Active Seismic Tomography Inversion with the Self-Adaptive Wavelet Parametrisation: Algorithm and its Application to Vesuvius Volcano

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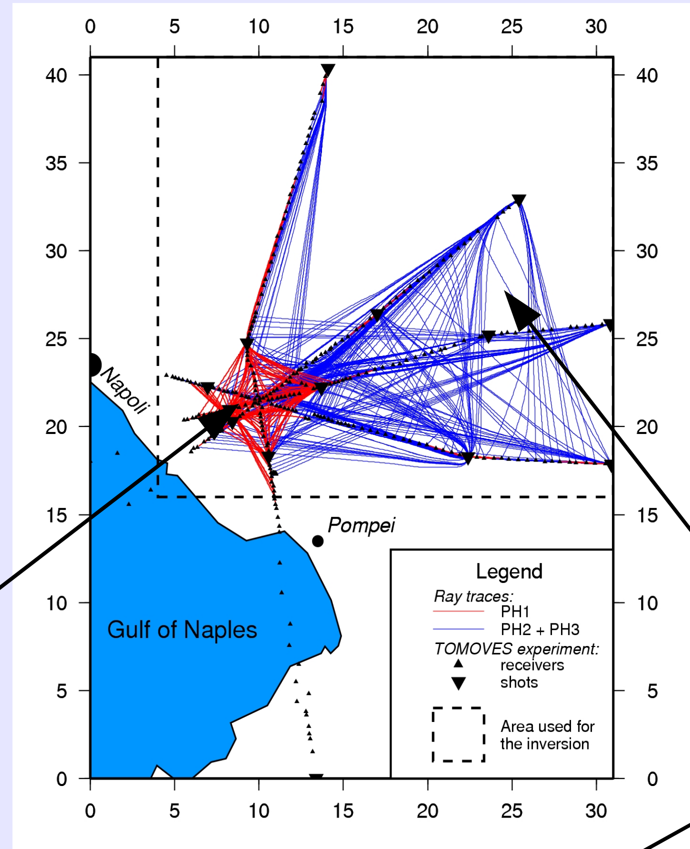
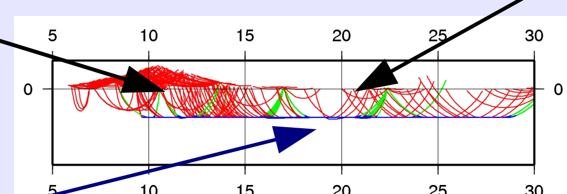
In most active tomography experiments resolution is very irregular because of the uneven ray coverage

TOMOVES experiment ray coverage



High resolution possible

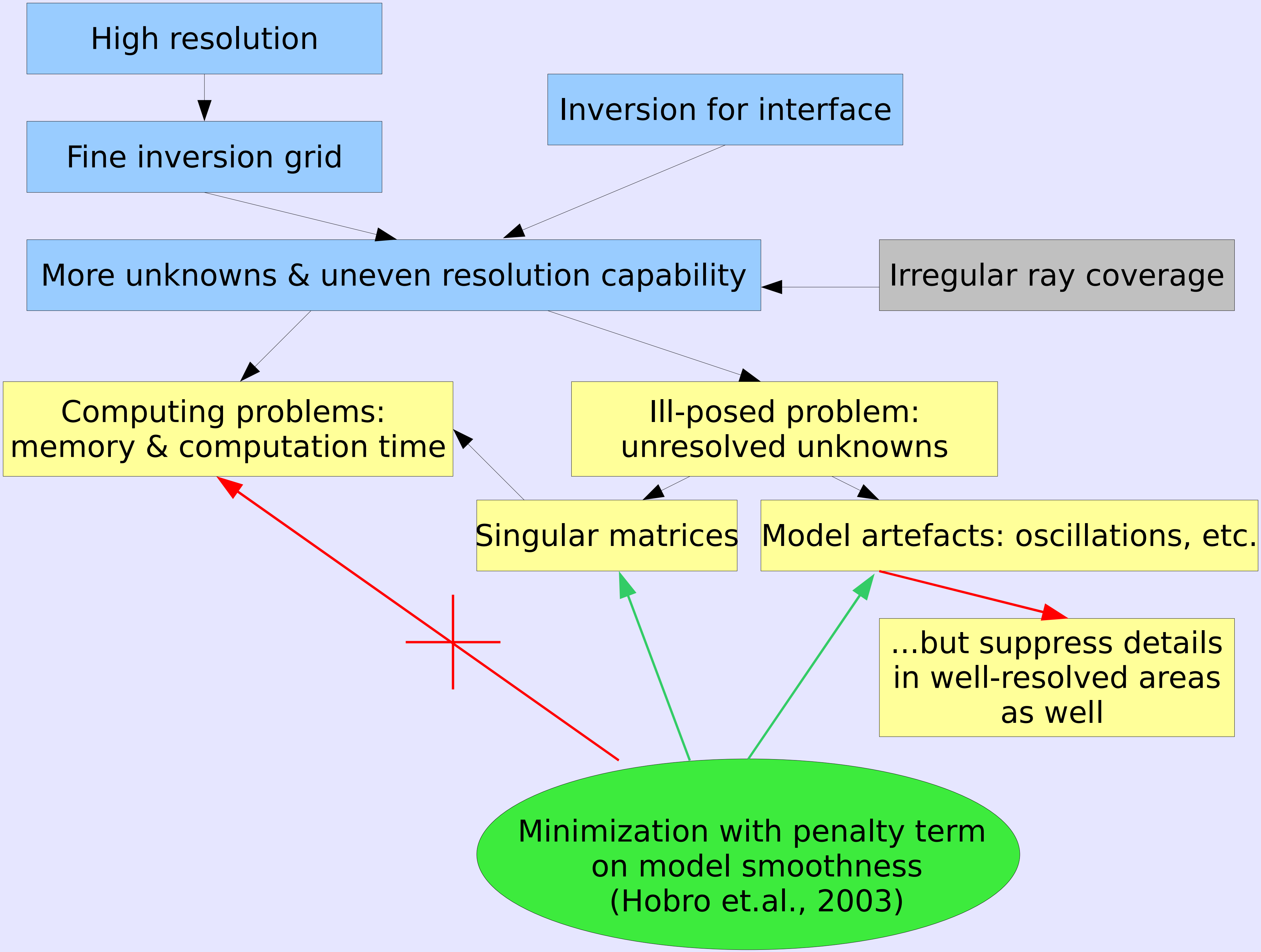
Only rough pattern can be reconstructed



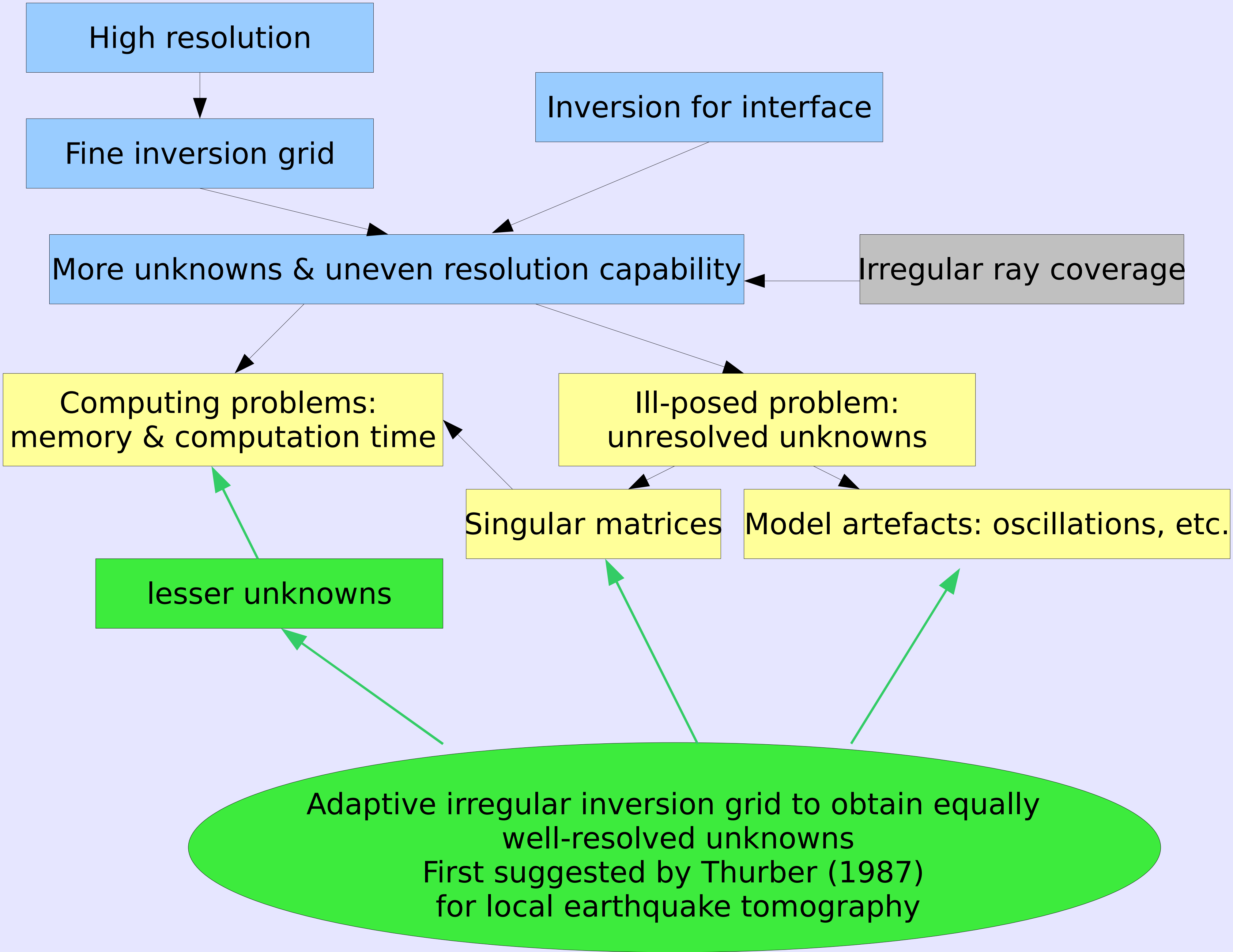
Limestone basement with the velocity increase from 4.0 to 5.5 km/s
and an unknown topography is present

Two major points to be addressed in the current study:

- ✔ Adaptive irregular parametrization of the 3D seismic model to achieve the reasonably resolved solution and avoid inversion artefacts.
- ✔ Explicit introduction of the refracting/reflecting interfaces as the first-order velocity discontinuities.



