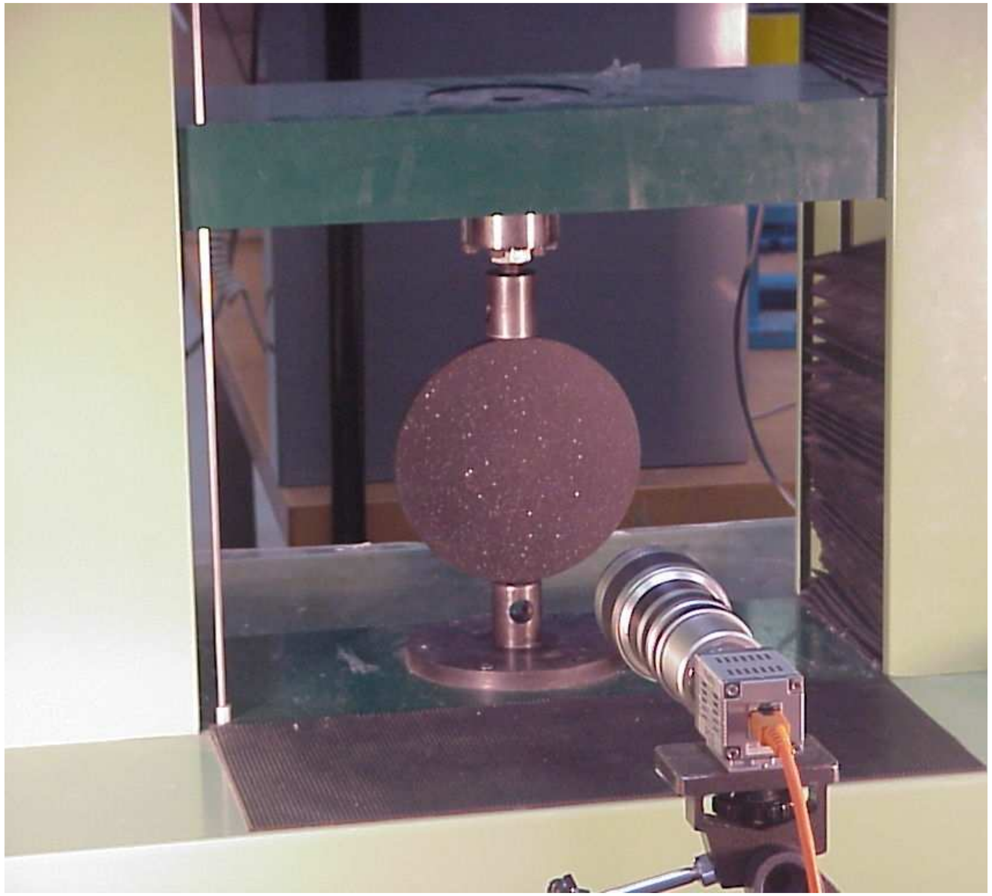


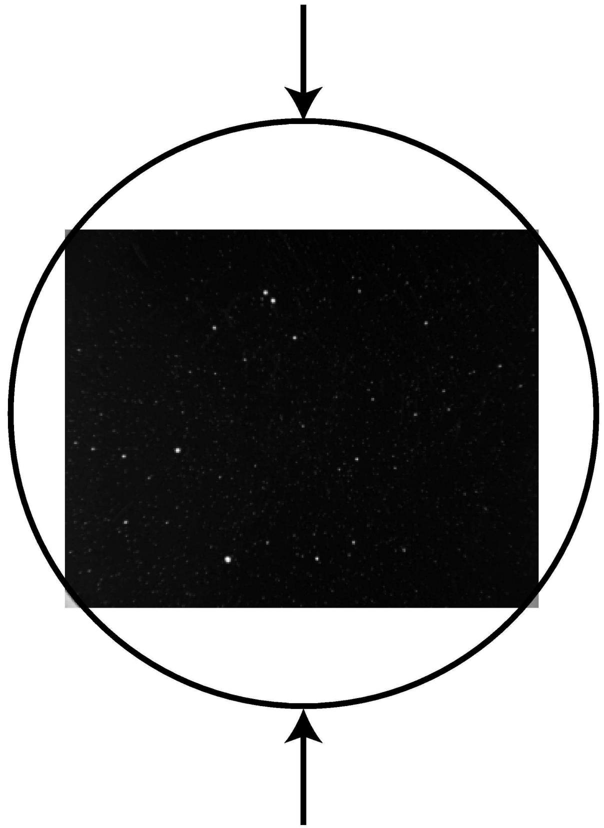
$$\log \left\langle \{\delta a_x\}\{\delta a_x\}^t \right\rangle_e$$



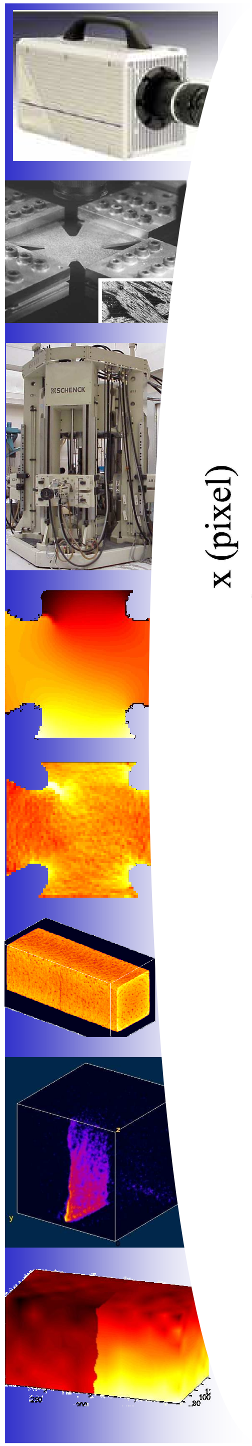
$$\log \left\langle \{\delta a_y\}\{\delta a_y\}^t \right\rangle_e$$

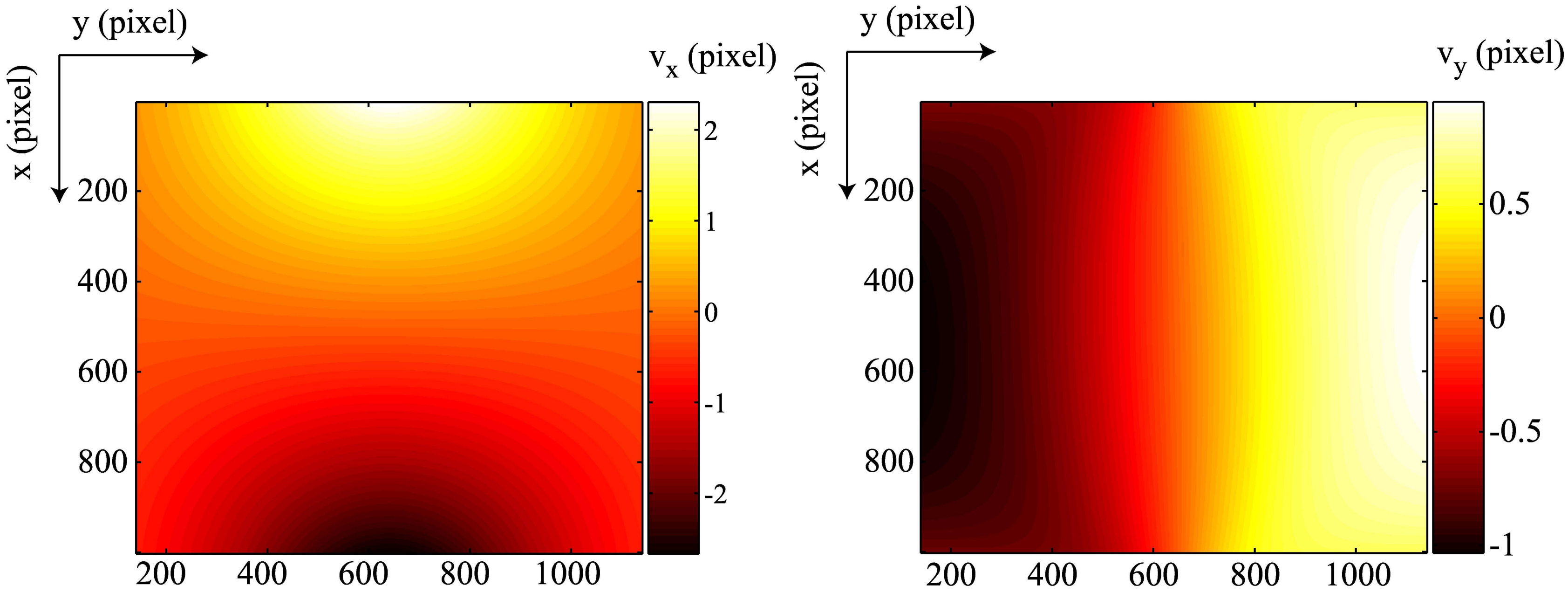
Elastic properties?





Elastic properties

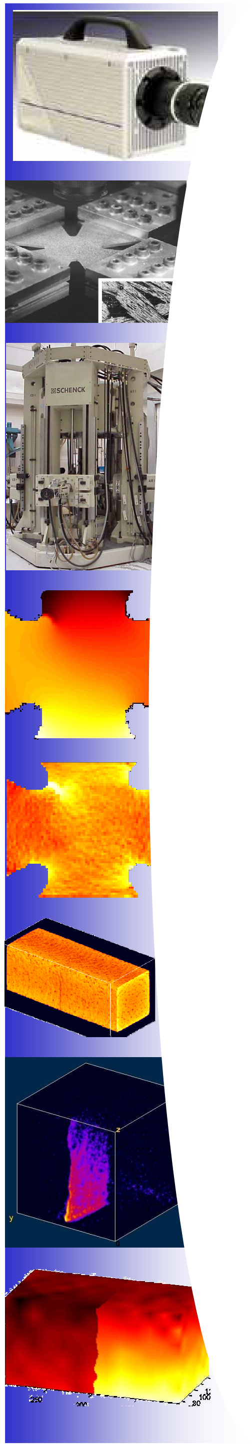


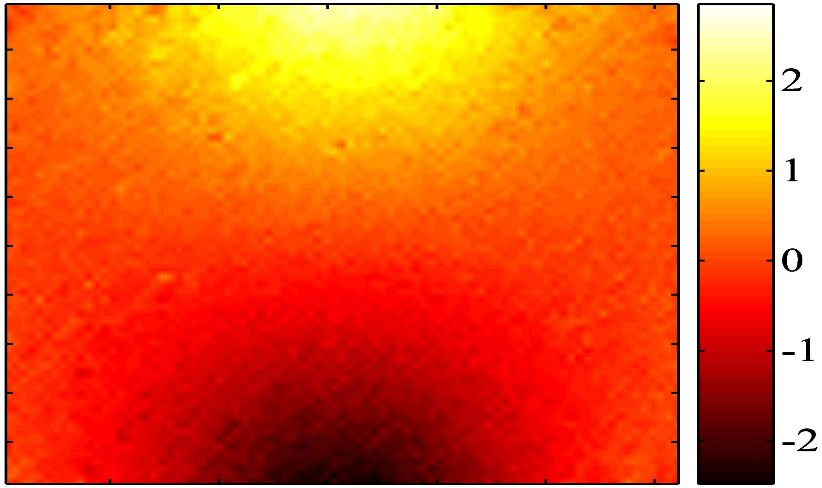


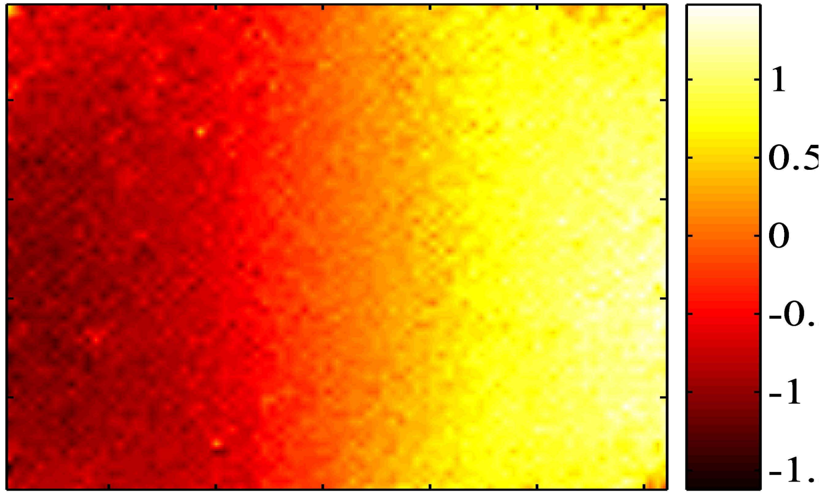
I-DIC (ℓ = 1000 pixels)

1 pixel ↔ 98 µm

Elastic properties



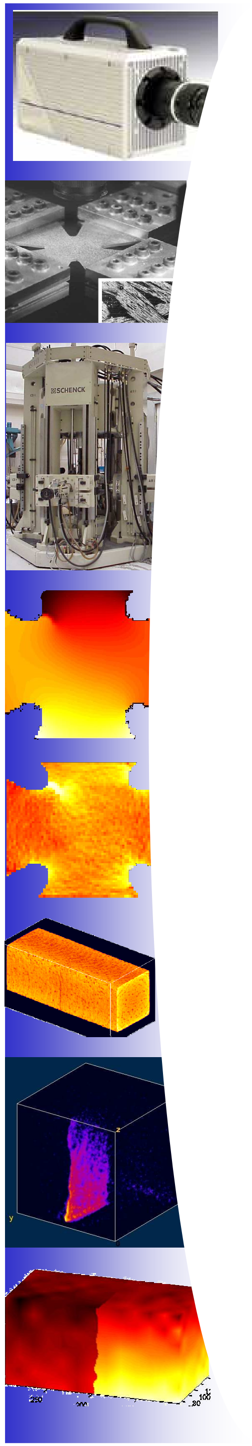




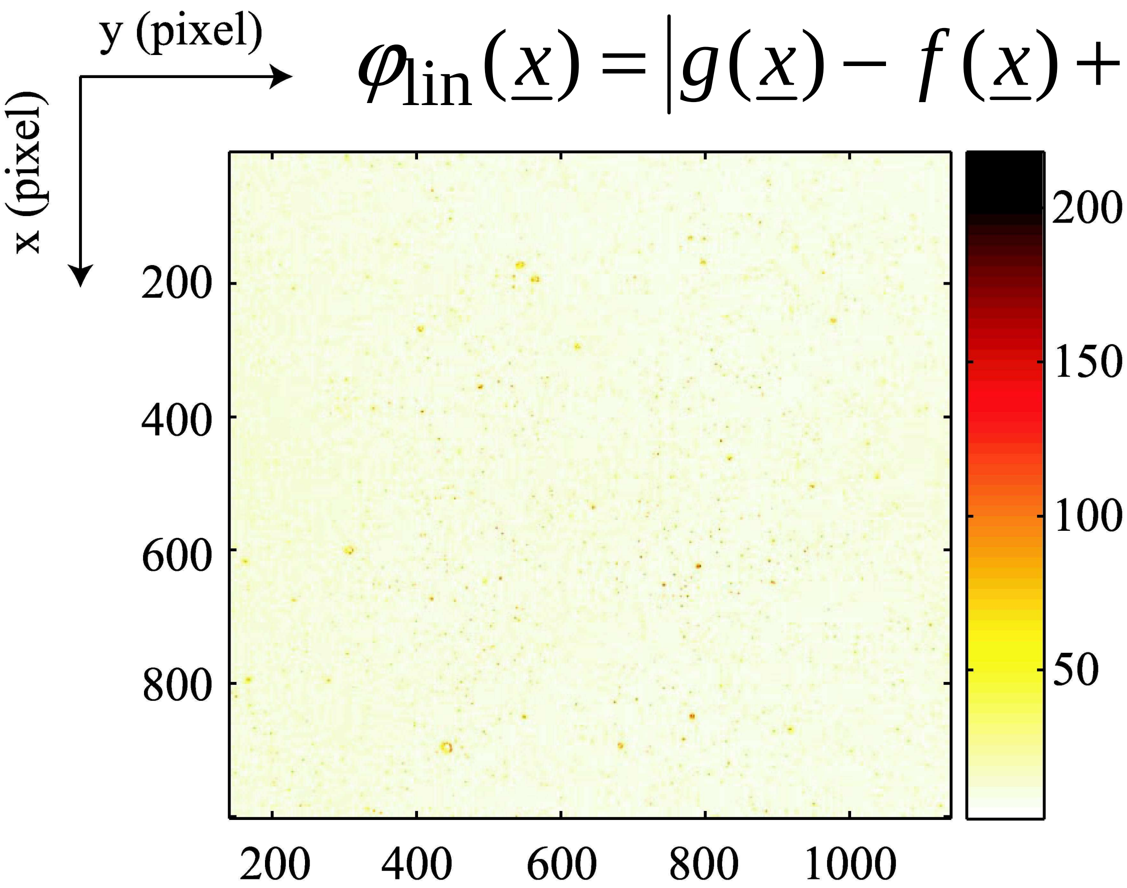
Q4-DIC (ℓ = 16 pixels)

1 pixel ↔ 98 µm

Elastic properties



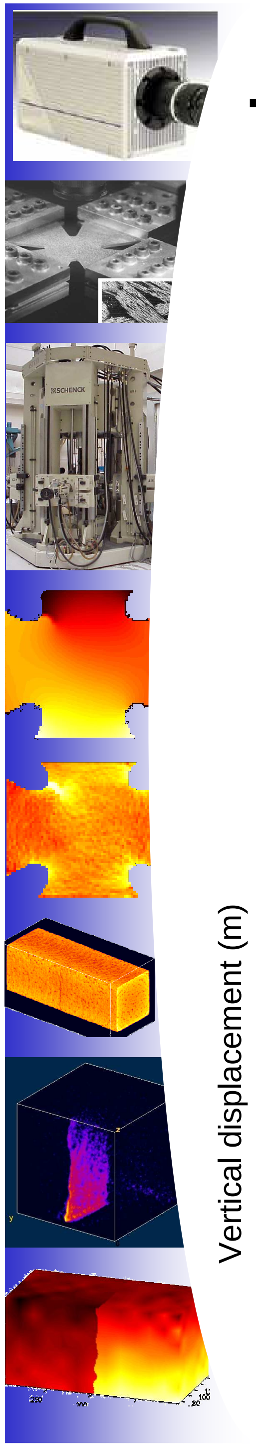
$$\varphi_{\text{lin}}(\underline{x}) = \left| g(\underline{x}) - f(\underline{x}) + \underline{u}(\underline{x}).\underline{\nabla} f(\underline{x}) \right|$$

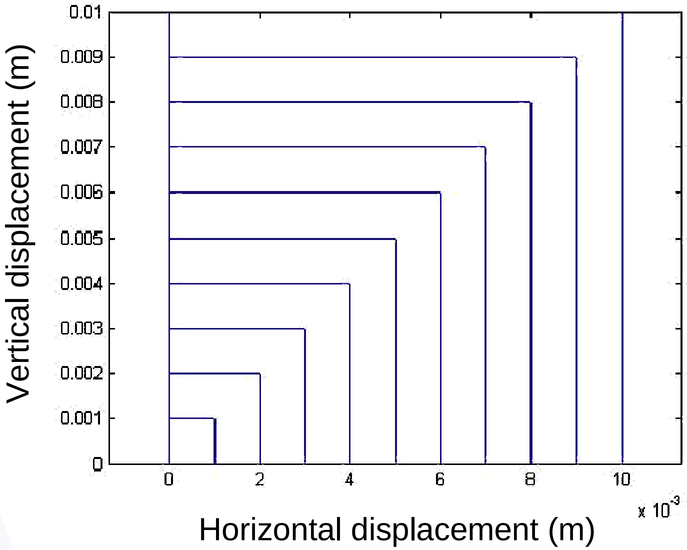


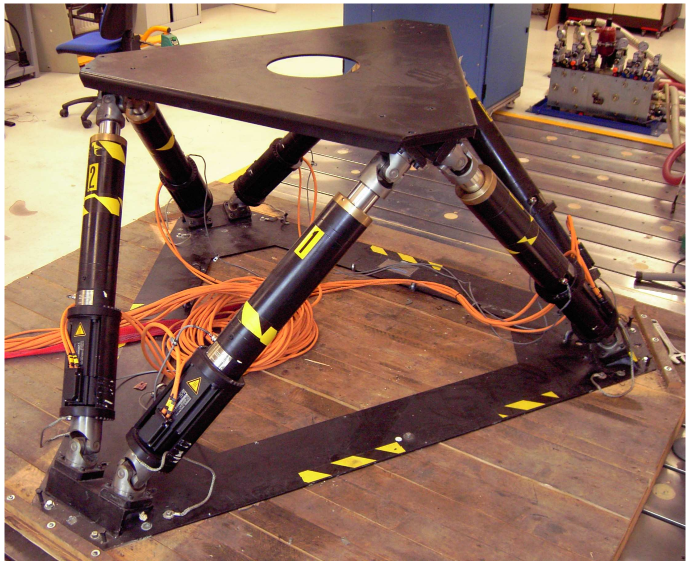
I-DIC (ℓ = 1000 pixels)

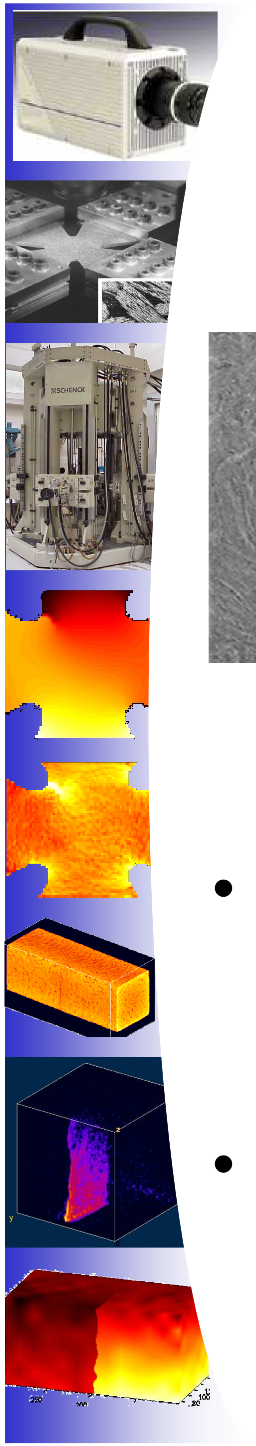
1 pixel ↔ 98 µm

Texture / Mechanical properties?

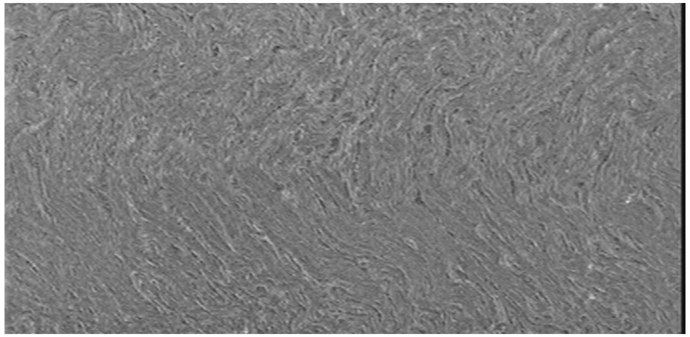




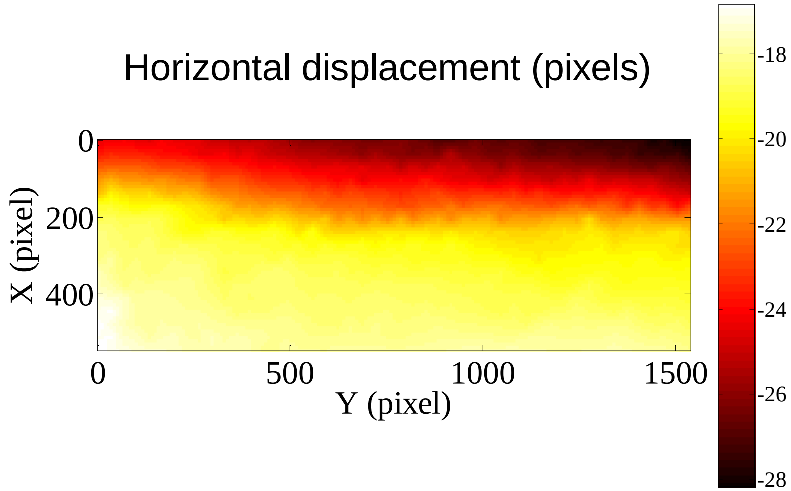




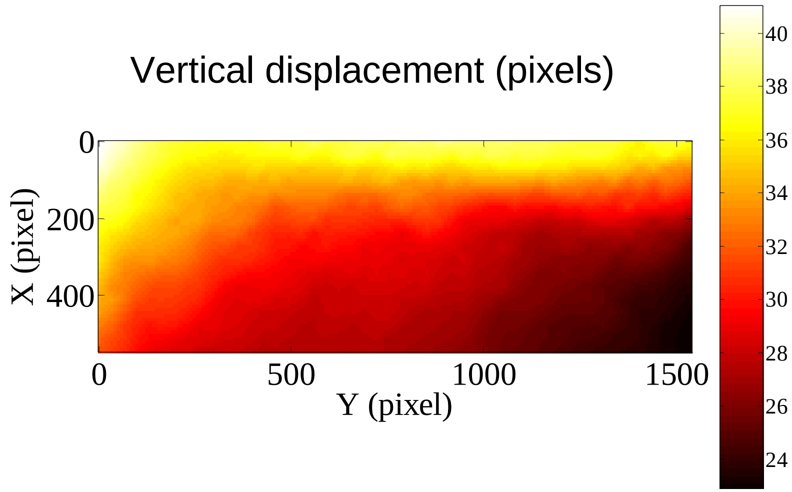
Texture / Mechanical properties



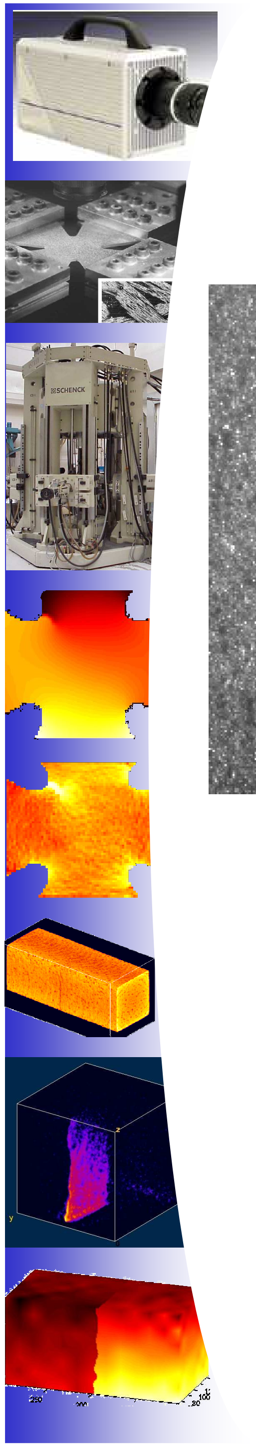
- Relationship between texture / mechanical properties?
- Identification of elastic parameters



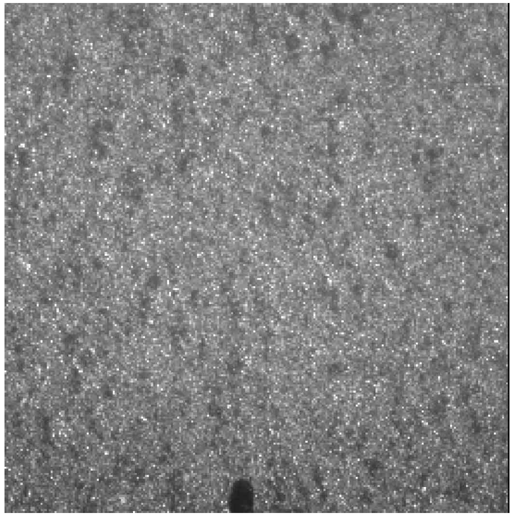
Q4-DIC (ℓ = 32 pixels)

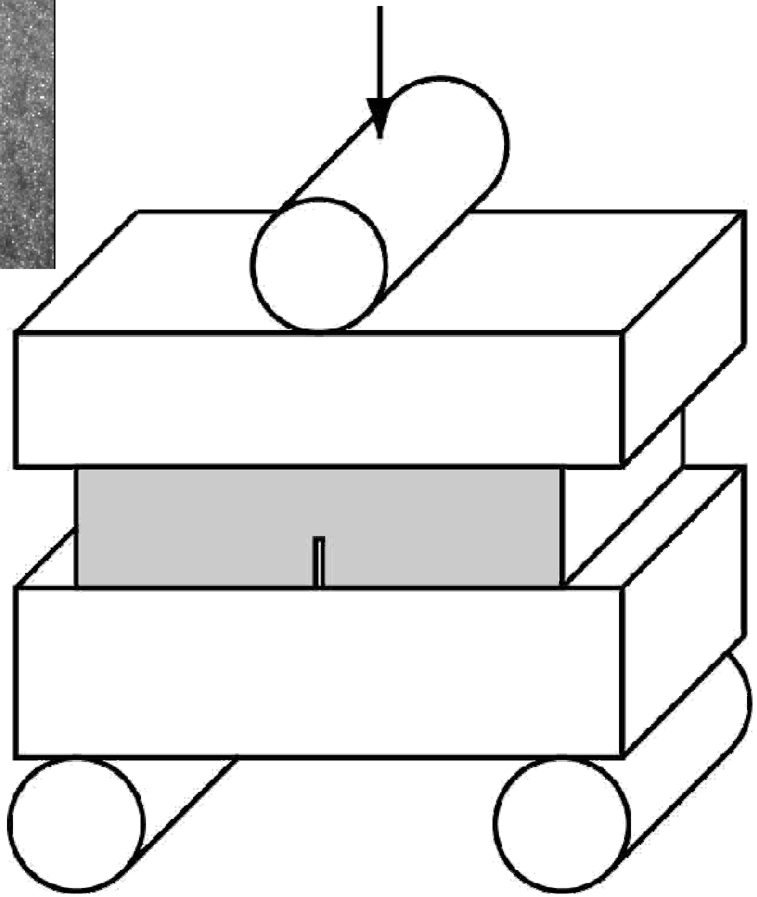


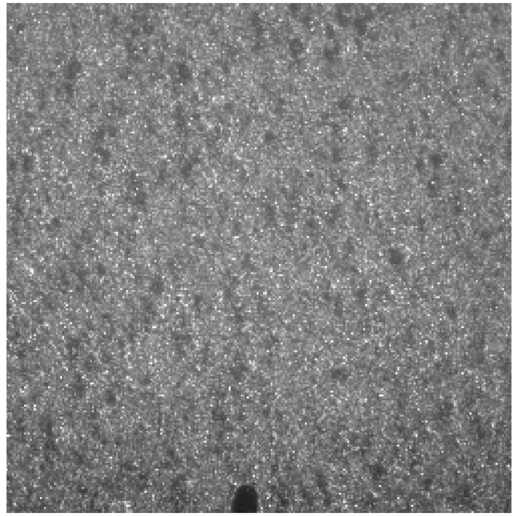
1 pixel ↔ 130 µm



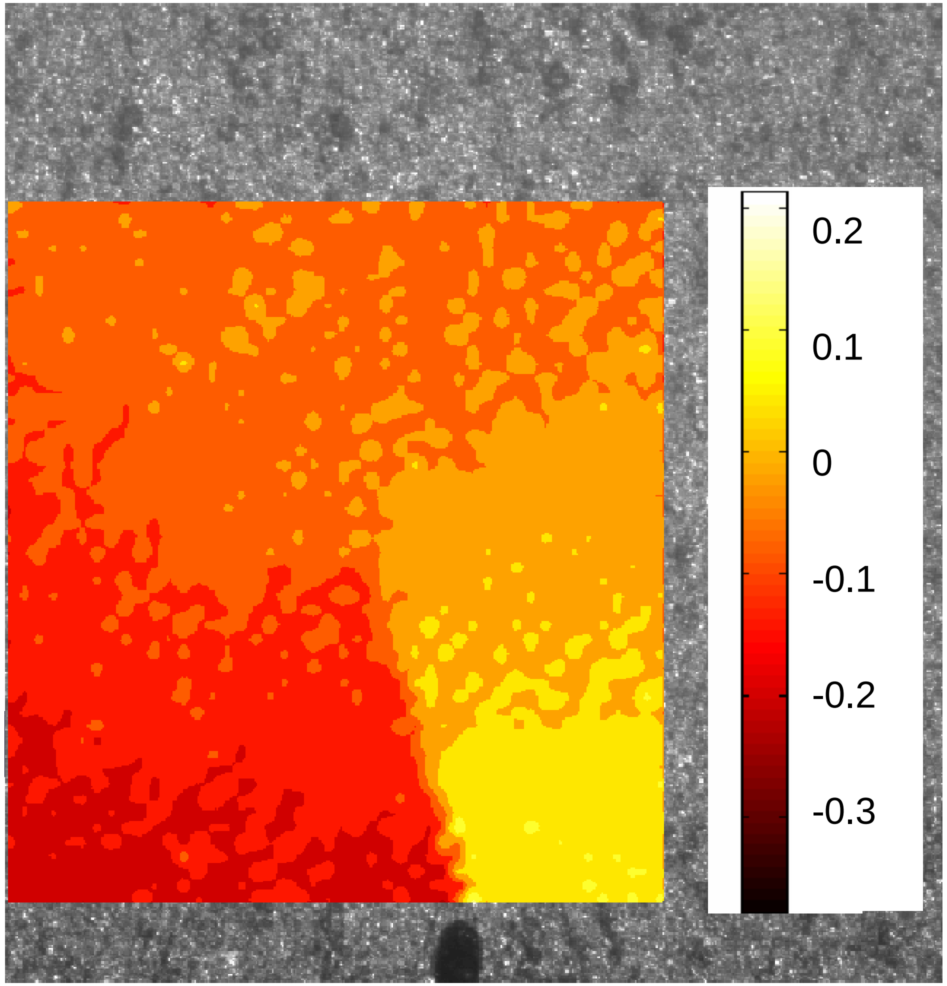
Toughness of brittle materials?



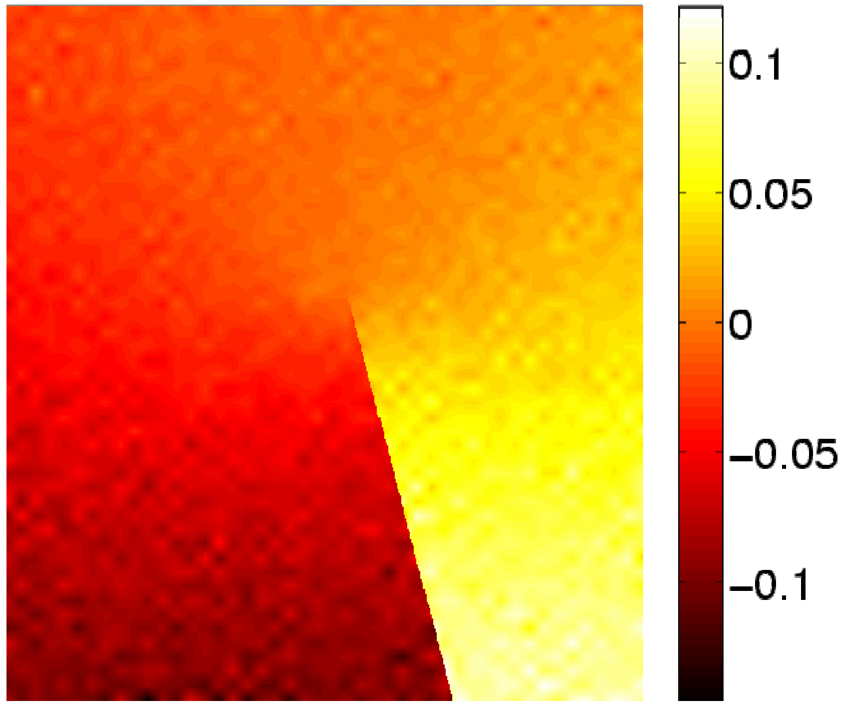




Toughness of ceramics



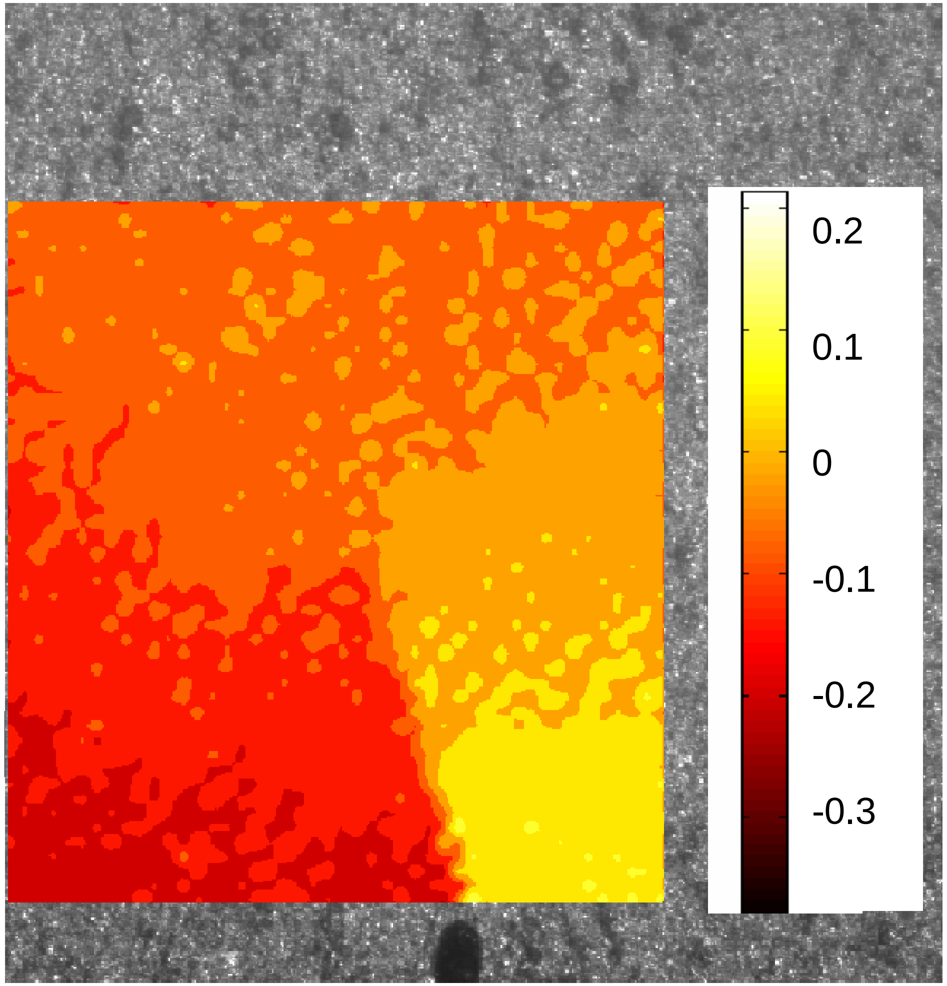
Q4-DIC (ℓ = 16 pixels)



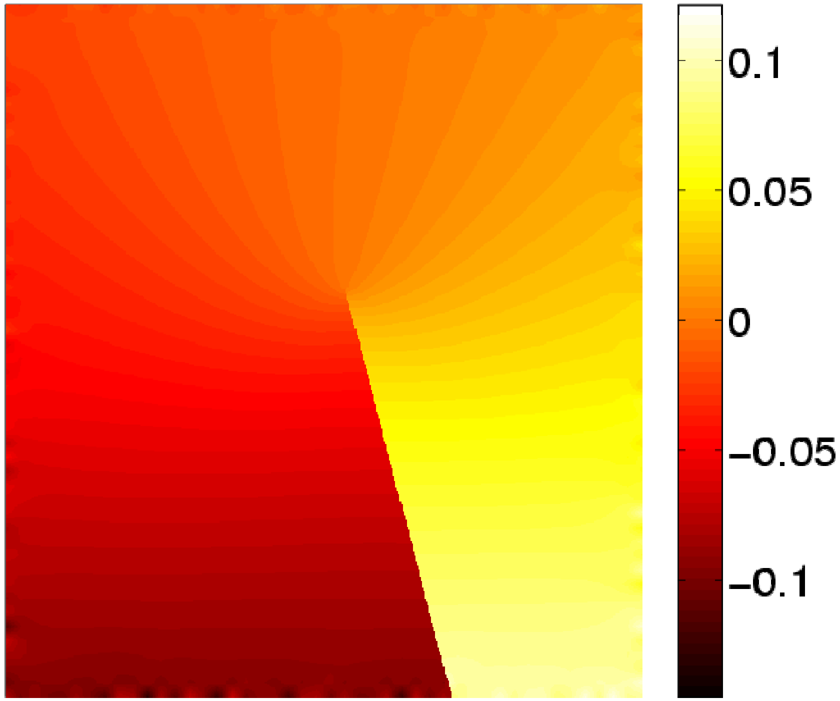
XQ4-DIC (ℓ = 16 pixels)

1 pixel ↔ 1.85 µm

Toughness of ceramics

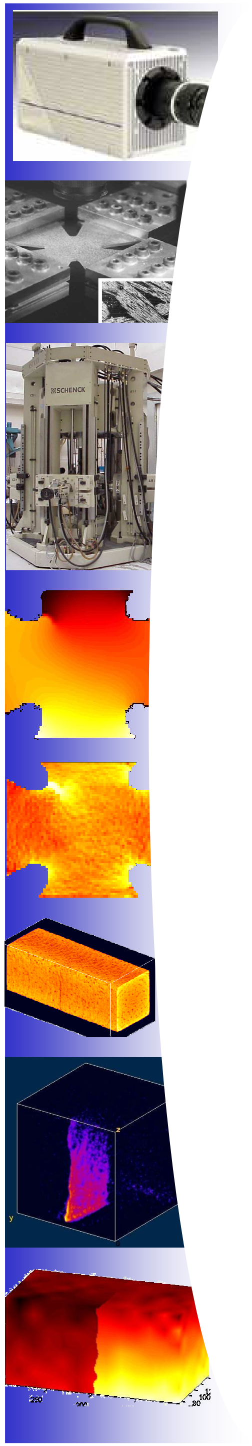


Q4-DIC (ℓ = 16 pixels)

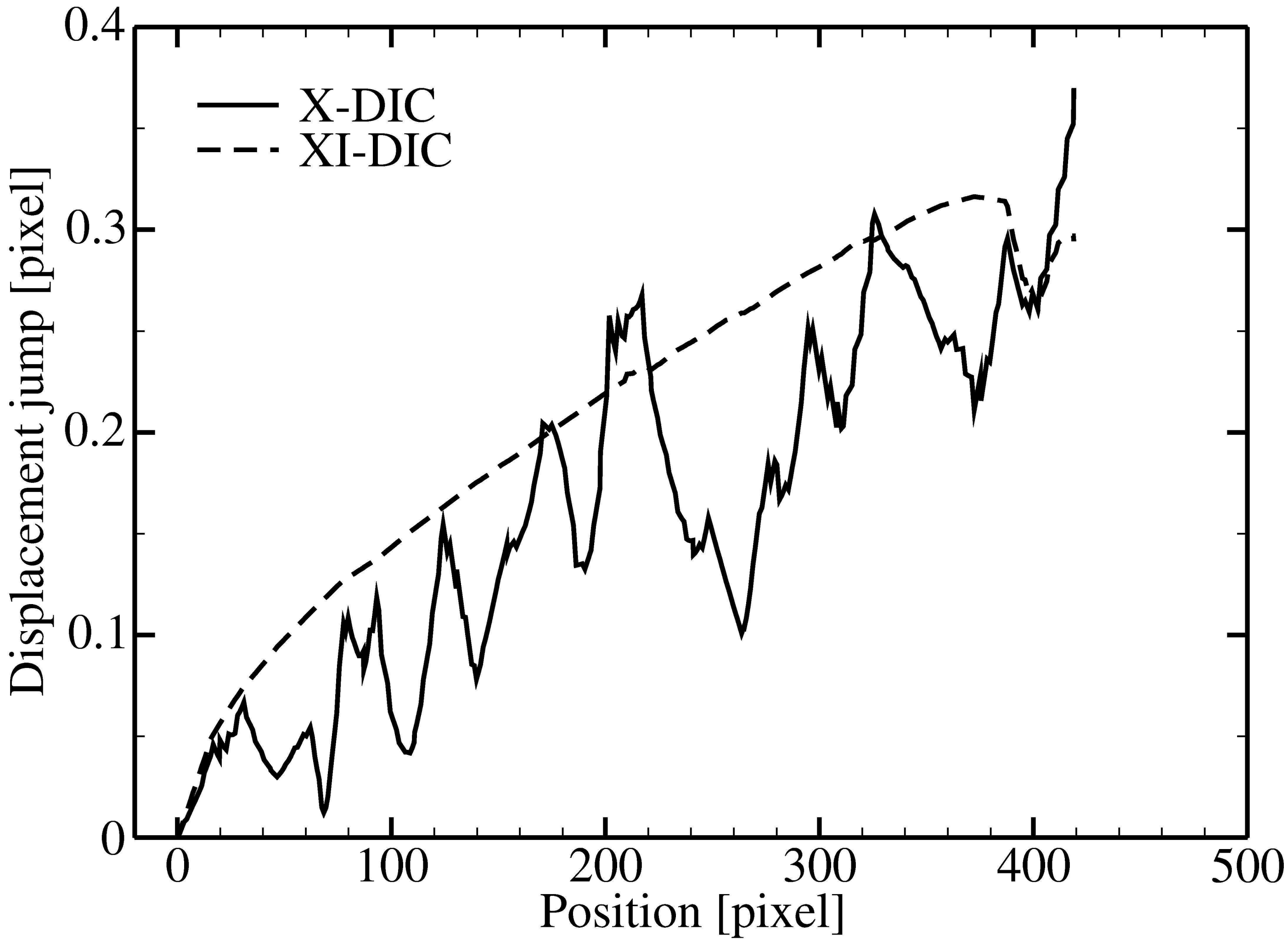


XIQ4-DIC (ℓ = 16 pixels)

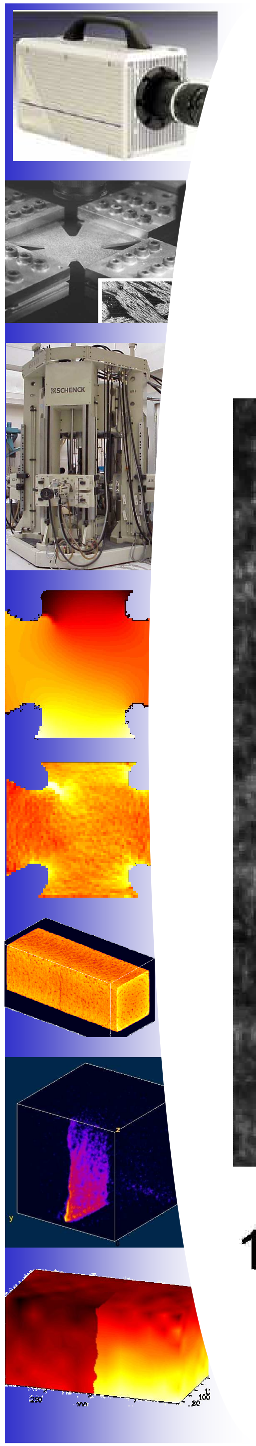
1 pixel ↔ 1.85 µm



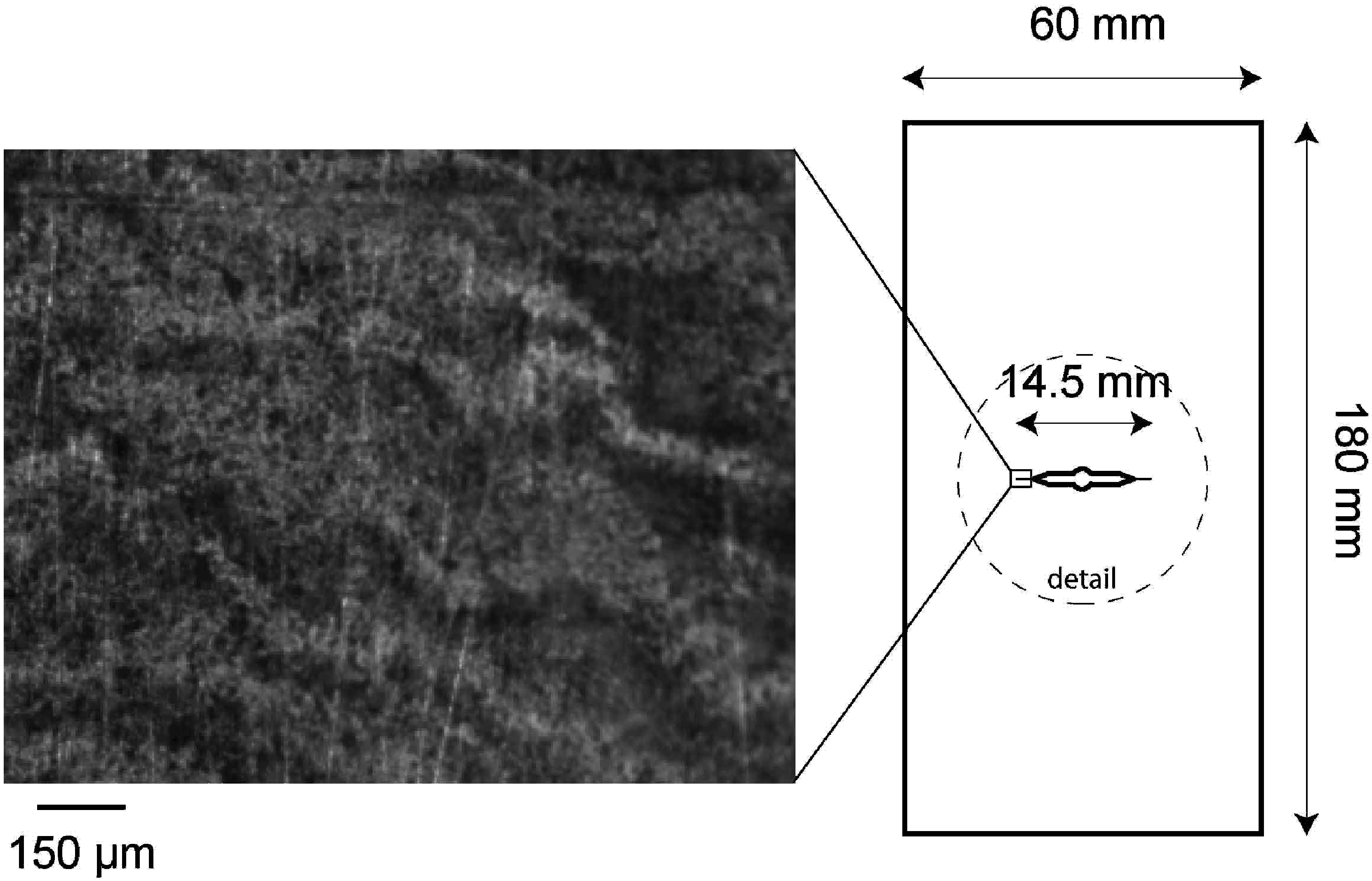
Toughness of ceramics



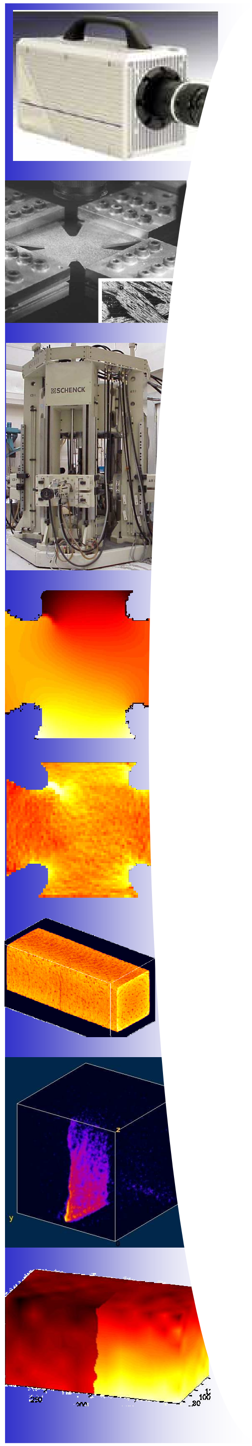
1 pixel ↔ 1.85 µm



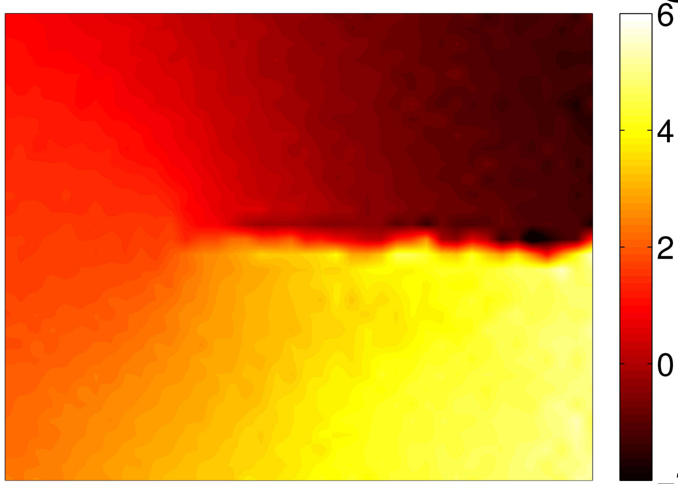
Crack growth law?



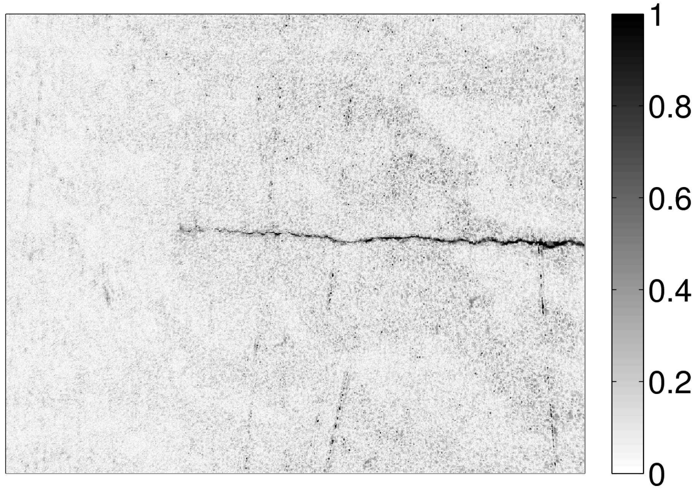
Long distance microscope: 1 pixel ↔ 2.08 µm



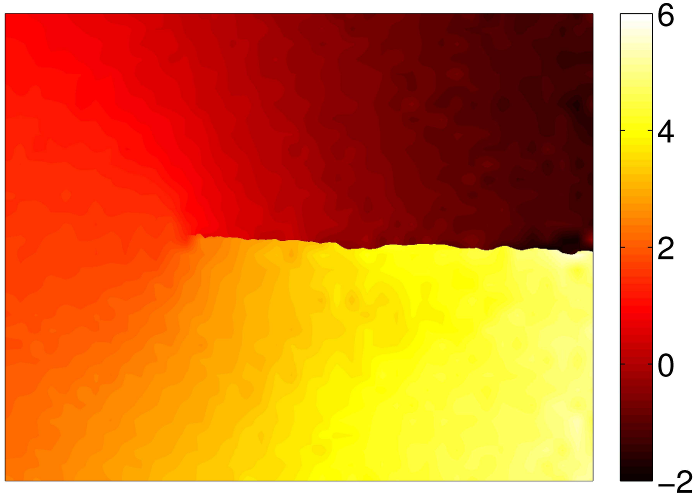
Crack growth law



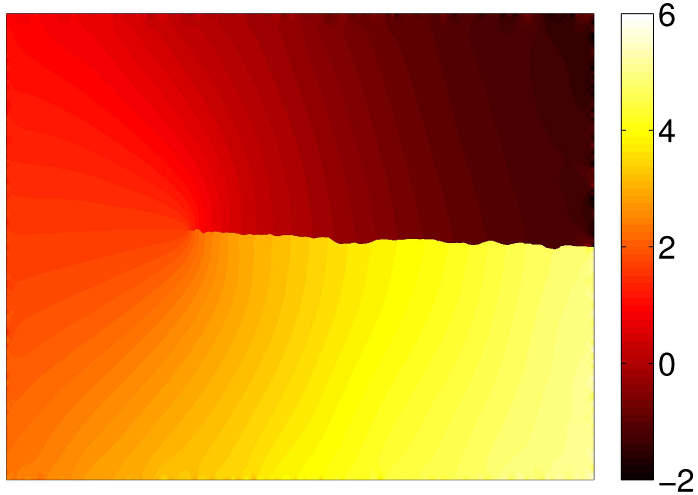
Q4-DIC (ℓ = 32 pixels)



Correlation residuals



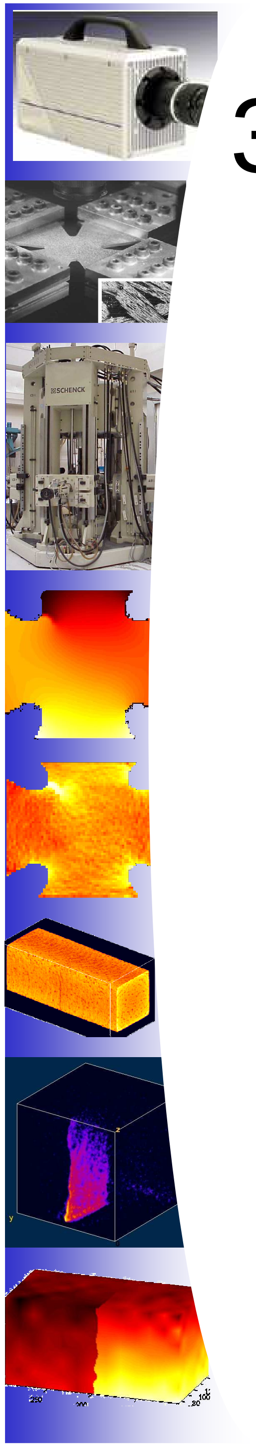
XQ4-DIC (ℓ = 32 pixels)

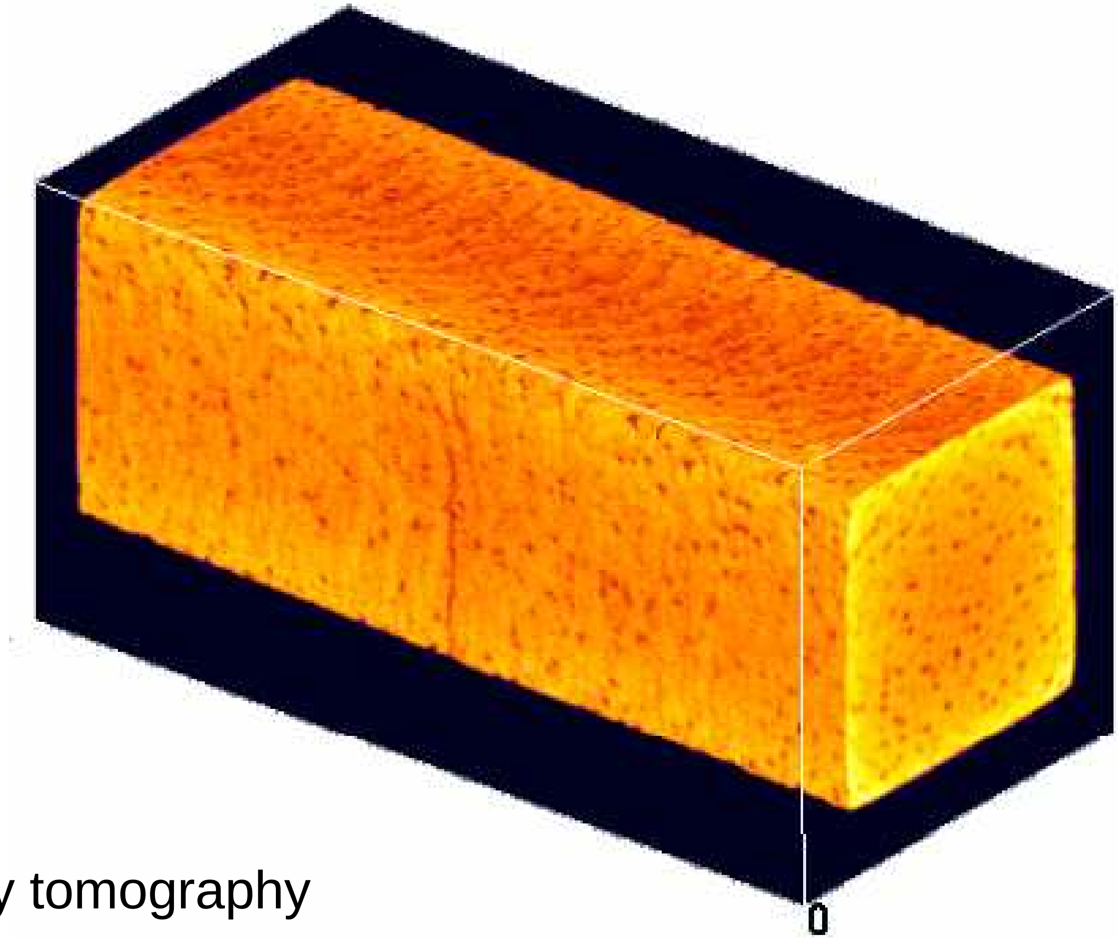


XIQ4-DIC (ℓ = 32 pixels)

1 pixel ↔ 2.08 µm

3D aspects of crack propagation?