High resolution
Fine inversion grid
Inversion for interface
More unknowns & uneven resolution capability
Irregular ray coverage
Computing problems:
memory & computation time
Ill-posed problem:
unresolved unknowns
Singular matrices
Model artefacts: oscillations, etc.
...but suppress details
in well-resolved areas
as well
Minimization with penalty term
on model smoothness
(Hobro et.al., 2003)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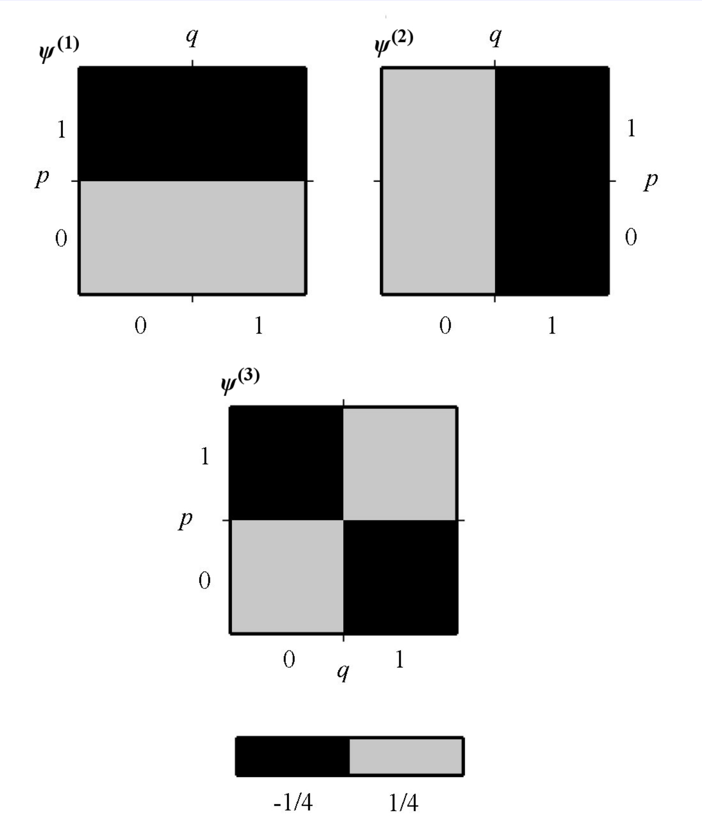
Model artefacts: oscillations, etc.
lesser unknowns
Adaptive irregular inversion grid to obtain equally
well-resolved unknowns
First suggested by Thurber (1987)
for local earthquake tomography

Major problem: How to construct this adaptive irregular inversion grid in 3D?

Suggested approach: wavelet series expansion



Slowness field expansion (as well as the expansion of the interface depth):

$$ds(i,j) = c_0 + \sum_{l=1}^{J} \sum_{m=0}^{2^{J-1}-1} \sum_{n=0}^{2^{J-1}-1} \sum_{k=1}^{3} c_{l,m,n}^{k} \cdot \psi_{l,m,n}^{k}(i,j) \qquad (1)$$

where particular Haar wavelets:

$$\psi_{l,m,n}^{k}(i,j) = 2^{-l}\tilde{\psi}^{(k)}(\lfloor (i-1)/2^{l-1} \rfloor - 2m, \lfloor (j-1)/2^{l-1} \rfloor - 2n), \qquad (2)$$
$$i,j = 1,..,N = 2^J$$

($\lfloor x \rfloor$ means truncated integer part of x) are obtained by scaling (as defined by index l) and shift (indices m,n) of the "mother"wavelets:

$$\tilde{\psi}^{(1)}(p,q) = \begin{cases} 1/4, & q=1, p=0,1 \\ -1/4, & q=0, p=0,1 \\ 0, & otherwise \end{cases} \qquad (3)$$

$$\tilde{\psi}^{(2)}(p,q) = \begin{cases} 1/4, & p=1, q=0,1 \\ -1/4, & p=0, q=0,1 \\ 0, & otherwise \end{cases} \qquad (4)$$

$$\tilde{\psi}^{(3)}(p,q) = \begin{cases} 1/4, & q=0, p=0 \text{ or } q=1, p=1 \\ -1/4, & q=0, p=1 \text{ or } q=1, p=0 \\ 0, & otherwise \end{cases} \qquad (5)$$

Series (1) can be made sparse by rejecting those functions that are not “well defined” by data

How to select “well defined” wavelet coefficients?

• Look for the equally well-resolved unknowns as determined by the resolution matrix elements – most accurate, but time consuming

We suggest to select wavelets on the basis of:

1. Hit counts, i.e. number of rays that cross the particular wavelet support area
2. Angular coverage

Let $\vec{d_j}$ be the vector with its origin at the point where a jth ray enters the wavelet support area and with its endpoint where the ray leaves the support area. We define the direction of the $\vec{d_j}$ as a jth ray direction inside the wavelet support. Let a_j, b_j, c_j be the direction cosines of the $\vec{d_j}$. Then the confidence angle is defined as:

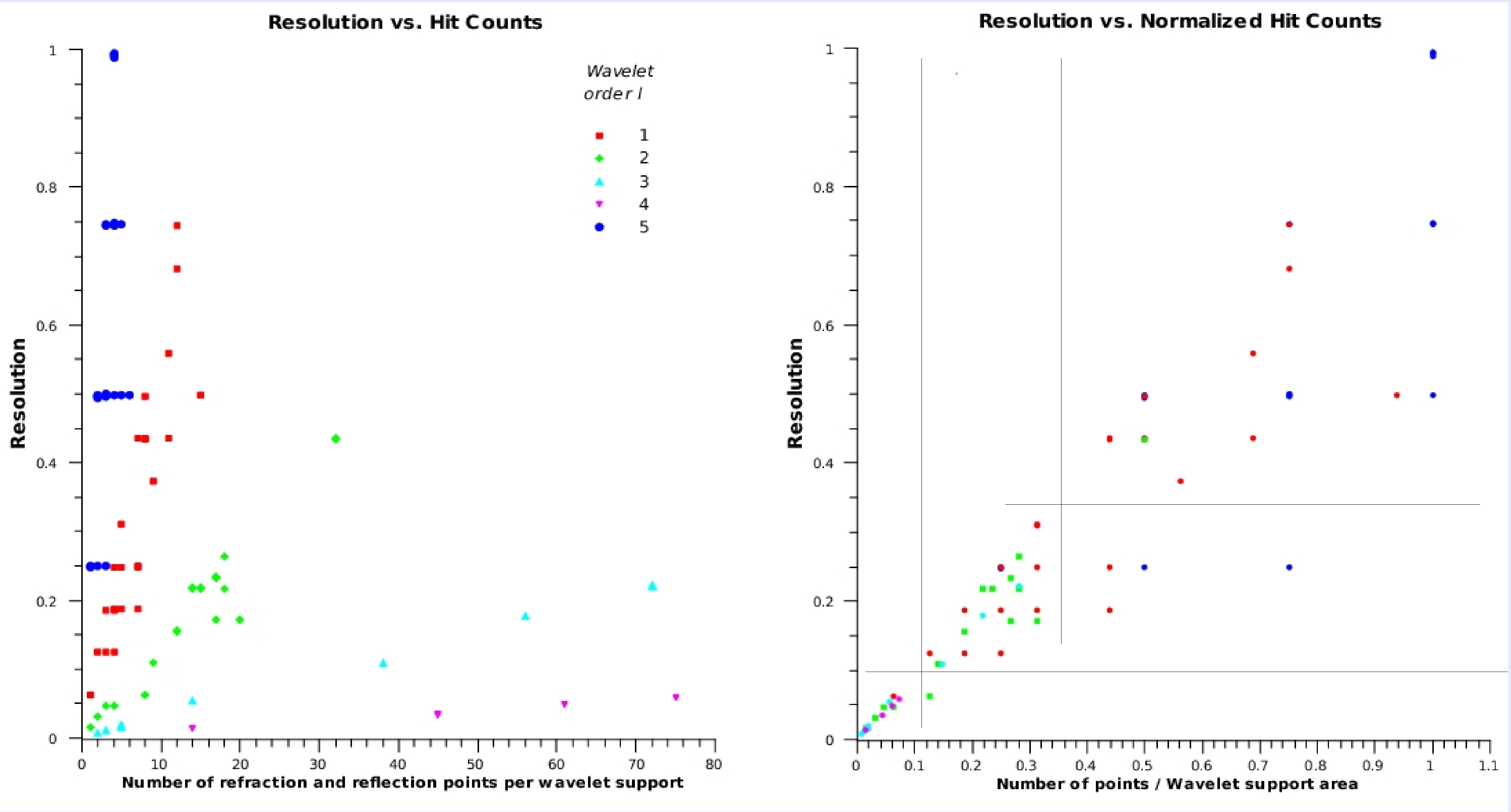
$$\theta_r = \arccos\left(\frac{R}{n_r}\right), \tag{1}$$

where

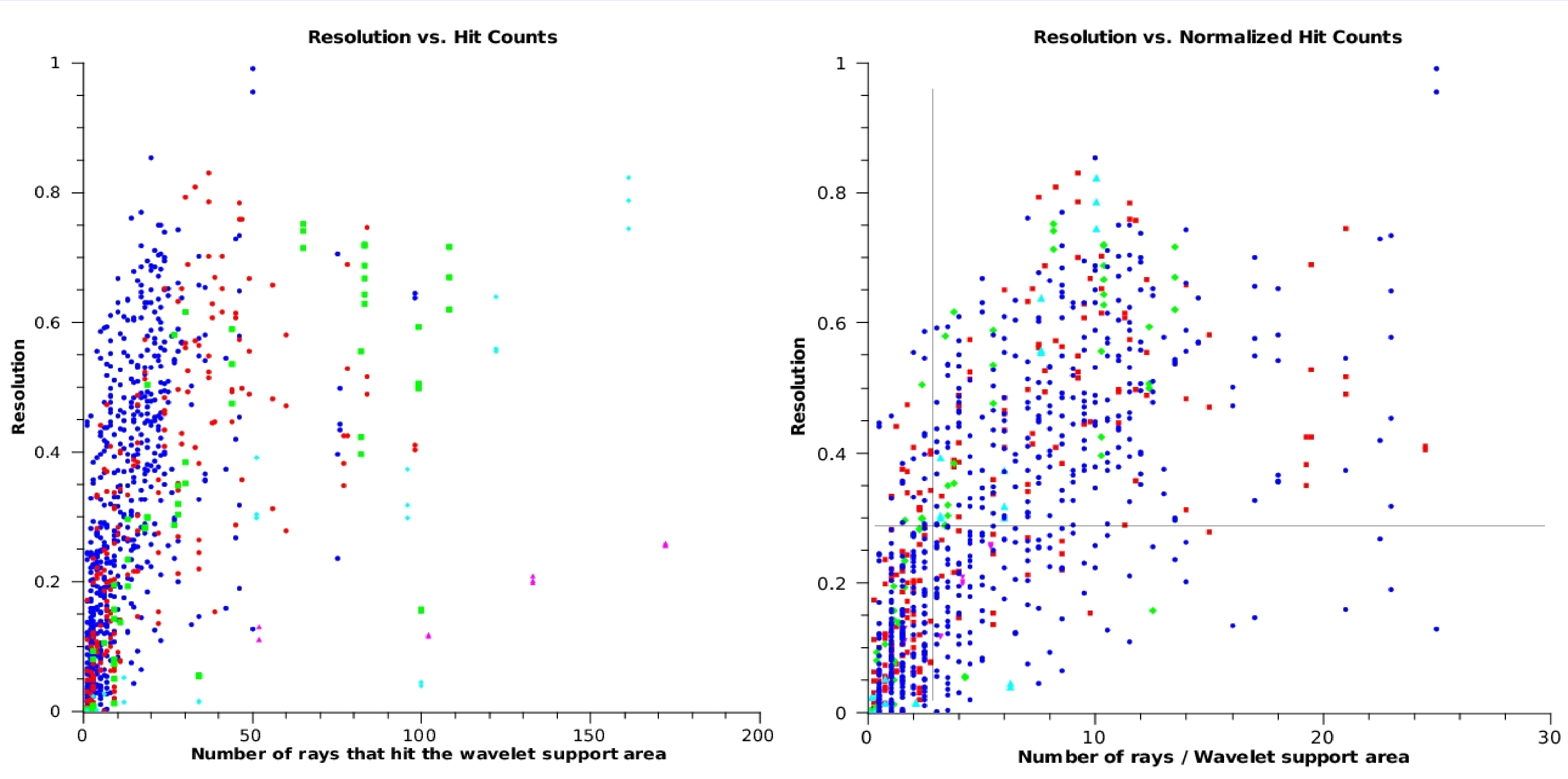
$$R = \sqrt{\left(\sum_1^{n_r} a_j\right)^2 + \left(\sum_1^{n_r} b_j\right)^2 + \left(\sum_1^{n_r} c_j\right)^2}. \tag{2}$$

The θ_r is an equivalent of the root-mean-square error for the scalar random quantities and represents the solid angle of the probable deviation of the vectors from their mean direction.

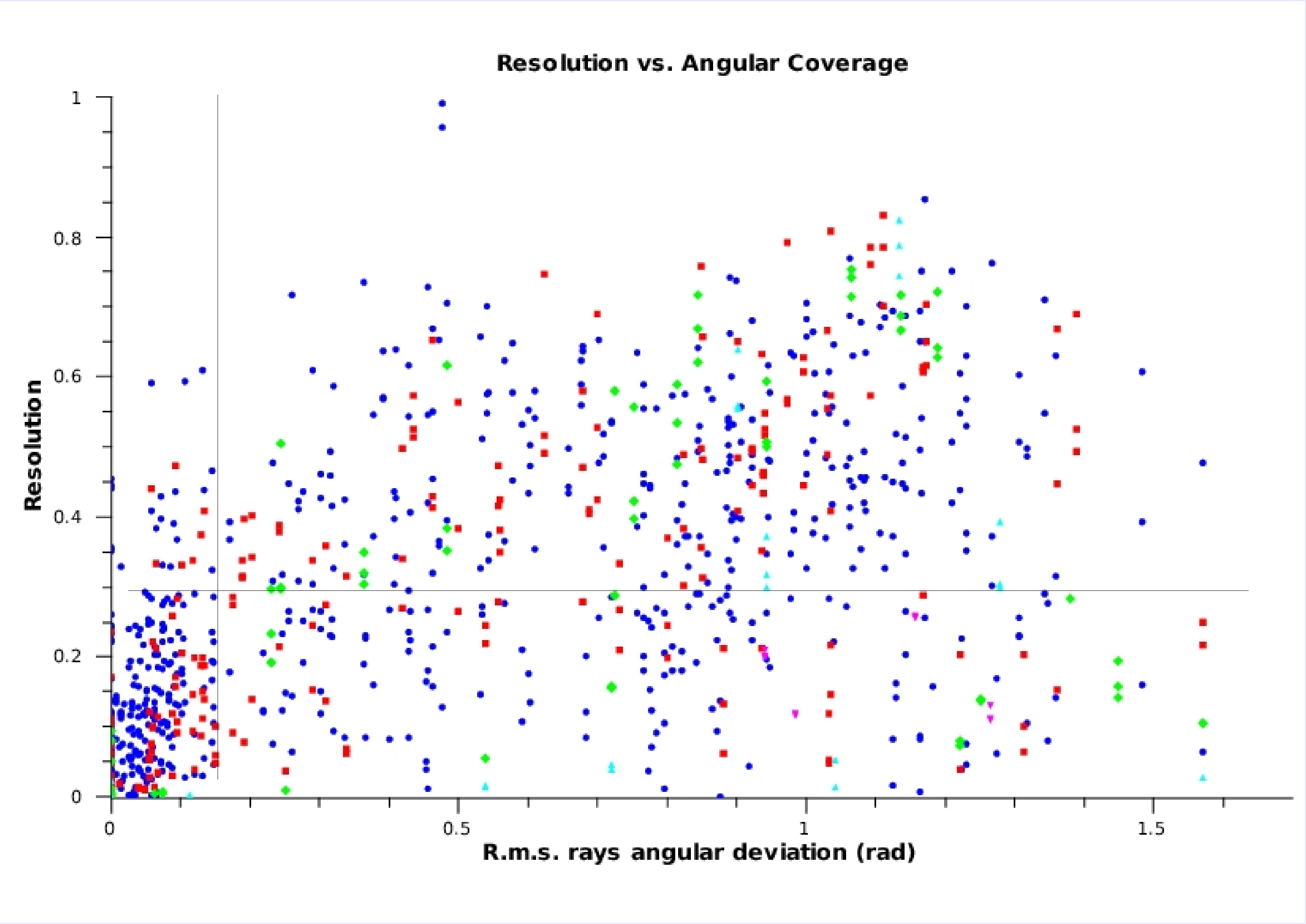
Resolution of the Interface Wavelet Series Coefficients
w.r.t. ***Number of Refraction/Reflection Points*** that Hit the **supp** $\psi_{l,m,n}$