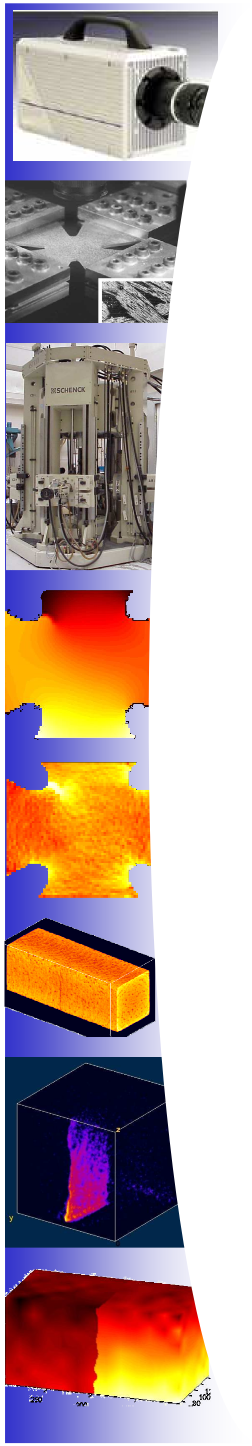




X-Ray tomography

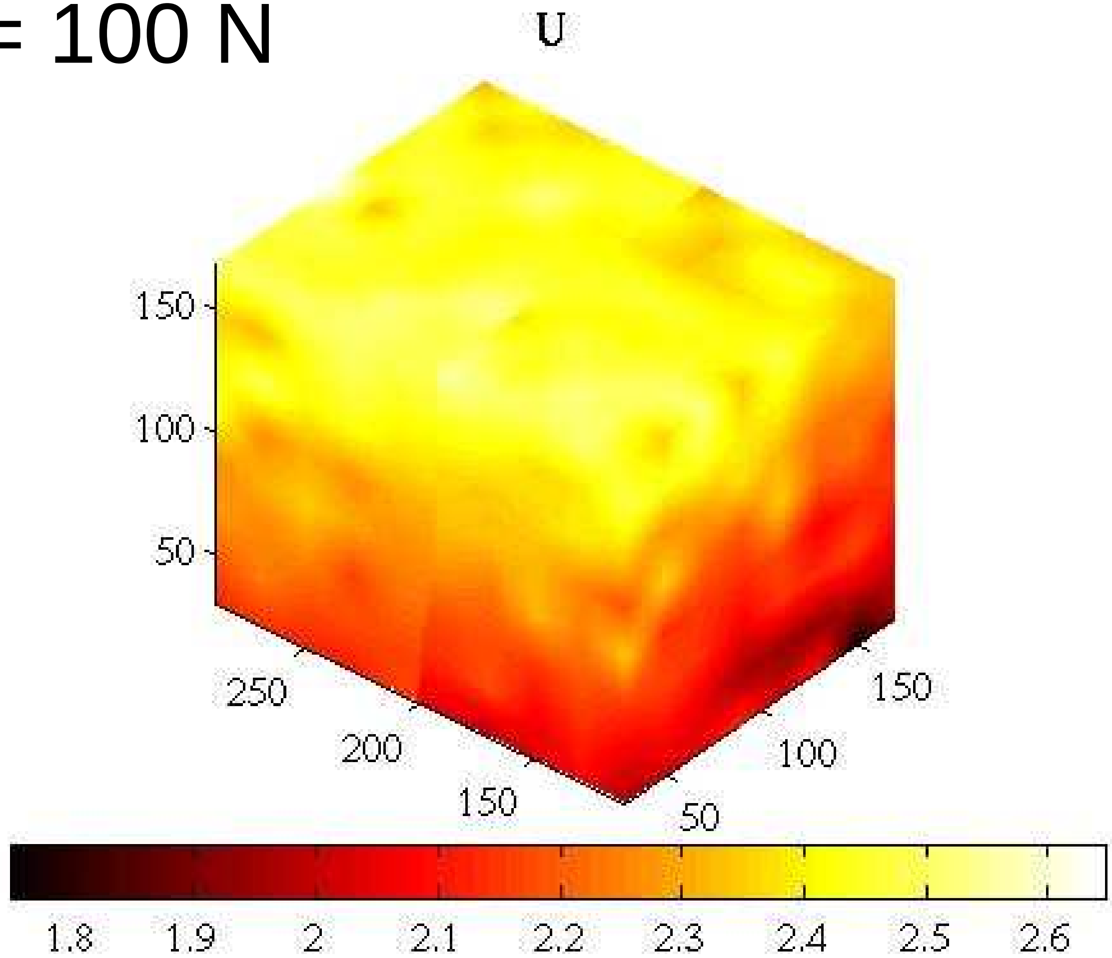
J.-Y. Buffière, J.-P. Tinnes (MATEIS)