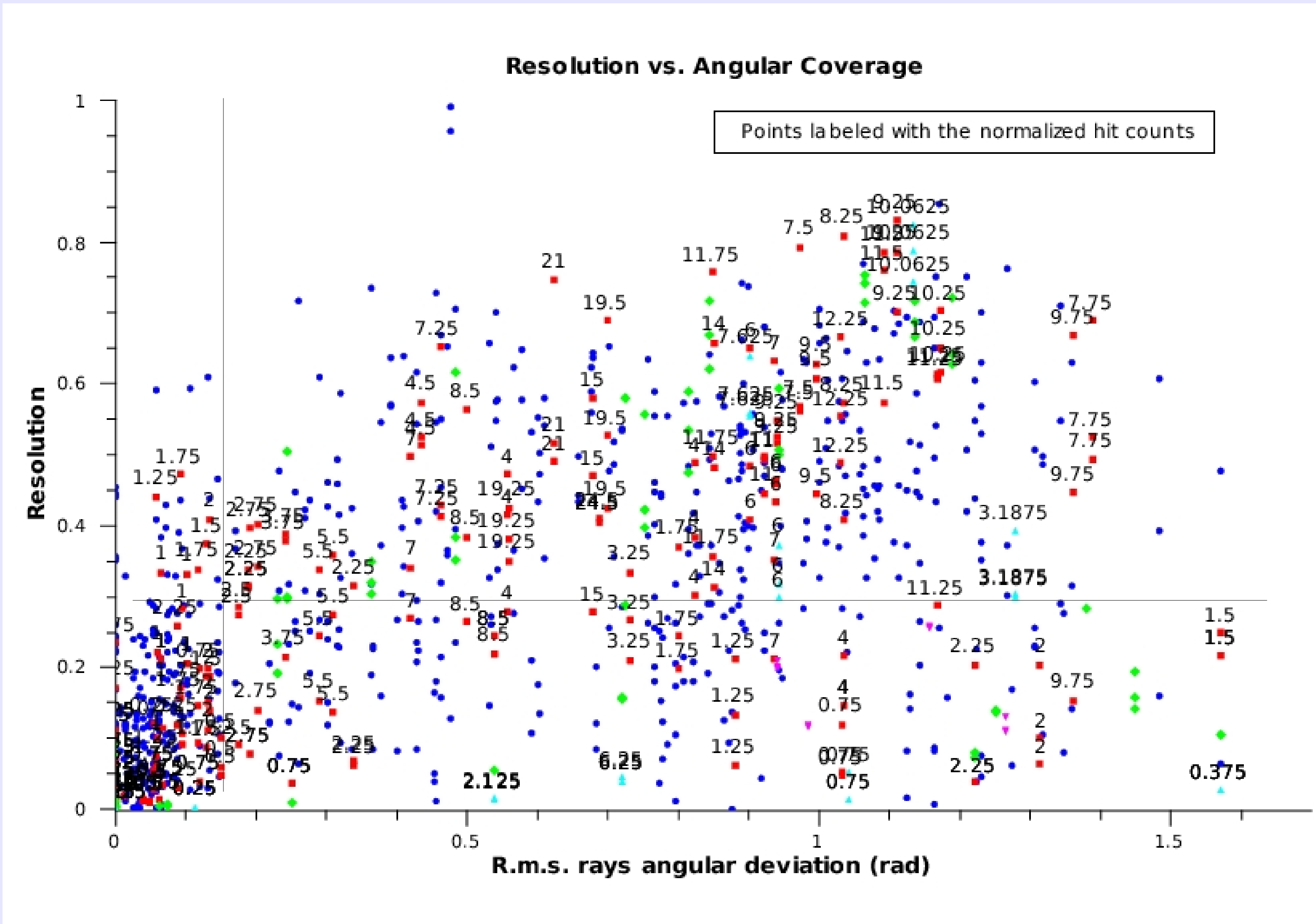
Resolution of the Vp Slowness Wavelet Series Coefficients w.r.t. ***Number of Rays*** that Cross the **supp** $\psi_{l,m,n}$



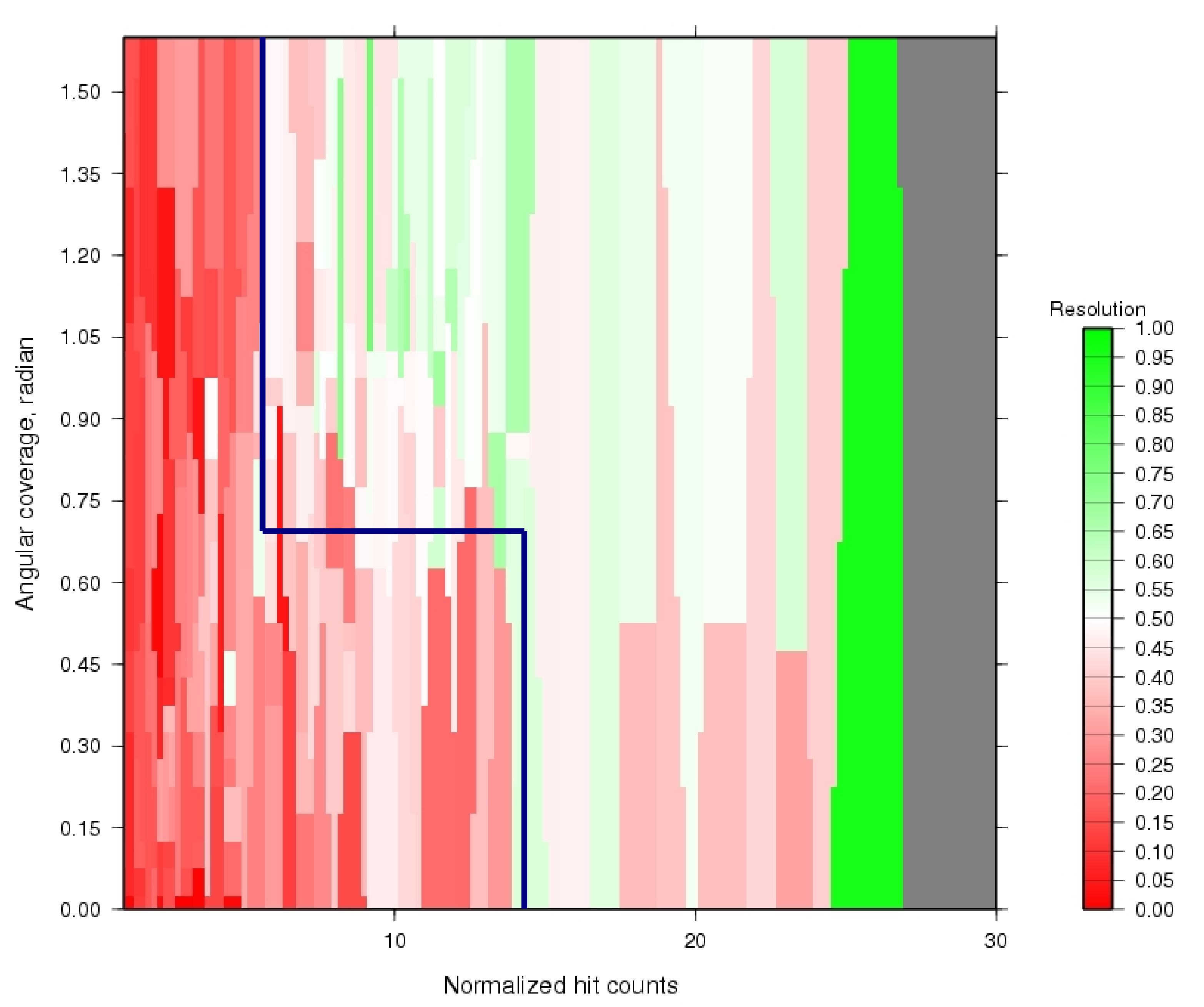
Resolution of the Vp Slowness Wavelet Series Coefficients
w.r.t. ***Angular Coverage*** of Rays that Cross the **supp** $\psi_{l,m,n}$



Resolution of the Vp Slowness Wavelet Series Coefficients
w.r.t. ***Angular Coverage*** of Rays that Cross the **supp** $\psi_{l,m,n}$



Resolution of the Vp Slowness Wavelet Series Coefficients
w.r.t. ***Normalized Hit Counts & Angular Coverage*** of Rays that Cross the **supp** $\psi_{l,m,n}$



- ✔ The pattern of the resolution dependence on the hit counts and angular coverage is generally stable with respect to experiment geometry and subsurface structure.
- ✔ The particular optimal threshold values of the hit counts and angular coverage depend on the experiment geometry and subsurface structure but almost does not change between the iterations of the iterative non-linear inversion.

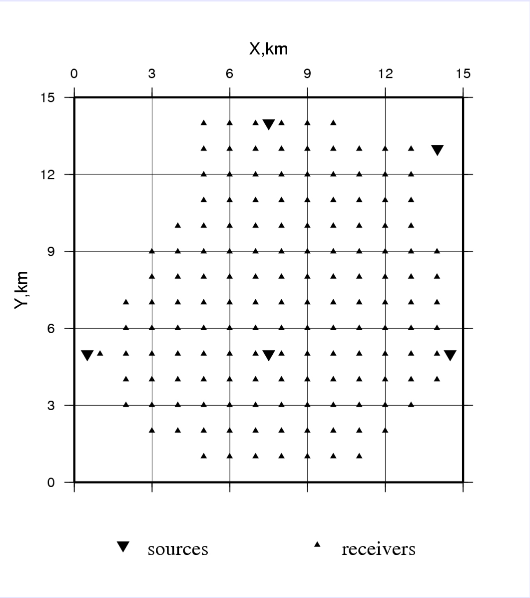
How to estimate threshold values for the specific problem?

a) Use trial-and-error modelling

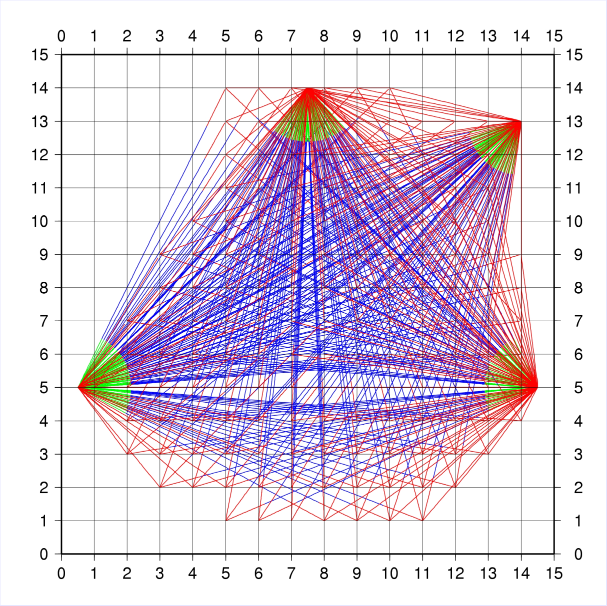
b) Estimate the resolution matrix once (using the PROPACK algorithm it takes ~100 x time needed for the inversion with the LSQR algorithm) at the first iteration and then keep them for the subsequent iterations

c) Let the threshold values to be low (*we suggest normalized hit counts ~ 3-5 and angular coverage ~ 0.1-0.3 radian*) but sufficient enough to throw out most low-resolved wavelets and therefore decrease the number of unknowns. This will decrease the resources needed for the resolution matrix computation. Compute the resolution matrix for this reduced problem and then refine the threshold rule.

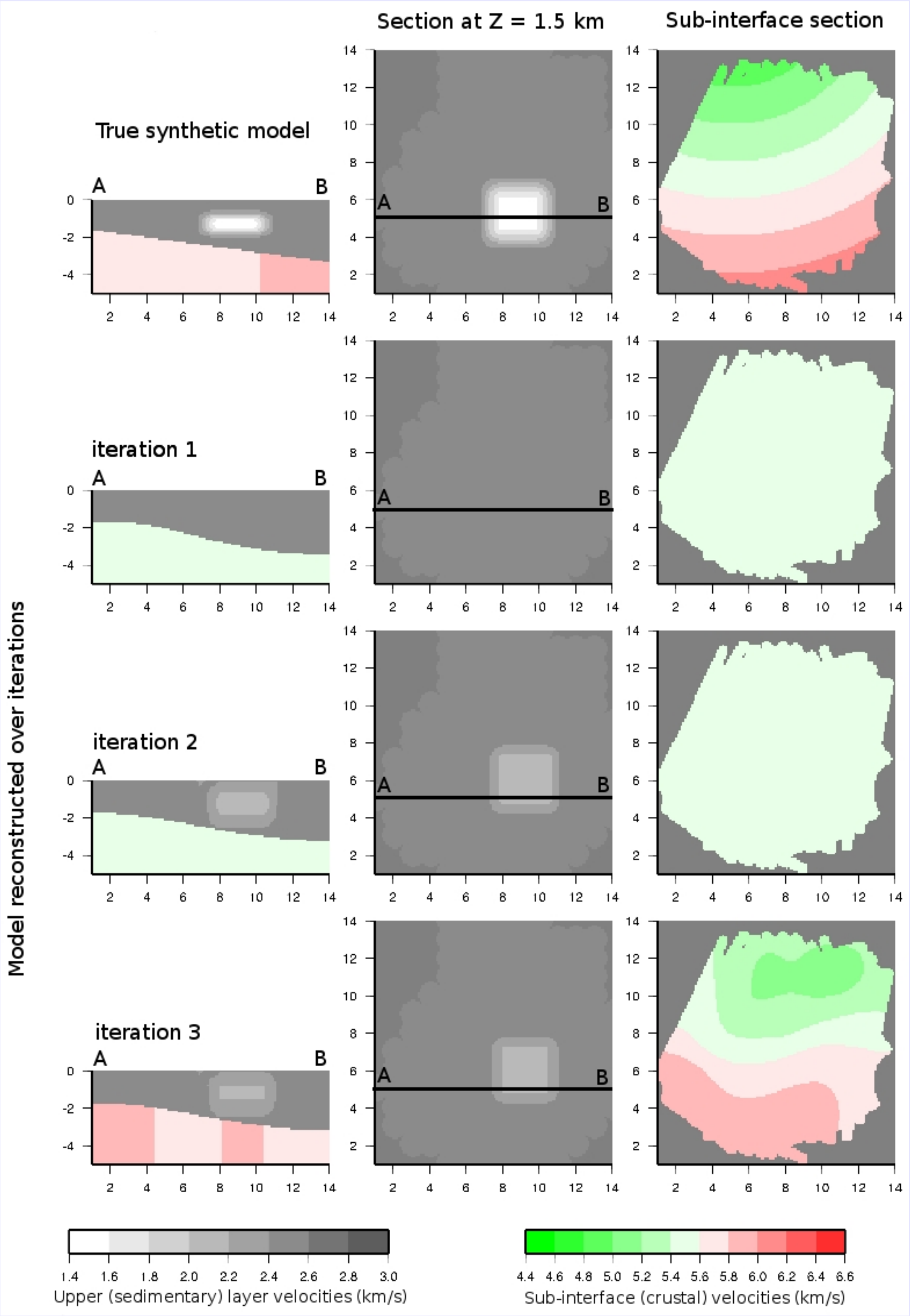
Synthetic example



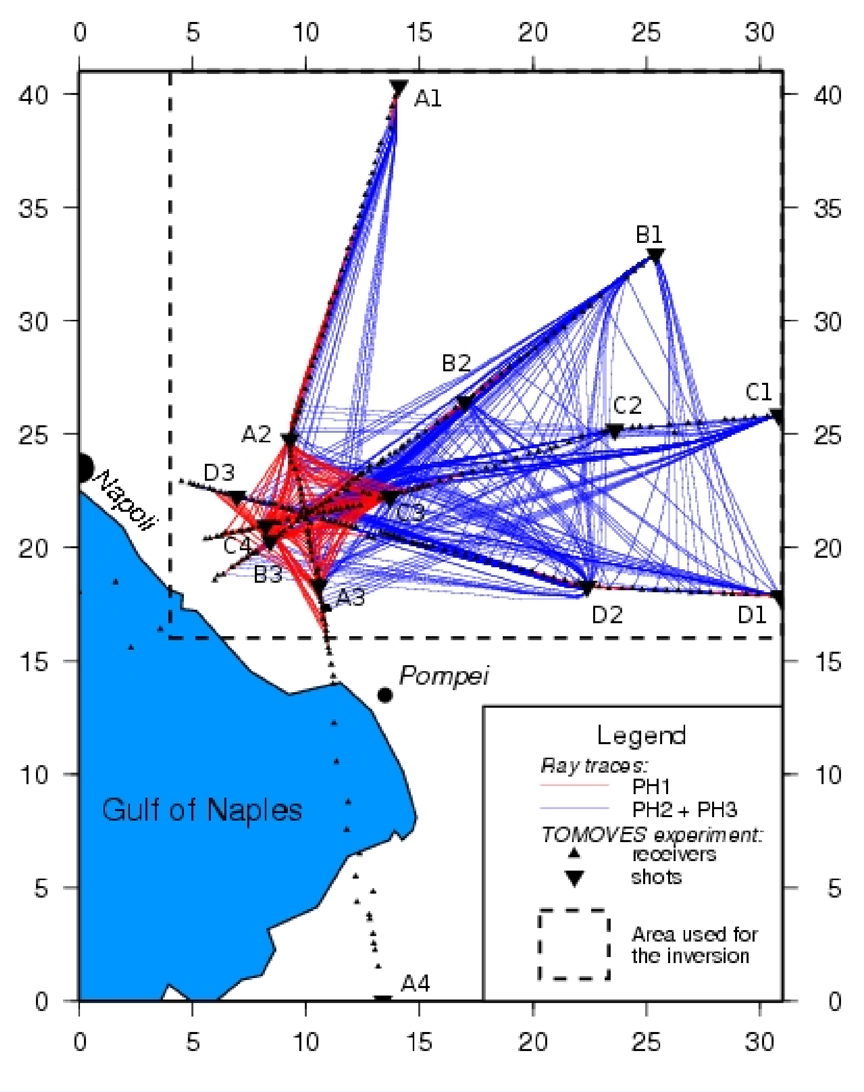
"Experiment" geometry

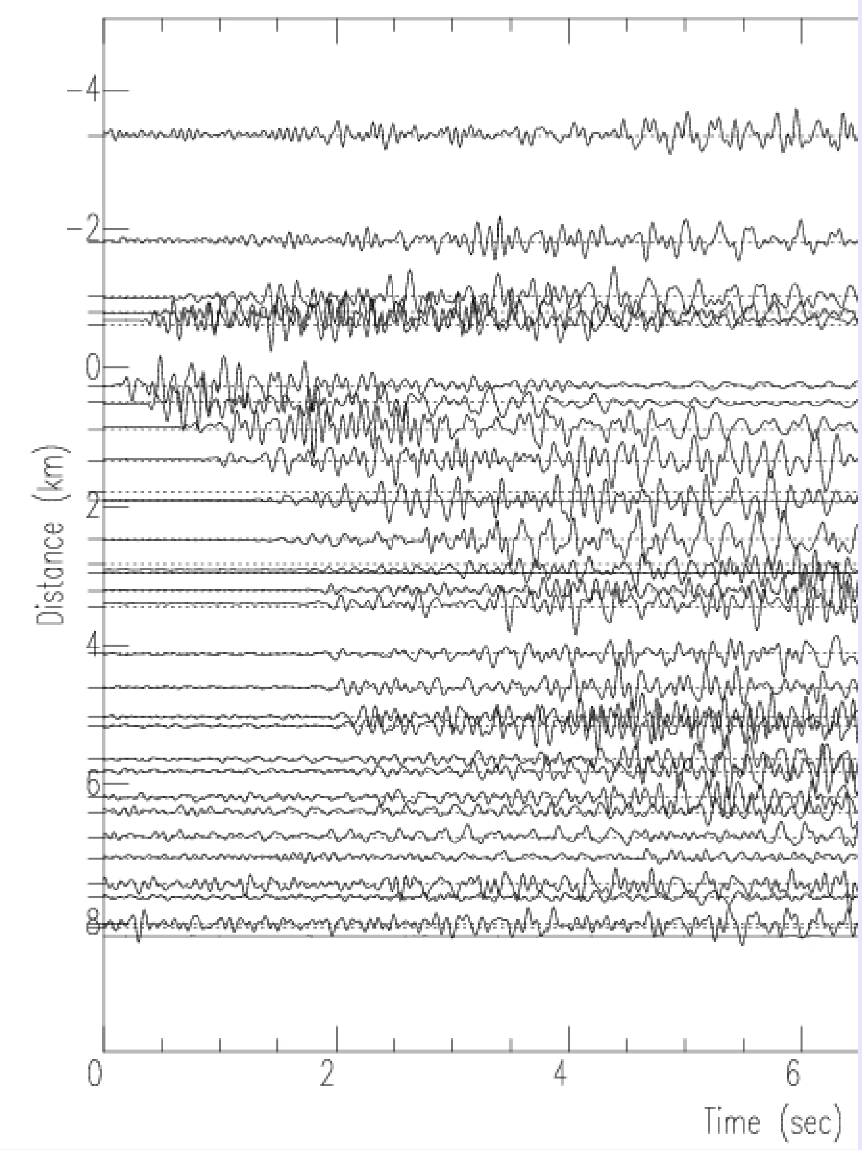