1 voxel ↔ 13.5 µm



3D aspects of crack propagation

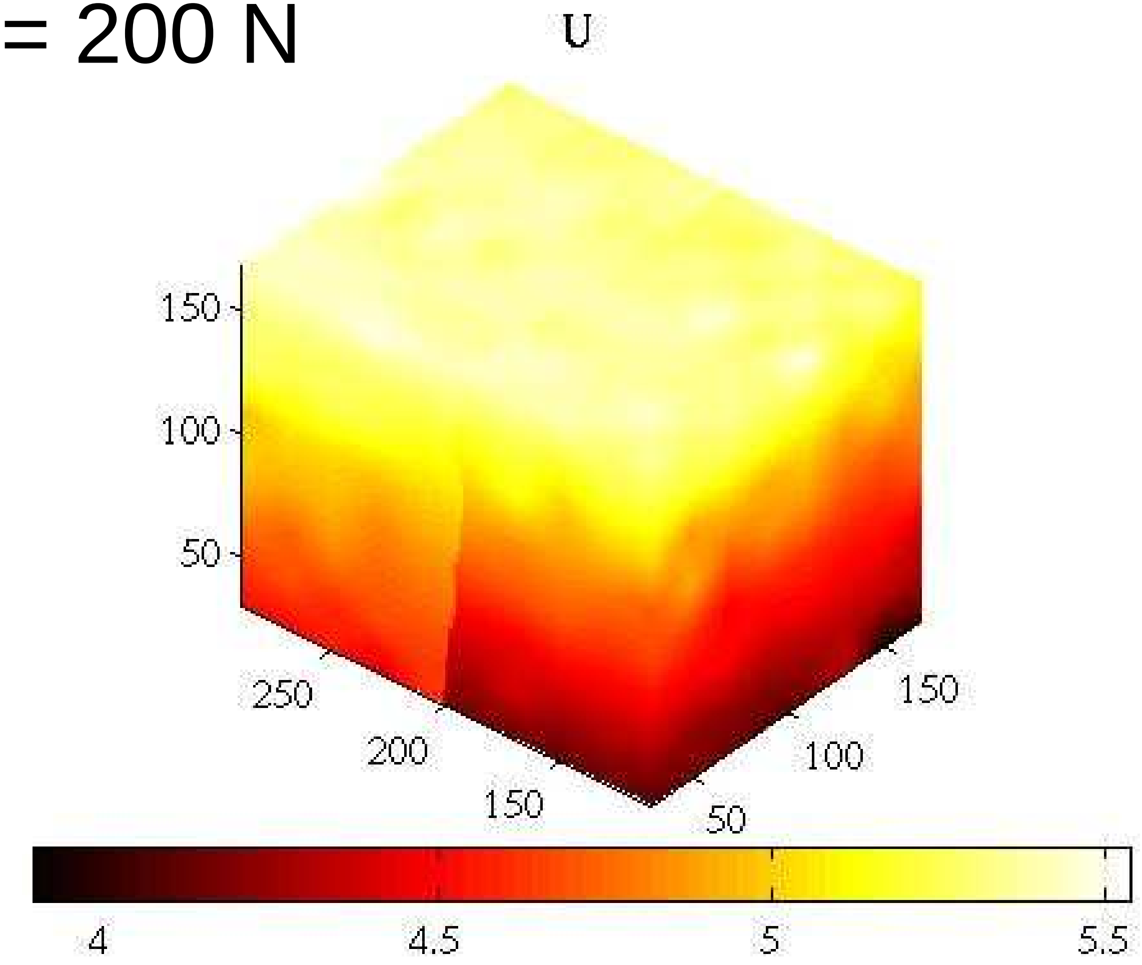
- F = 100 N



1 voxel ↔ 13.5 µm

3D aspects of crack propagation

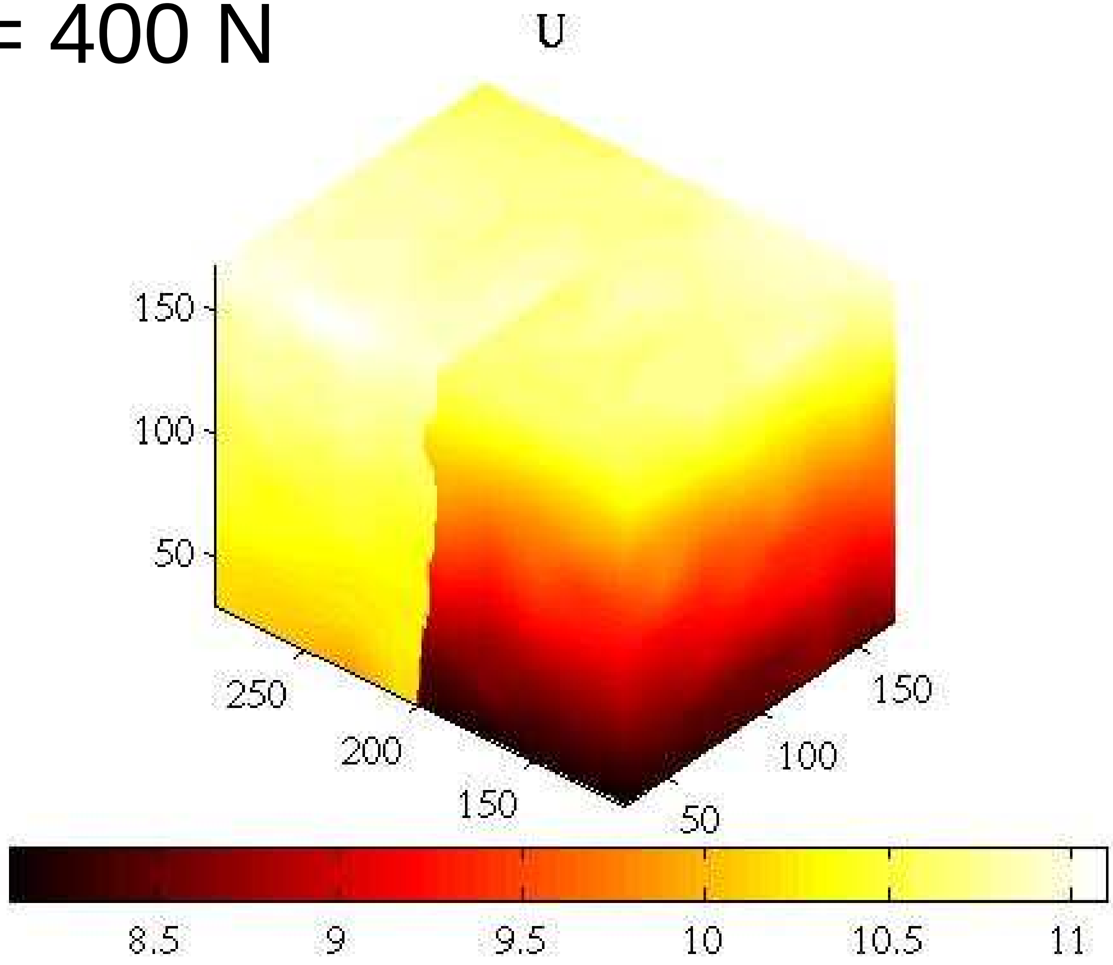
- F = 200 N



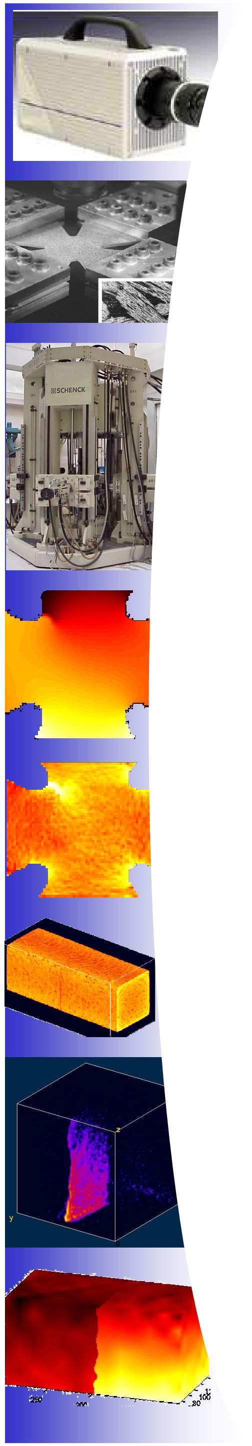
1 voxel ↔ 13.5 µm

3D aspects of crack propagation

- F = 400 N

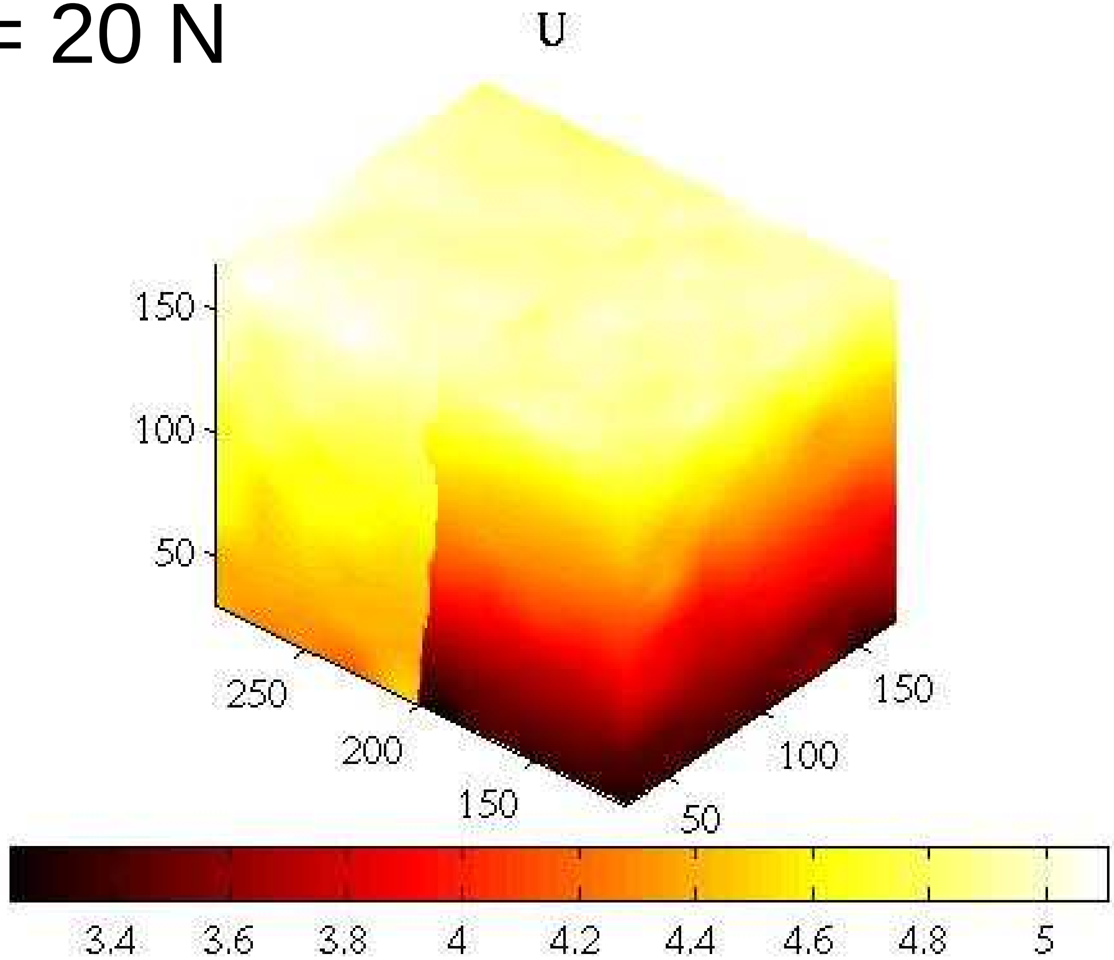


1 voxel ↔ 13.5 µm

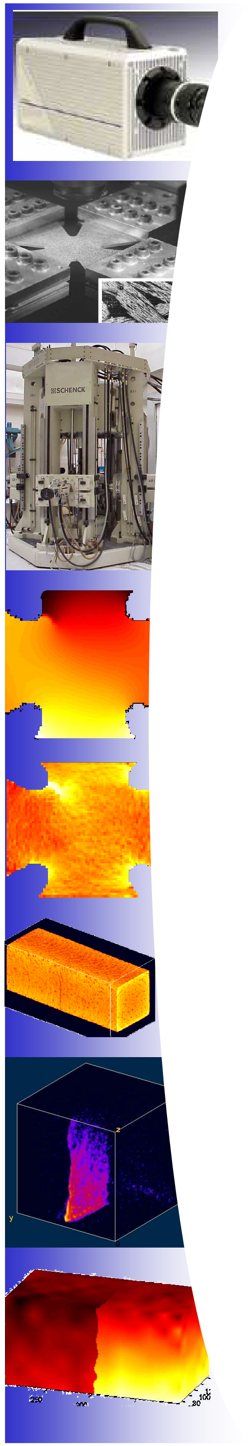


3D aspects of crack propagation

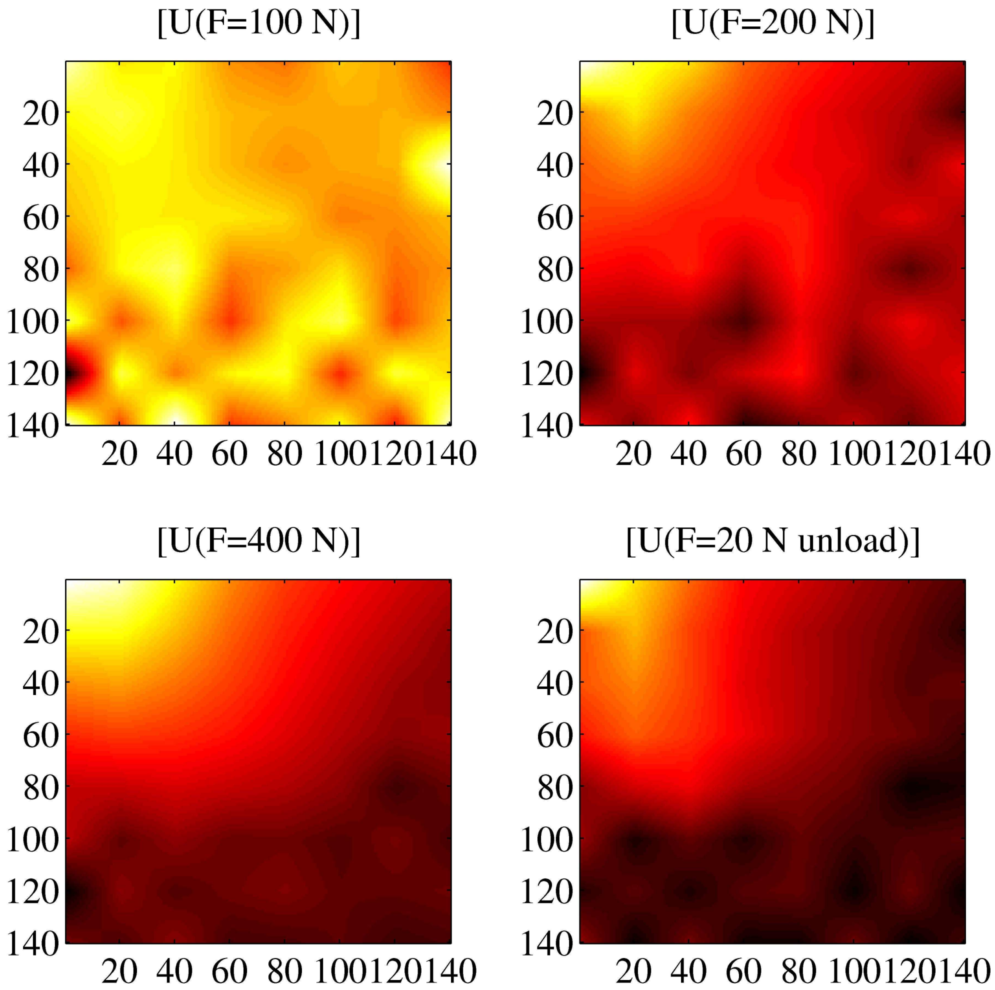
- F = 20 N



1 voxel ↔ 13.5 µm

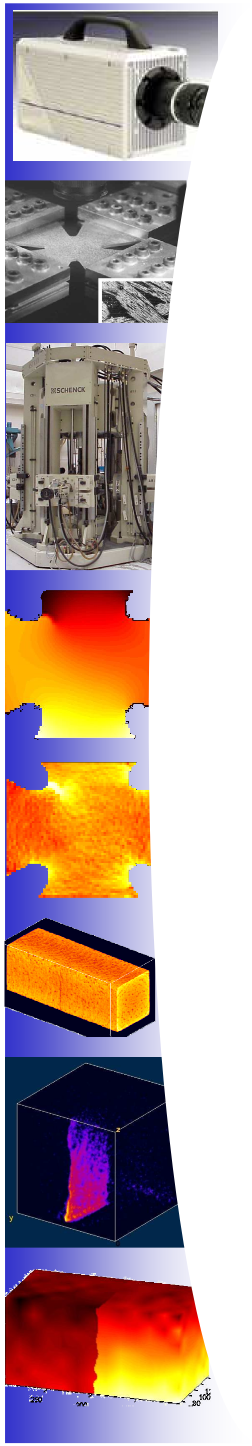


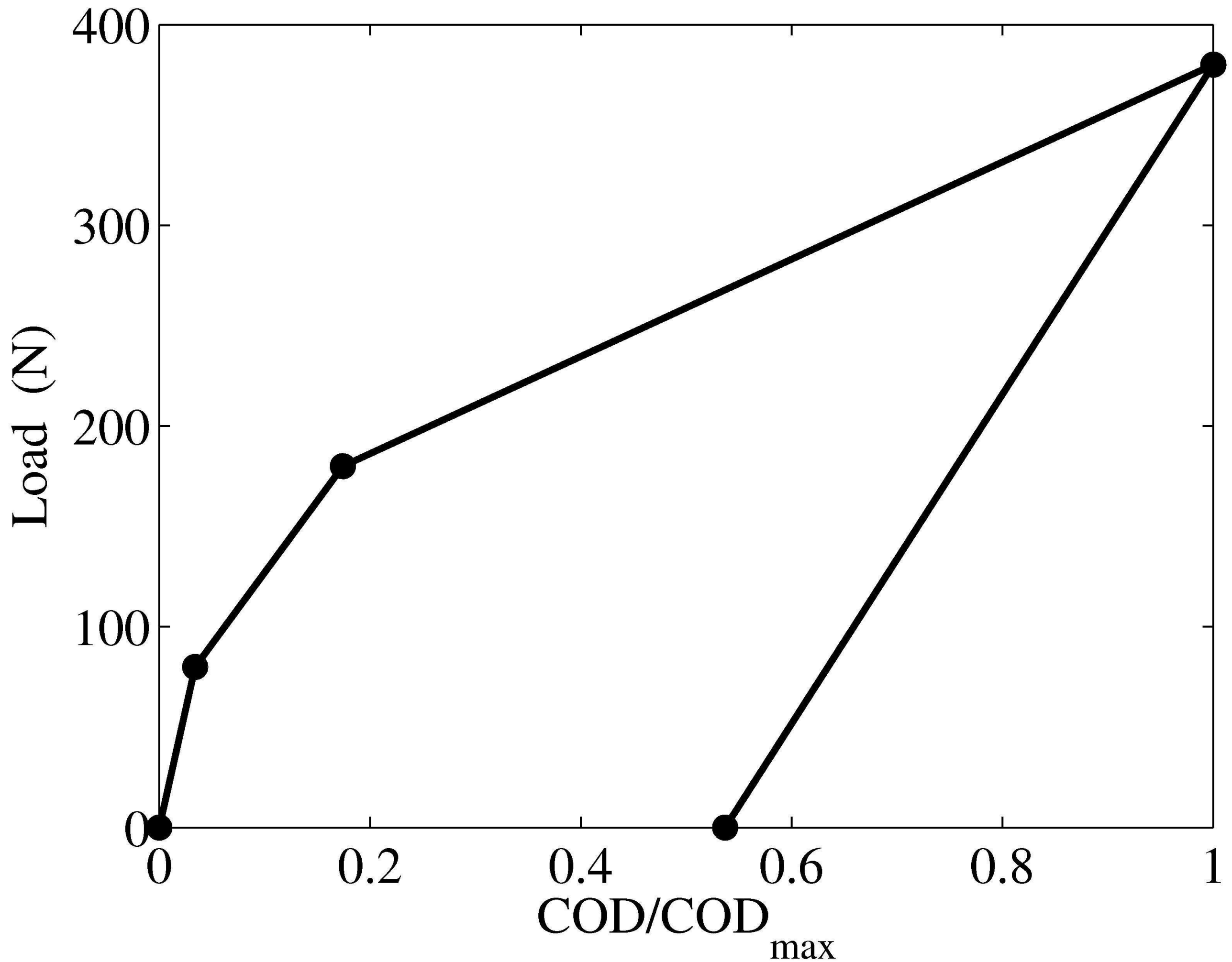
3D aspects of crack propagation



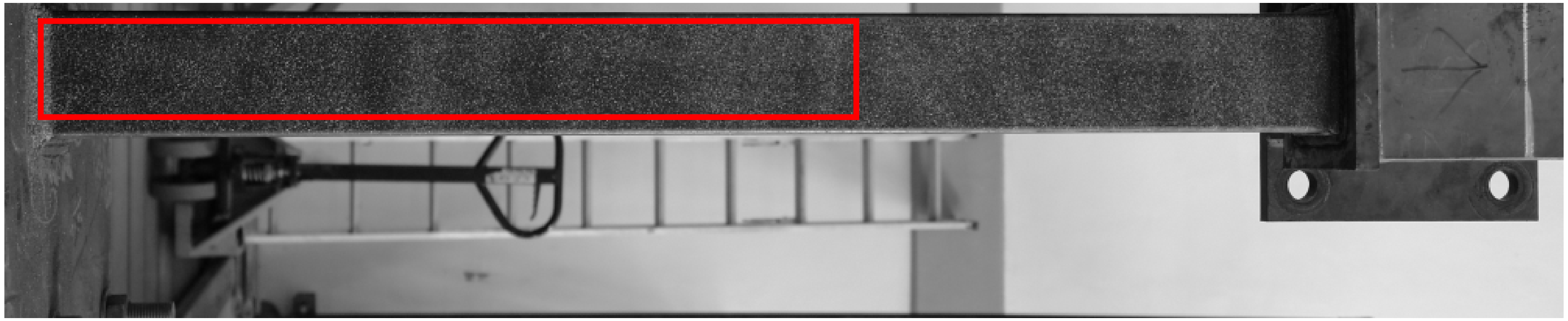
1 voxel ↔ 13.5 µm

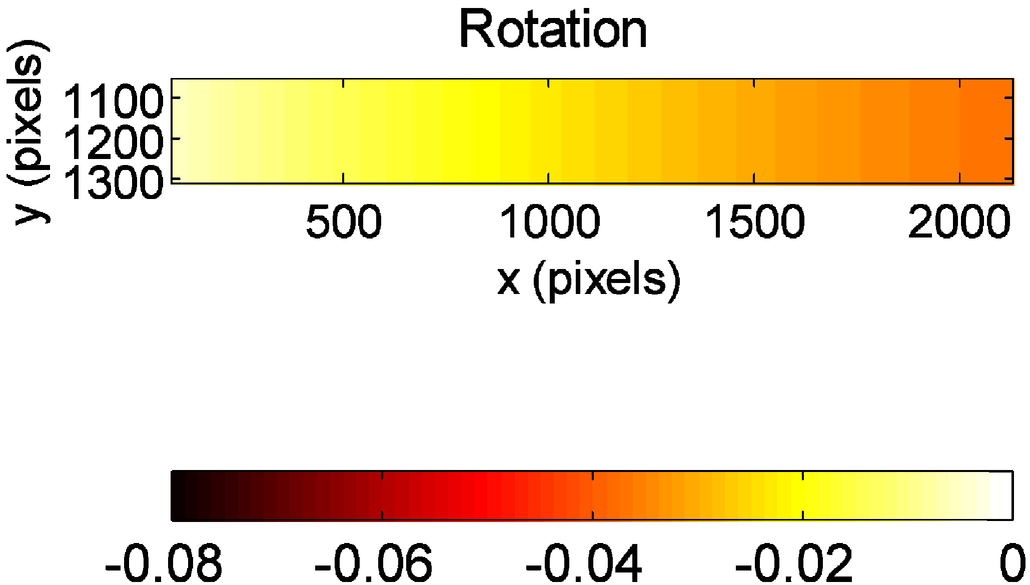
3D aspects of crack propagation





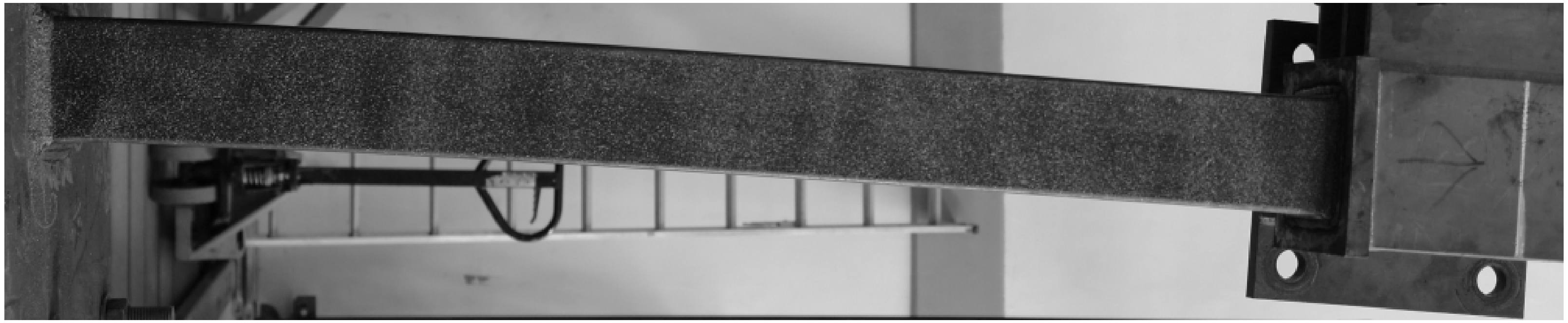
Analysis of local buckling?

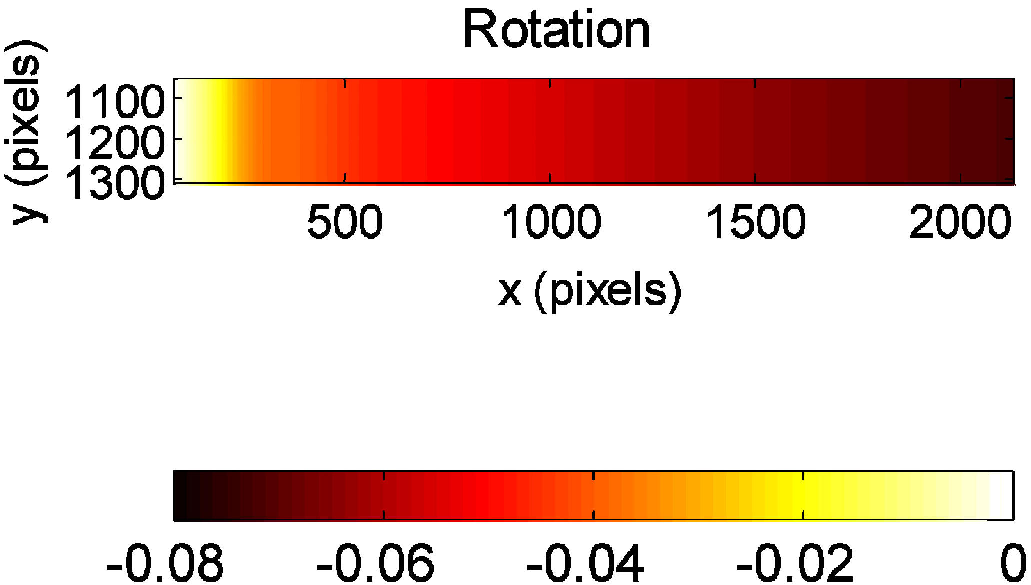




1 pixel ↔ 300 µm

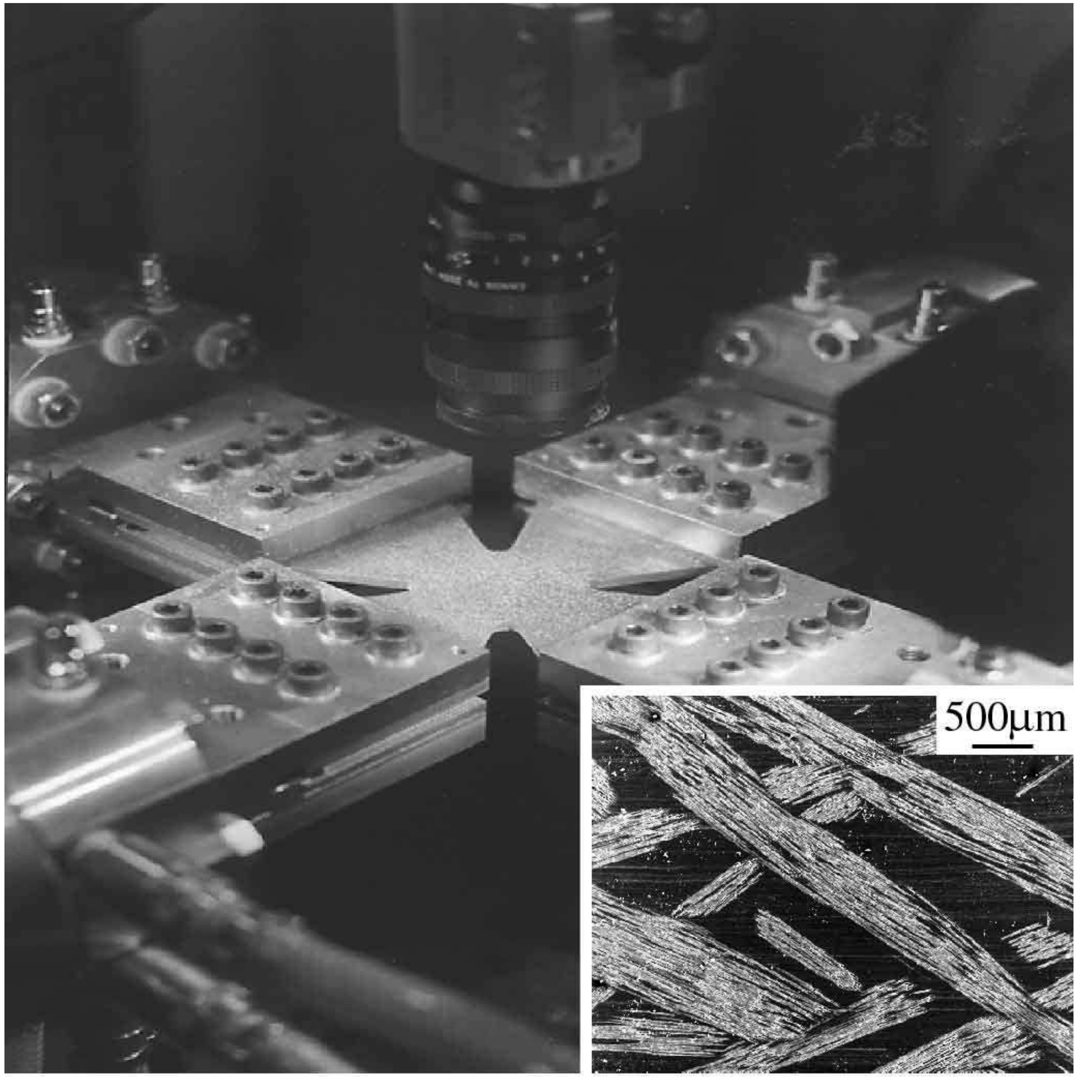
Analysis of local buckling

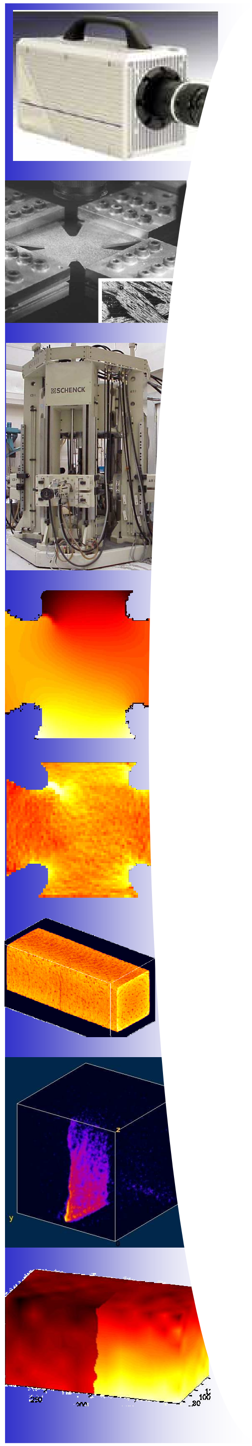




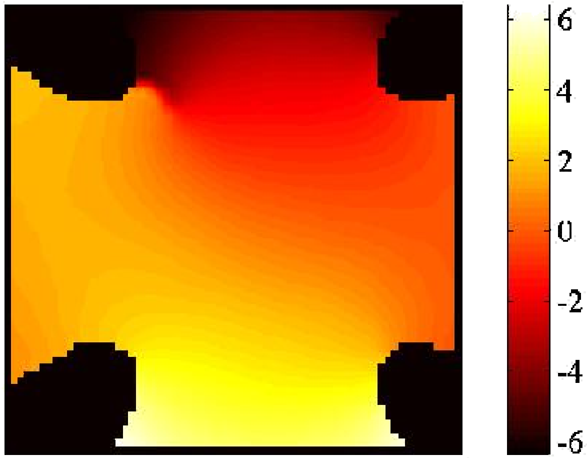
1 pixel ↔ 300 µm

Damage and fracture?

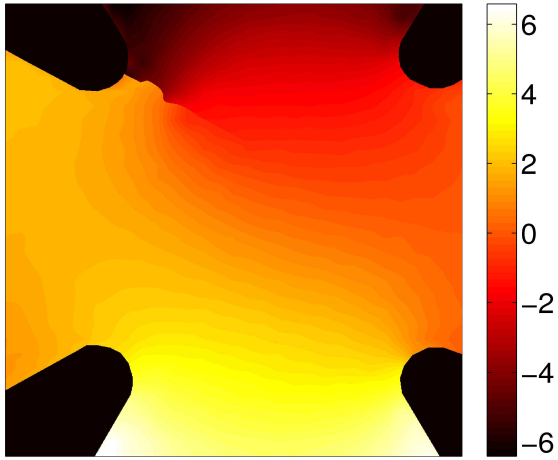




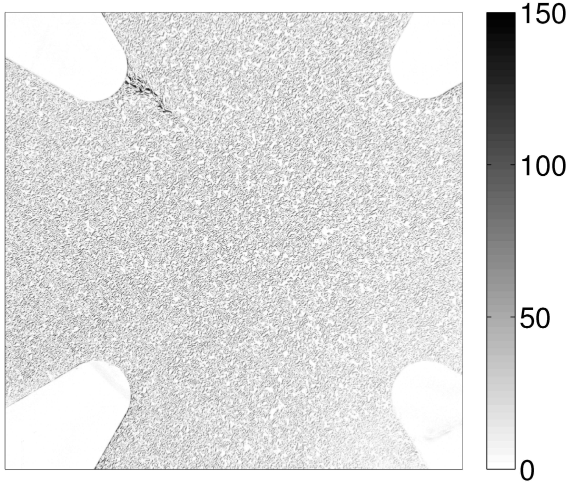
Damage and fracture

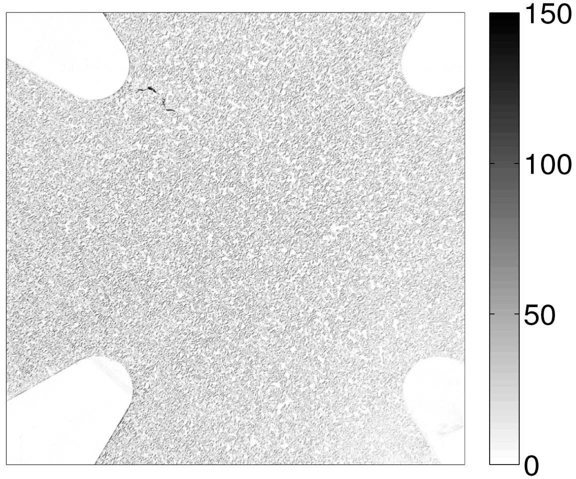


Q4-DIC (ℓ = 16 pixels)



XQ4-DIC (ℓ = 32 pixels)

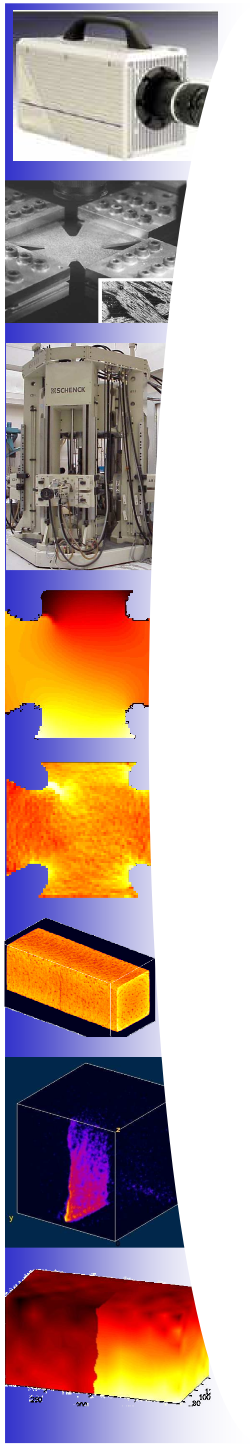


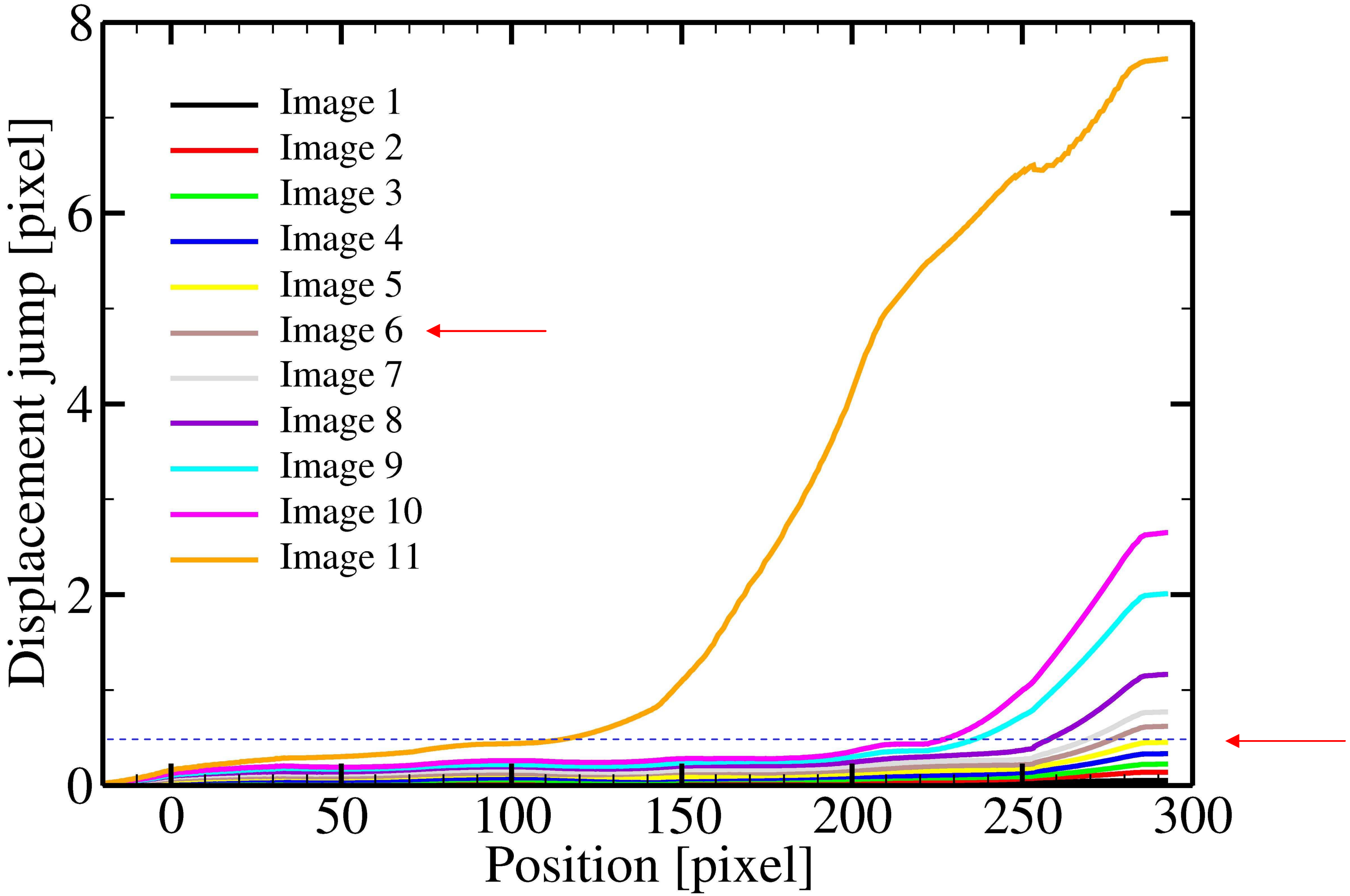


Correlation residuals

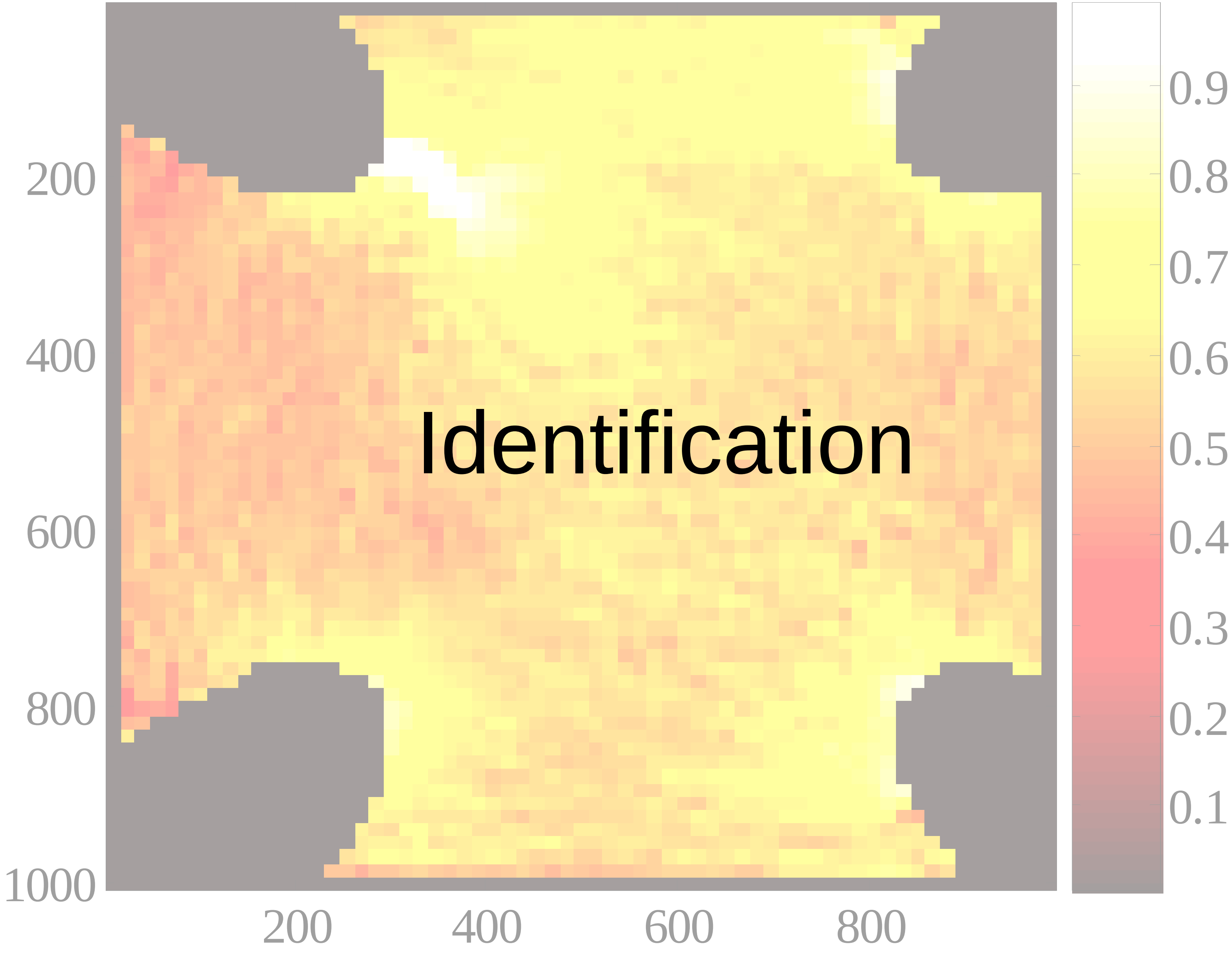
1 pixel $\leftrightarrow$ 68 µm

Damage and fracture

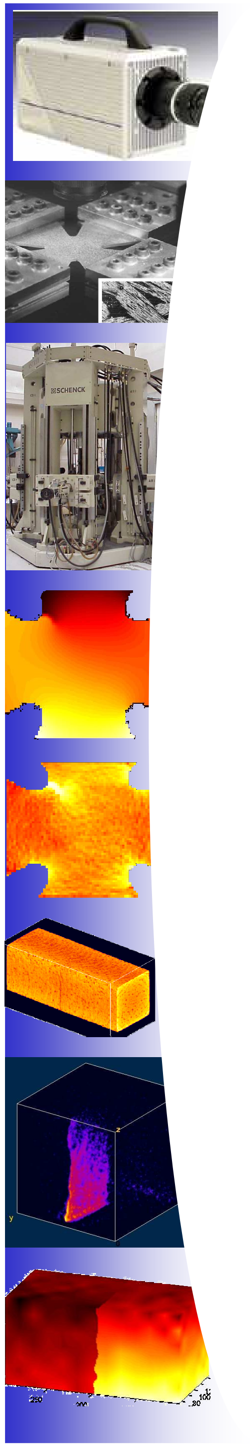


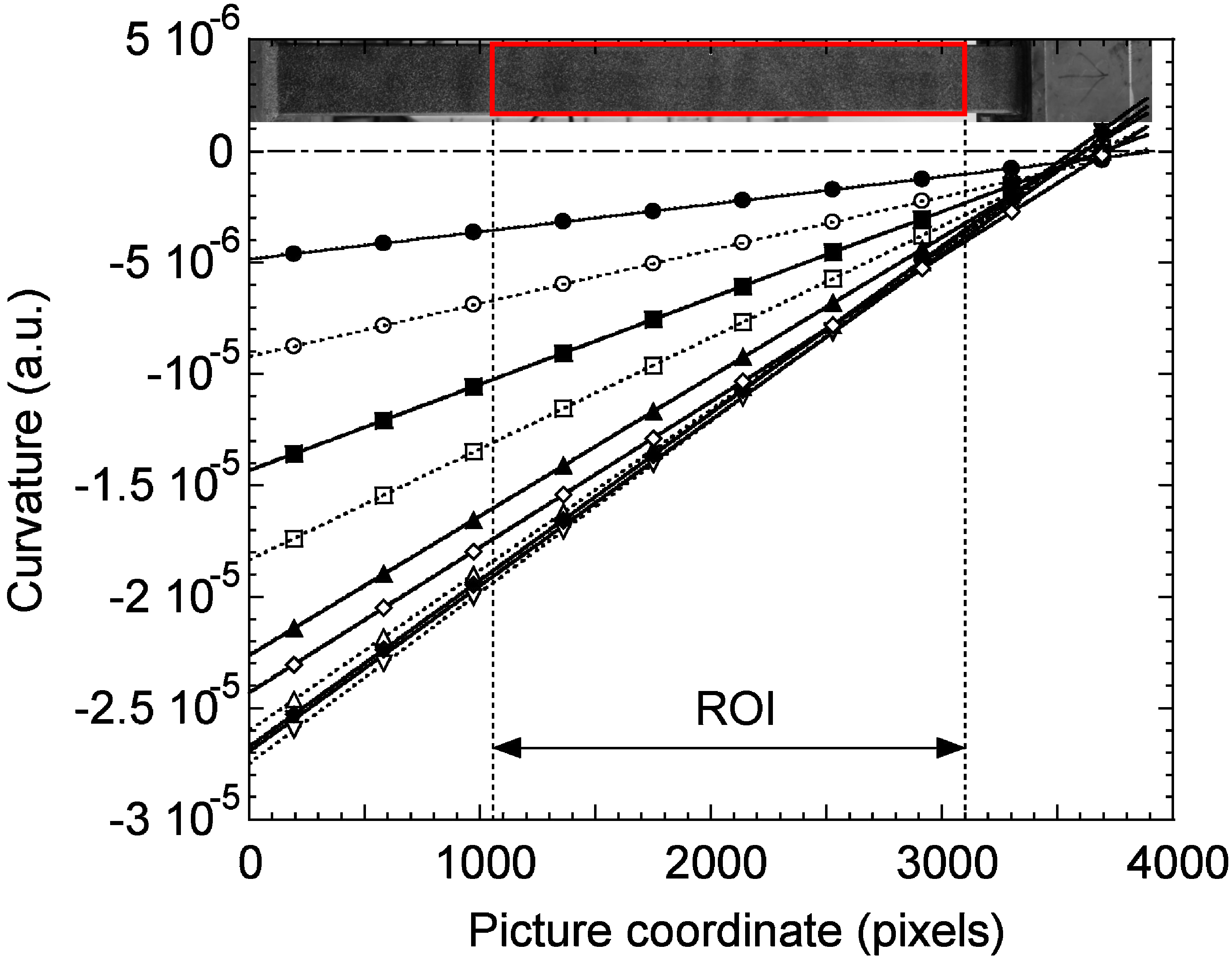


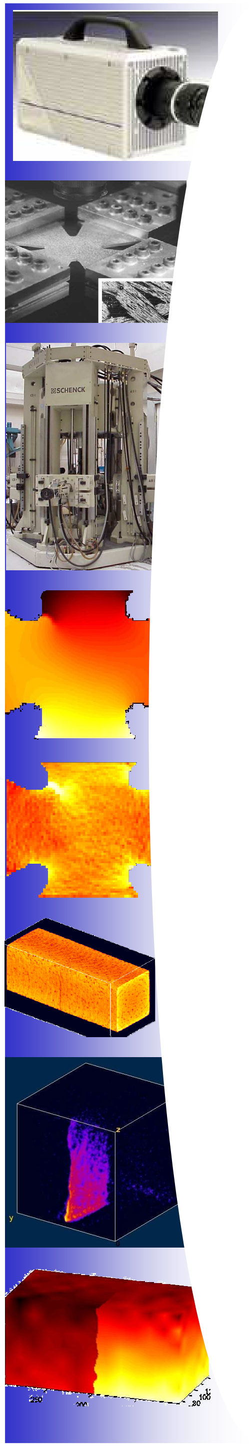
1 pixel ↔ 68 µm


Identification
200
400
600
800
1000
200
400
600
800
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1

Load measurement







Elastic parameters

- A suited framework:
  Kolossov-Muskhelichvili potentials in the complex plane z

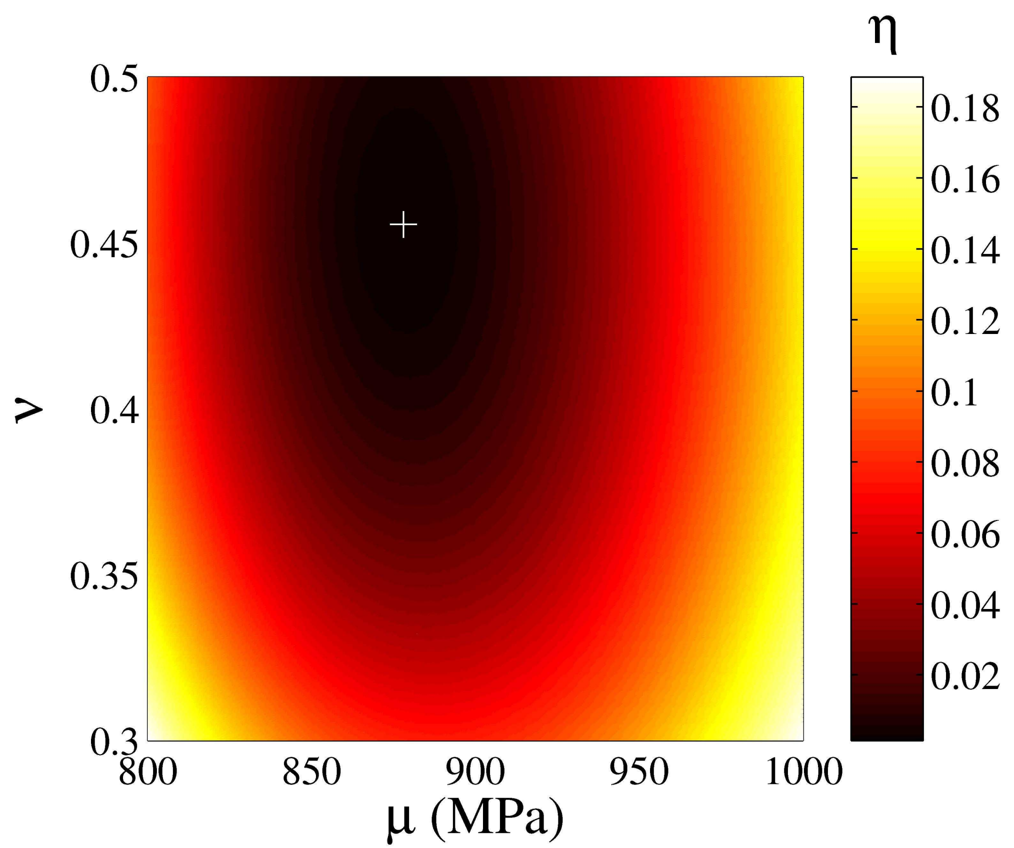
$$2\mu U(z) = \kappa\Phi(z) - z\overline{\Phi'(z)} - \overline{\Psi(z)}$$

with $$\kappa = \frac{(3-\nu)}{(1+\nu)}$$

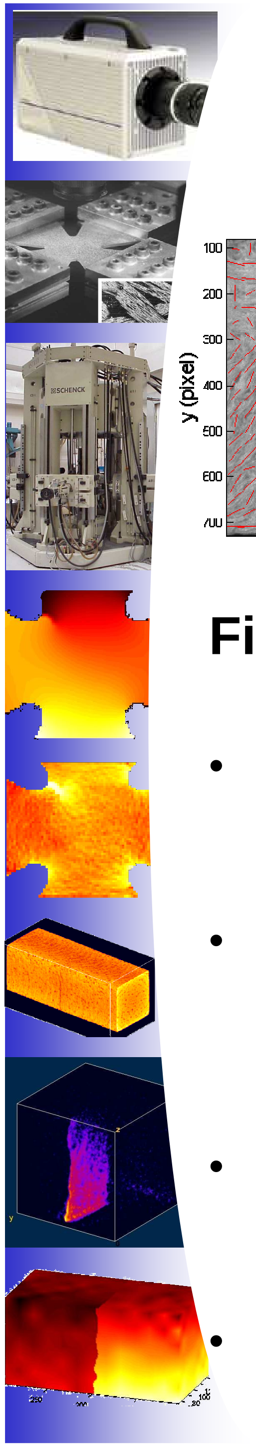
- Linear structure in $(1/\mu\,,\kappa/\mu)$

Elastic parameters

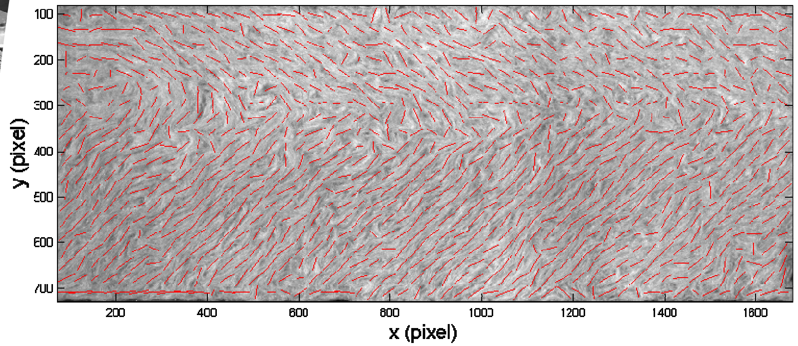
- Brazilian test



I-DIC (ℓ = 1000 pixels)

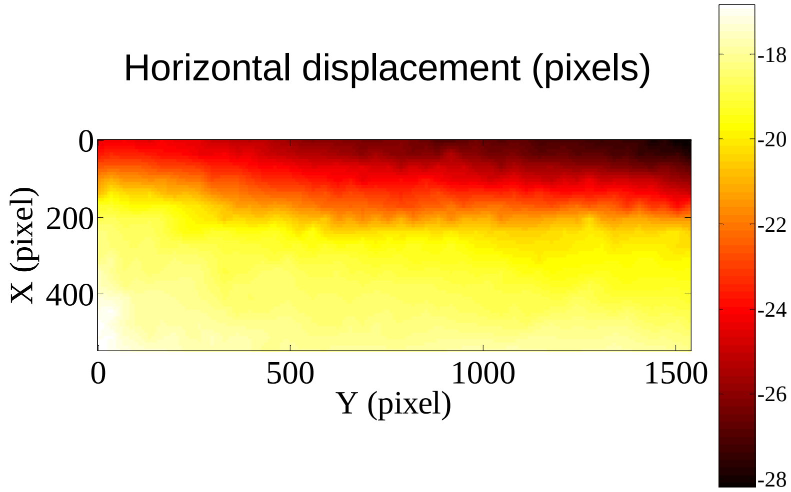


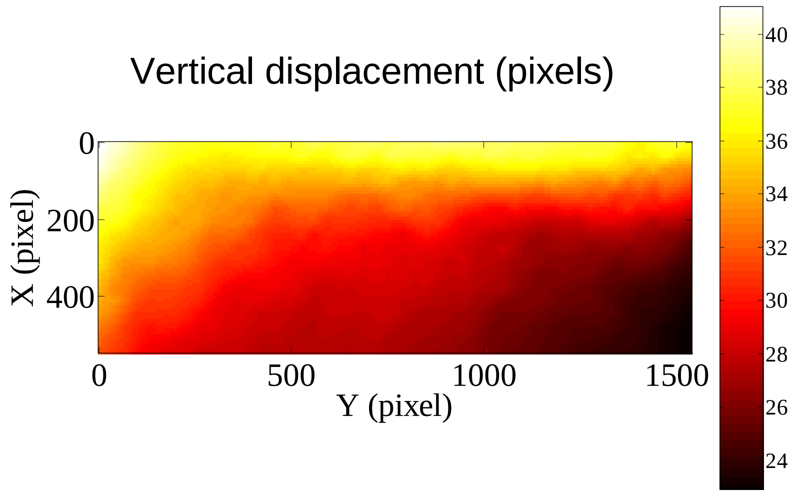
Field of elastic parameters



Finite element updating

- Initial parameters (tests on uncrimped mineral wool)
- FEA with measured BCs + auxiliary problems associated with elastic parameter changes
- Comparison between measured / computed displacement fields
- Update elastic parameters (S_{nn}, S_{tt}, S_{nt}, S_{uu})

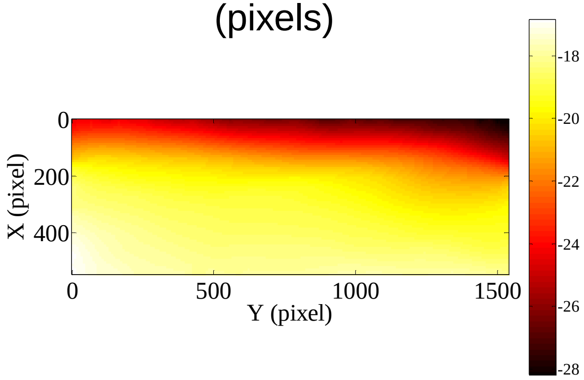




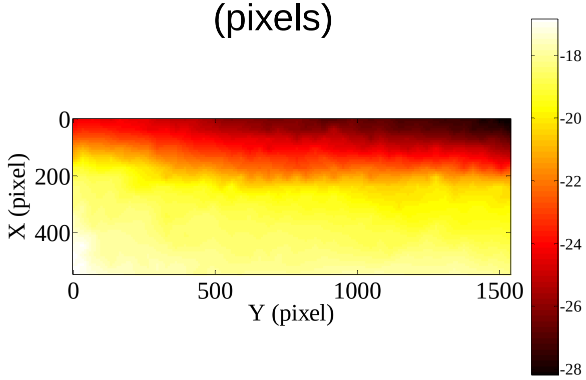
1 pixel ↔ 130 µm

Field of elastic parameters

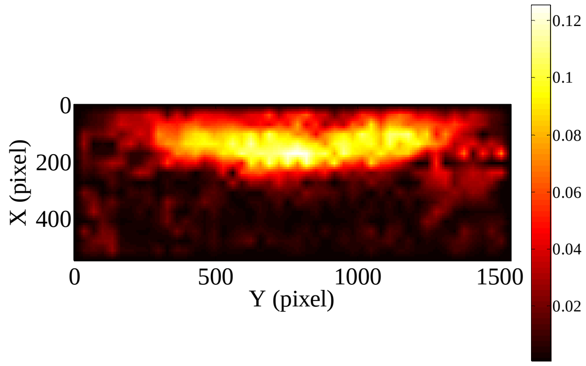
Computed horizontal displacement (pixels)



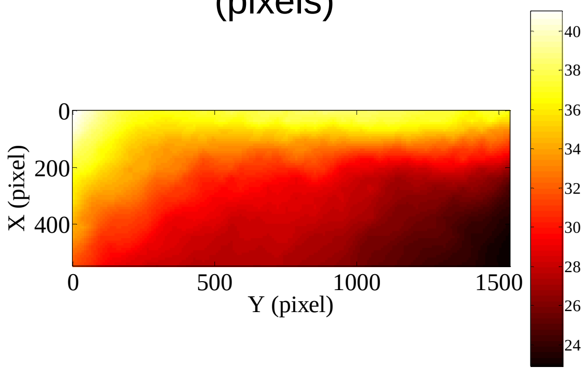
Measured horizontal displacement (pixels)



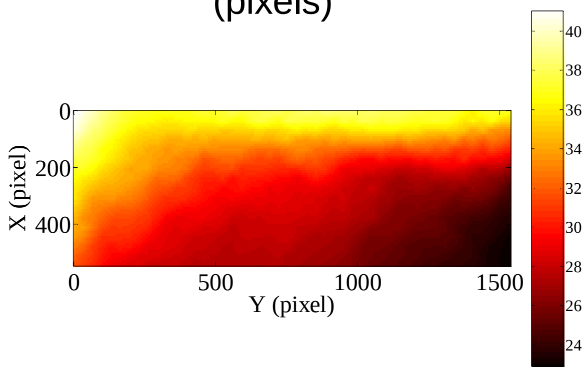
Residual



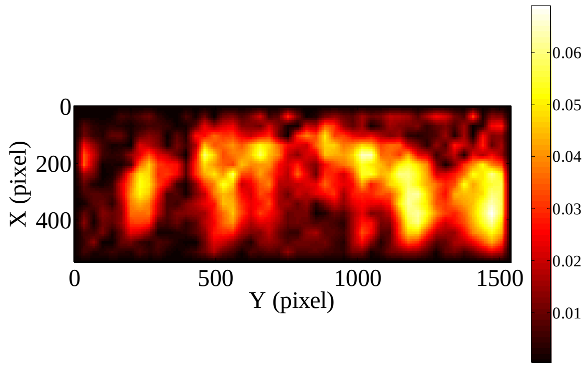
Computed vertical displacement (pixels)



Measured vertical displacement (pixels)



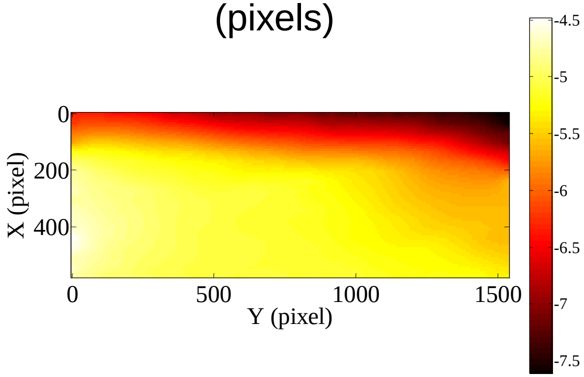
Residual



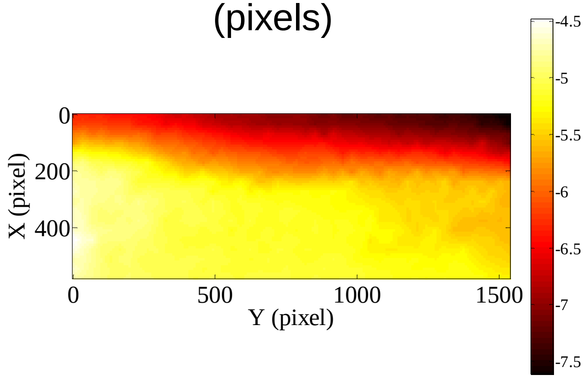
Global relative displacement error: 3.1 %

Field of elastic parameters

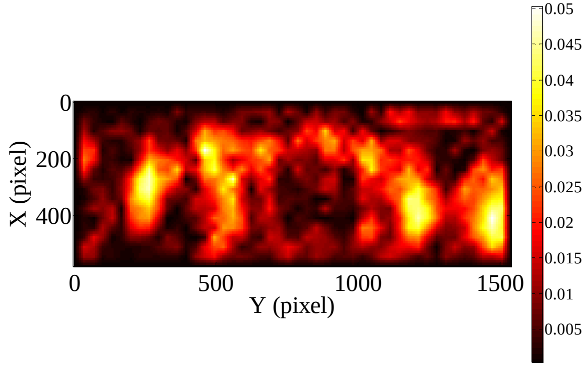
Predicted horizontal displacement (pixels)



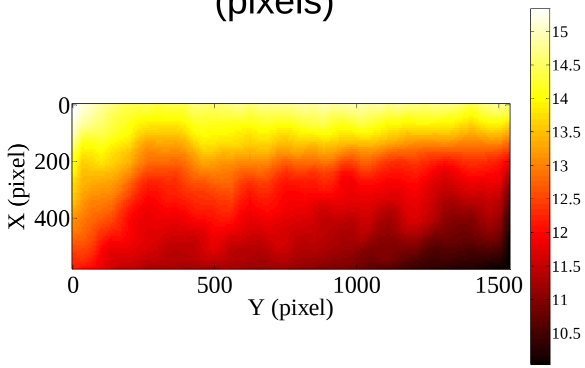
Measured horizontal displacement (pixels)



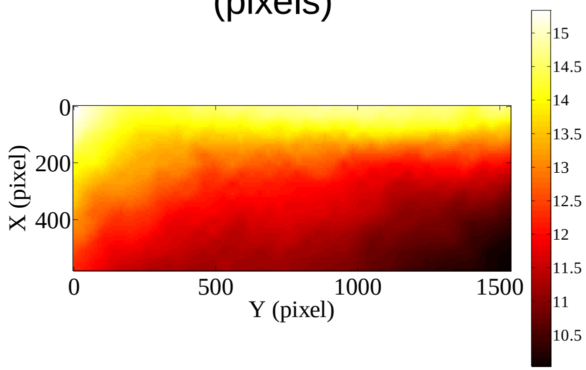
Residual



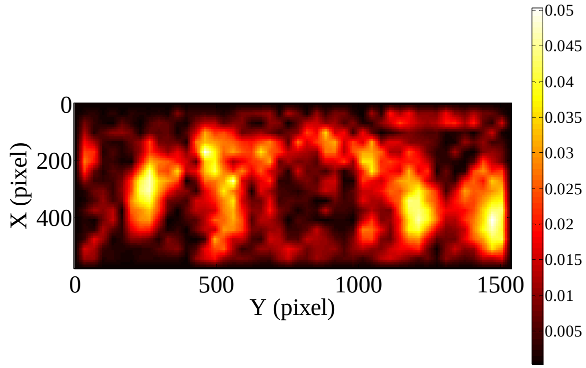
Predicted vertical displacement (pixels)



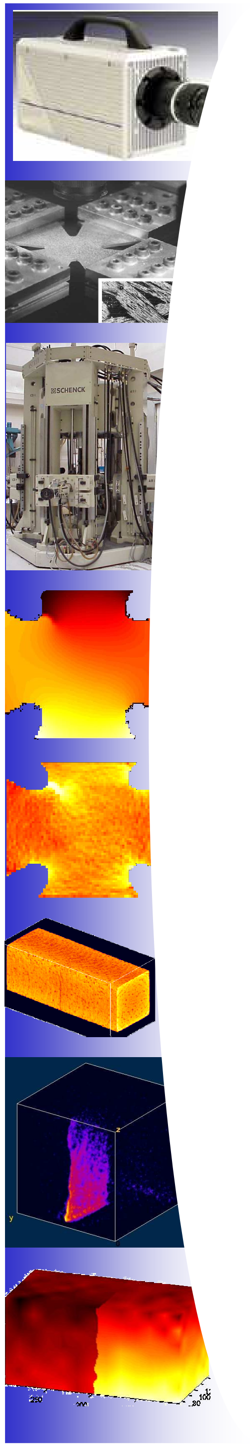
Measured vertical displacement (pixels)



Residual



Global relative displacement error: 2.3 %



SIF identification

Interaction integral

$$I^{\text{int}} = -\int_{\Omega} \left(\sigma_{ml}^{aux} u_{m,l} \delta_{kj} - \left(\sigma_{ij}^{aux} u_{i,k} + \sigma_{ij} u_{i,k}^{aux} \right) \right) q_{k,j}\, dV$$

- Auxiliary fields = Westergaard solution

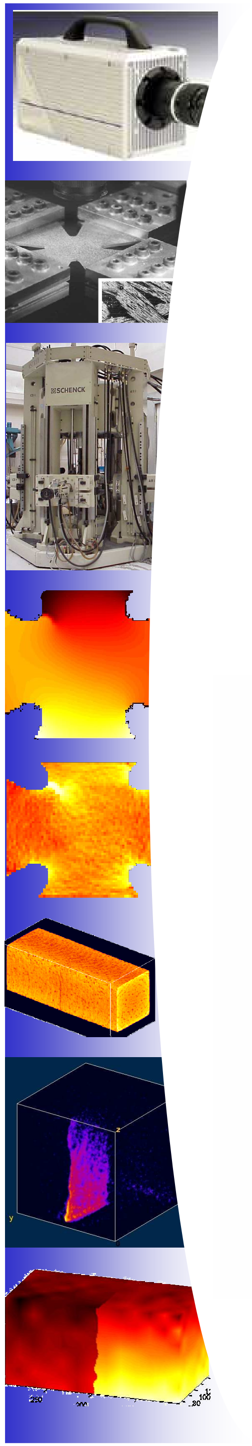
$$I^{\text{int}} = \frac{2}{E} \left(K_I K_I^{aux} + K_{II} K_{II}^{aux} \right)$$

- Virtual crack extension

$$\underline{q} = \underline{0} \quad (edge)$$

$$\underline{q} = \underline{e}_1 \quad (crack\ tip)$$

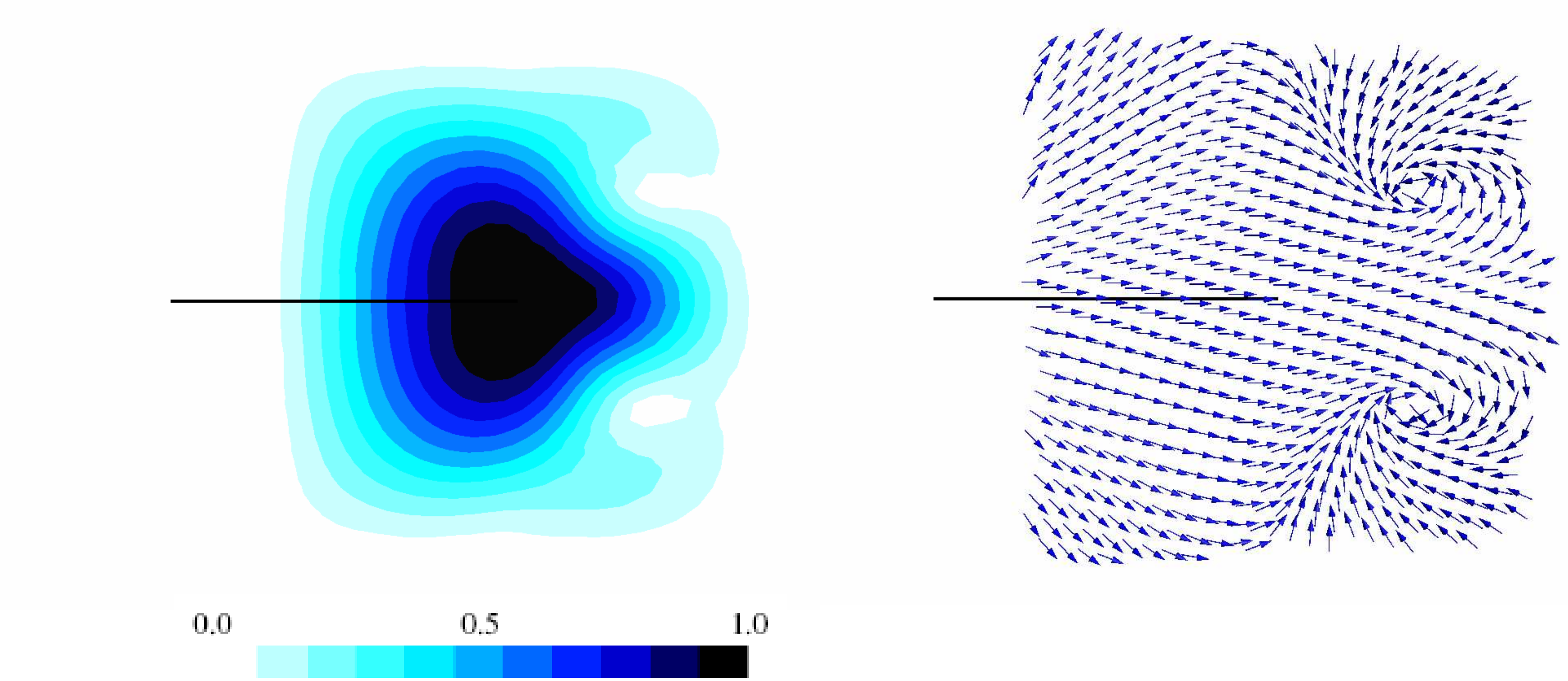
$$\underline{q}.\underline{n} = 0 \quad (crack\ \ face)$$

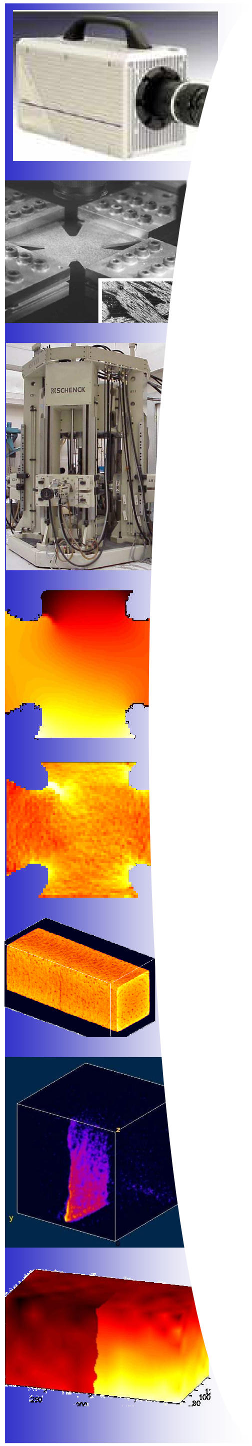


SIF identification

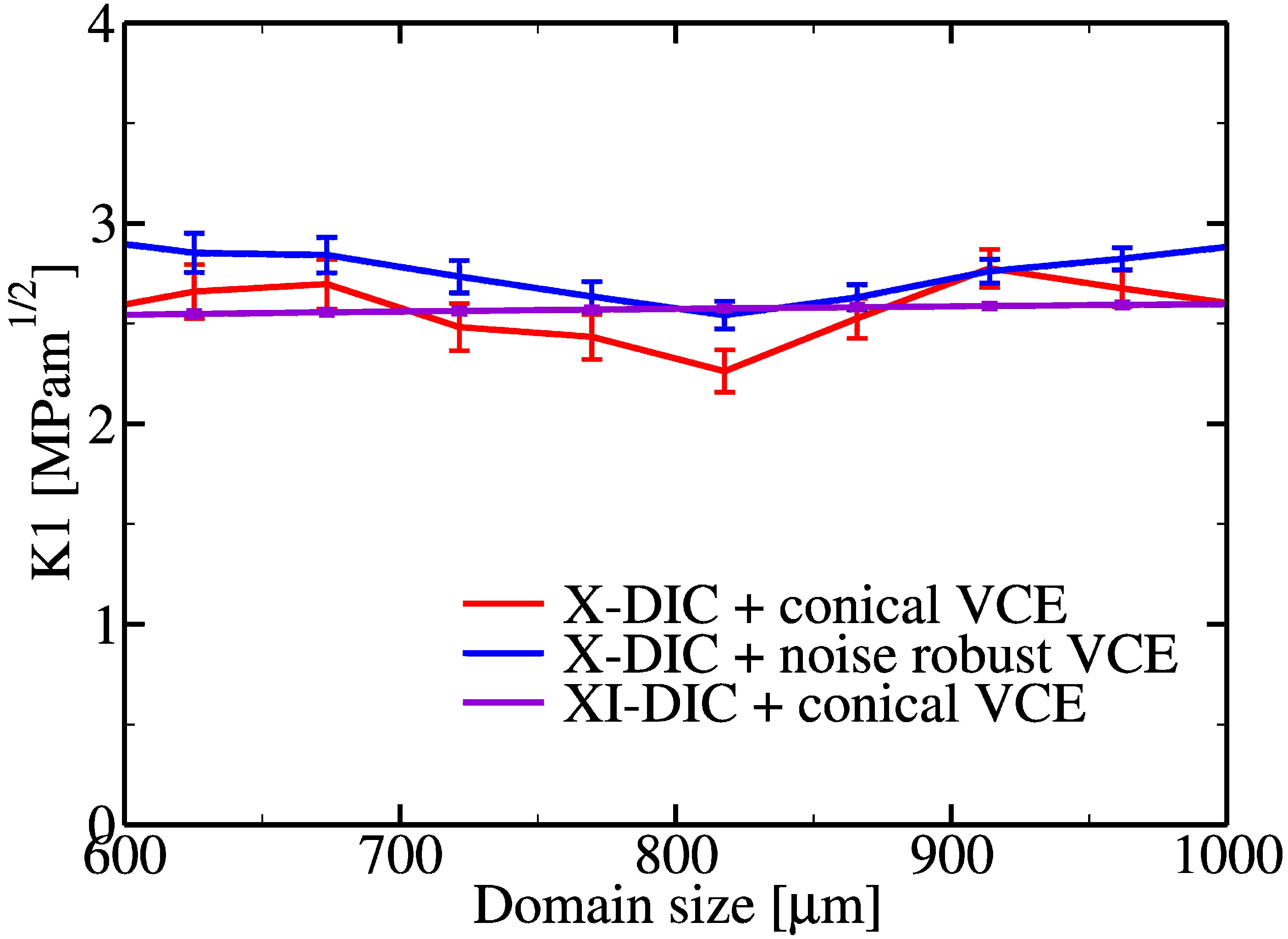
Noise-robust interaction integral

Optimize the virtual crack extension field so that I^{int} has smallest sensitivity to noise