Ray traces



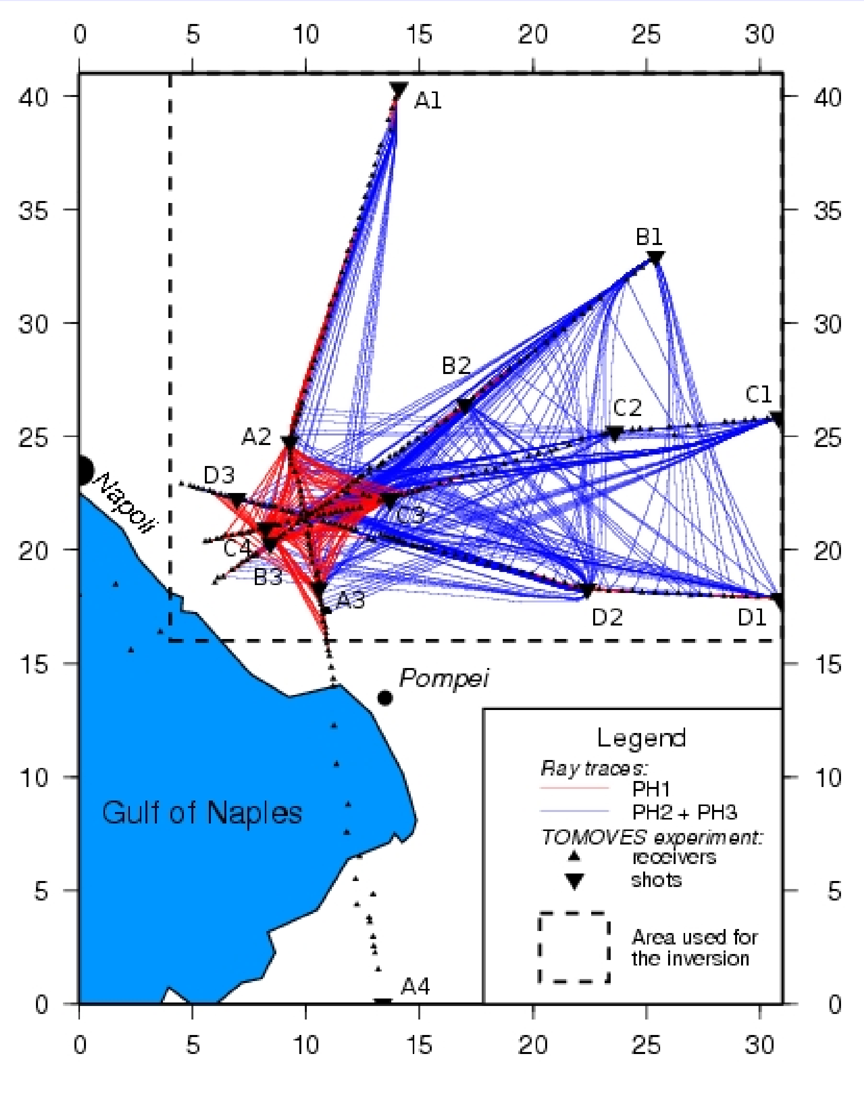
TOMOVES experiment

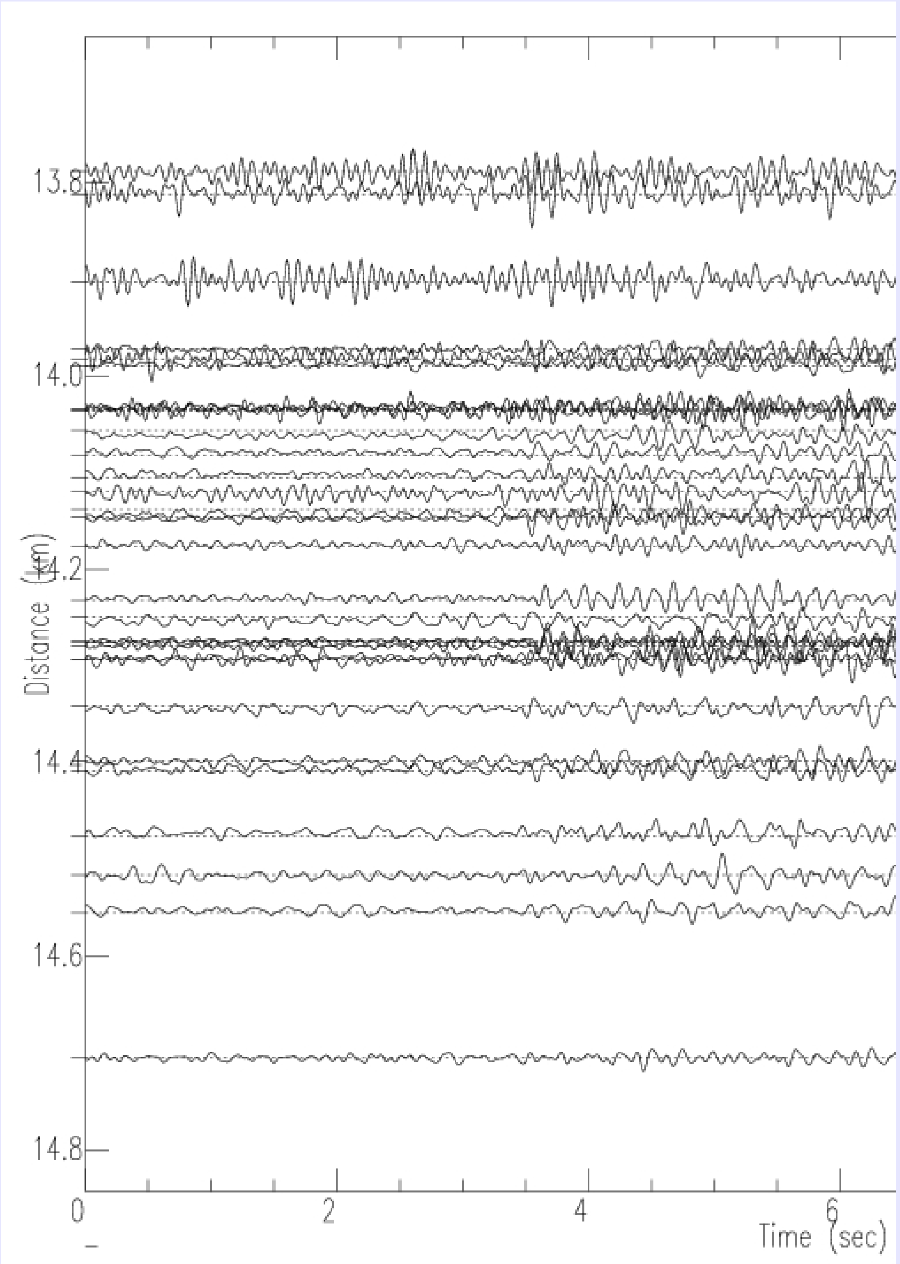




Traces recorded from shot C3 over profile C