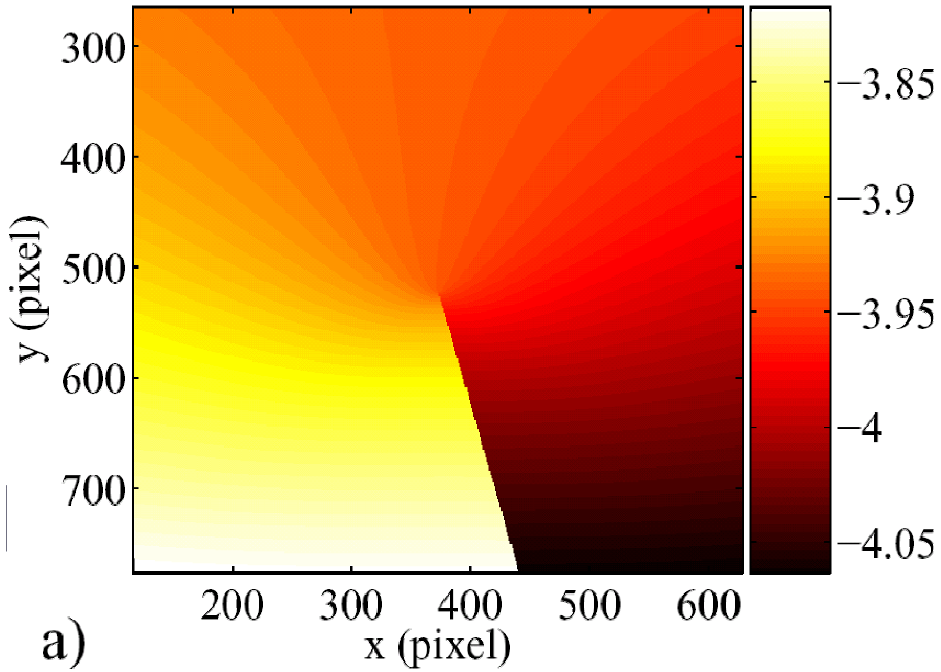


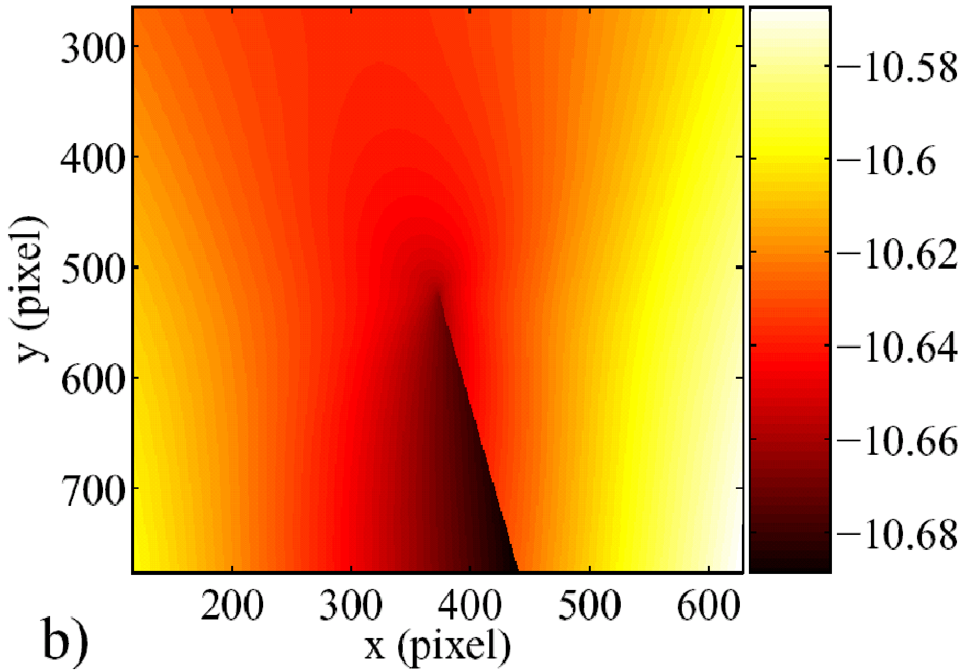
SIF identification



SIF *measurement**

Horizontal displacement (pixel) Vertical displacement (pixel)





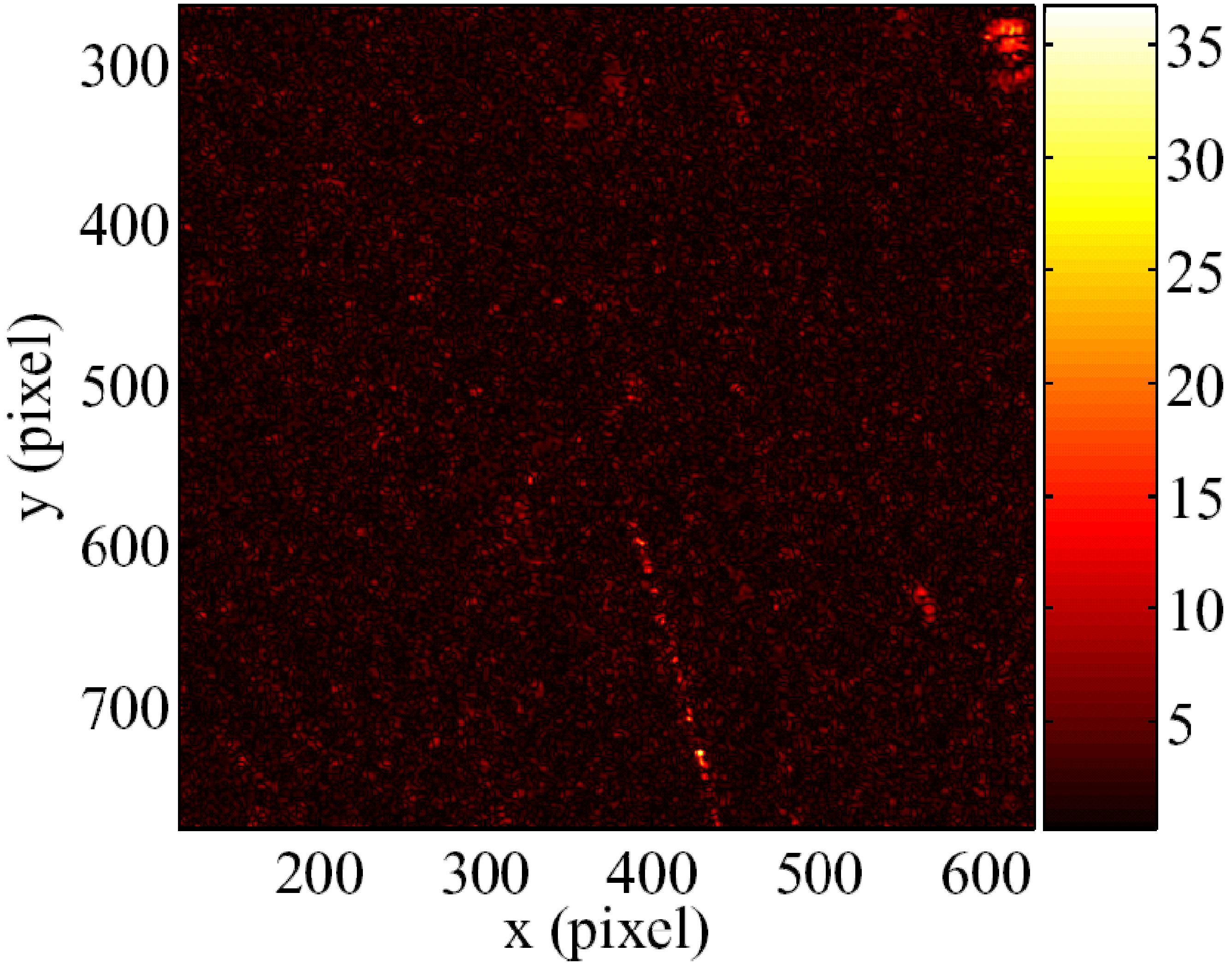
$K_I = 2.8 \pm 0.1$ MPa $m^{1/2}$ $K_{II} = 0.0 \pm 0.1$ MPa $m^{1/2}$

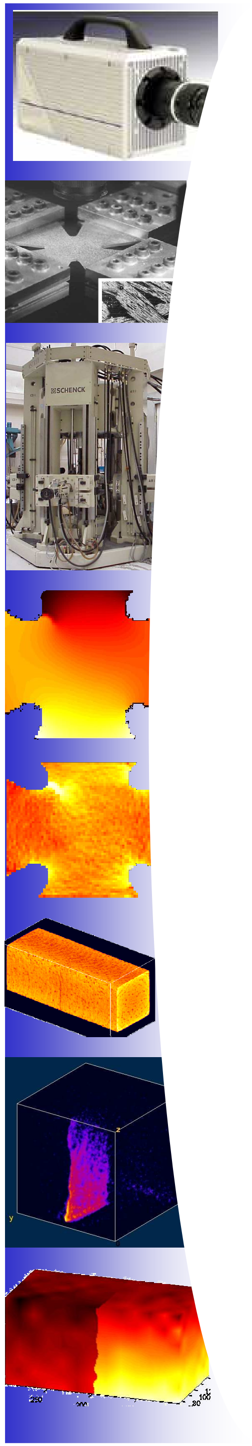
*I-DIC (ℓ = 500 pixels)

1 pixel $\leftrightarrow$ 1.85 µm

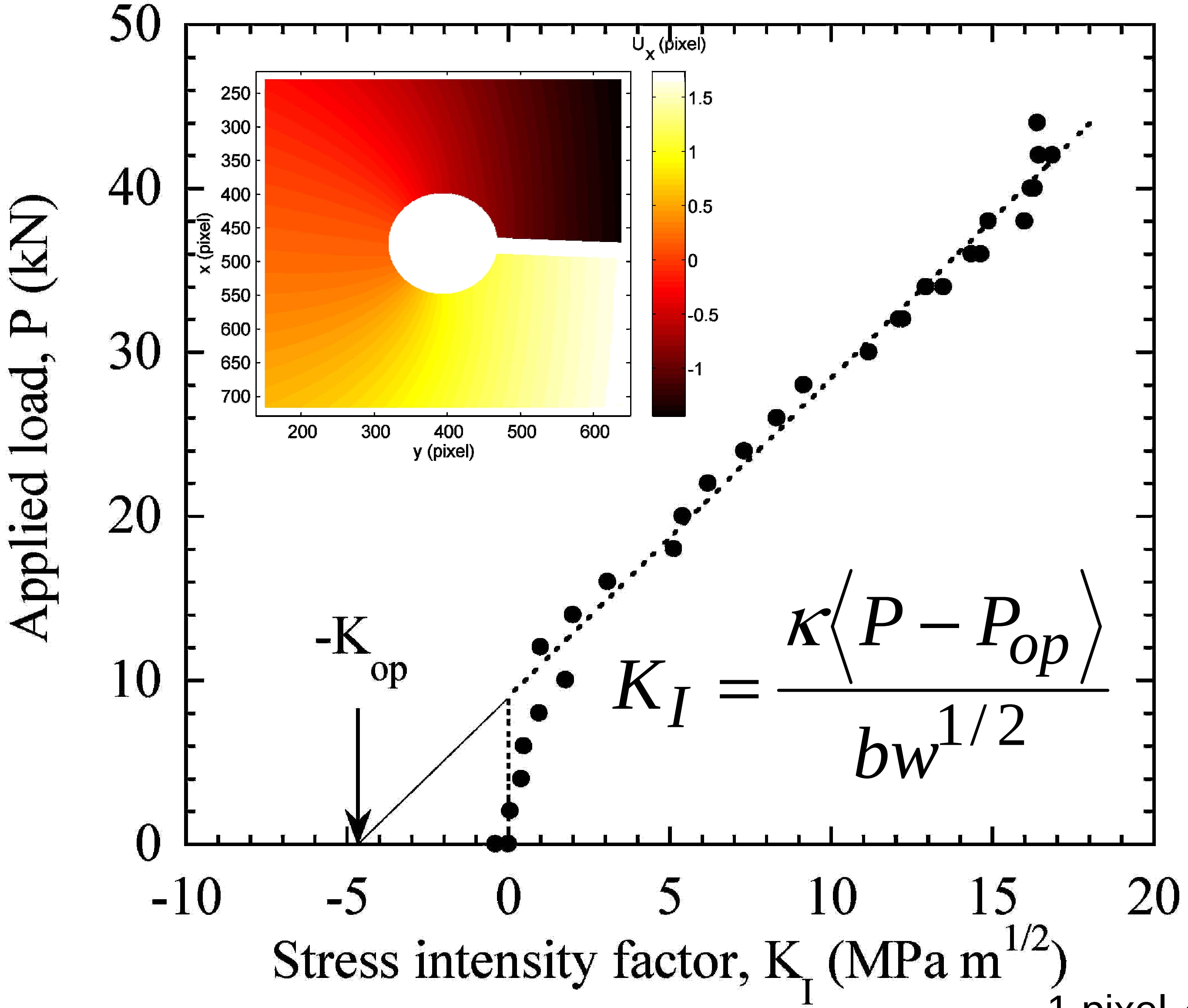
SIF *measurement*

$$\varphi_{\text{lin}}(\underline{x}) = \left| g(\underline{x}) - f(\underline{x}) + \underline{u}(\underline{x}).\underline{\nabla} f(\underline{x}) \right|$$

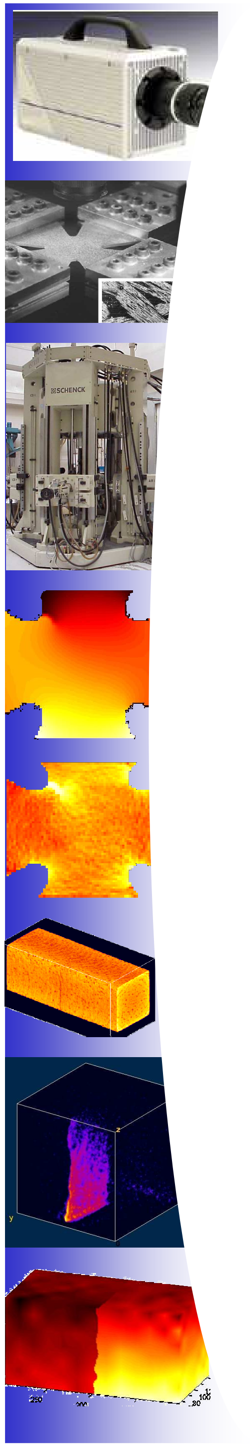




SIF *measurement*

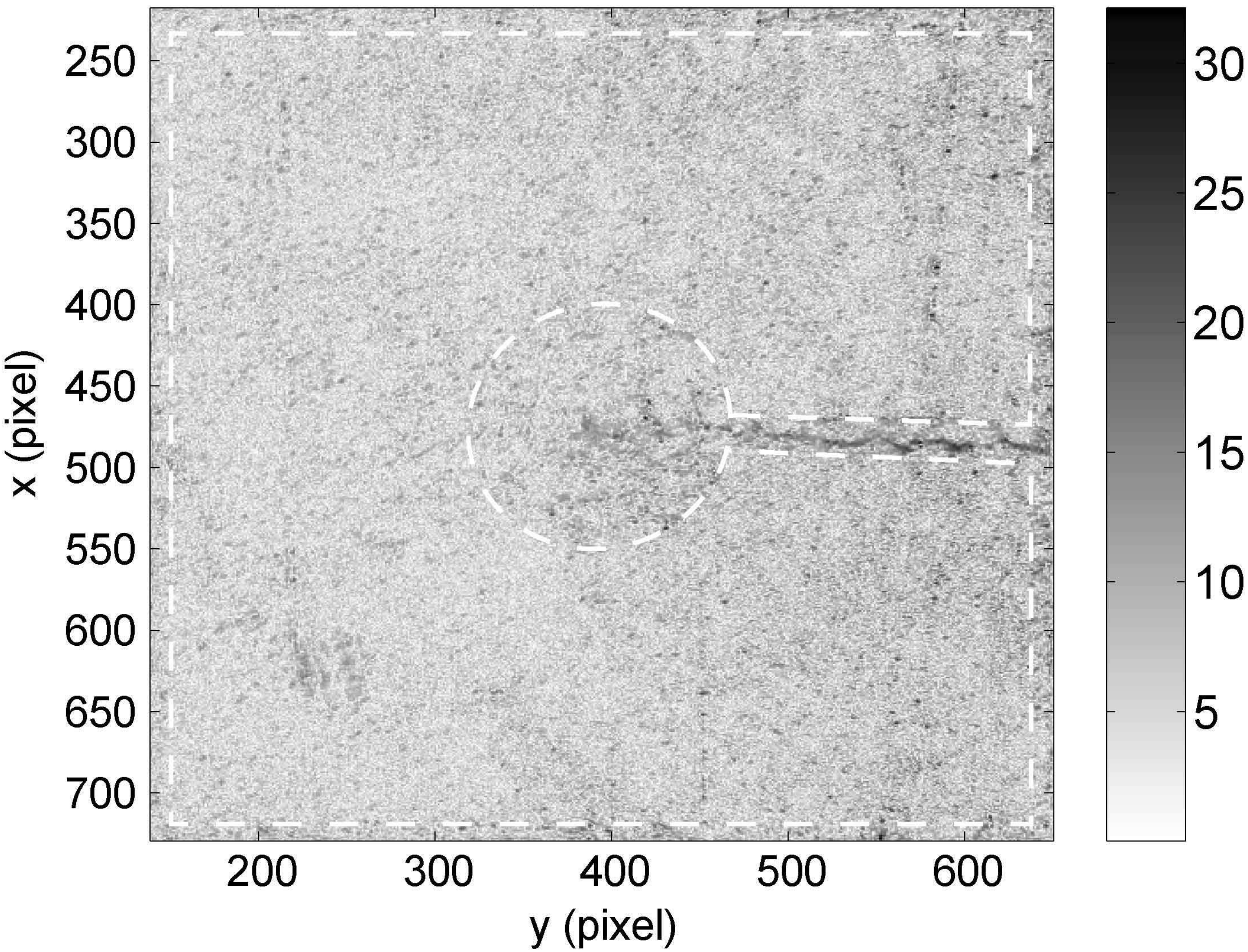


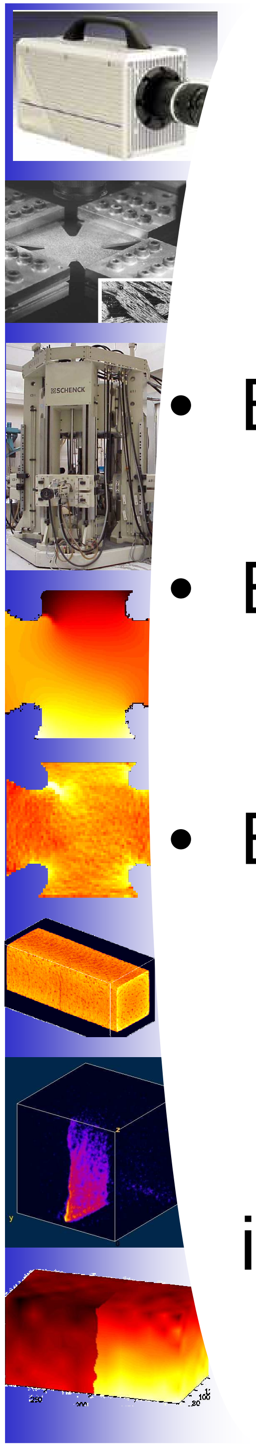
1 pixel ↔ 2.05 µm



SIF *measurement*

$$\varphi_{\mathrm{lin}}(\underline{x}) = \left|g(\underline{x}) - f(\underline{x}) + \underline{u}(\underline{x}).\underline{\nabla} f(\underline{x})\right|$$





Equilibrium gap method

- Behavior: Incremental (linearized) problem

$$\dot{\underline{\underline{\sigma}}} = \mathbf{C}(\underline{x}) : \dot{\underline{\underline{\varepsilon}}}$$

- Equilibrium

$$\mathrm{div}(\underline{\underline{\sigma}}) + \underline{f} = 0$$

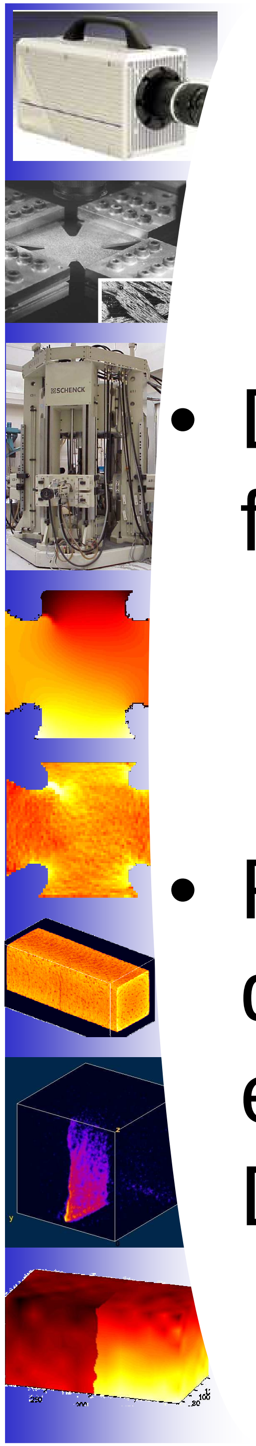
- Basic formulation: find $\mathbf{C}(x)$ such that

$$\Omega = \iint_D \left\{ \mathrm{div}(\mathbf{C}(\underline{x}) : \underline{\nabla} \otimes \underline{u}) + \underline{f} \right\}^2 d\,\underline{x}$$

known

measured

is minimized



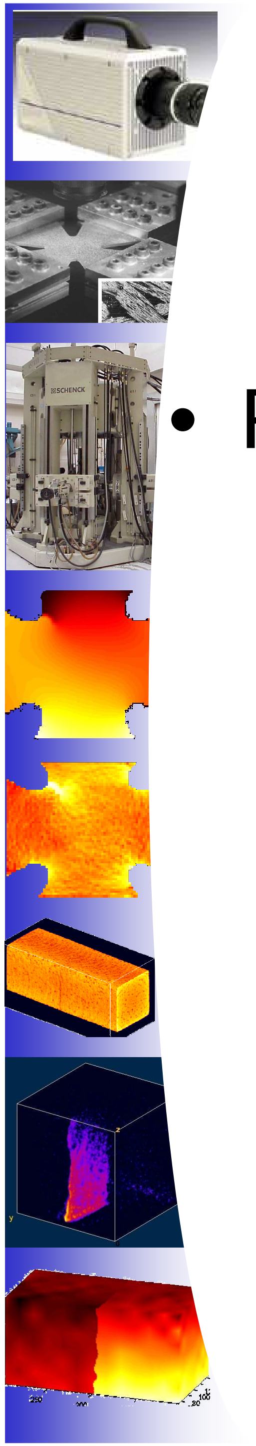
Equilibrium gap method

- Difficulty: sensitivity to short wavelength fluctuations

$$\Omega = \iint_D \left\{ \mathrm{div}\left(\mathbf{C}(\underline{x}) : \underline{\nabla} \otimes \underline{u}\right) + \underline{f} \right\}^2 d\,\underline{x}$$