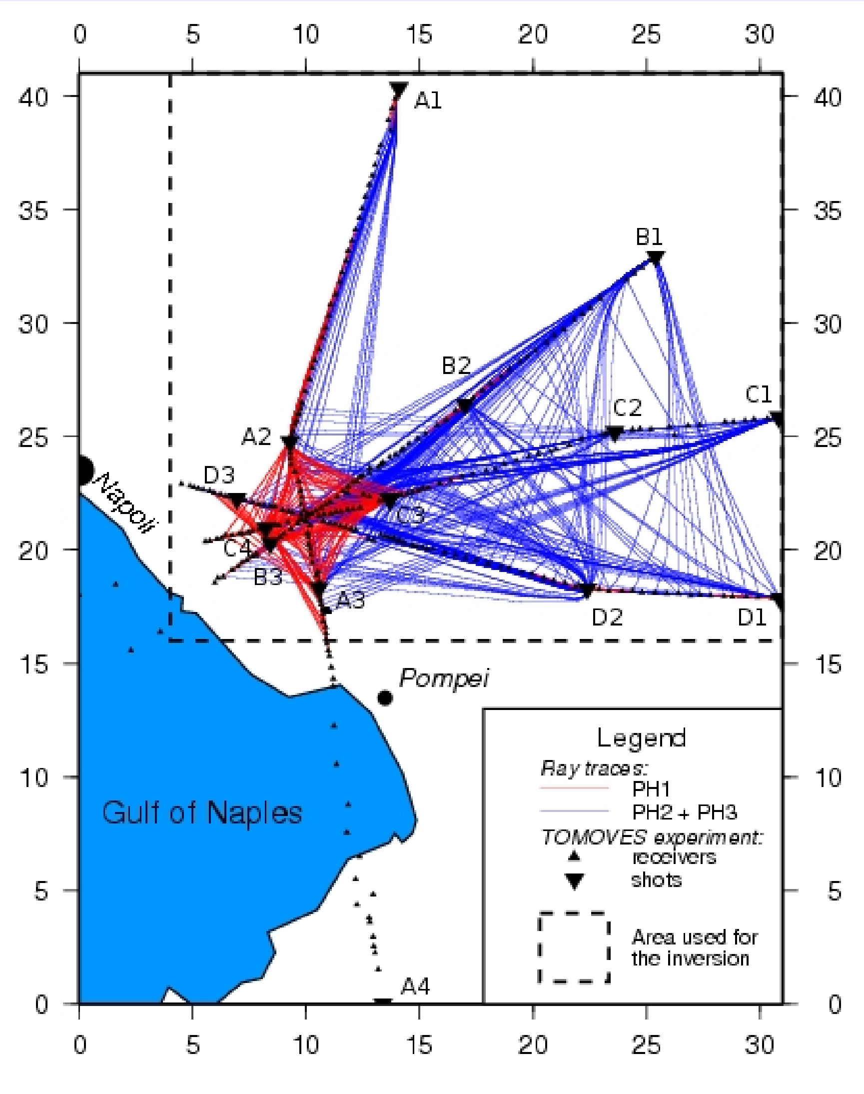
TOMOVES experiment

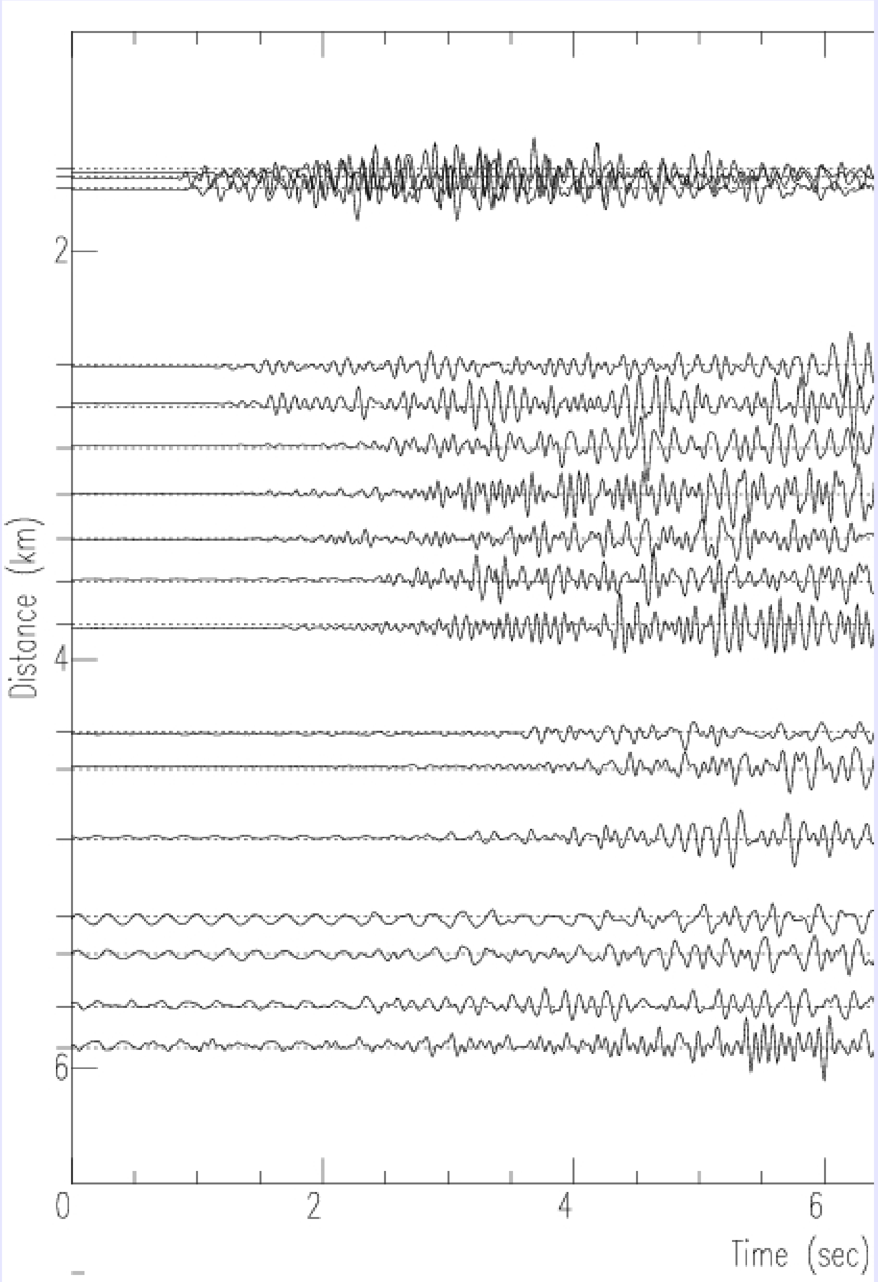




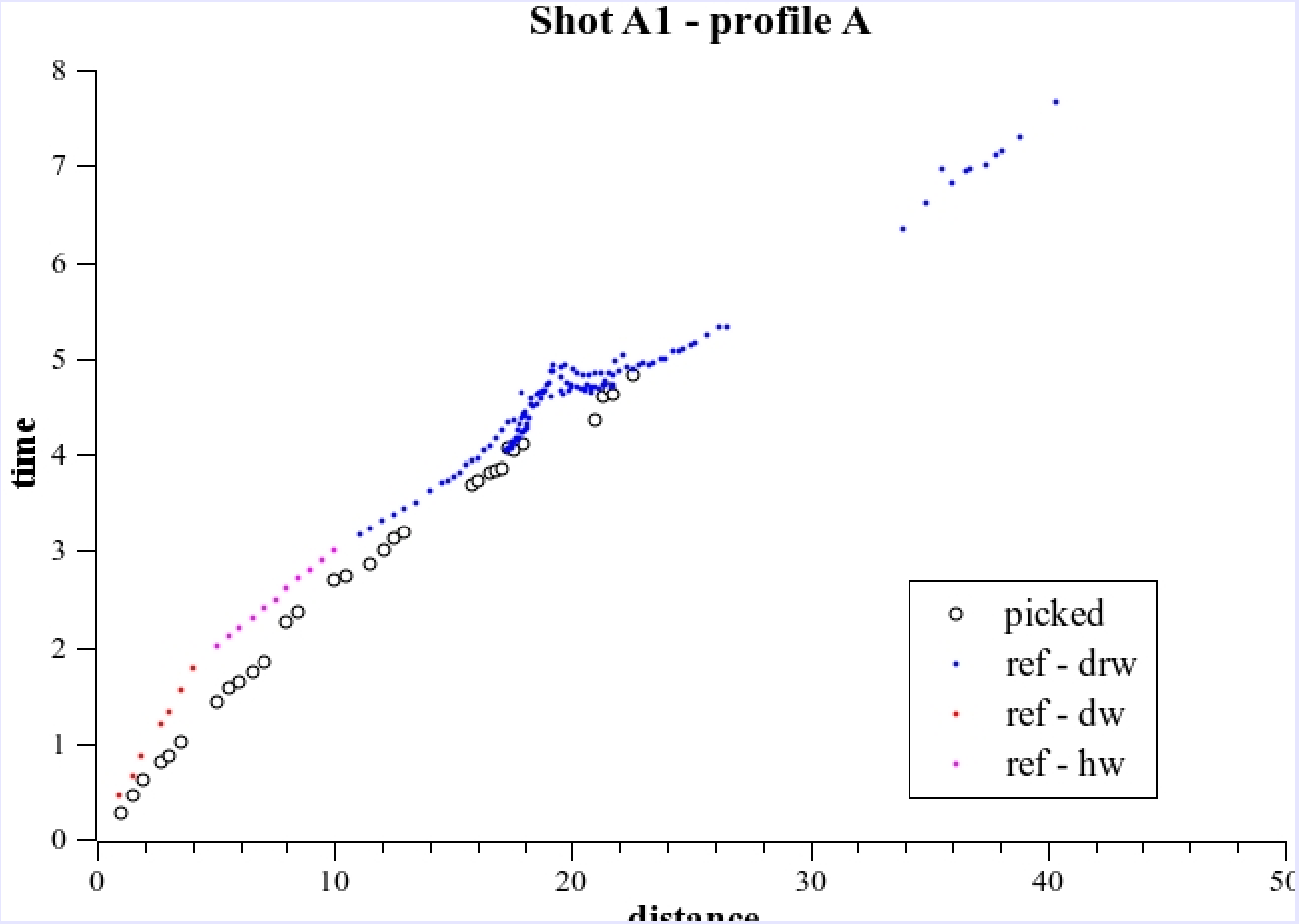
Traces recorded from shot C3 over profile A