- Reconditioning: Solve for the displacement $\underline{v}(\underline{x})$ of a homogeneous elastic solid with body force g, and Dirichlet b.c.s

$$v = L_0^{-1}[g]$$



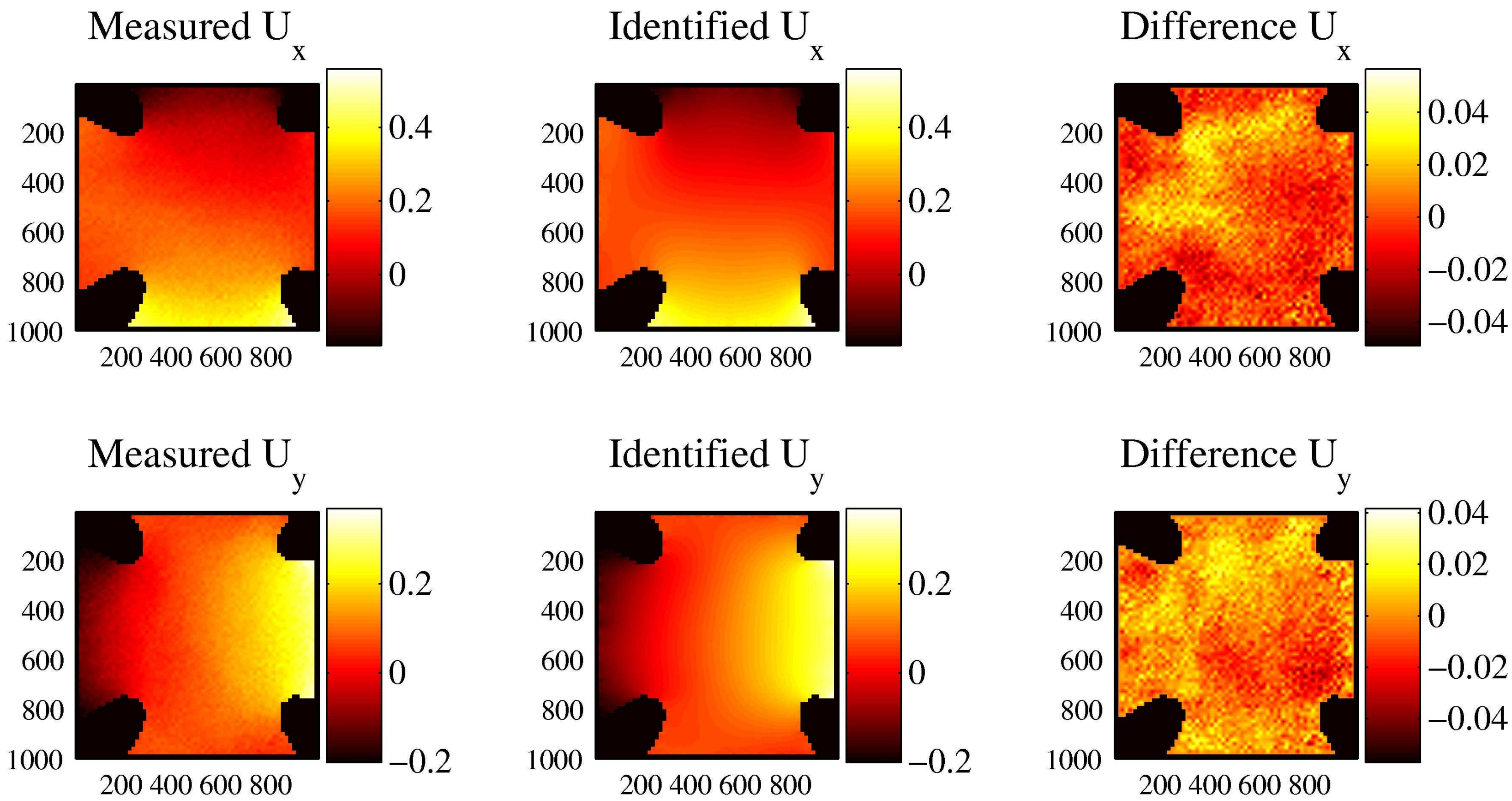
Equilibrium gap method

- Reconditioning:
  - find $\mathbf{C}(\underline{x})$ such that

$$\Omega_{rec} = \iint_D \left\{ \mathrm{L}_0^{-1} \left[\mathrm{div}\left(\mathbf{C}(\underline{x}) : \underline{\nabla} \otimes \underline{u} \right) + \underline{f} \right] \right\}^2 d\,\underline{x}$$

is minimized

Poisson's ratio identification



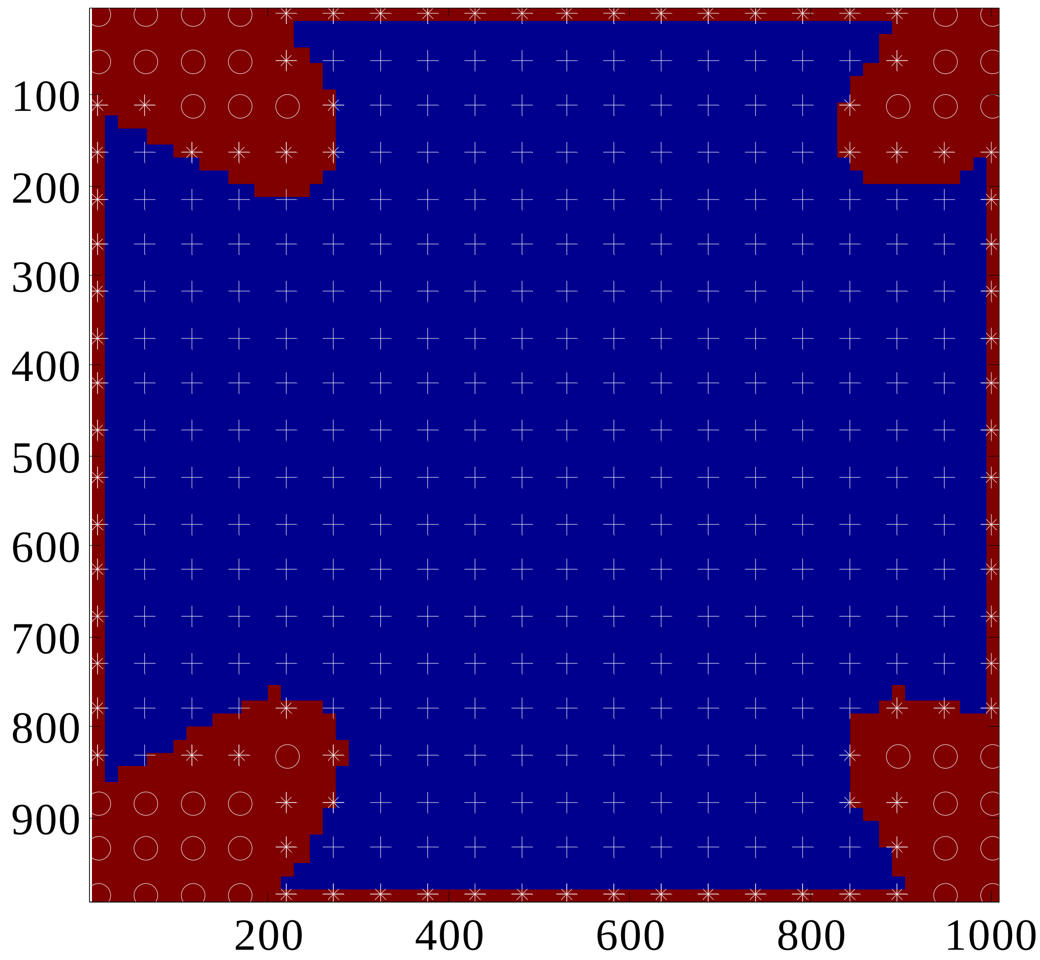
$\nu = 0.31$

Global relative displacement error: 12 %

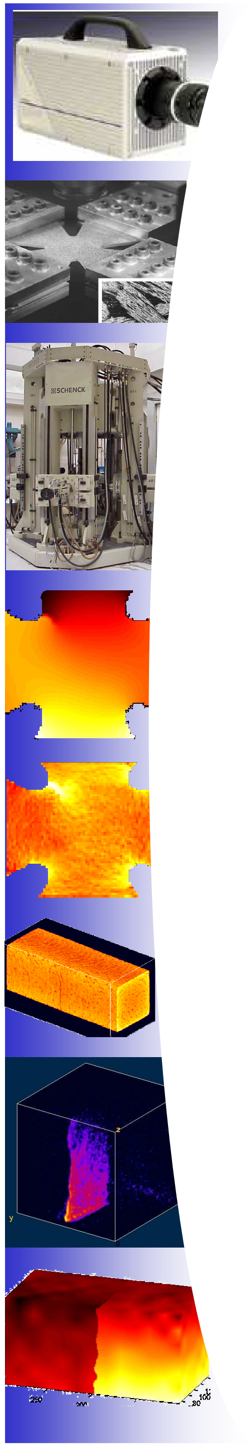
1 pixel ↔ 68 µm

Young's modulus field

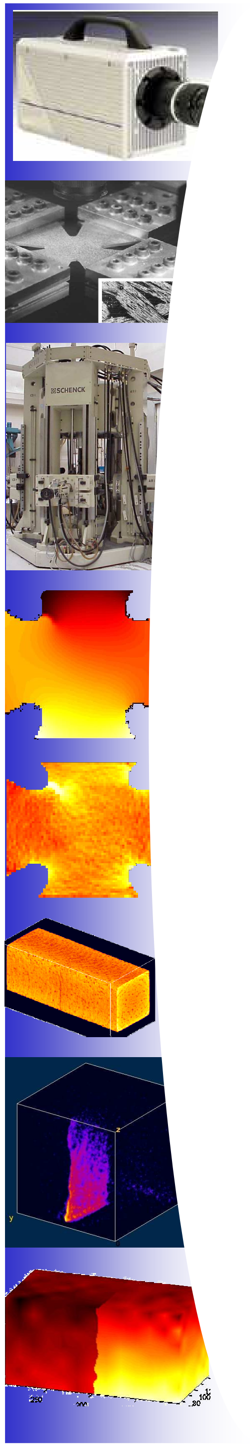
Measurement mesh: 66 x 66 elements
Contrast mesh: 20 x 20 elements

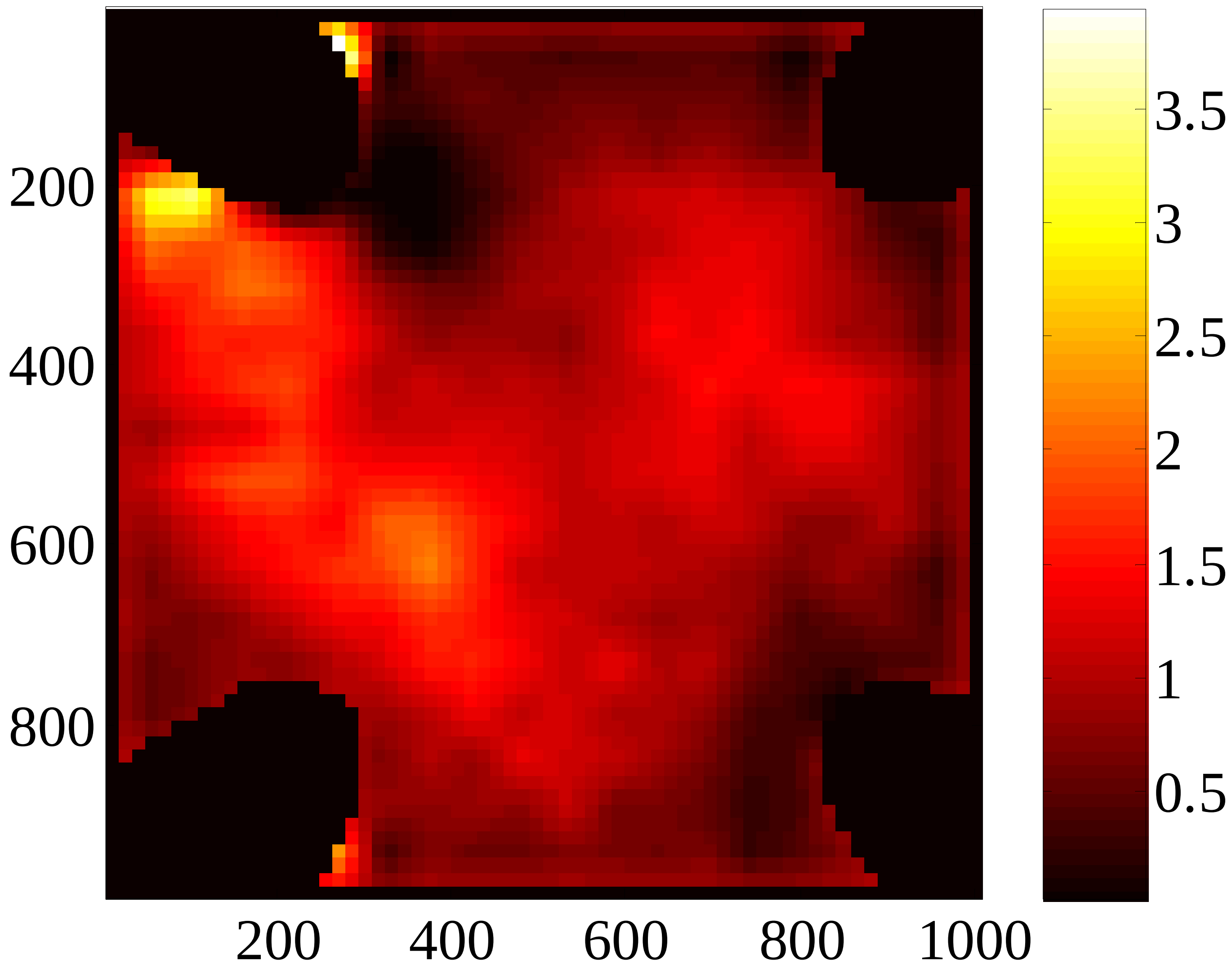


1 pixel ↔ 68 µm

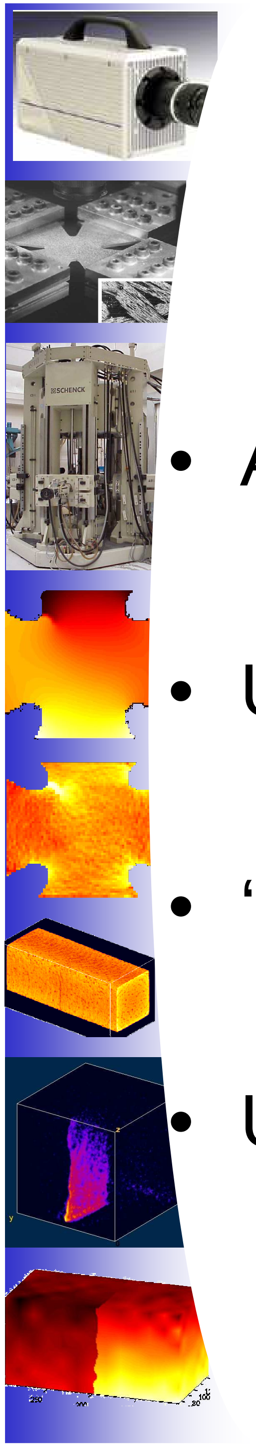


Young's modulus field





1 pixel ↔ 68 µm



Application to simple damage law

- Assume $\mathbf{C}(\underline{x}) = (1 - \mathrm{D}(\varepsilon_{\mathrm{eq}}(u_{meas}(\underline{x})))) \, \mathbf{C}_0$
- Unknown is a scalar function $D(\varepsilon_{eq})$
- ‘Discretize’ $D(\varepsilon_{eq}) = \sum_i c_i \Phi_i(\varepsilon_{eq})$
- Use a poor man Laplace transform

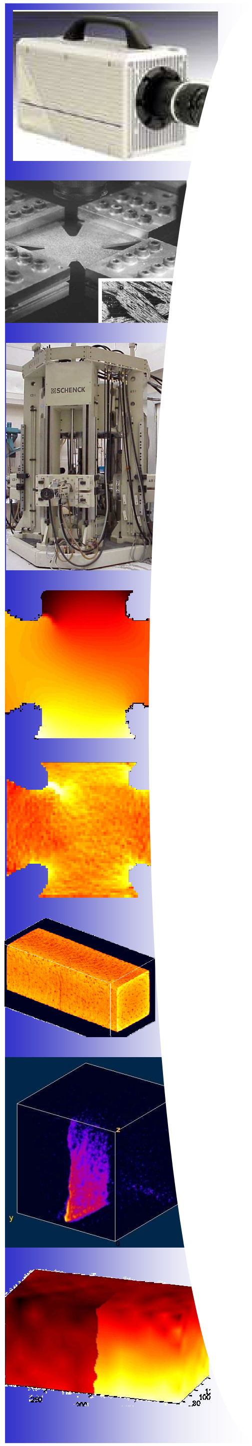
$$\Phi_i(\varepsilon_{eq}) = 1 - \exp(-\varepsilon_{eq} / \varepsilon_i)$$

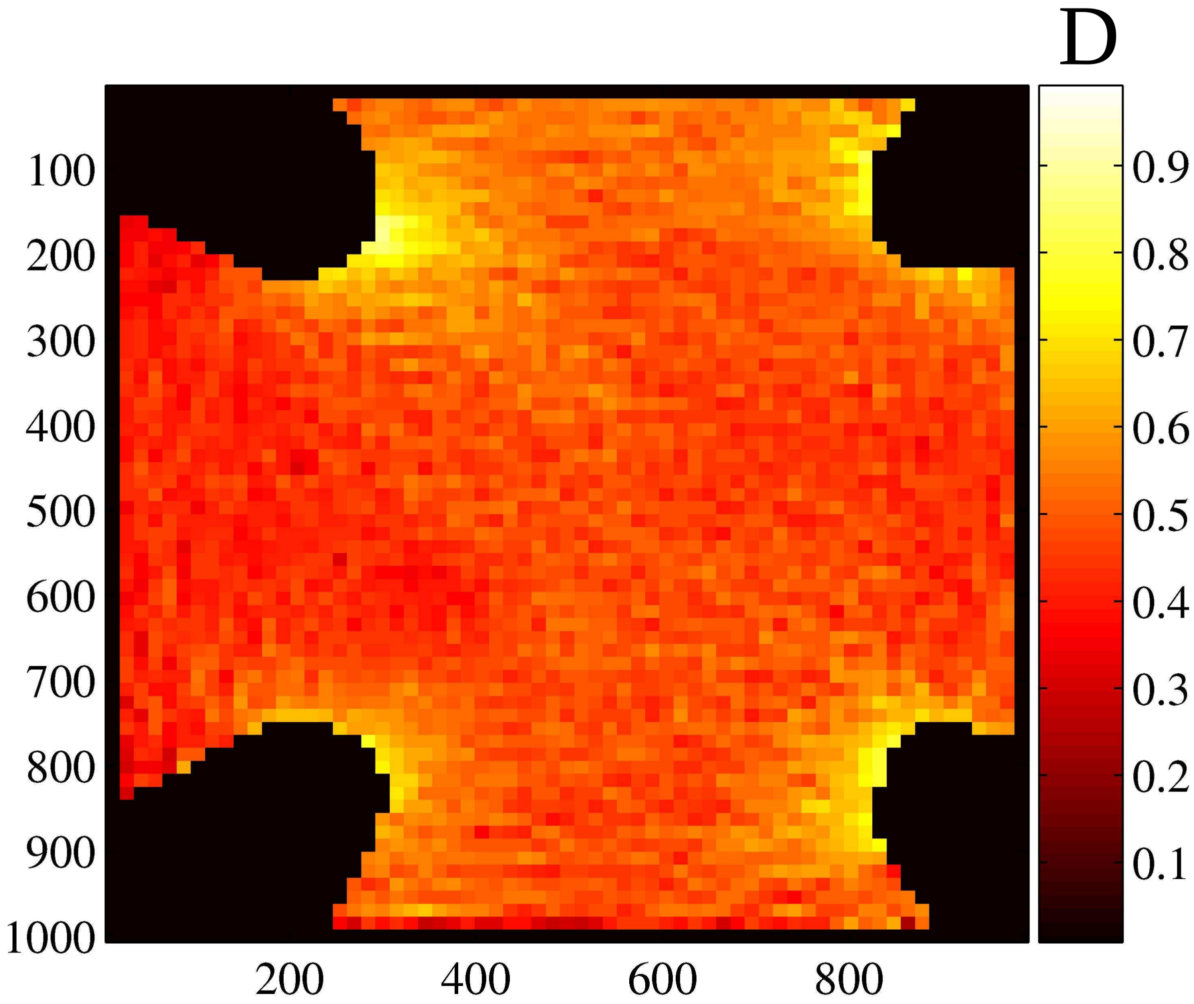
Application to simple damage law

- Functional to minimize is **quadratic** in the unknowns $\{c_i\}$
- Computation cost: as many homogeneous elastic problems as

  (# unknowns)*(# exploited images)