TOMOVES experiment

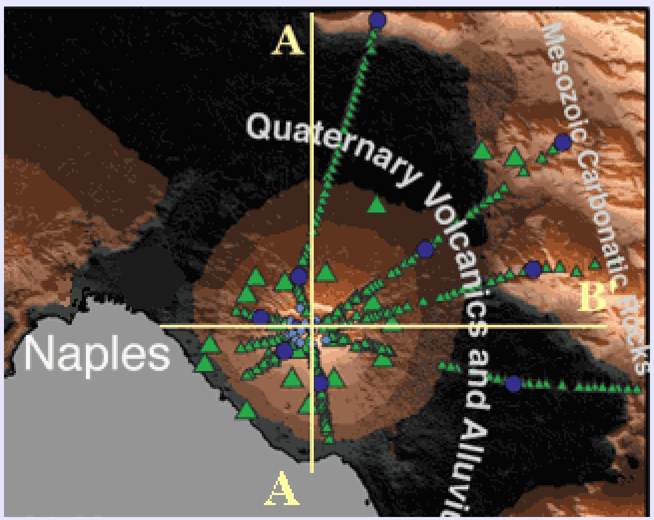


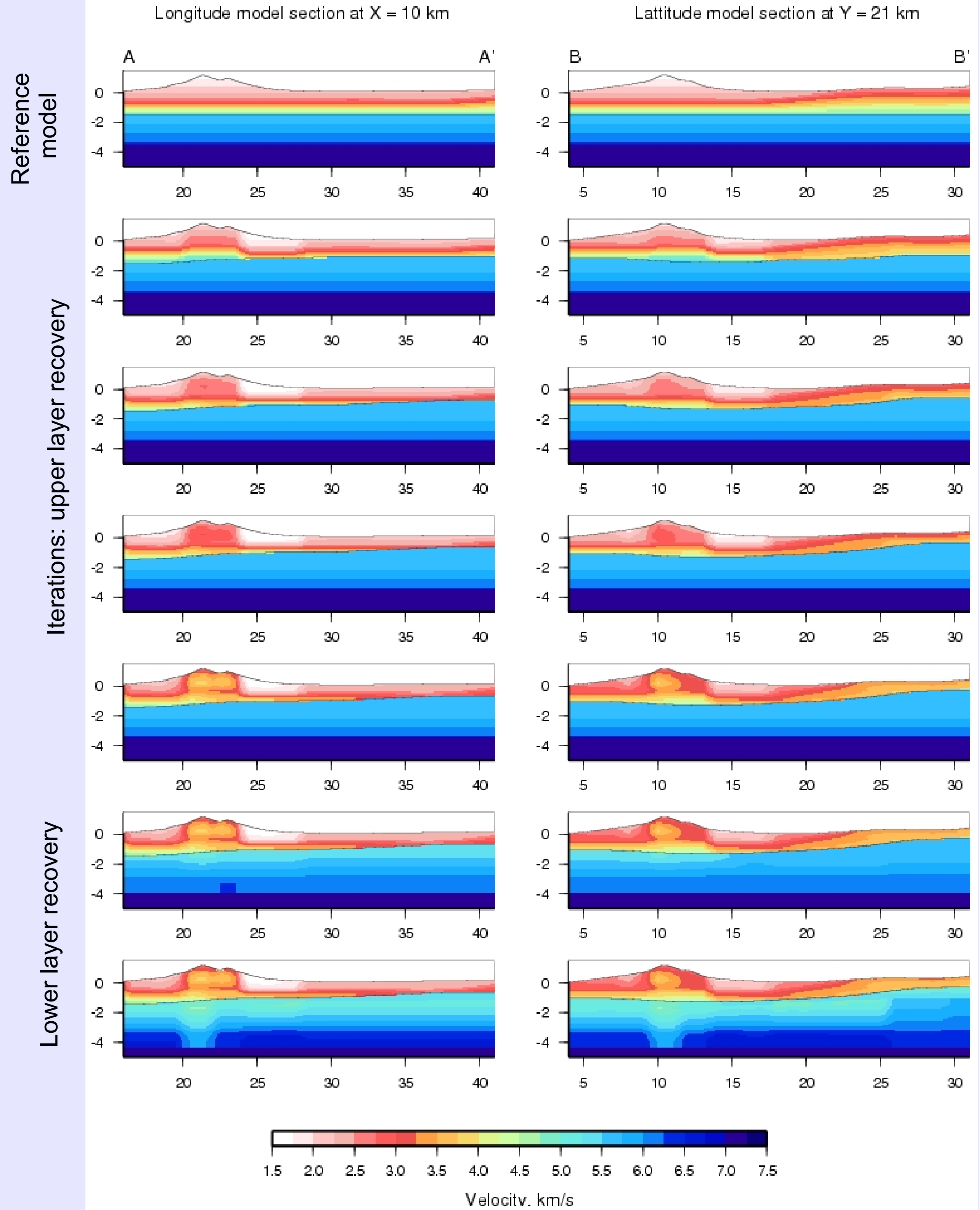


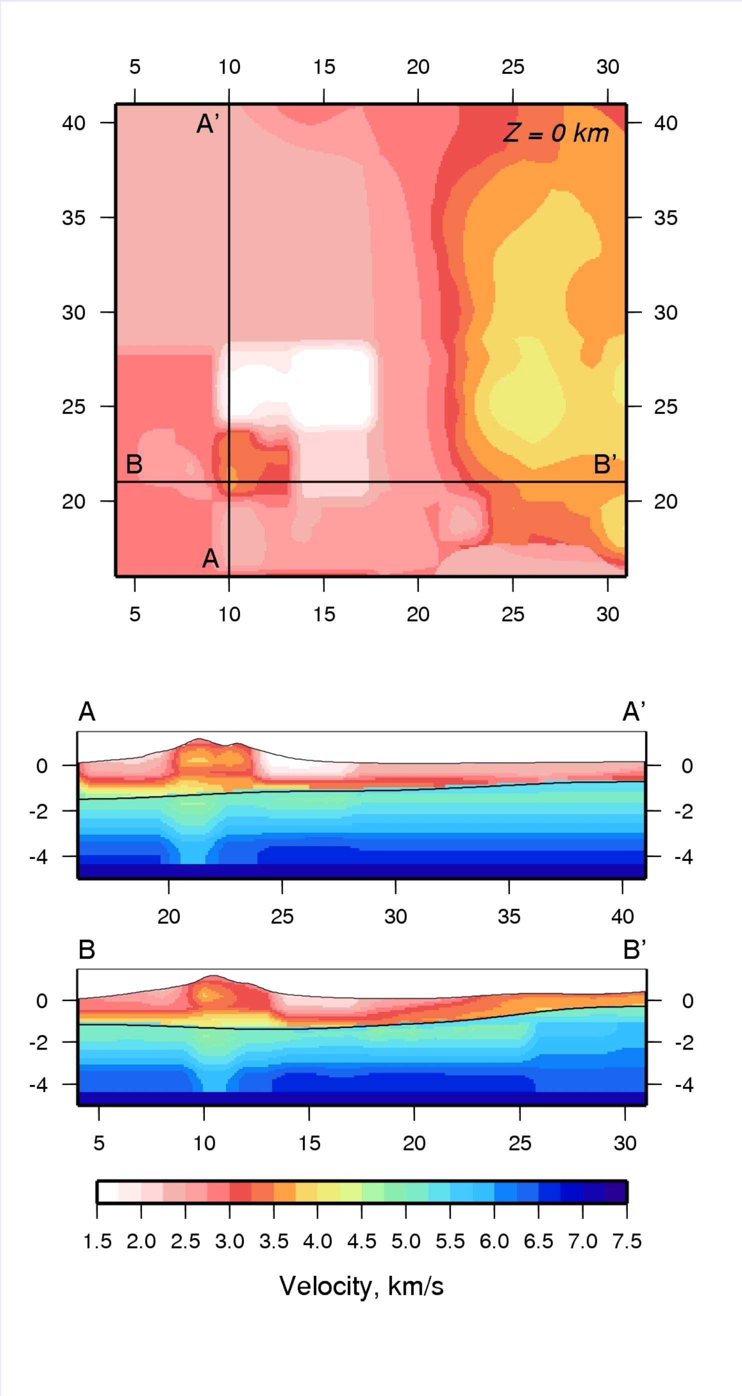
Traces recorded from shot D3 over profile B

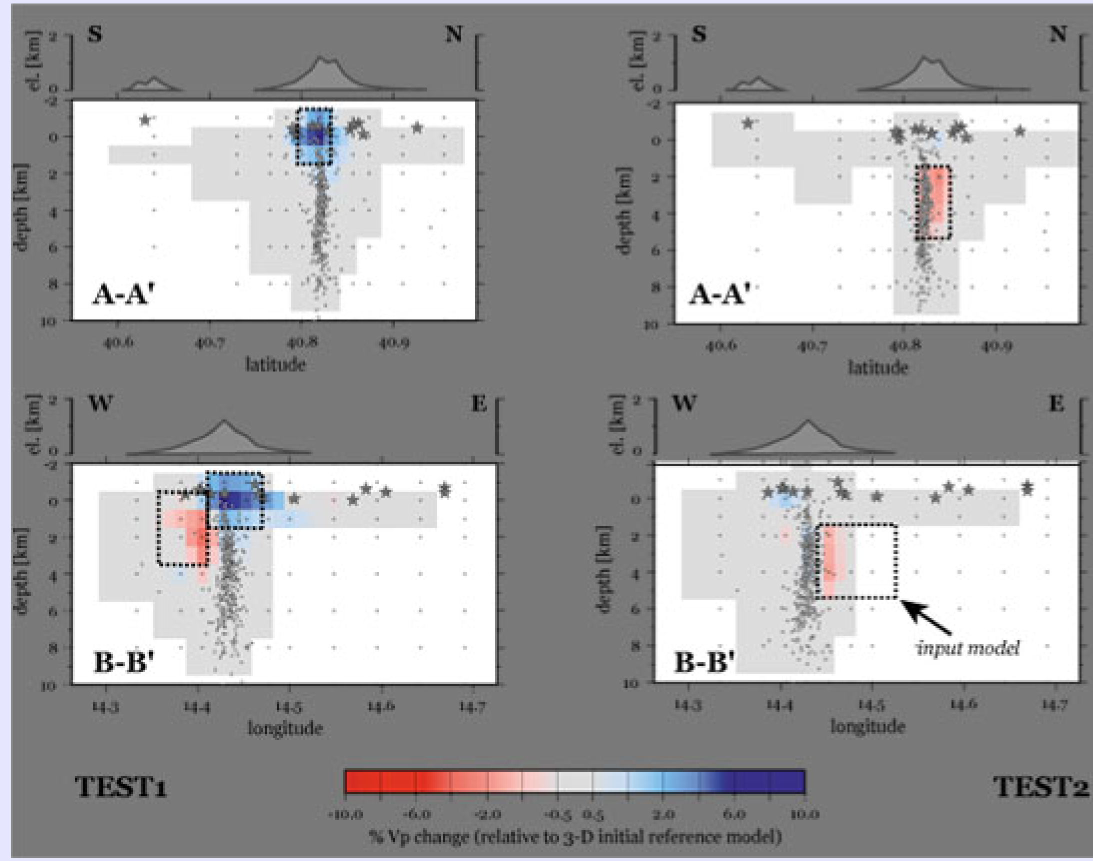


Shot A1 - profile A
time
distance
0
1
2
3
4
5
6
7
8
0
10
20
30
40
picked
ref - drw
ref - dw
ref - hw


A
Quaternary Volcanics and
Mesozoic Carbonatic Rocks
B'
Naples
A


Longitude model section at X = 10 km
Lattitude model section at Y = 21 km
A
A'
B
B'
Reference model
Iterations: upper layer recovery
Lower layer recovery
0
-2
-4
20 25 30 35 40
5 10 15 20 25 30
1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5
Velocity, km/s





From: *E. Kissling, S. Husen, G. De Natale, C. Chiarabba, and TomoVes Working Group*, Internal and Regional Structure of Mt. Vesuvius by Local Earthquake and Active Source Seismic Tomography. (Poster presented at AGU Fall Meeting, San Francisco, 2000)

Thank you!!!