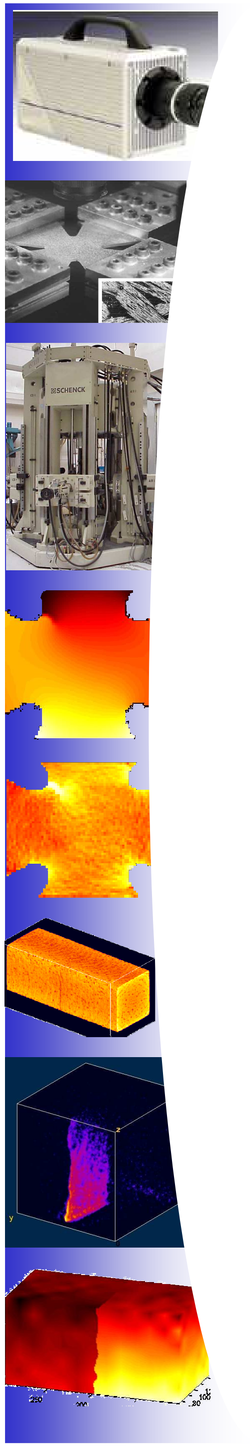
Identification of damage law

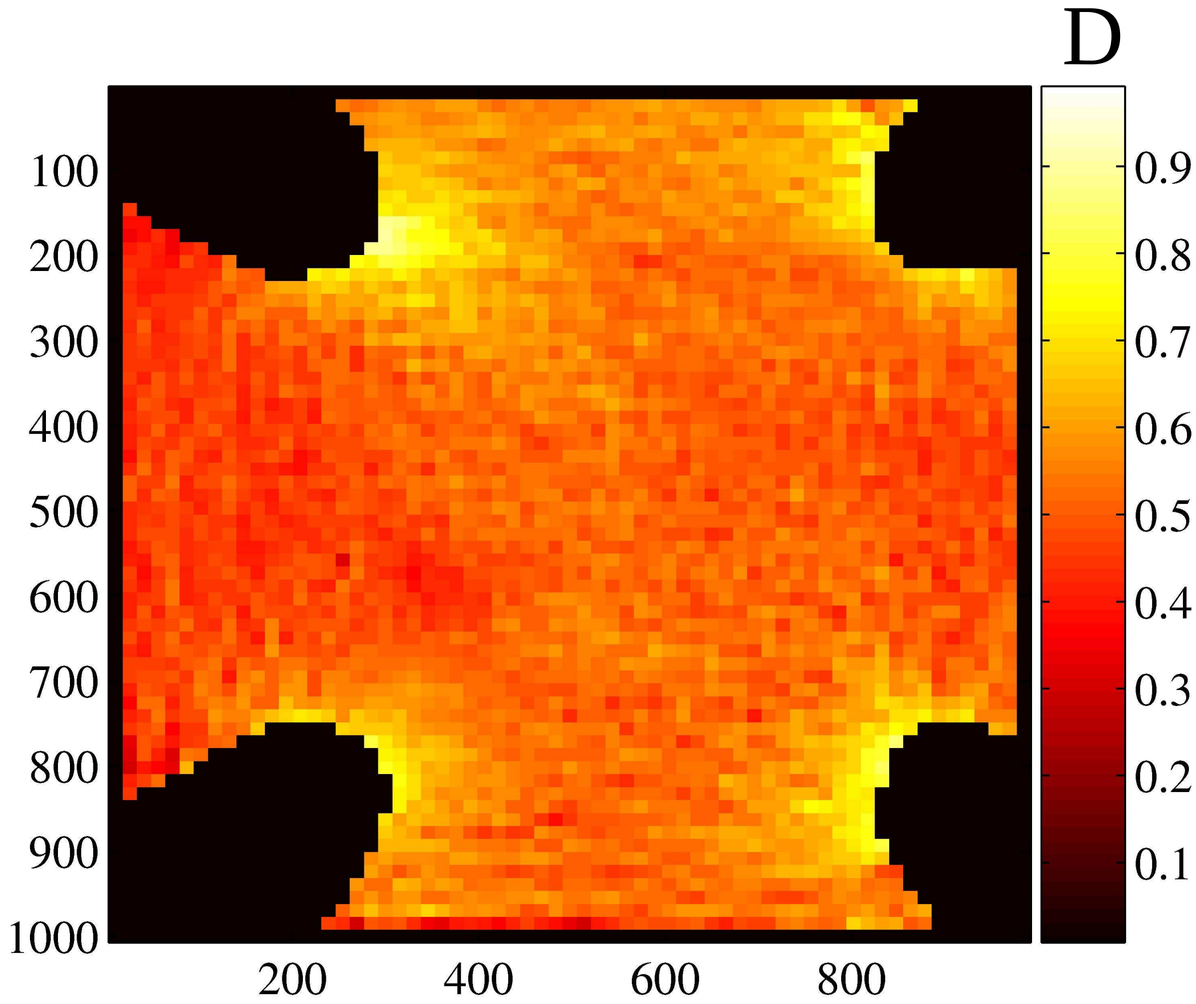




1 pixel ↔ 68 µm

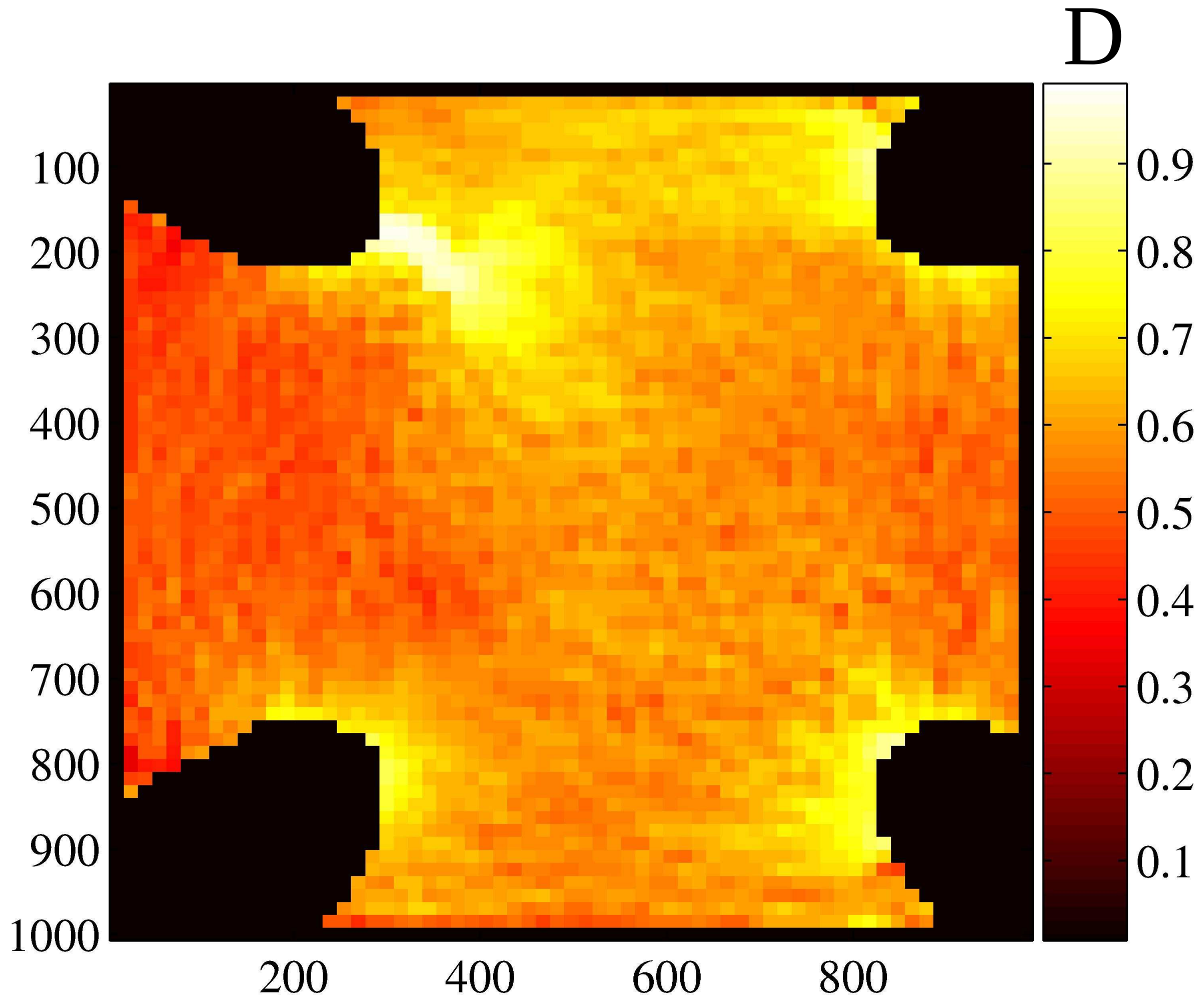
Identification of damage law



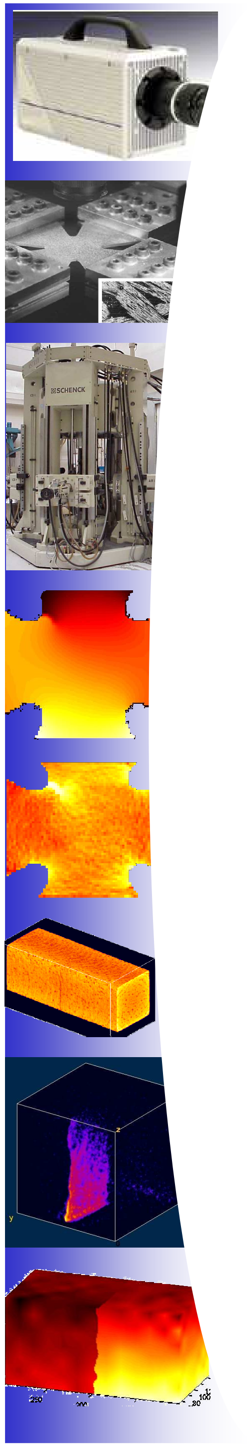


1 pixel ↔ 68 µm

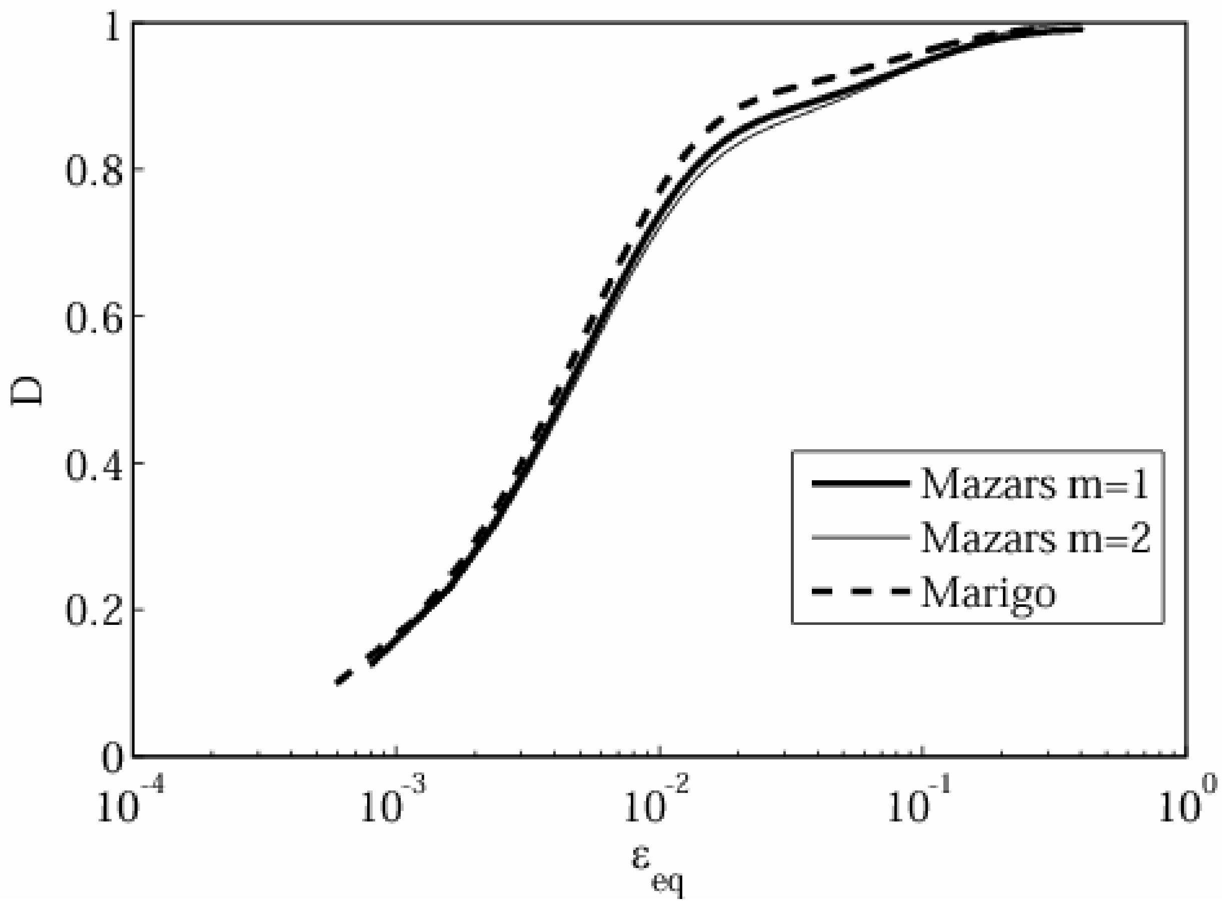
Identification of damage law



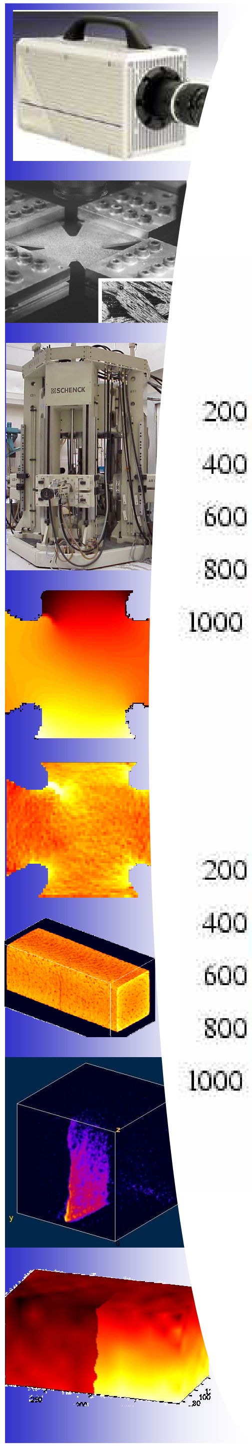
1 pixel ↔ 68 µm



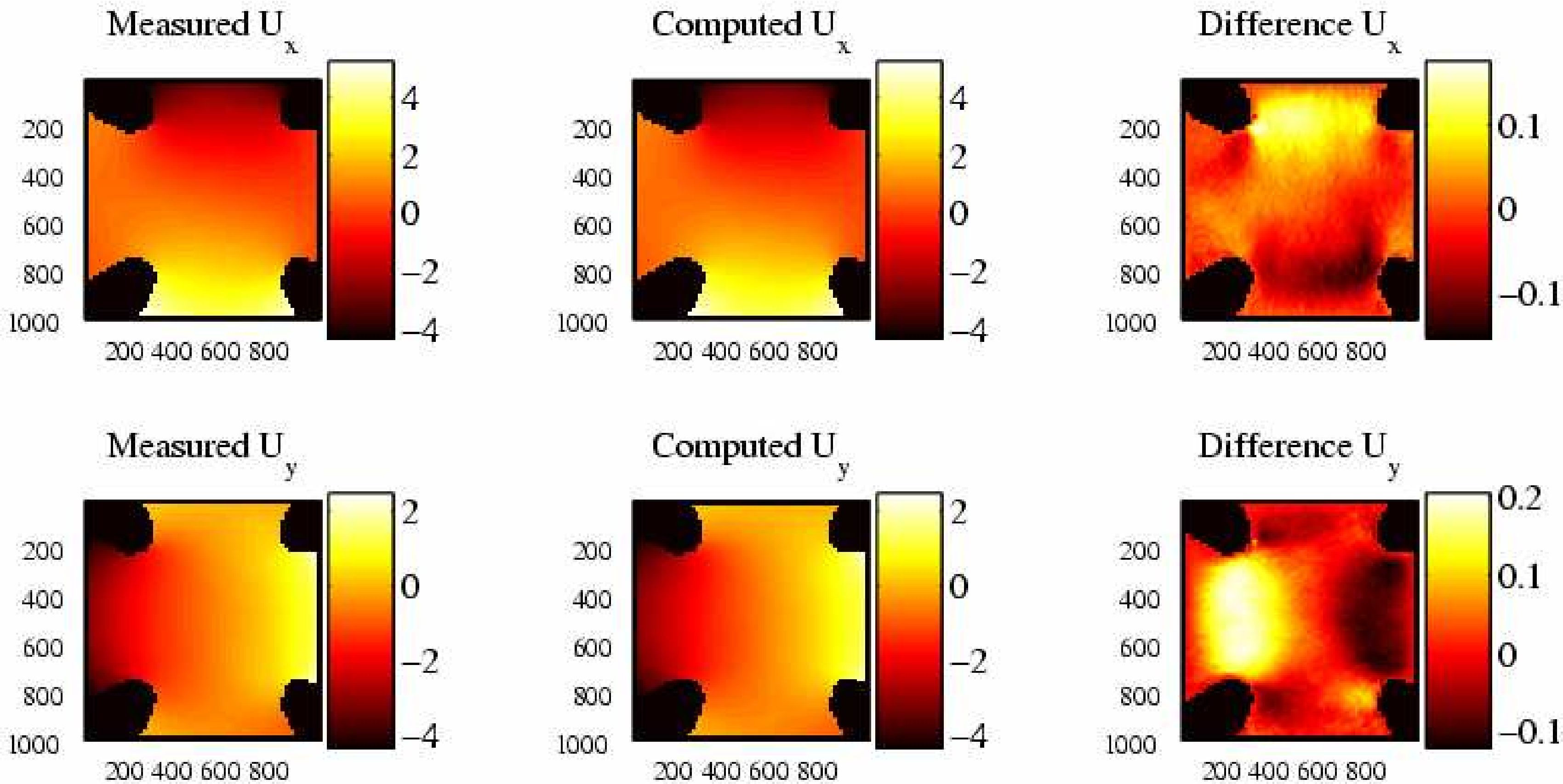
Identification of damage law



1 pixel ↔ 68 µm

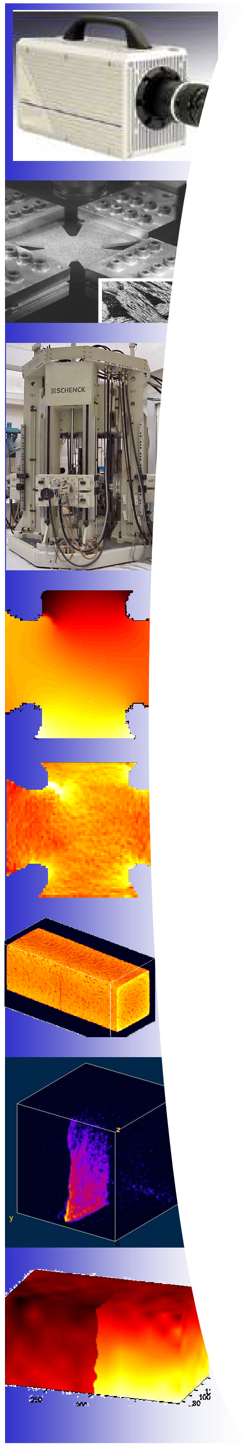


Identification of damage law

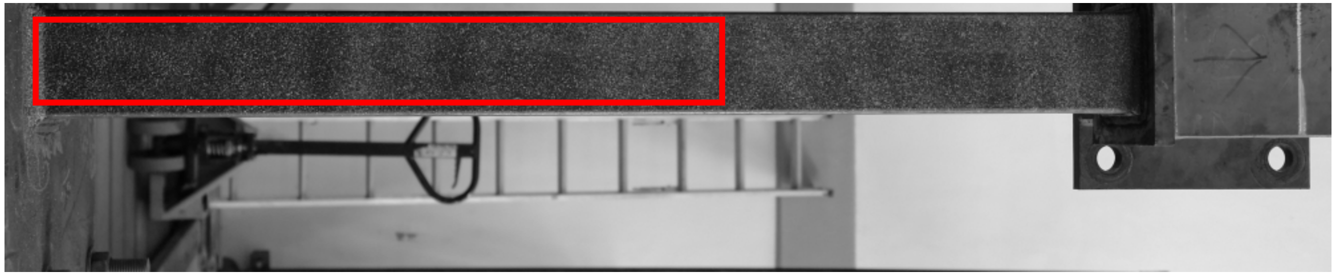


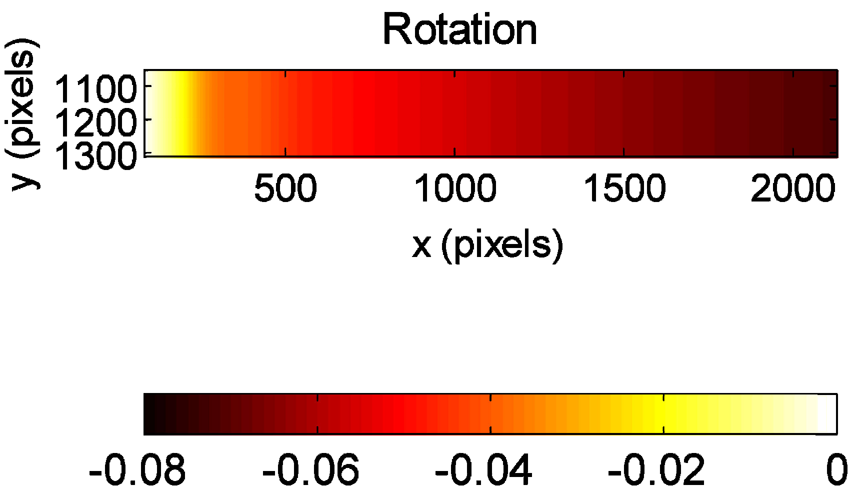
Global relative displacement error: 5 %

1 pixel ↔ 68 µm



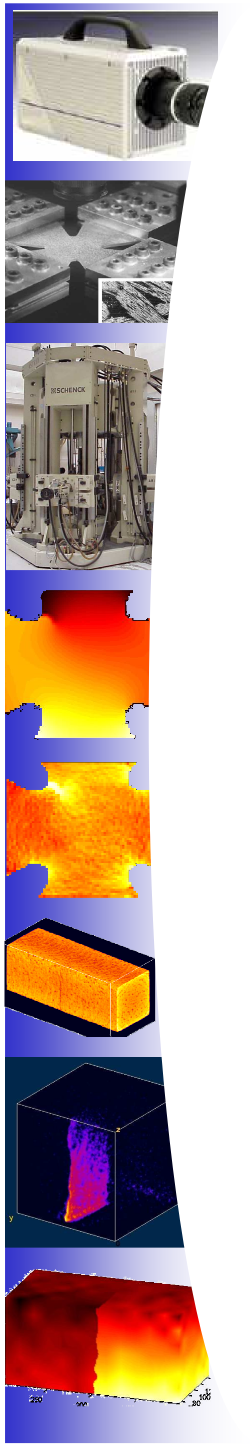
Analysis of local buckling



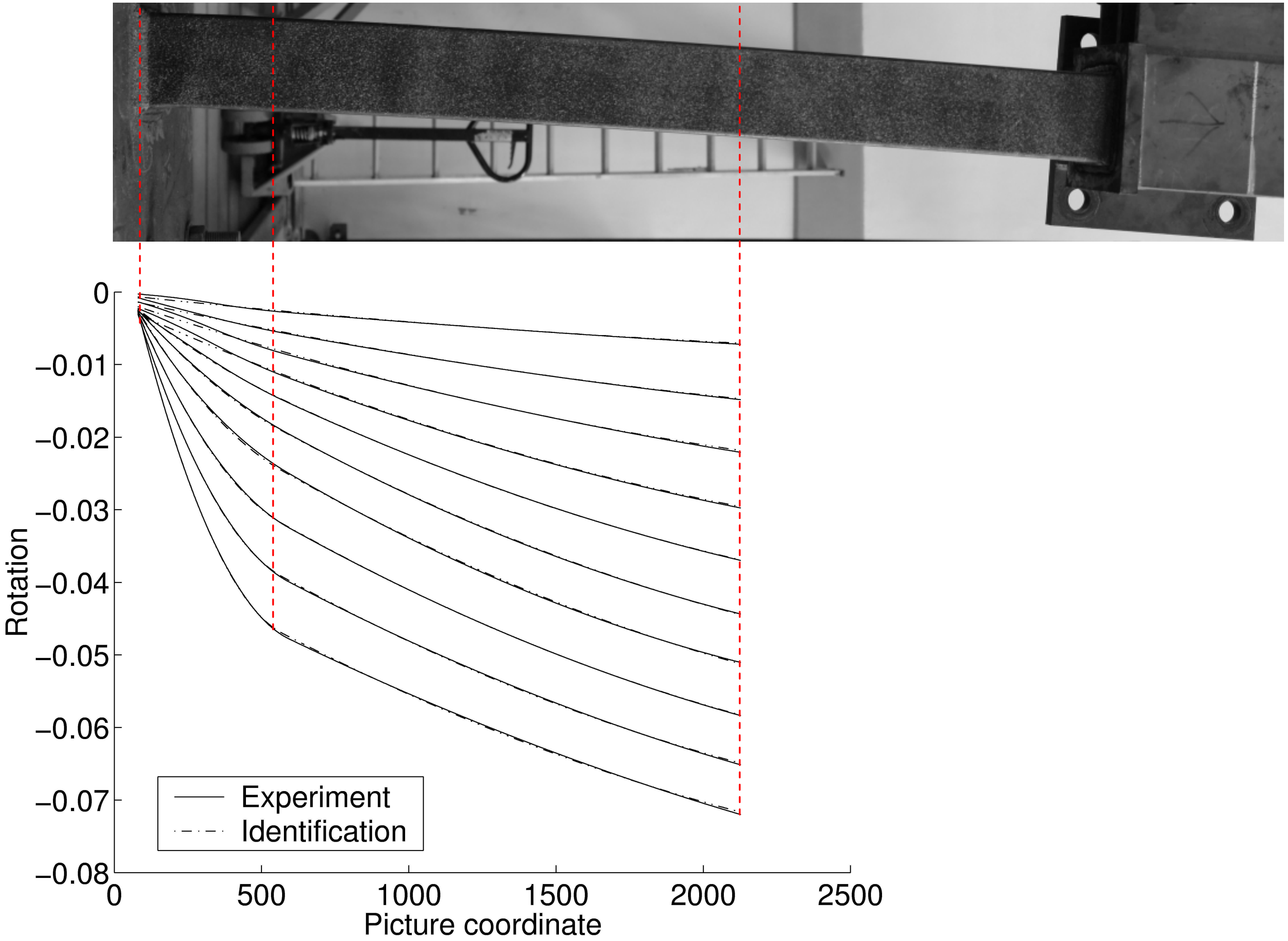


- Beam-DIC
  (ℓ = 2048 pixels, h = 256 pixels)

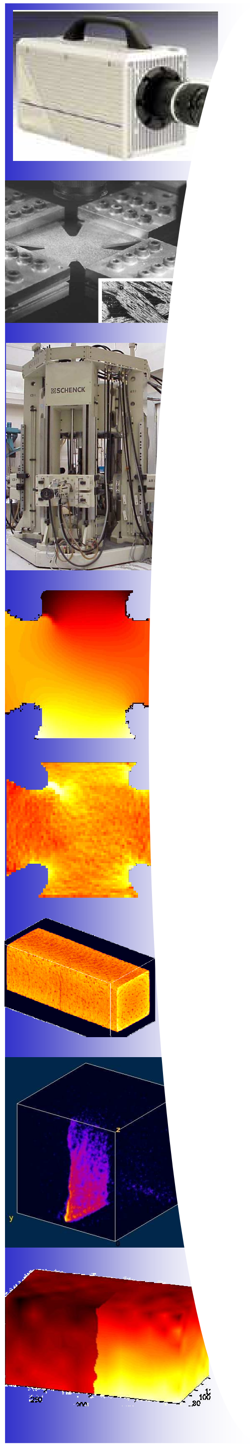
1 pixel ↔ 300 µm

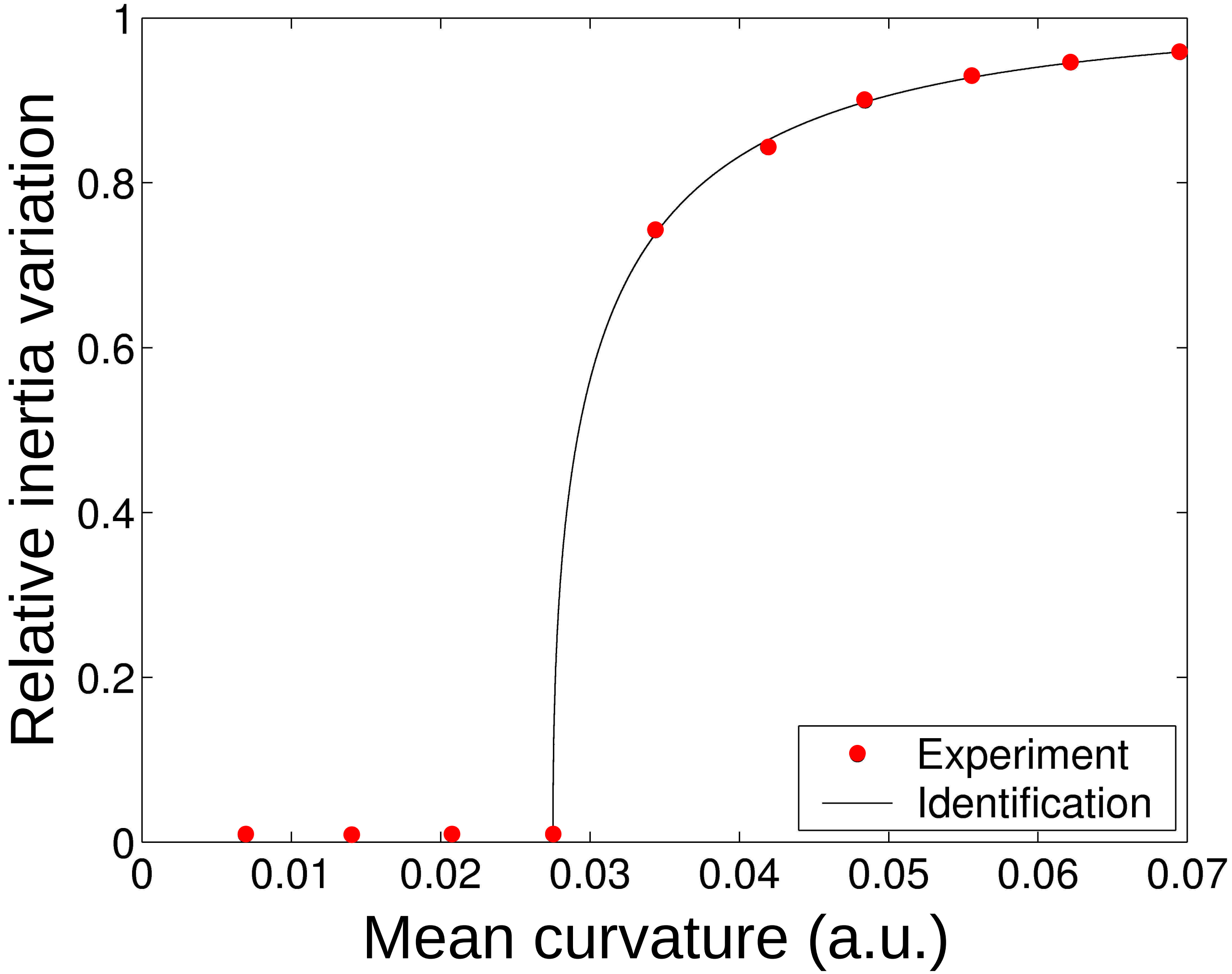


Analysis of local buckling

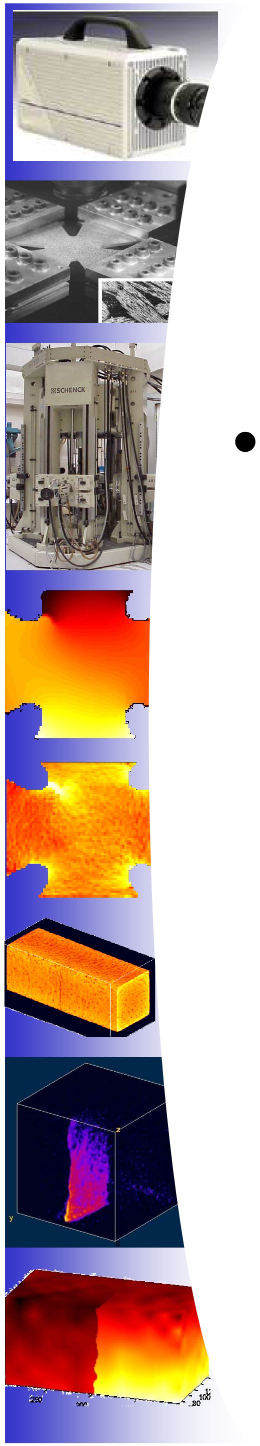


Analysis of local buckling






Future?

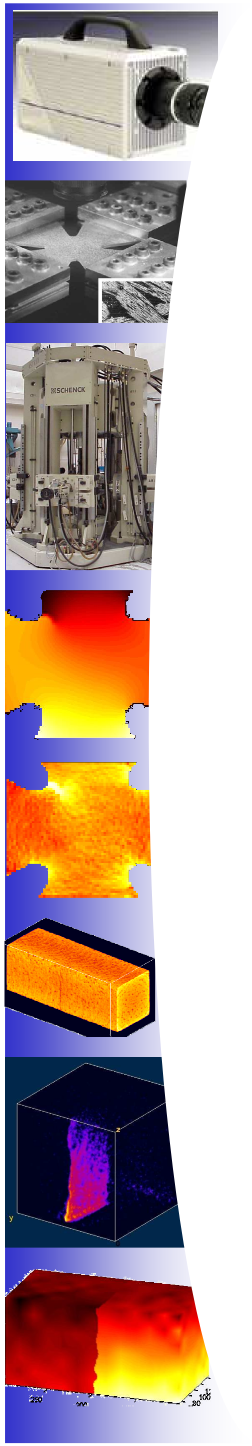


Conclusion

- A continuous pathway from images to constitutive laws is open:

  “Digital Image Mechanical Identification” (DIMI)

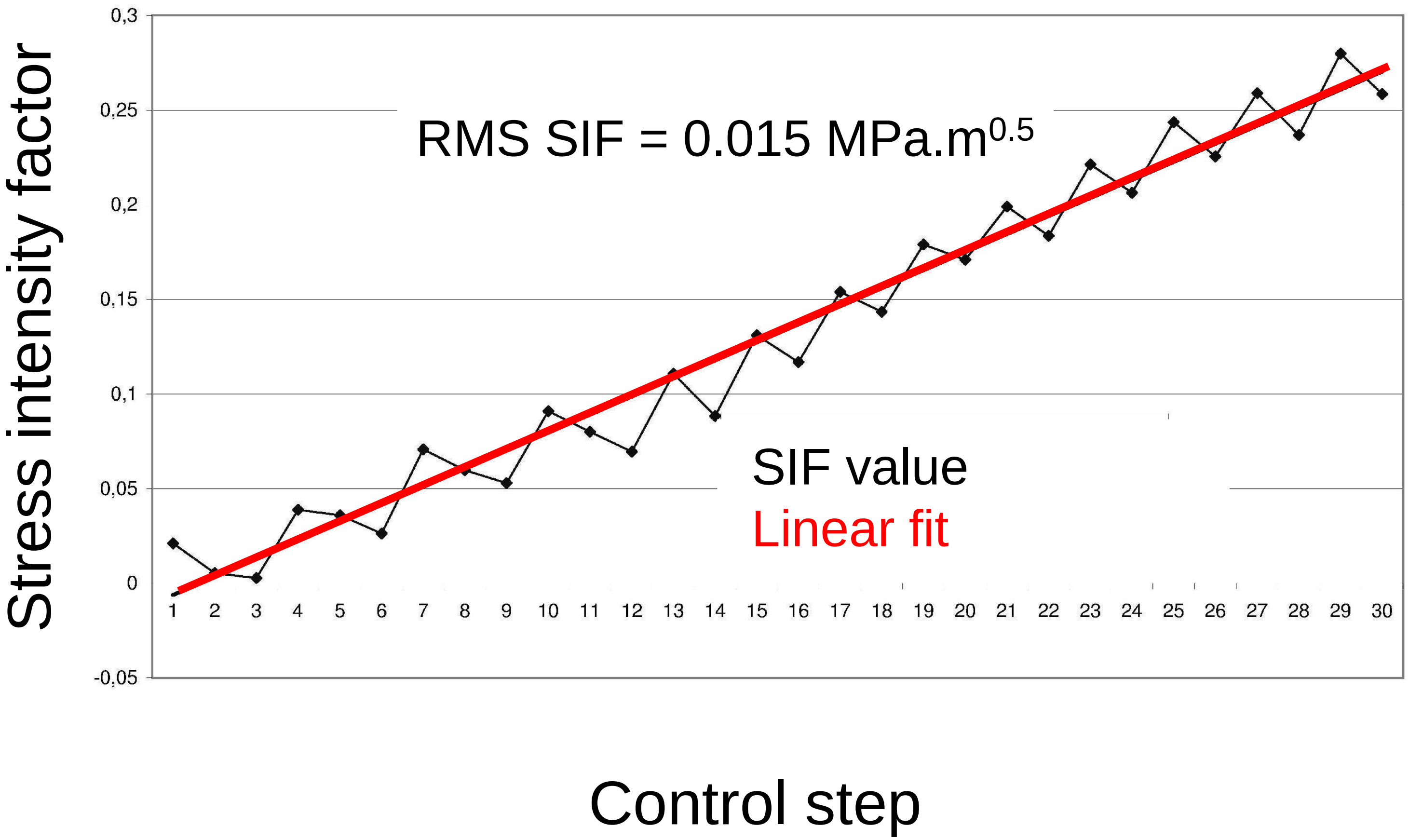
  taking advantage of full field measurements



Perspectives

- Experiment
  - DIC control (e.g. SIF, crack advance monitoring)
  - Hybrid techniques (e.g., pseudo-dynamic tests)
  - New design of “smart” experiments
- DIC
  - Different imaging (e.g., AFM, stereo-correlation, movies)
  - Combination with other measurement techniques
- Identification
  - Other non-linear constitutive laws
  - 2D displacement fields / 3D numerical modelling
- Validation of constitutive laws and numerical tools

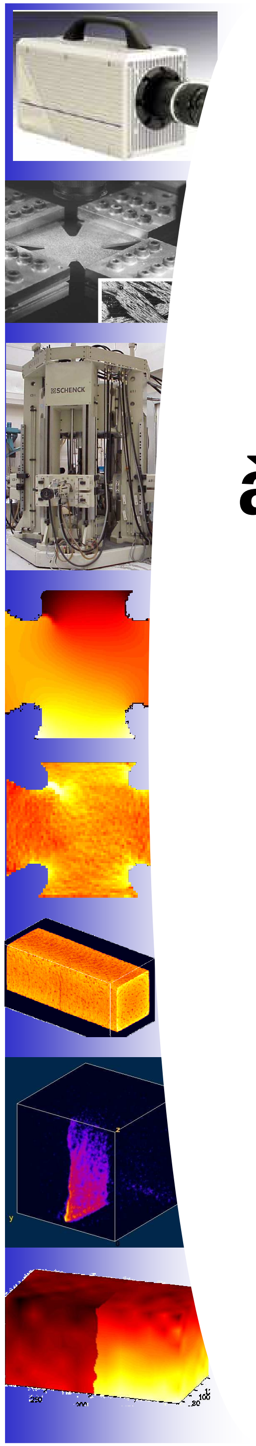
Example: SIF control



[Fayolle, 2007]

‘All too often, experimental work in applied mechanics is thought of only as a check on existing theories or as a convenient substitute for analysis. This is a valid but rather inferior function of experiment. The greater and essential contribution is to guide the development of theory, by providing the fundamental basis for an understanding of the real world.’

D. Drucker (1967)



LMT Cachan

Progrès récents en corrélation d'images : de la mesure de champs à l'identification de propriétés mécaniques

F. Hild, J. Réthoré & S. Roux

FEDERATION FRANCILIENNE DE MECANIQUE

Matériaux•Structures•Procédés